

CBSE SAMPLE PAPER

Class XII Mathematics

Paper 1(Questions)

Time: 3 hours

Total Marks: 100

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Find the inverse of $g: R \rightarrow R, g(x) = 4e^{2x} + 3$ and the domain of the $g^{-1}(x)$
2. What should be the value of n for which the points $(2, -3)$, $(3, 1)$ and $(5, n)$ are collinear?
3. If X is a set of the factors of 30, $a, b \in X$ and $f(a, b, c) = (a + b) * (b' + c)$ where
 - (i) $a + b$ is defined such that $a + b = \text{LCM}(a, b)$
 - (ii) $a * b$ is defined such that $a * b = \text{GCD}(a, b)$
 - (iii) $a' = 30/a$

What is the value of $f(2, 5, 15)$?

4. A parallelogram is constructed on $3\vec{a} + \vec{b}$ and $\vec{a} - 4\vec{b}$ where $\vec{a} = 6$ and $\vec{b} = 8$ and \vec{a} and \vec{b} are anti-parallel. What is the length of the longer diagonal?

(OR)

If $a = \hat{i} + \hat{j} + \hat{k}$, $b = \hat{i} - \hat{j} + 2\hat{k}$ and $c = x\hat{i} + (x - 2)\hat{j} - \hat{k}$, and if the vector lies in the plane of vectors a and b , then what is the value of x ?

SECTION – B

5. Express the matrix $A = \begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew symmetric.

(OR)

Given that $X = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $g(x) = x^2 - 3x + 7$, find the value of $g(X) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$

6. If $f(\alpha) = \begin{cases} \frac{1-\sin^3\alpha}{3\cos^2\alpha}, & \text{if } \alpha < \pi/2 \\ i, & \text{if } \alpha = \pi/2 \\ j\left(\frac{1-\sin\alpha}{(\pi-2\alpha)^2}\right), & \text{if } \alpha > \pi/2 \end{cases}$ so that $f(\alpha)$ is continuous at $\alpha = \pi/2$, then what are the values of i and j ?

7. What is the value of $\frac{dv}{du}$ if $u = e^{v+e^{v+e^{v+\dots}}}$?

8. Peter starts from his office at 8 PM to his home walking through a garden at a uniform velocity of 6 km/hr. While he walks away from a 6-metre-high garden lamp post, he finds his shadow's length increasing at a rate of ' u ' km/hr? If Peter's height is 2 m, then what is the value of u ?

9. Show that the relation R in the set of integers, defined as

$R = \{(m, n) : m, n \in \mathbb{Z} \text{ and } 5 \text{ divides } (m - n)\}$ is an equivalence relation

10. There is an auditorium filled with students of the school. A coordinator is collecting on a random basis the list of students who is born in a leap year. If he aims to achieve the probability of getting at least one such student more than 0.5. What should be the minimum number of students to be surveyed to achieve this level of probability?

(OR)

On his way to home from office, almost every evening Jack has to face traffic chaos at a particular traffic signal located at Junction A. The signal would be red 20% of the time Jack approaches it during the evening commute. Assuming that each evening would represent an independent trial, over five evenings, determine the probability that the signal appears red on exactly one day.

11. What type of quadrilateral will be formed by the points if their respective position vectors are $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j}$, $3\hat{i} + 5\hat{j} - 2\hat{k}$, $\hat{k} - \hat{j}$?

(OR)

A right circular cylinder has a radius of 5 cm and a height of 16 cm. In each of these dimensions there may be error by ± 0.2 cm. What is the relative percentage error in the volume of the cone?

12. Determine the maximum value of $Z = 3x + 2y$ subject to the given constraints:

- (i) $x + 2y \leq 10$
- (ii) $3x + y \leq 15$
- (iii) Both x and y are whole numbers.

SECTION-C

13. Find the direction ratio of the line passing through the origin if the angle between this line $x - 2y - z + 3 = 0$ is 60° .

14. If $2k = l + m + n$ and $X = \begin{bmatrix} l^2 & (k-l)^2 & (k-l)^2 \\ (k-m)^2 & m^2 & (k-m)^2 \\ (k-n)^2 & (k-n)^2 & n^2 \end{bmatrix}$, what is the value of the determinant of X ?

15. The dollar cost of producing ' m ' cameras is given by the function $P(m) = 300 + 0.25m - 0.5\left(\frac{m}{1000}\right)^3$. Which of the following is the estimate of the cost of the 2001st camera?

(OR)

The coordinates of point Y is (i, j, k) which is the image of point $X(-6, 2, 3)$ in the plane $3x - 4y + 5z - 9 = 0$. What is the value of $i + j + k$?

16. Let $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4} + x\right) \right]^{\frac{1}{x}}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$. What should be the value of k such that $f(x)$ is continuous at $x=0$?

(OR)

A curve passes through the point $\left(1, \frac{\pi}{3}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Determine the equation of the curve.

17. $y = 2 - \frac{x}{2}$ and $y = 2 - \frac{3z}{2}$ are the equation of lines forming a tetrahedron which is bounded by four different planes. The four planes include the three coordinate planes (XY plane, YZ plane and ZX plane) and the plane $2x + 4y + 6z = 8$. What is the distance of the given plane from the origin?

18. The curve $y = 4ax^3 + 6bx^2 + 8cx + 10$ touches the x-axis at a point S(-2,0) and cuts the y-axis at a point R where its gradient is 24. Find the values of a, b and c.

19. What is the approximate value of the probability of getting not more than 4 successes if getting a doublet when a pair of dice is thrown which is regarded as success given that the pair of dice is thrown six times?

20. There are two cardboard boxes. Each of them has six tiles numbered as 1, 1, 2, 3, 3 and 4. Find the variance of the distribution of the expected values of getting a total of 2, 3, 4, 5, 6, 7 and 8 when one picks two tiles at a single event and this event repeats for thirty six times assuming the picked tiles are replaced on every repetition.

21. Jill is a student who always guesses answers for any objective type exam he attends. He has sat down to attend a test containing 25 objective type questions. Each question has four options from which Jill has to choose one option as his answer. What is the probability that Jill could have ticked the right options for more than 20 questions would be approximately?

22. If v be an implicit function of u defined $u^{2u} - 2u^u \cot v - 1 = 0$. What is the value of $v'(1)$?

23. What are the steps involved in the Corner Point Method?

(OR)

What is the value of the integral $\int \left(1 + a - \frac{1}{a}\right) e^{a + \frac{1}{a}} da$?

SECTION-D

24. Find the equation of a plane containing the lines $\frac{x-3}{4} = \frac{y-4}{1} = \frac{(z-1)}{1}$ and $\frac{x+1}{12} = \frac{y-7}{6} = \frac{z-5}{3}$ and this plane is parallel to plane $x - 4y + 4z + 9 = 0$

(OR)

A line segment joins the points P(15,-12, 17) and Q(0,3,6). What is the length of its projection (component) on the plane $13x - 9y + 16z - 69 = 0$?

25. Determine the area of the largest parallelogram that could be inscribed in an ellipse $\frac{x^2}{25} + \frac{y^2}{49} = 1$. Given that the inscribed parallelogram has equal diagonals bisecting each other at angle other than 90° .

26. Pooja has Rs 10 lakhs to invest in a stock mutual fund, a bond mutual fund and a money market fund for which the expected annual returns are 10%, 7% and 5% respectively. In order to obtain an overall annual return of 8%, a financial planner advises Pooja to invest as much in stocks as in bonds and the money market combined. How much should Pooja invest in stock fund?

27. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos 3\alpha + 1}{2\cos\alpha - 1} d\alpha$

(OR)

In the first week of March 2010, ABC, a television manufacturing company sent out three shipments. The first shipment order was for 4 LED TVs, 6 Smart TVs and 10 Flat Screen TVs and this order contained a bill for 114,000 Euros. The second shipment order was 8 LED TVs, 3 Smart TVs and 5 Flat Screen TVs and this order contained a bill for 72,000 Euros. The third shipment order which contained a bill for 81,000 Euros was for 2 LED TVs, 9 Smart TVs and 5 Flat Screen TVs. What had ABC charged for Smart TVs?

28. What is the value of the integral $\int_0^{\pi} \frac{u \tan u}{\sec u + \tan u} du$?

(OR)

A line $x = \frac{7}{\sqrt{2}}$ is drawn such that it cuts off the circle $x^2 + y^2 = 49$. Find the area of the larger part that has been cut off?

29. If α lies in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, then show that $\tan^{-1}\left(\frac{1}{\sec\alpha(1+\sin\alpha)}\right) = \frac{\pi}{4} - \frac{\alpha}{2}$

(OR)

What is the maximum value of $(\cos^{-1} \alpha)^2 + (\sin^{-1} \alpha)^2$?