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GMAT

Quant Section Test
[COORDINATE GEOMETRY]
- Solutions

1. Solution:

Topic: Geometry

Concept Tested: Coordinate Geometry (Distance Formula)

Type of Question: Problem Solving (PS)

In an isosceles triangle we know that two sides of the triangle are equal.

Considering AC and BC to be equal in length, we come across at below mentioned algebraic equations using distance formula.

$$\sqrt{(x-10)^2 + 2^2} = \sqrt{(x+4)^2 + 16^2}$$

$$x^2 + 100 - 20x + 4 = x^2 + 16 + 8x + 256$$

$$104 - 20x = 272 + 8x$$

$$-28x = 168$$

$$x = -6$$

Hence, the answer is C.

Alternate solution:

This problem is solvable without algebra once we identify that the points A and B are reflection on the line y = -x. (Reflection of y = -x: (x ,y) = (-y ,-x)

And any point on the line y = -x will be equidistant from the points A and B. Hence, if y coordinate = 6, then x coordinate = -6.

Hence, the answer is C.

2. Solution:

Topic: Geometry

Concept Tested: Coordinate Geometry (Circles)

Type of Question: Problem Solving (PS)

Given: The circle equation $= 4x^2 + 4y^2 = 20^2$

Approach: $4x^2 + 4y^2 = 20^2$,

Divide the entire equation by "4". We get;

$x^2 + y^2 = 10^2$

This is the equation of circle with Center (0, 0) and radius = 10.

So we have 4 points on the circumference that are integers → (10, 0), (-10, 0), (0, 10), (0, -10).

For any point (a, b) on the circumference, distance from center = radius

$(x - a)^2 + (y - b)^2 = 10^2$

Substituting (x, y) = (0, 0) and the equation becomes

$a^2 + b^2 = 10^2$

Integer value pairs satisfying the above equation are (a, b) →

(6, 8), (-6, 8), (6, -8), (-6,-8), (8,6), (-8,6), (8,-6), and (-8,-6).

Total number of points = 4 + 8 = 12 points.

Hence, the answer is D.

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3. Solution:

Topic: Geometry

Concept Tested: Coordinate Geometry (Slopes)

Type of Question: Problem Solving (PS)

Slope of X must fall between the numeric values of the other two lines' slopes.

1) Slope A = Lower limit, NOT inclusive, of Line X's slope

Find the slope of Line X as IF it passed through points (3,3) and (20,10)
Call that "Slope A."

$$Slope\ A = \frac{y2 - y1}{x2 - x1} = \frac{10 - 3}{20 - 3} = \frac{7}{17}$$

$$Slope\ A = \frac{7}{17}$$

Slope A (lower, non-inclusive limit of Slope X) < Slope of Line X.

2) Slope B = Upper limit, NOT inclusive, of Line X's slope

Upper limit, Slope B, is between points (3, 3) and (15, 10).

$$Slope\ B = \frac{y2 - y1}{x2 - x1} = \frac{10 - 3}{15 - 3} = \frac{7}{12}$$

Line X can fall only in the area is defined by the slope inequality below:

$$\frac{7}{17} = 0.411 < Slope\ of\ line\ X < \frac{7}{12} = 0.583$$

The answer that falls between the lower and upper limit of the compound inequality is the answer.

- A. $\frac{1}{4} = 0.25$
- B. $\frac{2}{7} = 0.28$
- C. $\frac{1}{2} = 0.5$
- D. $\frac{7}{8} = 0.875$
- E. $\frac{12}{5} = 2.4$

So, it's clear that option C falls between the lower and upper limit of the compound inequality.

Hence, the answer is C.

4. Solution:

Topic: Geometry

Concept Tested: Coordinate Geometry (Equations of Lines, Slopes)

Type of Question: Data Sufficiency (DS)

Given: Two lines: $y_r = m_1 x + b_1$ and $y_s = m_2 x + b_2$.

Where, m_1 and m_2 are the slopes of two lines r and s respectively.

Question: Is $m_1 < m_2$?

Statement I is insufficient:

Given: Lines r and s intersects at the point (5, 1). Plug in (x, y) as (5, 1)

$1 = 5m_1 + b_1 = 5m_2 + b_2 \rightarrow 5(m_1 - m_2) = b_2 - b_1$.

From this equation we cannot decide whether $m_1 < m_2$.

Therefore, Statement I by itself is insufficient to answer the question asked.

So, eliminate A and D.

The answer is either B, C or E.

Statement II is insufficient:

Given: The y-intercept of line 'r' is greater than the y-intercept of line 's'.

We know that y-intercept is the value of y for x = 0

$\implies b1 > b2 \ or \ b2 - b1 < 0$

From this equation we cannot decide whether $m_1 < m_2$.

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate B.

The answer is either C or E.

Combine both statements:

$5(m1 - m2) = b2 - b1$ From statement 1:

$b2 - b1 < 0$; From statement 2.

So,
$5(m1 - m2) < 0$

$m1 - m2 < 0$

$m1 < m2$.

Therefore, combining the statements I and II is sufficient to answer the question asked.

Hence, the answer is C.

5. Solution:

Topic: Geometry

Concept Tested: Coordinate Geometry (Intercepts and Slopes)

Type of Question: Data Sufficiency (DS)

Given: Line 'p' is drawn in xy plane.

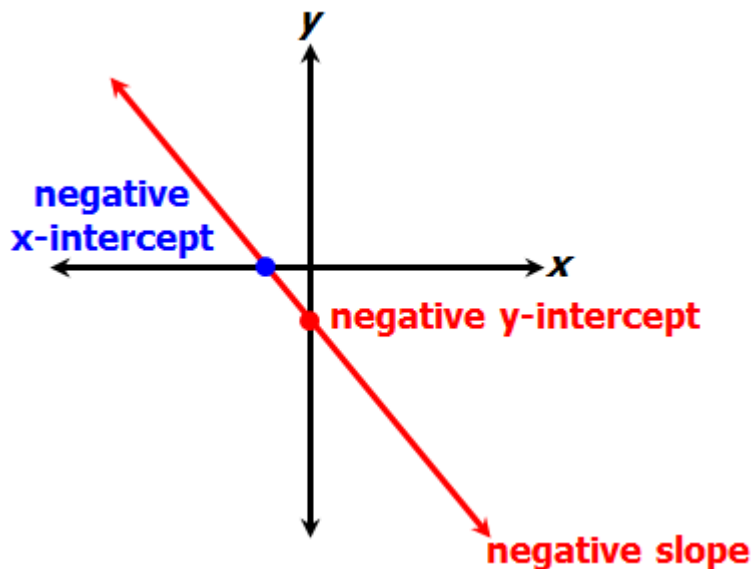**Question:** Does line 'p' have positive x intercept?

Statement I is sufficient:

Given: (y intercept of line p) (slope of line p) > 0

There are can be two cases possible here:

A. The slope and y intercept are both negative.
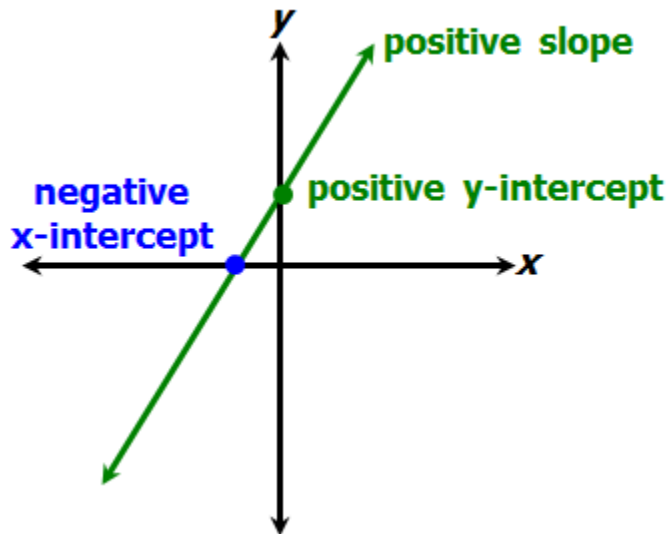B. The slope and y intercept are both positive.

Let's examine each case:

Case A: The slope and y intercept are both negative.

As you can see, the answer to the target question is NO; line 'p' does NOT have a positive x intercept.

Case B: The slope and y intercept are both positive.

As you can see, the answer to the target question is a definite NO; i.e. line p does NOT have a positive x intercept.

In both cases, the answer to the target question is the SAME.

Therefore, Statement I by itself is sufficient to answer the question asked.
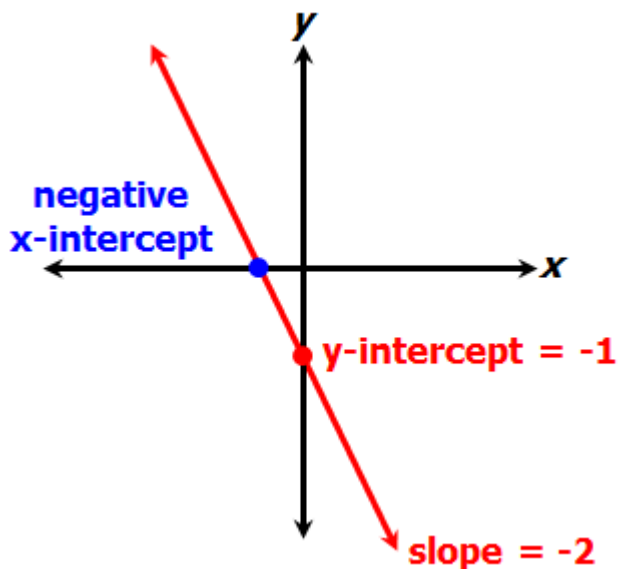
So, eliminate B, C and E.

The answer is either A or D.

Statement II is insufficient:
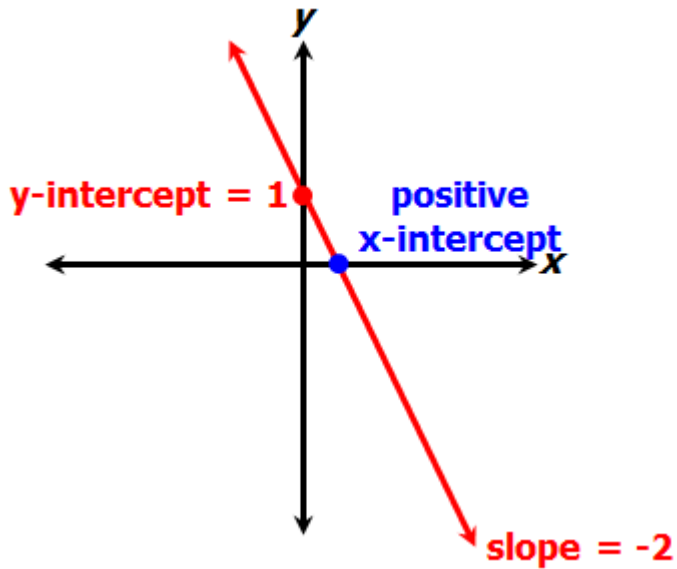
Given: (Slope of line p) – (y intercept of line p) < 0

There are several lines that satisfy statement II. Here are two:

Case A: The line has a slope of -2 and y intercept of -1.

In this case, the answer to the target question is NO; i.e. line p does NOT have a positive x-intercept.

Case B: The line has a slope of -2 and y intercept of 1.

In this case, the answer to the target question is YES; i.e. line p DOES have a positive x intercept.

We cannot answer the question with certainty.

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate D.

Hence, the answer is A.