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GMAT

Quant Section Test

[NUMBER THEORY]

- Solutions

1. **Solution:**

Topic: Number Theory

Concept Tested: Unit Digit Cyclicity, Remainders

Type of Question: Problem Solving (PS)

Given: r is a positive integer $\Rightarrow r > 0$

Question: What is the remainder when 138^{13r+4} is divided by 10?

Approach: If we come to know the unit digit of 138^{13r+4}, it is nothing but the remainder when 138^{13r+4} is divided by 10. Also use plugging in.

Plug in $r = 1$

$\Rightarrow 138^{13r+4} = 138^{13+4} = 138^{17}$

It is same as working with 8^{17}

So now, the question has reduced to, what is the remainder when 8^{17} is divided by 10.

If we find the unit digit of 8^{17}, we will come to know the reminder 8^{17} is divided by 10.

Unit digit of 8^1 is 8

Unit digit of 8^2 is 4

Unit digit of 8^3 is 2

Unit digit of 8^4 is 6

Unit digit of 8^5 is again 8.

$\Rightarrow$ The unit digit will repeat after every four values that means, the cyclicity is 4.

$\Rightarrow$ For8^{16}, unit digit is 6, as 16 is divisible by the cyclicity 4.

$\Rightarrow$ For 8^{17}, unit digit is 8.

Therefore, any number ending with 8, when divided by 10 gives the remainder as 8.

Hence, the answer is E.

2. Solution:

Topic: Number Theory

Concept Tested: Patterns and Sequences and Approximation

Type of Question: Problem Solving (PS)

Given: $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots\ldots\ldots}}}} = k$

Question: What is the value of k?

Approach: Approximate $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots\ldots\ldots}}}}$ as k

$\Rightarrow \sqrt{6+k} = k$ Squaring on both sides and solving we get,

$$k^2 - k - 6 = 0$$

Solve using factorization.

We get $k = 3 \; and \; k = -2$

Ignore negative. So, $k = 3$

Hence, the answer is B.

3. **Solution:**

Topic: Number Theory

Concept Tested: Even and Odd, Prime Factorization, Exponents and Roots.

Type of Question: Data Sufficiency (DS)

Given: a, b and c are integers

Question: Is a even?

Approach: As it is a Yes/No Data Sufficiency Question, if we get a definite Yes or Definite No for the questions using the statements, then it is sufficient. Otherwise, it is insufficient.

Statement I is sufficient:

Given that $15^a = 9^b 25^c$

Do prime factorization to represent using same base.

$\Longrightarrow 3^a 5^a = 3^{2b} 5^c$

$\Longrightarrow a = c \; and \; a = 2b$

Since, b is an integer and a is the multiple of 2 always, a must be an even number.

It's a definite Yes scenario.

Therefore, Statement I by itself is sufficient to answer the question asked.

So, eliminate B, C and E.

The answer will be either A or D.

Statement II is insufficient:

Given that $4^{a+5} = 64^{b+1}$

Do factorization to represent using same base.

$\Longrightarrow 4^{a+5} = 4^{3(b+1)}$

$\Longrightarrow 4^{a+5} = 4^{3b+3}$

As bases are same, equate the powers.

We get $a + 5 = 3b + 3$

Solving further, we get $a = 3b - 2$

Now, if we plug in $b = 1$, which is odd, we get $a = 1$

$\Rightarrow a$ is an odd number.

But, if we plug in $b = 2$, which is even, we get $a = 4$

$\Rightarrow a$ is an even number.

It is a contradiction.

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate D.

Hence, the answer is A.

4. **Solution:**

Topic: Number Theory

Concept Tested: Prime Factorization, Multiples and Factors

Type of Question: Data Sufficiency (DS)

Given: m is a variable which can hold any value.

Question: Is m a multiple of 588?

$\Rightarrow Is\ \frac{m}{588} = integer?$

Approach: As it is a Yes/No Data Sufficiency Question, if we get a definite Yes or Definite No for the questions using the statements, then it is sufficient. Otherwise, it is insufficient.

As the question contains a big number, let us do prime factorization.

So, 588 can be uniquely expressed as follow.

$588 = 2^2 3^1 7^2$ Now the question reduced to is $\frac{m}{2^2 3^1 7^2} = integer?$

To get a definite Yes to the given question, we need at least two 2's, one 3's and two 7's.

Let us evaluate each statement and see if we can get.

Statement I is insufficient:

Given that m is a multiple of 21

$\Rightarrow \frac{m}{21} = integer$

$\Rightarrow \frac{m}{3\times 7} = integer$

From this we will get only one 3 and one 7.

Therefore, Statement I by itself is insufficient to answer the question asked.

So, eliminate A and D.

The answer will be either B, C or E.

Statement II is insufficient:

Given that m is a multiple of 84

$\Rightarrow \frac{m}{84} = integer$

$\Rightarrow \frac{m}{2^2\times 3\times 7} = integer$

From this we will get exactly two 2's but only one 3 and one 7.

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate B.

The answer will be either C or E.

Combine both Statements:

$\Longrightarrow$ m is a multiple of both 21 and 84.

So, if we plug in $m = 2^2 3^1 7^1 = 84$, a multiple of both 21 and 84.

The answer to the main question is a 'No' as 84 is not the multiple of 588(because we don't have at least two 2's, one 3's and two 7's.

But, if we plug in $m = 2^2 3^1 7^2 = 588$, a multiple of both 21 and 84.

The answer to the main question is a 'Yes' as 588 is a multiple of 588(because we have at least two 2's, one 3's and two 7's.

Therefore, even after combining two statements, it is insufficient to answer the question asked.

So, eliminate C.

Hence, the answer is E.

5. **Solution:**

Topic: Number Theory

Concept Tested: Prime Factorization, Multiples and Factors, Remainders

Type of Question: Data Sufficiency (DS)

Given: p is a two digit prime number.

Question: What is the value of p?

Approach: As variables are involved, lets us plug in according to the statements. Then answer the main question with the same value.

Statement I is insufficient:

Given that when p is divided by 4, the remainder is 3.

$\Rightarrow p = 4a + 3$, where a is an integer.

If we plug in $a = 2 \Rightarrow p = 11$,which is prime.

If we plug in $a = 5 \Rightarrow p = 23$, which is prime.

So, we are not getting a unique value for p.

Therefore, Statement I by itself is insufficient to answer the question asked.

So, eliminate A and D.

The answer will be either B, C or E.

Statement II is insufficient:

Given that p is 2 more than multiple of 7.

$\Rightarrow p = 7b + 2$, where b is an integer.

If we plug in $b = 3 \Rightarrow p = 23$,which is prime.

If we plug in $b = 5 \Rightarrow p = 37$, which is prime.

So, we are not getting a unique value for p.

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate B.

The answer will be either C or E.

Combine both Statements:

Recycle the numbers which you have got while evaluating two statements separately.

It has to satisfy both the following conditions.

$p = 4a + 3$, where a is an integer.

$p = 7b + 2$, where b is an integer

$p = 23$ will satisfy both equations given above.

Now, if $b = 11 \Longrightarrow p = 79 \Longrightarrow a = 19$, an integer.

So, $p = 79$

So, we are not getting a unique value for p.

Therefore, even after combining two statements, it is insufficient to answer the question asked.

So, eliminate C.

Hence, the answer is E.

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