INEQUALITIES AND MODULUS SOLUTIONS

1. **Solution:**

Topic: Algebra
Concept Tested: Properties of Inequalities
Type of Question: Data Sufficiency.

Given:

No information provided in the question stem.

Question:

Is $A^2 > B^2$?

Approach:

Plugging in or using the rules of inequalities.

Statement I is insufficient:

$A^3 > B^3$

If A > B, then $A^3 > B^3$ for all A and B.

Similarly, if $A^3 > B^3$, then A > B for all A and B.

We know that, A > B but we can't say whether $A^2 > B^2$

Because,

If A > B, then $A^2 > B^2$ only if A and B are positive.

So its insufficient.

We can Plug in values too,

For example,

If A = 3 and B = 2

$A^3 > B^3$ and also $A^2 > B^2$. So, answer to the question is YES.

But

If A = -2 and B = -3

$A^3 > B^3$ but $A^2 < B^2$. So, answer to the question is NO.

Therefore, Statement I by itself is insufficient to answer the question asked.

So, eliminate A and D.

The answer will be either B, C or E.

Statement II is insufficient:

A > 0

This statement doesn't tell anything about "B"

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate B.

The answer will be either C or E.

Combine both Statements:

From statement we know that,

$A > B$,

And statement II, says $A > 0$

So, B can be either positive or negative.

For example,

If $A = 3$ and $B = 2$

$A^3 > B^3$ and also $A^2 > B^2$. So, answer to the question is YES.

But

If $A = 2$ and $B = -3$

$A^3 > B^3$ but $A^2 < B^2$. So, answer to the question is NO.

Therefore, even after combining two statements, it is insufficient to answer the question asked.

So, eliminate C.

Hence, the answer is E.

2. **Solution:**

Topic: Algebra
Concept Tested: Definition of Modulus
Type of Question: Data Sufficiency.

Given:

No information provided in the question stem.

Question:

Is $x > 0$?

Approach: Using the definition of Modulus.

Definition of Modulus:

$|x| = x$, if $x \geq 0$

$|x| = -x$, if $x < 0$

Statement I is sufficient:

$|x + 3| = 2x - 1$

By definition,

x+3 = 2x-1 whenever $x \geq -3$

Solving it we get,

x = 4

OR

x+3 = -2x+1 whenever $x < -3$

Solving it we get

x = -2/3, this doesn't satisfy the condition of $x < -3$.

$\Longrightarrow$ Only one value of "x" i.e., x = 4

Therefore, Statement I by itself is sufficient to answer the question asked.

So, eliminate B, C and E.

The answer will be either A or D.

Statement II is sufficient:

$| x + 2 | = 4x – 10$

By definition,

x+2 = 4x-10 whenever $x \geq -2$

Solving it we get,

x = 4

OR

x+2 = -4x+10 whenever $x < -2$

Solving it we get

x = 8/5, this doesn't satisfy the condition of $x < -2$.

Therefore, only one value of "x" i.e., x = 4

Therefore, Statement II by itself is sufficient to answer the question asked.

So, eliminate A.

Hence, the answer is D.

3. **<u>Solution:</u>**

Topic: Algebra
Concept Tested: Properties of Inequalities
Type of Question: Data Sufficiency.

Given:

"a" is not equal to "b".

Question:

Is $\frac{(b+a)}{(b-a)} > 0$?

Approach:

First mistake, students end up doing here is, cross multiplying the actual question. It is wrong.

We can cross in the inequality without changing its order if both numerator and denominator are positive.

For this question, plugging in would be the best approach.

Statement I is insufficient:

$a < 0$

This doesn't say anything about "b"

Therefore, Statement I by itself is insufficient to answer the question asked.

So, eliminate A and D.

The answer will be either B, C or E.

Statement II is insufficient:

$b > 0$

This doesn't say anything about "a"

Therefore, Statement II by itself is insufficient to answer the question asked.

So, eliminate B.

The answer will be either C or E.

Combine both Statements:

We know that,

$a < 0$ and $b > 0$, but we don't their magnitudes.

For example,

If a= -2 and b = 3

$\frac{(b+a)}{(b-a)} > 0$

$\frac{(3-2)}{(3-(-2))} > 0$

$1/5 > 0$

So answer to the question is YES.

But if

If a= -3 and b = 2

$\frac{(b+a)}{(b-a)}$

$\frac{(2-3)}{(2-(-3))}$

-1/5 < 0

So, answer to the question is No.

Therefore, even after combining two statements, it is insufficient to answer the question asked.

So, eliminate C.

Hence, the answer is E.

4. **Solution:**

Topic: Algebra
Concept Tested: Properties of Modulus
Type of Question: Problem Solving (PS)

Given:

"x" is an integer greater than zero.

|x - 3.5| < |x – 7.5|

Question:

Number of values of "x"?

Approach:

Properties of Modulus.

We know that Modulus is a function which is always non-negative.

|x - 3.5| < |x – 7.5|

So, we can square on both sides for the above inequality.

$(x-3.5)^2 < (x-7.5)^2$

Let's bring everything one side,

$(x-3.5)^2 - (x-7.5)^2 < 0$

Using $a^2 - b^2 = (a+b) * (a-b)$

$(x-3.5+x-7.5)(x-3.5-x+7.5) < 0$

$(2x-11)(4) < 0$

So,

$(2x-11) < 0$

i.e.,

x < 5.5

Since "x" is a positive integer, the values which satisfy the above inequality is 1,2,3,4,5

Hence, the answer is C.

5. **Solution:**
Topic: Algebra

Concept Tested: Properties of inequalities.
Type of Question: Problem Solving (PS)

Given:

$-\frac{1}{2} \le x \le -\frac{1}{10}$ and $-\frac{1}{3} \le y \le -\frac{1}{16}$

Question:

Maximum possible value of $x^2 * y$?

Approach:

Using the properties of Inequalities.

For "x" and "y" both are negative then, if x < y then $x^2 > y^2$

$$-\frac{1}{2} \le x \le -\frac{1}{10}$$

Using the above rule,

$$\frac{1}{4} \ge x^2 \ge \frac{1}{100}$$

i.e.,

$$\frac{1}{100} \le x^2 \le \frac{1}{4}$$

Now, we need to find $x^2 * y$?

$$-\frac{1}{3} \le y \le -\frac{1}{16}$$

We need to find product all four extremes to find the least value.

$$\frac{1}{100} * \left(-\frac{1}{3}\right) = -\frac{1}{300}$$

$$\frac{1}{100} * \left(-\frac{1}{16}\right) = -\frac{1}{1600}$$

$$\frac{1}{4} * \left(-\frac{1}{3}\right) = -\frac{1}{12}$$

$$\frac{1}{4} * \left(-\frac{1}{16}\right) = -\frac{1}{64}$$

$$-\frac{1}{12} \le x^2 * y \le -\frac{1}{1600}$$

So, the maximum possible value is $-\frac{1}{1600}$

Hence, the answer is B.