(English Version)

Instructions : 1. *The question paper has five Parts namely A, B, C, D and E. Answer **all** the **Parts**.*

2. *Use the Graph Sheet for the question on Linear Programming problem in Part-E.*

PART – A

Answer **all** the **ten** questions : **(10 × 1 = 10)**

1) Define Binary Operation.

2) Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

3) Define a scalar matrix.

4) Find a value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

5) If $y = \sin(x^2 + 5)$ then find $\frac{dy}{dx}$.

6) Find $\int (1-x)\sqrt{x} \cdot dx$.

7) Find a value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

8) If a line has direction ratios $2, -1, -2$ then determine its direction cosines.

9) Define objective function in Linear Programming Problem.

10) If $P(E)=0.6$, $P(F)=0.3$ and $P(E\cap F)=0.2$ then find $P(F/E)$.

PART – B

Answer **any ten** questions : **(10 × 2 = 20)**

11) Show that the function $f:N\rightarrow N$ given by $f(x)=2x$ is one-one but not onto.

12) Prove that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $x\in[-1, 1]$.

13) Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $x>1$ in the simplest form.

14) Find the area of the triangle with vertices $(2, 7)$, $(1, 1)$ and $(10, 8)$ using determinant method.

15) Find $\frac{dy}{dx}$, if $y=(\log x)^{\cos x}$.

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16) If $ax+by^2=\cos y$ then find $\frac{dy}{dx}$.

17) Find the approximate change in the volume V of a cube of side x meters caused by increasing side by 2%.

18) Find $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx$.

19) Find $\int \sin 2x.\cos 3x\, dx$.

20) Find the order and degree (if defined) of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$.

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21) If $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$ then find $\left|\vec{b}\right|$.

22) Find the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

23) Find the distance of the point (3, –2, 1) from the plane $2x-y+2z+3=0$.

24) Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

PART – C

Answer **any ten** questions : **(10 × 3 = 30)**

25) Check whether the relation R in $\mathbb{R}$ of real numbers defined by $R=\{(a,b): a\le b^3\}$ is reflexive, symmetric or transitive.

26) Prove that $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$.

27) By using elementary transformations, find the inverse of the matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

28) If $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ then show that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$.

29) Verify Rolle's theorem for the function $f(x)=x^2+2x-8$, $x\in[-4, 2]$.

30) Find the intervals in which the function f given by $f(x)=2x^3-3x^2-36x+7$ is strictly increasing.

31) Find $\int x\cdot\log x\,dx$.

32) Evaluate : $\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx$.

33) Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

34) Form the differential equation of the family of curves $y = ae^{3x} + be^{-2x}$ by eliminating arbitrary constants a and b.

35) Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

36) Show that the four points with position vectors; $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ are coplanar.

37) Find the vector equation of the plane passing through the points $R(2, 5, -3)$, $S(-2, -3, 5)$ and $T(5, 3, -3)$.

38) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

PART – D

Answer **any six** questions : **(6 × 5 = 30)**

5 × 5 – 25

39) Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Show that f is invertible. Find the inverse of f.

40) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then show that $A^3 - 23A - 40I = 0$.

41) Solve the following system of linear equations by matrix method :

$3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4.$

42) If $y = \sin^{-1} x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right) = 0$.

43) The length x of a rectangle is decreasing at the rate of 3 cm/min. and the width y is increasing at the rate of 2 cm/min. When $x = 10\ cm$ and $y = 6\ cm$, find the rates of change of

i) the perimeter and

ii) the area of the rectangle.

44) Find the integral of $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{1}{x^2-16} \cdot dx$.

45) Using the method of integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

46) Find the general solution of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x$, $\left(0\leq x<\frac{\pi}{2}\right)$.

47) Derive the equation of a line in space passing through a given point and parallel to a given vector in both vector and Cartesian form.

48) Five cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

i) all the five cards are spades?

ii) only three cards are spades?

iii) none is a spade?

PART – E

Answer **any one** of the following question : **(1 × 10 = 10)**

49) a) Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^{\frac{\pi}{2}}\frac{\cos^5 x}{\sin^5 x+\cos^5 x}\,dx$. **(6)**

b) Show that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. **(4)**

50) a) Minimise and Maximise $z = 5x + 10y$

subject to constraints : $x + 2y \leq 120$,
$x + y \geq 60$,
$x - 2y \geq 0$,
$x \geq 0$ and $y \geq 0$

by graphical method. **(6)**

b) Find the value of K, if $f(x) = \begin{cases} Kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$ is continuous at $x = 5$.

(4)