(English Version)

Instructions : *1. The question paper has five Parts namely A, B, C, D and E. Answer **all** the five **Parts**.*

2. Use the Graph Sheet for the question on Linear Programming problem in Part-E.

PART – A

Answer **all** the **ten** questions : **(10 × 1 = 10)**

1) Define bijective function.

2) Write the principal value branch of $\cos^{-1} x$.

3) Construct a 2 × 2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$.

4) If A is an invertible matrix of order 2 then find $|A^{-1}|$.

5) If $y = e^{x^3}$, find $\frac{dy}{dx}$.

6) Find $\int \frac{x^3 - 1}{x^2} dx$.

7) Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

8) If a line makes angle 90°, 60° and 30° with the positive direction of x, y and z – axis respectively, find its direction cosines.

9) Define optimal solution in a linear programming problem.

10) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, find $P(A/B)$.

PART – B

Answer **any ten** questions : **(10 × 2 = 20)**

11) Let $*$ be a binary operation on Q, defined by $a * b = \frac{ab}{2}, \forall a, b \in Q$. Determine whether $*$ is associative or not.

12) If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ then find the value of x.

13) Write the simplest form of $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$, $0<x<\frac{\pi}{2}$.

14) Find the area of the triangle whose vertices are (–2, –3), (3, 2) and (–1, –8) by using determinant method.

15) Differentiate $x^{\sin x}, x>0$ with respect to x.

16) Find $\frac{dy}{dx}$, if $x^2+xy+y^2=100$.

17) Find the slope of the tangent to the curve $y=x^3-x$ at $x=2$.

18) Integrate $\frac{e^{\tan^{-1}x}}{1+x^2}$ with respect to x.

19) Evaluate : $\int_2^3 \frac{x\,dx}{x^2+1}$.

20) Find the order and degree of the differential equation :

$$\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^4+y^5=0.$$

21) Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

22) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.

23) Find the angle between the planes whose vector equations are $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$.

23) Find the angle between the planes whose vector equations are $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$.

24) A random variable X has the following probability distribution :

X	0	1	2	3	4
$P(X)$	0.1	k	$2k$	$2k$	k

Determine :

(i) k

(ii) $P(X \geq 2)$.

PART – C

Answer **any ten** questions : **(10 × 3 = 30)**

25) Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation.

26) Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

27) By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.

28) If $x = \sin t$, $y = \cos 2t$ then prove that $\frac{dy}{dx} = -4\sin t$.

29) Verify Rolle's theorem for the function $f(x) = x^2 + 2$, $x \in [-2, 2]$.

30) Find two numbers whose sum is 24 and whose product is as large as possible.

31) Find $\int \frac{x\,dx}{(x+1)(x+2)}$.

32) Find $\int e^x \sin x\,dx$.

33) Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

34) Form the differential equation representing the family of curves $y = a\sin(x+b)$, where a, b are arbitrary constants.

35) Show that the position vector of the point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m:n$ is $\frac{m\vec{b}+n\vec{a}}{m+n}$.

36) Find x such that the four points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

37) Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1).

38) A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART – D

Answer **any six** questions : **(6 × 5 = 30)**

39) Let R_+ be the set of all non-negative real numbers. Show that the function $f : R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$ is invertible and write the inverse of f.

40) If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, calculate AC, BC and $(A+B)C$. Also, verify that $(A+B)C = AC + BC$.

41) Solve the following system of linear equations by matrix method.

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5.$$
$$2x - y + 3z = 12$$

42) If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.

43) Sand is pouring from a pipe at the rate of $12\ cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

44) Find the integral of $\frac{1}{x^2+a^2}$ with respect to x and hence find $\int \frac{1}{x^2-6x+13}dx$.

45) Using integration find the area of the region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

46) Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$.

47) Derive the equation of the line in space passing through two given points, both in vector and Cartesian form.

48) If a fair coin is tossed 10 times, find the probability of

i) exactly six heads

ii) atleast six heads.

PART – E

Answer **any one** of the following questions : **(1 × 10 = 10)**

49) a) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$. **(6)**

b) Prove that :

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$$ **(4)**

50) a) Solve the following problem graphically :

Minimise and Maximise

$z = 3x + 9y$

Subject to the constraints :

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0,\ y \geq 0.$ **(6)**

b) Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1, & \text{if} \quad x \leq 3 \\ bx + 3, & \text{if} \quad x > 3 \end{cases}$$

is continuous at $x = 3$. **(4)**