Code No. 1018

Second Year – March 2016

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പരിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) The function $f : N \rightarrow N$, given by $f(x) = 2x$ is

(i) one-one and onto

(ii) one-one but not onto

(iii) not one-one and not onto

(iv) onto, but not one-one **(Score : 1)**

(b) Find $gof(x)$, if $f(x) = 8x^3$ and $g(x) = x^{1/3}$. **(Scores : 2)**

(c) Let $*$ be an operation such that $a * b = \text{LCM}$ of a and b defined on the set $A = \{1, 2, 3, 4, 5\}$. Is $*$ a binary operation ? Justify your answer. **(Scores : 2)**

2. (a) If $xy < 1$, $\tan^{-1}x + \tan^{-1}y =$ ______. **(Score : 1)**

(b) Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$. **(Scores : 3)**

3. (a) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, then BA =

(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **(Score : 1)**

(b) Write $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **(Scores : 3)**

(c) Find the inverse of $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$. **(Scores : 2)**

4. (a) The value of $\begin{vmatrix} x & x-1 \\ x+1 & x \end{vmatrix}$ is

(i) 1 (ii) x

(iii) x^2 (iv) 0 **(Score : 1)**

(b) Using properties of determinants, show that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$$ **(Scores : 4)**

5. (a) Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x + 3, & x \leq 2 \\ 2x - 3, & x > 2 \end{cases}$$ **(Scores : 2)**

(b) If $e^{x-y} = x^y$, then prove that

$$\frac{dy}{dx} = \frac{\log x}{[\log ex]^2}$$ **(Scores : 4)**

6. (a) The slope of the tangent to the curve given by

$x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{2}$ is

(i) 0 (ii) -1

(iii) 1 (iv) Not defined **(Score : 1)**

(b) Find the intervals in which the function $f(x) = x^2 - 4x + 6$ is strictly decreasing. **(Scores : 2)**

(c) Find the minimum and maximum value, if any, of the function $f(x) = (2x - 1)^2 + 3$. **(Scores : 2)**

OR

(a) Which of the following functions has neither local maxima nor local minima ?

(i) $f(x) = x^2 + x$ (ii) $f(x) = \log x$

(iii) $f(x) = x^3 - 3x + 3$ (iv) $f(x) = 3 + |x|$ **(Score : 1)**

(b) Find the equation of the tangent to the curve $y = 3x^2$ at (1, 1). **(Scores : 2)**

(c) Use differential to approximate $\sqrt{36.6}$. **(Scores : 2)**

7. (a) The angle between the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = 1$ is

(i) $\frac{\pi}{2}$ (ii) $\frac{\pi}{3}$

(iii) $\frac{\pi}{4}$ (iv) 0 (Score : 1)

(b) Find the unit vector along $\vec{a} - \vec{b}$, where $\vec{a} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$.

(Scores : 2)

8. (a) If the points A and B are (1, 2, –1) and (2, 1, –1) respectively, then $\overrightarrow{AB}$ is

(i) $\hat{i} + \hat{j}$ (ii) $\hat{i} - \hat{j}$

(iii) $2\hat{i} + \hat{j} - \hat{k}$ (iv) $\hat{i} + \hat{j} + \hat{k}$ (Score : 1)

(b) Find the value of λ for which the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} - \lambda\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar. (Scores : 2)

(c) Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. (Scores : 2)

9. (a) Prove that $\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$. (Scores : 2)

(b) Find $\int \frac{dx}{\sqrt{2x - x^2}}$ (Scores : 2)

(c) Find $\int x \cos x \, dx$. (Scores : 2)

10. Evaluate $\int_0^{\pi} \log(1 + \cos x) \, dx$. (Scores : 4)

OR

Find $\int_0^5 (x + 1) \, dx$ as limit of a sum. (Scores : 4)

11. (a) The area bounded by the curve $y = f(x)$, above the x-axis, between $x = a$ and $x = b$ is

(i) $\int_{f(a)}^{b} y\,dy$ (ii) $\int_{a}^{f(b)} x\,dx$

(iii) $\int_{a}^{b} x\,dy$ (iv) $\int_{a}^{b} y\,dx$ **(Score : 1)**

(b) Find the area of the circle $x^2 + y^2 = 4$ using integration. **(Scores : 5)**

12. (a) $y = a\cos x + b\sin x$ is the solution of the differential equation.

(i) $\frac{d^2y}{dx^2} + y = 0$ (ii) $\frac{d^2y}{dx^2} - y = 0$

(iii) $\frac{dy}{dx} + y = 0$ (iv) $\frac{dy}{dx} + x\frac{dy}{dx} = 0$ **(Score : 1)**

(b) Find the solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ given that $y = 0$ when $x = 1$. **(Scores : 5)**

13. Find the shortest distance between the lines

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ **(Scores : 4)**

14. (a) Equation of the plane with intercepts 2, 3, 4 on the x, y and z axis respectively is

(i) $2x + 3y + 4z = 1$ (ii) $2x + 3y + 4z = 12$

(iii) $6x + 4y + 3z = 1$ (iv) $6x + 4y + 3z = 12$ **(Score : 1)**

(b) Find the Cartesian equation of the plane passing through the points A(2, 5, –3), B(–2, –3, 5) and C(5, 3, –3). **(Scores : 3)**

15. Consider the following L.P.P.

Maximize $Z = 3x + 2y$

Subject to the constraints

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$

(a) Draw its feasible region. **(Scores : 3)**

(b) Find the corner points of the feasible region. **(Scores : 2)**

(c) Find the maximum value of Z. **(Score : 1)**

16. (a) If $P(A) = 0.3$, $P(B) = 0.4$, then the value of $P(A \cup B)$ where A and B are independent events is

(i) 0.48 (ii) 0.51

(iii) 0.52 (iv) 0.58 **(Score : 1)**

(b) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be diamonds. Find the probability of the lost card being a diamond. **(Scores : 4)**

OR

A pair of dice is thrown 4 times. If getting a doublet is considered as a success,

(1) find the probability of getting a doublet. **(Score : 1)**

(2) hence, find the probability of two successes. **(Scores : 4)**