: No. 9018

Name : ..........................................

**Second Year – March 2018**

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

# MATHEMATICS (SCIENCE)

Maximum : 80 Scores

***General Instructions to Candidates :***

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

***വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :***

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

**Questions 1 to 7 carry 3 scores each. Answer any Six questions.**

**(Scores : $6 \times 3 = 18$)**

1. If $f(x) = \frac{x}{x-1}, x \neq 1$
   (a) Find $f \circ f(x)$ **(Scores : 2)**
   (b) Find the inverse of f. **(Score : 1)**

2. Using elementary row operations, find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. **(Scores : 3)**

3. (a) f(x) is a strictly increasing function, if f'(x) is ________
   (i) positive
   (ii) negative
   (iii) 0
   (iv) None of these **(Score : 1)**
   (b) Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$ is strictly increasing. **(Scores : 2)**

4. (a) $\int_0^a f(a-x)dx =$ ________. **(Score : 1)**
   $\left[ \text{(i)} \int_0^{2a} f(x)\,dx, \quad \text{(ii)} \int_{-a}^{a} f(x)\,dx, \quad \text{(iii)} \int_0^{a} f(x)\,dx, \quad \text{(iv)} \int_a^{0} f(x)\,dx \right]$
   (b) Find the value of $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$. **(Scores : 2)**

5. Find the area of the region bounded by the Curve $y^2 = x$, $x$-axis and the lines $x = 1$ and $x = 4$. **(Scores : 3)**

6. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$. **(Scores : 3)**

7. A manufacturer produces nuts and bolts. It takes 1 hour of work on Machine A and 3 hours on Machine B to produce a package of nuts. It take 3 hours on Machine A and 1 hour on Machine B to produce a package of bolts. He earns a profit of ₹ 17.50 per package on nuts and ₹ 7.00 per package on bolts. Formulate the above L.P.P., if the machines operates for at most 12 hours a day. **(Scores : 3)**

**Questions 8 to 17 carry 4 Scores each. Answer any eight. (Scores : 8 × 4 = 32)**

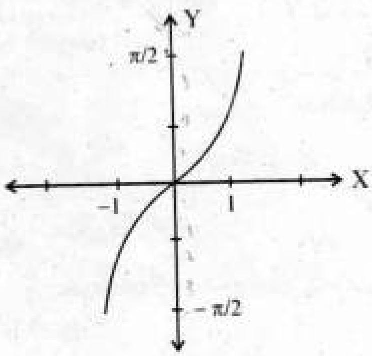
8. Let A = N × N and '*' be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d)

(a) Find (1, 2) * (2, 3) **(Score : 1)**

(b) Prove that '*' is commutative **(Score : 1)**

(c) Prove that '*' is associative. **(Scores : 2)**

9.




(a) Identify the function from the above graph.

(i) $\tan^{-1}x$

(ii) $\sin^{-1}x$

(iii) $\cos^{-1}x$

(iv) $\operatorname{cosec}^{-1}x$ **(Score : 1)**

(b) Find the domain and range of the function represented in above graph. **(Score : 1)**

(c) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$. **(Scores : 2)**

10. (a) $\frac{d(a^x)}{dx}$ = ______

(i) $a^x$

(ii) $\log(a^x)$

(iii) $a^x \log a$

(iv) $xa^{x-1}$ **(Score : 1)**

(b) Find $\frac{dy}{dx}$ if $x^y = y^x$. **(Scores : 3)**

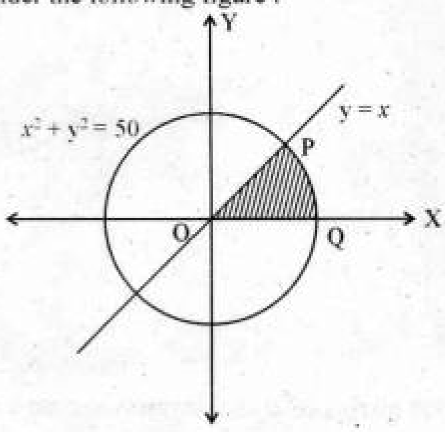
11. (a) Find the slope of the tangent to the curve $y = (x - 2)^2$ at $x = 1$. **(Score : 1)**

(b) Find a point at which the tangent to the curve $y = (x - 2)^2$ is parallel to the chord joining the points A(2, 0) and B(4, 4). **(Scores : 2)**

(c) Find the equation of the tangent to the above curve and parallel to the line AB. **(Score : 1)**

12. $\int_0^2 (x^2 + 1)\, dx$ as the limit of a sum. **(Scores : 4)**

13. Consider the following figure :




(a) Find the point of intersection 'P' of the circle $x^2 + y^2 = 50$ and the line $y = x$. **(Score : 1)**

(b) Find the area of the shaded region. **(Scores : 3)**

14. (a) The degree of the differential equation $xy\left(\frac{d^2y}{dx^2}\right)^2 + x^4\left(\frac{dy}{dx}\right)^3 - y\frac{dy}{dx} = 0$ is _____.

(i) 4
(ii) 3
(iii) 2
(iv) 1

**(Score : 1)**

(b) Find the general solution of the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$ **(Scores : 3)**

15. (a) Prove that for any vectors $\vec{a}, \vec{b}, \vec{c}$, $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$. **(Scores : 3)**

(b) Show that if $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar then $\vec{a}, \vec{b}, \vec{c}$ are also coplanar. **(Score : 1)**

16. (a) Find the equation of a plane which makes $x$, y, z intercepts respectively as 1, 2, 3. **(Scores : 2)**

(b) Find the equation of a plane passing through the point (1, 2, 3) which is parallel to above plane. **(Scores : 2)**

17. Solve the L.P.P. given below graphically :

Minimise $Z = -3x + 4y$

Subject to $x + 2y \leq 8$,

$3x + 2y \leq 12$,

$x \geq 0, \ y \geq 0$ **(Scores : 4)**

**Questions from 18 to 24 carry 6 scores each. Answer any five.**

**(Scores : 5 × 6 = 30)**

18. (a) Find $x$ and y if

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$ **(Scores : 2)**

(b) Express the matrix $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. **(Scores : 4)**

19. (a) Prove that $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0.$ **(Scores : 2)**

(b) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$,

(i) Prove that $B = A^{-1}$.

(ii) Using $A^{-1}$ solve the system linear equations given below.

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$ **(Scores : 4)**

20. (a) Prove that the function defined by $f(x) = \cos(x^2)$ is a continuous function. **(Scores : 2)**

(b) (i) If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$, show that $\dfrac{dy}{dx} = \dfrac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$. **(Score : 1)**

(ii) Hence, prove that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0.$ **(Scores : 3)**

21. Evaluate the following :

(a) $\int \sin mx \, dx$. **(Score : 1)**

(b) $\int \frac{1 \, dx}{\sqrt{x^2 + 2x + 2}}$ **(Scores : 3)**

(c) $\int \frac{x \, dx}{(x+1)(x+2)}$ **(Scores : 2)**

22. (a) If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

(i) Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. **(Scores : 2)**

(ii) Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. **(Scores : 2)**

(b) Consider the points A(1, 2, 7), B (2, 6, 3), C(3, 10, –1).

(i) Find $\overrightarrow{AB}$, $\overrightarrow{BC}$ **(Score : 1)**

(ii) Prove that A, B, C are collinear points. **(Score : 1)**

23. (a) Find the angle between the lines

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$ **(Scores : 2)**

(b) Find the shortest distance between the pair of lines

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$ **(Scores : 4)**

24. (a) The probability distribution of a random variable is given by $P(x)$. What is $\Sigma P(x)$ ? **(Score : 1)**

(b) The following is a probability distribution function of a random variable.

| $x$ | –5 | –4 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(x) | k | 2k | 3k | 4k | 5k | 7k | 8k | 9k | 10k | 11k | 12k |

(i) Find k **(Scores : 2)**

(ii) Find $P(x > 3)$ **(Score : 1)**

(iii) Find $P(-3 < x < 4)$ **(Score : 1)**

(iv) Find $P(x < -3)$ **(Score : 1)**