Tamilnadu Board Class 12 Mathematics Previous year Question Paper March 2016

PART - III

கலினிய / MATHEMATICS

(தமிழ் மற்றும் ஆங்கில நூல் / Tamil & English Versions)

[பொதுத்துணர்வு] 3 வளை [மத்திய முக்கியத்துவம்] : 200
Time Allowed : 3 Hours [மத்திய முக்கியத்துவம்] : 200

Instructions: (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and pencil to draw diagrams.

பகுதி - A / PART - A

சொத்து: (i) அதிசத்து விளையாட்டுக்குரிய விளையாட்டுக்கு.

(ii) அதிசத்து விளையாட்டுக்குரிய விளையாட்டுக்கு வரையறுக்கப்பட்ட விளையாட்டுக்கு இன்று உள் விளையாட்டுக்கு.

Note: (i) Answer all the questions.

(ii) Choose the most suitable answer from the given four alternatives and write the option code and corresponding answer.

[துருக்கு] / Turn over
1. \( y = 2x, x = 0 \) and \( x = 2 \) define a region which about \( x \)-axis is:

(1) \( 8\sqrt{5} \pi \)  
(2) \( 2\sqrt{5} \pi \)  
(3) \( \sqrt{5} \pi \)  
(4) \( 4\sqrt{5} \pi \)

The surface area of the solid of revolution of the region bounded by \( y = 2x, x = 0 \) and \( x = 2 \) about \( x \)-axis is:

(1) \( 8\sqrt{5} \pi \)  
(2) \( 2\sqrt{5} \pi \)  
(3) \( \sqrt{5} \pi \)  
(4) \( 4\sqrt{5} \pi \)

2. \( E(X + C) = 8 \) and \( E(X - C) = 12 \) then the value of \( C \) is:

(1) \(-2\)  
(2) \(4\)  
(3) \(-4\)  
(4) \(2\)

3. \( (2, -3) \) and \( x = 4 \) define a region which length of the latus rectum of the parabola whose vertex is \( (2, -3) \) and the directrix \( x = 4 \) is:

(1) \(2\)  
(2) \(4\)  
(3) \(6\)  
(4) \(8\)

4. \( z \)-axis is \( \vec{i} - \vec{j} \) define a vector:

(1) \(0\)  
(2) \(1\)  
(3) \(-1\)  
(4) \(2\)

The projection of \( \vec{i} - \vec{j} \) on \( z \)-axis is:

(1) \(0\)  
(2) \(1\)  
(3) \(-1\)  
(4) \(2\)

A
5. \[ x = 0 \quad x = \frac{\pi}{4} \] The area of the region bounded by the graphs of \( y = \sin x \) and \( y = \cos x \) between \( x = 0 \) and \( x = \frac{\pi}{4} \) is:

(1) \( \sqrt{2} + 1 \) (2) \( \sqrt{2} - 1 \) (3) \( 2\sqrt{2} - 2 \) (4) \( 2\sqrt{2} + 2 \)

6. \( f(x) = x^2 \) has:

(1) a maximum value at \( x = 0 \)
(2) minimum value at \( x = 0 \)
(3) finite number of maximum values
(4) infinite number of maximum values

7. \[ x^2 - 4(y - 3)^2 = 16 \] The directrices of the hyperbola \( x^2 - 4(y - 3)^2 = 16 \) are:

(1) \( y = \pm \frac{8}{\sqrt{5}} \) (2) \( x = \pm \frac{8}{\sqrt{5}} \) (3) \( y = \pm \frac{\sqrt{5}}{8} \) (4) \( x = \pm \frac{\sqrt{5}}{8} \)
8. A missile fired from ground level rises $x$ metres vertically upwards in “t” seconds and $x = t(100 - 12.5t)$. Then the maximum height reached by the missile is:

(1) 100 m  (2) 150 m  (3) 250 m  (4) 200 m

A missile fired from ground level rises $x$ metres vertically upwards in “t” seconds and $x = t(100 - 12.5t)$. Then the maximum height reached by the missile is:

(1) 100 m  (2) 150 m  (3) 250 m  (4) 200 m

9. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is:

(1) $\sqrt{7}$  (2) 4  (3) 3  (4) 5

The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is:

(1) $\sqrt{7}$  (2) 4  (3) 3  (4) 5

10. \[
\int_0^a f(x) \, dx = 2 \int_0^a f(x) \, dx \quad \text{if:}
\]

(1) $f(2a - x) = f(x)$  
(2) $f(a - x) = f(x)$  
(3) $f(x) = -f(x)$  
(4) $f(-x) = f(x)$

$\int_0^a f(x) \, dx = 2 \int_0^a f(x) \, dx$ if:

(1) $f(2a - x) = f(x)$  
(2) $f(a - x) = f(x)$  
(3) $f(x) = -f(x)$  
(4) $f(-x) = f(x)$

11. \[
\left[ \frac{-1 + i\sqrt{3}}{2} \right]^{100} + \left[ \frac{-1 - i\sqrt{3}}{2} \right]^{100}
\]

(1) 2  (2) 0  (3) -1  (4) 1

The value of $\left[ \frac{-1 + i\sqrt{3}}{2} \right]^{100} + \left[ \frac{-1 - i\sqrt{3}}{2} \right]^{100}$ is:

(1) 2  (2) 0  (3) -1  (4) 1

A
12. \[ \overrightarrow{a} + \overrightarrow{b}, \overrightarrow{b} + \overrightarrow{c}, \overrightarrow{c} + \overrightarrow{a} \] = 8 \text{ then } \[ \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \] is equal to the following:
(1) 4 (2) 16 (3) 32 (4) -4

If \[ \overrightarrow{a} + \overrightarrow{b}, \overrightarrow{b} + \overrightarrow{c}, \overrightarrow{c} + \overrightarrow{a} \] = 8 then \[ \overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c} \] is:
(1) 4 (2) 16 (3) 32 (4) -4

13. \( z \) lies in the third quadrant then \( z \) lies in the:
(1) first quadrant (2) second quadrant (3) third quadrant (4) fourth quadrant

14. The set of positive even integers, with usual addition forms:
(1) a finite group (2) only a semi group (3) only a monoid (4) an infinite group

A [ திறந்து / Turn over
16. A random variable $X$ has the following p.d.f.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>0</td>
<td>$k$</td>
<td>$2k$</td>
<td>$2k$</td>
<td>$3k$</td>
<td>$k^2$</td>
<td>$2k^2$</td>
<td>$7k^2+k$</td>
</tr>
</tbody>
</table>

The value of $k$ is

(1) $\frac{1}{8}$ (2) $\frac{1}{10}$ (3) 0 (4) $-1$ or $\frac{1}{10}$

17. The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D-a) g(D)$, $g(a) \neq 0$ is:

(1) $m e^{ax}$ (2) $\frac{e^{ax}}{g(a)}$ (3) $g(a) e^{ax}$ (4) $\frac{x e^{ax}}{g(a)}$

The marks secured by 400 students in a Mathematics test were normally distributed with mean 65. If 120 students got marks above 85, the number of students securing marks between 45 and 65 is:

(1) 120 (2) 20 (3) 80 (4) 160
19. The locus of the point of intersection of perpendicular tangents to the parabola \( y^2 = 4ax \) is:
(1) latus rectum
(2) directrix
(3) tangent at the vertex
(4) axis of the parabola

20. Identify the incorrect statement:
(1) The order of a differential equation is the order of the highest order derivative occurring in it.
(2) The degree of the differential equation is the degree of the highest order derivative which occurs in it (the derivatives are free from radicals and fractions).
(3) \( \frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)} \) is the first order first degree homogeneous differential equation.
(4) \( \frac{dy}{dx} + xy = e^x \) is a linear differential equation in \( x \).
21. \[ \sqrt{1 + \left( \frac{dy}{dx} \right)^3} = \frac{d^2y}{dx^2} \] is only possible when \[ y = x^3 + C \] (where \( C \) is a constant).

Options:
(1) 1  
(2) 2  
(3) 3  
(4) 6

The degree of the differential equation \[ \sqrt{1 + \left( \frac{dy}{dx} \right)^3} = \frac{d^2y}{dx^2} \] is:

Options:
(1) 1  
(2) 2  
(3) 3  
(4) 6

22. \[ \frac{x - 3}{4} = \frac{y - 1}{2} = \frac{z - 5}{-3} \] and \[ \frac{x - 1}{4} = \frac{y - 2}{2} = \frac{z - 3}{-3} \] are parallel lines.

The shortest distance between the parallel lines:

Options:
(1) 3  
(2) 2  
(3) 1  
(4) 0

23. The points \( z_1, z_2, z_3, z_4 \) in the complex plane are the vertices of a parallelogram if and only if:

Options:
(1) \( z_1 + z_4 = z_2 + z_3 \)  
(2) \( z_1 + z_3 = z_2 + z_4 \)  
(3) \( z_1 + z_2 = z_3 + z_4 \)  
(4) \( z_1 - z_2 = z_3 - z_4 \)
24. \( y = e^{mx} \) and \( y = e^{-mx} \), \( m > 1 \) determine the acute angle between the curves.

\[
\begin{align*}
(1) & \quad \tan^{-1}\left( \frac{2m}{m^2 - 1} \right) \\
(2) & \quad \tan^{-1}\left( \frac{2m}{1 - m^2} \right) \\
(3) & \quad \tan^{-1}\left( \frac{-2m}{1 + m^2} \right) \\
(4) & \quad \tan^{-1}\left( \frac{2m}{m^2 + 1} \right)
\end{align*}
\]

The angle between the curve \( y = e^{mx} \) and \( y = e^{-mx} \) for \( m > 1 \) is

\[
\begin{align*}
(1) & \quad \tan^{-1}\left( \frac{2m}{m^2 - 1} \right) \\
(2) & \quad \tan^{-1}\left( \frac{2m}{1 - m^2} \right) \\
(3) & \quad \tan^{-1}\left( \frac{-2m}{1 + m^2} \right) \\
(4) & \quad \tan^{-1}\left( \frac{2m}{m^2 + 1} \right)
\end{align*}
\]

25. Which of the following is a contradiction?

(1) \( p \vee q \)  
(2) \( p \wedge q \)  
(3) \( p \vee \neg p \)  
(4) \( p \wedge \neg p \)

26. \( ay^2 = x^2(3a - x) \) determines the curve that intersects the y-axis. The points of intersection are:

\[
\begin{align*}
(1) & \quad x = -3a, x = 0 \\
(2) & \quad x = 0, x = 3a \\
(3) & \quad x = 0, x = a \\
(4) & \quad x = 0
\end{align*}
\]
The curve \( ay^2 = x^2(3a - x) \) cuts the y-axis at:

\[
\begin{align*}
(1) & \quad x = -3a, x = 0 \\
(2) & \quad x = 0, x = 3a \\
(3) & \quad x = 0, x = a \\
(4) & \quad x = 0
\end{align*}
\]
27. \( ax + y + z = 0; x + by + z = 0; x + y + cz = 0 \) तीन रेखांकितों के अभाज्यकार मूलें हैं।

\[ \frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = \]

(1) 1  (2) 2  (3) -1  (4) 0

The system of equations \( ax + y + z = 0; x + by + z = 0; x + y + cz = 0 \) has a non-trivial solution then.

\[ \frac{1}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = \]

(1) 1  (2) 2  (3) -1  (4) 0

28. \( a \) एक रेखाकार वेक्टर है, \( m \) एक रेखाकार स्केलर है। वेक्टर \( ma \) एक नॉर्मल वेक्टर है तब वेक्टर का मान है:

(1) \( m = \pm 1 \)  (2) \( a = |m| \)  (3) \( a = \frac{1}{|m|} \)  (4) \( a = 1 \)

If \( \vec{a} \) is a non-zero vector and \( m \) is a non-zero scalar then \( m \vec{a} \) is a unit vector if:

(1) \( m = \pm 1 \)  (2) \( a = |m| \)  (3) \( a = \frac{1}{|m|} \)  (4) \( a = 1 \)

29. \( |z - z_1| = |z - z_2| \) तथा \( z - z_1 \) एक रेखाकार वेक्टर है:

(1) \( z_1 \) के लिए एक रेखाकार वेक्टर है।
(2) \( z_1 \) के लिए एक रेखाकार वेक्टर है।
(3) \( z_1 \) के लिए एक रेखाकार वेक्टर है।
(4) \( z_1 \) के लिए एक रेखाकार वेक्टर है।

If \( |z - z_1| = |z - z_2| \) then the locus of \( z \) is:

(1) a circle with centre at the origin
(2) a circle with centre at \( z_1 \)
(3) a straight line passing through the origin
(4) is a perpendicular bisector of the line joining \( z_1 \) and \( z_2 \)
30. For a homogeneous system $\rho(A) < n$, where $n$ is the number of unknowns, then the system has:

(1) only trivial solution
(2) trivial solution and infinitely many non-trivial solutions
(3) only non-trivial solutions
(4) no solution

31. Which of the following are statements?

(i) May God bless you.
(ii) Rose is a flower.
(iii) Milk is white
(iv) 1 is a prime number.

(1) (i), (ii), (iii)
(2) (i), (ii), (iv)
(3) (i), (iii), (iv)
(4) (ii), (iii), (iv)
32. \[ \vec{a} + \vec{j} - \vec{k} \] is a vector at an angle with \( \vec{i} - \vec{3j} + 4\vec{k} \). The magnitude of \( \vec{a} + \vec{j} - \vec{k} \) is:

(1) \[ 10\sqrt{3} \] 
(2) \[ 6\sqrt{3} \] 
(3) \[ \frac{3}{2} \sqrt{30} \] 
(4) \[ 3\sqrt{30} \]

The area of the parallelogram having a diagonal \( \vec{a} + \vec{j} - \vec{k} \) and a side \( \vec{i} - \vec{3j} + 4\vec{k} \) is:

(1) \[ 10\sqrt{3} \] 
(2) \[ 6\sqrt{3} \] 
(3) \[ \frac{3}{2} \sqrt{30} \] 
(4) \[ 3\sqrt{30} \]

33. \[ a^2y^2 = x^2(a^2 - x^2) \] is a curve defined for:

(1) \[ x \leq a \text{ and } x \geq -a \] 
(2) \[ x < a \text{ and } x > -a \] 
(3) \[ x \leq -a \text{ and } x \geq a \] 
(4) \[ x < a \text{ and } x > -a \]

34. \( f(a) = 2; f'(a) = 1; g(a) = -1; g'(a) = 2 \) and \( \lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a} \)

(1) \[ 5 \] 
(2) \[ -5 \] 
(3) \[ 3 \] 
(4) \[ -3 \]

If \( f(a) = 2; f'(a) = 1; g(a) = -1; g'(a) = 2 \) then the value of \( \lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a} \) is:

(1) \[ 5 \] 
(2) \[ -5 \] 
(3) \[ 3 \] 
(4) \[ -3 \]

35. A matrix \( A \) of order 3 has \( \det (kA) \) equals:

(1) \[ k^3 \det (A) \] 
(2) \[ k^2 \det (A) \] 
(3) \[ k \det (A) \] 
(4) \[ \det (A) \]

If \( A \) is a matrix of order 3, then \( \det (kA) \) is:

(1) \[ k^3 \det (A) \] 
(2) \[ k^2 \det (A) \] 
(3) \[ k \det (A) \] 
(4) \[ \det (A) \]
36. In 16 throws of a die getting an even number is considered a success, then the variance of the successes is

(1) 4  (2) 6  (3) 2  (4) 256

37. $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

(a) Adjoint $(\text{adj} \ A) A =$

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (3) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$  (4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then $(\text{adj} \ A) A =$

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (3) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$  (4) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

38. The differential equation satisfied by all the straight lines in $xy$-plane (not parallel to $y$-axis) is:

(1) $\frac{dy}{dx} = \text{a constant}$  (2) $\frac{d^2 y}{dx^2} = 0$

(3) $y + \frac{dy}{dx} = 0$  (4) $\frac{d^2 y}{dx^2} + y = 0$

The differential equation satisfied by all the straight lines in $xy$-plane (not parallel to $y$-axis) is:

(1) $\frac{dy}{dx} = \text{a constant}$  (2) $\frac{d^2 y}{dx^2} = 0$

(3) $y + \frac{dy}{dx} = 0$  (4) $\frac{d^2 y}{dx^2} + y = 0$
39. \[ \mathbf{r} = \left( -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \right) + t \left( -2\mathbf{i} + \mathbf{j} + \mathbf{k} \right) \]

\[ \mathbf{r} = \left( 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \right) + s \left( \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \right) \]

The point of intersection of the lines \( \mathbf{r} = \left( -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \right) + t \left( -2\mathbf{i} + \mathbf{j} + \mathbf{k} \right) \) and

\[ \mathbf{r} = \left( 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \right) + s \left( \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \right) \] is:

(1) (2, 1, 1) (2) (1, 2, 1) (3) (1, 1, 2) (4) (1, 1, 1)

40. The order of \(-i\) in the multiplicative group of 4th roots of unity is:

(1) 4 (2) 3 (3) 2 (4) 1

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(1) 4 (2) 3 (3) 2 (4) 1

PART - B

10x6=60

Note: (i) Answer any ten questions.

(ii) Question No. 55 is compulsory and choose any nine from the remaining.

41. Solve the system of equations

\[ x + y + 2z = 4; \quad 2x + 2y + 4z = 8; \quad 3x + 3y + 6z = 10 \]

by using determinant.

A
42. \[ A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \, \text{ Bài tập:} \, A \text{ là diagonal.} \]

Show that the adjoint of \( A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \) is \( A \) itself.

43. \[ \vec{r} = (\vec{i} + 2\vec{j} - 5\vec{k}) + t(2\vec{i} - 3\vec{j} + 4\vec{k}) \text{ và } \vec{r} \cdot (2\vec{i} + 4\vec{j} - \vec{k}) = 3, \text{ và } \vec{r} \cdot (2\vec{i} + 4\vec{j} - k) = 3. \]

Find the co-ordinates of the point where the line
\[ \vec{r} = (\vec{i} + 2\vec{j} - 5\vec{k}) + t(2\vec{i} - 3\vec{j} + 4\vec{k}) \]
meets the plane \( \vec{r} \cdot (2\vec{i} + 4\vec{j} - \vec{k}) = 3. \)

44. (i) \[ 2\vec{i} - 2\vec{j} + \vec{k} \, \text{ và } \vec{A} = (1, 2, 3) \rightarrow \vec{B} = (5, 3, 7). \]

(ii) \[ x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0, \, \text{ và } \vec{A} = (-1, 4, -3) \rightarrow \vec{B}. \]

(i) A force of magnitude 5 units acting parallel to \( 2\vec{i} - 2\vec{j} + \vec{k} \) displaces the point of application from \( (1, 2, 3) \) to \((5, 3, 7)\). Find the work done by the force.

(ii) If \( A(-1, 4, -3) \) is one end of a diameter \( AB \) of the sphere \( x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0 \), then find the coordinates of \( B. \)

[\( A \)] [ ศักยภาพ / Turn over
45. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, if one of its roots is $2 + \sqrt{3}i$.

46. Prove that $\tan^{-1}x < x$, for all $x > 0$.

47. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/sec. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed lengths is $\frac{\pi}{3}$.

48. If $V = 3e^{ax} + by$ and $z$ is a homogeneous function of degree $n$ in $x$ and $y$, prove that

$$x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} = (ax + by + n)V$$.

49. Evaluate:\n
$$\int \sin^6 x \, dx.$$
50. The temperature $T$ of a cooling object drops at a rate proportional to the difference $(T-S)$, where $S$ is constant temperature of surrounding medium. If initially $T = 150^\circ C$, find the temperature of the cooling object at any time $t$.

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51. $[(\neg q) \land p] \land q$ என்பதாயிருக்கும் கள்பகுதிக் கோணத்துக்கு இந்த புள்ளிக் கோட்டாது.

Show that $[(\neg q) \land p] \land q$ is a contradiction.

52. எந்த குறுகிய வரைப்பட்ட வட்டம் அதன் சுற்று மத்தியாகக் குறுகியேதாமே, அகலத்தை ஒன்றுக்கு ஒன்றின் சுற்றின்று அல்ல என்பது எச்சொல்லும் ஏன் கிடைக்கும்.

If every element of a group is its own inverse then prove that the group is abelian.

53. கோல்லாப் 52 பிள்ளைகளையான பிள்ளைத் தொகுப்பு ஒன்றாக நிற்பர்க்கப்பட்டுள்ள கற்று சுற்றியுள்ள பிள்ளைகள் என்போவை. என்ன சுற்றியுள்ள பிள்ளைகளைத் தொகுப்பு, பருப்புப்பட்டு காணாது.

Two cards are drawn with replacement from a well shuffled deck of 52 cards. Find the mean and variance for the number of aces.
20% of the bolts produced in a factory are found to be defective. Find the probability that in a sample of 10 bolts chosen at random exactly 2 will be defective using:

(i) Binomial distribution

(ii) Poisson distribution \( [e^{-2} = 0.1353] \)

(a) Find the equation of the hyperbola if its centre is \((2, 1)\); one of the foci is \((8, 1)\) and the corresponding directrix is \(x = 4\).

OR

(b) (i) Find the least positive integer \(n\) such that \(\left(\frac{1 + i}{1 - i}\right)^n = 1\).

(ii) Find the values of \((i)^3\).
PART - C

Note: (i) Answer any ten questions.

(ii) Question No. 70 is compulsory and choose any nine from the remaining.

56. Examine the consistency of the system

\[ \begin{align*}
x - 3y - 8z &= -10 \\
3x + y - 4z &= 0 \\
2x + 5y + 6z &= 13 \\
\end{align*} \]

by using rank method and hence solve the system.

57. If \( \vec{a} = \vec{i} + \vec{j} + \vec{k}, \) \( \vec{b} = 2\vec{i} + \vec{k}, \)

\( \vec{c} = 2\vec{i} + \vec{j} + \vec{k}, \) \( \vec{d} = \vec{i} + \vec{j} + 2\vec{k} \)

then verify that

\[ (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d} \]
58. \[ \frac{-2}{2} = \frac{y - 2}{3} = \frac{z - 1}{-2} \] \text{and the equation of the plane containing the line}\ \[ \frac{x - 2}{2} = \frac{y - 2}{3} = \frac{z - 1}{-2} \] \text{and passing through the point}\ \((-1, 1, -1)\) \\

Find the vector and Cartesian equations of the plane containing the line \[ \frac{x - 2}{2} = \frac{y - 2}{3} = \frac{z - 1}{-2} \] and passing through the point \((-1, 1, -1)\).

59. \[ \frac{-2}{2} = \frac{y - 2}{3} = \frac{z - 1}{-2} \] \text{and the expression for the variable complex number}\ \(z\). \text{Find the locus of} \ P \text{if} \ \text{Re}\left(\frac{z + 1}{z + i}\right) = 1. \]

60. \[ y^2 + 8x - 6y + 1 = 0 \] \text{Find the axis, vertex, focus, equation of directrix, latus rectum, length of latus rectum for the parabola}\ \(y^2 + 8x - 6y + 1 = 0\) \text{and also draw the diagram.}

61. The orbit of the planet Mercury around the Sun is in elliptical shape with Sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find:

(i) How close the Mercury gets to Sun?

(ii) The greatest possible distance between Mercury and Sun.

(iii) The magnitude of the speed of Mercury at its perihelion and aphelion.

(iv) The angular momentum of Mercury about the Sun at its perihelion and aphelion.

(v) The gravitational force between the Sun and Mercury at its perihelion and aphelion.
62. \( x - y + 4 = 0 \) is tangent to the ellipse \( x^2 + 3y^2 = 12 \). Find the point of contact.

63. \( x = a(\theta + \sin \theta), \quad y = a(1 + \cos \theta) \). Find the equations of the tangent and normal at \( \theta = \frac{\pi}{2} \) to the curve \( x = a(\theta + \sin \theta), \quad y = a(1 + \cos \theta) \).

64. \( u = \sin 3x \cos 4y \). Verify \( \frac{2u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \).

65. \( x = a(t + \sin t), \quad y = a(1 + \cos t) \). Find the surface area of the solid generated by revolving one arc of the cycloid \( x = a(t + \sin t), \quad y = a(1 + \cos t) \) about its base (x-axis).
66. \[ \frac{x^2}{9} + \frac{y^2}{5} = 1 \] 

Find the area of the region bounded by the ellipse \( \frac{x^2}{9} + \frac{y^2}{5} = 1 \) between the two latus rectums.

67. 

\[(x^2 + y^2)\,dx + 3xy\,dy = 0\]

Solve:

\[(x^2 + y^2)\,dx + 3xy\,dy = 0\]

68. \((36D^2 - 24D + 13)y = 2\sin^2x - e^{-x} + 2\) 

Solve the differential equation \((36D^2 - 24D + 13)y = 2\sin^2x - e^{-x} + 2\).

69. 

The mean score of 1000 students for an examination is 34 and S.D. is 16. 

(i) 30 students are below 60 marks. Determine the marks of the central 70% of the candidates. 

\[ P[0 < z < 0.25] = 0.0987 \]
\[ P[0 < z < 1.63] = 0.4484 \]
\[ P[0 < z < 1.04] = 0.35 \]

(ii) 70% of the candidates scored above 70 marks. Determine the normality of the distribution and the limit of marks of the central 70% of the candidates:

\[ P[0 < z < 0.25] = 0.0987 \]
\[ P[0 < z < 1.63] = 0.4484 \]
\[ P[0 < z < 1.04] = 0.35 \]
70. (a) Find the intervals of convexity and concavity of the Gaussian curve \( y = e^{-x^2} \) and also find the points of inflection.

OR

(b) Show that \((Z, \ast)\) is an infinite abelian group, where \('\ast'\) is defined as \(a \ast b = a + b + 2\) and \(Z\) is the set of all integers.

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