

Tamilnadu Board Half Yearly Question Paper 2018

Class 11 Mathematics

COMMON HALF YEARLY EXAMINATION - DECEMBER 2018

STANDARD - XI

Time : 2.30 hours

MATHEMATICS

Marks: 90

Instructions: 1. Check the question paper for fairness of printing. If there is any lack of fairness, Inform the Hall Supervisor immediately. 2. Use Blue or Black Ink to write and underline and pencil to draw diagrams.

PART - I

20x1=20

Answer all the questions:

- 1) For any non-empty sets A and B, if $A \subset B$ then, $(A \times B) \cap (B \times A)$ is equal to
 a) $A \cap B$ b) $A \times A$ c) $B \times B$ d) none of these
- 2) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 1 - |x|$, then the range of f is
 a) \mathbb{R} b) $(1, \infty)$ c) $(-1, \infty)$ d) $(-\infty, 1)$
- 3) The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is
 a) 1 b) 2 c) 3 d) 4
- 4) The value of $\sin\left(\cos^{-1} \frac{5}{13}\right)$
 a) $\frac{12}{13}$ b) $\frac{5}{13}$ c) $\frac{5}{12}$ d) 1
- 5) If $\pi < 2\theta < \frac{3\pi}{2}$ then $\sqrt{2} + \sqrt{2} + 2\cos 4\theta$ equals to
 a) $-2\cos\theta$ b) $-2\sin\theta$ c) $2\cos 2\theta$ d) $2\sin\theta$
- 6) In a ΔABC , if i) $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} > 0$ ii) $\sin A \sin B \sin C > 0$ then
 a) Both i) and ii are true b) Only i) is true
 c) Only ii) is true d) Neither i) nor ii) is true
- 7) If $2nC_3 : nC_3 = 11 : 1$ then n is
 a) 5 b) 6 c) 11 d) 7
- 8) If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then the value of A is
 a) 7^2 b) 8^2 c) 9^2
- 9) The sum up to n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \dots$ is
 a) $\frac{n(n-1)}{2}$ b) $2n(n+1)$ c) $\frac{n(n+1)}{2}$ d) 1
- 10) The coefficient of x^5 in the series e^{-2x} is
 a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $-\frac{4}{15}$ d) $\frac{4}{15}$
- 11) Which of the following equation is the locus of $(at^2, 2at)$
 a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ c) $x^2 + y^2 = a^2$ d) $y^2 = 4ax$

12) θ is the acute angle between the lines $x^2 - xy - 6y^2 = 0$, then $\frac{2 \cos \theta + 3 \sin \theta}{4 \sin \theta + 5 \cos \theta}$ is

- a) 1 b) $-\frac{1}{9}$ c) $\frac{5}{9}$ d) $\frac{1}{9}$ ✓

13) If $\begin{bmatrix} 4 & 3 \\ -2 & x \end{bmatrix}$ is singular then the value of x is

- a) $\frac{3}{2}$ b) $-\frac{3}{2}$ ✓ c) 3 d) -2

14) If the points $(x - 2)$, $(5, 2)$, $(8, 8)$ are collinear, then x is equal to

- a) -3 b) $1/3$ c) 1 d) 3

15) If $m(2\hat{i} + \hat{j} - \hat{k})$ is a unit vector then the value of m is

- a) $\pm \frac{1}{\sqrt{3}}$ b) $\pm \frac{1}{\sqrt{5}}$ ✓ c) $\pm \frac{1}{\sqrt{6}}$ d) $\pm \frac{1}{\sqrt{2}}$

16) If $|\vec{a}| = 13, |\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60^\circ$ then $|\vec{a} \times \vec{b}|$ is

- a) 15 b) 35 c) 45 d) 25 ✓

17) The value of $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x^2}}$ is

- a) 1 ✓ b) -1 c) 0 d) ∞

18) $\lim_{x \rightarrow 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$

- a) $2 \log 2$ b) $2(\log 2)^2$ ✓ c) $\log 2$ d) $3 \log 2$

19) If $f(x) = x \tan^{-1} x$ then $f'(1)$ is

- a) $1 + \frac{\pi}{4}$ b) $\frac{1}{2} + \frac{\pi}{4}$ ✓ c) $\frac{1}{2} - \frac{\pi}{4}$ d) 2

20) The derivative of $f(x) = x|x|$ at $x = -3$ is

- a) 6 b) -6 c) does not exist d) 0

PART-II

Answer any 7 questions. Question Number 30 is Compulsory. 7x2=14

21) Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B where x, y, z are distinct elements.

22) Solve $2|x + 1| - 6 \leq 7$ and graph the solution set in a number line. [Handwritten note: $x \in [-\frac{1}{2}, \frac{5}{2}]$]

23) Prove that $\tan 315^\circ \cot(-405^\circ) + \cot 495^\circ \tan(-585^\circ) = 2$

24) Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time? [Handwritten note: 7!]

25) Expand $(1 + x)^{\frac{2}{3}}$ up to four terms for $|x| < 1$ [Handwritten note: $\frac{2}{9}$]

26) Transform the equation $3x + 4y + 12 = 0$ in to normal form.

27) For what value of x , the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew, symmetric.

28) Differentiate : 2^x

29) Evaluate : $\lim_{x \rightarrow 0} \frac{e^{5x-1}}{x}$

30) Find the projection of the vector $\bar{i} + 3\bar{j} + 7\bar{k}$ on the vector $2\bar{i} + 6\bar{j} + 3\bar{k}$.

PART - III

Answer any 7 questions. Question number 40 is Compulsory.

7x3=21

31) Draw the curves of i) $y = x^2 + 1$ ii) $y = (x + 1)^2$ by using the graph of the curve $y = x^2$.

32) Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$

33) Find all the values of $x \neq 0$ in $(-\pi, \pi)$ satisfying the equation $8^1 + \cos x + \cos^2 x \pm \dots = 4^3$

34) If $nP_r = 11880$ and $nC_r = 495$, find n and r .

35) If a, b, c are in geometric progression and if $\frac{1}{a^x} = \frac{1}{b^y} = \frac{1}{c^z}$, then prove that x, y, z are in arithmetic progression.

36) Find the image of the point $(-2, 3)$ about the line $x + 2y - 9 = 0$

37) Prove that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

38) If G is the centroid of a triangle ABC , prove that $\overline{GA} + \overline{GB} + \overline{GC} = 0$

39) Evaluate : $\lim_{x \rightarrow 0} \frac{3^x - 1}{\sqrt{1+x} - 1}$

40) If $f(x) = 2x^2 + 3x - 5$, then prove that $f'(0) + 3f'(-1) = 0$

PART-IV

Note : Answer all the questions.

7x5=35

41) a) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 2x - 3$, then prove that f is a bijection and find its inverse.

(OR)

b) Solve : $\log_{5-x} (x^2 - 6x + 65) = 2$

42) a) If the function f is defined as $f(x) = \begin{cases} 3x - 2, & x > 3 \\ x^2 - 2, & -2 \leq x \leq 2 \\ 2x + 1, & x < -3 \end{cases}$ Then find the values, **(OR)**

if exists $f(4), f(-4), f(0), f(-7)$

b) If $A + B + C = \pi$, prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

43) a) Two trees A and B are on the same side of a river. From a point C in the river, the distances of the trees A and B are 250 m and 300 m respectively. If the angle C is 45° , find the distance between the trees. ($\sqrt{2} = 1.414$) **(OR)**

b) Prove that the sum of the first 'n' non-zero even numbers is $n^2 + n$.

44) a) Prove that $\sqrt[3]{x^3 + 7} - \sqrt[3]{x^3 + 4}$ is approximately equal to $\frac{3}{x^2}$ when x is large. **(OR)**

b) Show that the equations $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represent a pair of parallel lines. Find the distance between them.

45) a) Using factor theorem, show that $\begin{vmatrix} -2a & a+b & c+a \\ a+b & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(a+b)(b+c)(c+a)$

b) Show that the vectors $5\bar{i} + 6\bar{j} + 7\bar{k}, 7\bar{i} - 8\bar{j} + 9\bar{k}, 3\bar{i} + 20\bar{j} + 5\bar{k}$ are coplanar.

46) a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + KI = 0$, then find the value of k. **(OR)**

b) Let $\bar{a} = 2\bar{i} + \bar{j} - 2\bar{k}, \bar{b} = \bar{i} + \bar{j}$. If \bar{c} is a vector such that $\bar{a} \cdot \bar{c} = |\bar{c}|, |\bar{c} - \bar{a}| = 2\sqrt{2}$ and the angle between $\bar{a} \times \bar{b}$ and \bar{c} is 30° . Find the value of $|(\bar{a} \times \bar{b}) \times \bar{c}|$

47) a) Discuss the continuity of $f(x) = \sqrt{1-x^2}$, where $x \in [-1, 1]$ **(OR)**

b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $(1-x^2)y_2 - 3xy_1 - y = 0$.