The Questions number from 1 to 10 below carries 4 marks each:

Q1. Solve $\sin^2 x + \sin^2 2x = 1$

Q2. Find the value of $i^{30} + i^{40} + i^{60}$

Q3. Prove by mathematical induction that $11^{n+2} + 122^{n+1}$ *is divisible by* 133 for all positive integer values of *n*.

Q4. Determine whether the points (0,0) and (5,5) lie on different sides of the straight line $x + y - 8 = 0$ or on the same side of the straight line.

Q5. Prove that

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

Q6. A, B, C are 3 sets and U is the universal set such that

$n(U) = 800$, $n(A) = 200$, $n(B) = 300$, $n(A \cap B) = 100$ Find $n(A' \cap B')$

Q7. If P be the sum of the odd terms and Q the sum of the even terms in the expansion of $(x + a)^n$, prove that $P^2 - Q^2 = (x^2 - a^2)^n$

Q8. If α, β are the roots of the equation $x^2 - bx + c = 0$ find the value of $\alpha^2 + \beta^2$

Q9. Solve the inequality

$$\frac{x^2 - 3x + 6}{3 + 4x} < 0$$

Q10. Prove that:

$$\cot(A + 15) - \tan(A - 15) = \frac{4\cos 2A}{1 + 2\sin 2A}$$

The Questions number from 11 to 17 below carries 7 marks each:

Q11. The mean and variance of 7 observations are 8 and 19 respectively. If 5 of the observations are 2, 4, 12, 14, 11. Find the remaining observations.

Q12. How many 6 digits numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7 if the 10th, unit's places are always even and repetition is not allowed?

Q13. In the expansion $(1+x)^{40}$, the coefficients of T_{2r+1} and T_{r+2} are equal, find r?

Q14. Differentiate $\log_{10} x$ with respect to x.

Q15. On the average one person dies out of every 10 accidents find the probability that at least 4 will be safe out of 5 accidents.

Q16. Shift the origin to a suitable point so that the equation $x^2 + y^2 - 4x + 6y = 36$ representing a circle is transformed in to an equation of a circle with centre at origin in the new coordinate axes.

Q17. Prove that

$$\frac{1}{\log_a b}, \frac{1}{\log_{2a} b}, \frac{1}{\log_{4a} b}$$

form an AP.