MATHEMATICS

Standard 8

(Semester II)

- Born : 22 December, 1887, Erode, Tamilnadu
- At the age of 12 he studied all mathematics-books of college.
- He accepted Matriculation pass clerk job.
- At the age of 15 he solved a book of 6165 formulae named 'Synopsis of Pure-Mathematics".
- He invented methods to prepare Magic-Squares.
- He gave infinite series in Algebra.
- In 1912 his two essays were published in the journal of the Indian Mathematical Society.
- He met in 1914 with Dr. Hardy of Cambridge University.
- In 1915 his article about composite numbers was published in proceeding of London Mathematical Society. $6746328388800 = 2^6$, 3^4, 5^2, 5^2, 7^2, 11, 13, 17, 19, 23 which is registered as last in the list of largest prime number.
- He gave the theory of division of 'Zero'.
- He gave the principle of Division.
- His 21 articles were published in the Newspapers of Europe during 5 years.
- He was made a fellow of the Royal Society of London and Trinity College.
- He prepared 24 philosophical-articles in the Cambridge University.
- He returned back to Indian in 1919.
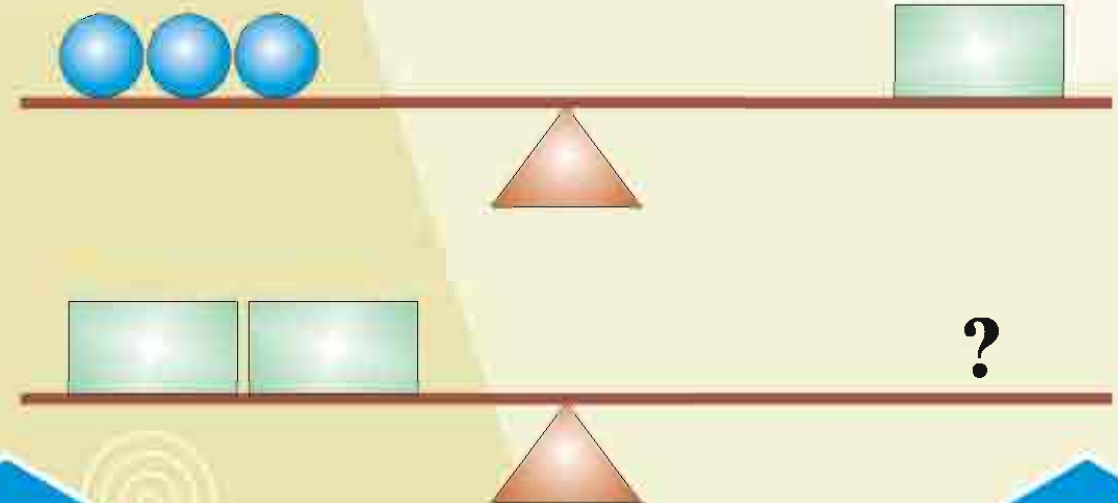- On 26 April, 1920 at the age of 32 he passed away in the 'World of Mathematics'.

Shriniwas Ramanujan

?

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$x^2 - y^2 = (x - y)(x + y)$$

$$\left\{\left(\frac{x}{y}\right)^{\frac{1}{7}}\right\}^{\frac{1}{3}}$$

Gujarat Council of Educational Research and Training
Gandhinagar

Gujarat State Board of School Textbooks
Gandhinagar

Website of Mathematics

www.mathsisfun.com

www.mathteaching.net

www.mathworksheetsgo.com

www.mattimath.com

www.medindia.com

www.puzzlemaster.com

www.puzzleplace.com

www.puzzles.com

www.megatrunks.com

www.mes-english.com

www.mozila.com

www.omniture.com

www.physlink.com

www.y-math.co.uk

www.aplusmath.com

www.aaamath.com

www.allmath.com

www.coolmath.com

www.mymath.co.uk

www.emaths.co.uk

www.mathsguru.co.uk

www.mathsroom.co.uk

Net Banking

In recent time a person can do money transactions without going to the bank at any time, from any place if he has internet facility with computer, laptop or cell phone (mobile.)

If the account holder wishes, this facility is provided by the bank. The bank provides a private PIN (Personal Identification Number) to the account holder. With the help of this number the account holder can open and see his account details. Also...

- He can transfer money from his own account to another account besides this he can transfer money to the account of other person.
- He can pay many types of bills/premiums.
- He can apply for loan.
- He can generate account statement.
- He can recharge mobile as well as DTH.

Merits : • Time is saved. • Work is done speedly.

Precautions : • Keep confidential PIN (Password)
• Follow the instructions given by the bank time to time.

How much time is taken to complete any definite work by two persons togather can be found out with the help of graphs.

Example, To do any definite work, Manish takes 6 hours and Nilay takes 12 hours. How much time will be taken of they do this work together ?
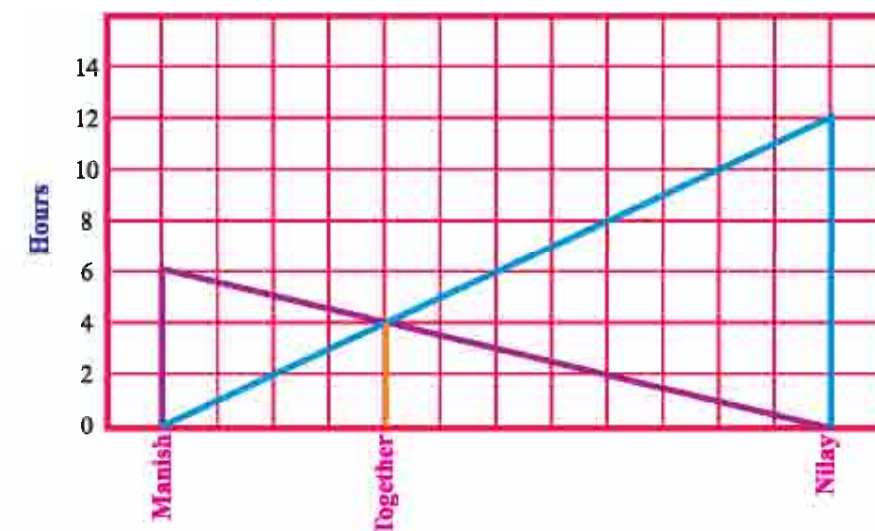
Solution : Both will take 4 hours to complete the work togather.

A 'Teacher's book' has been prepared for teachers and parents (separately). Kindly use this.

MATHEMATICS

Standard 8

(Semester II)

PLEDGE

India is my country.

All Indians are my brothers and sisters.

I love my country and I am proud of its rich and varied heritage.

I shall always strive to be worthy of it.

I shall respect my parents, teachers and all my elders and treat everyone with courtesy.

I pledge my devotion to my country and its people.

My happiness lies in their well-being and prosperity.

Price : ₹ 33.00

PREFACE

The National Curriculum Framework (NCF) 2005 and the Right to Education Act (RTE) 2009 recommends connecting knowledge that is provided in school to the life outside the school. This principle marks a departure from the legacy of book is learning which continues to shape our education system and is creating removes a huge gap between the school, home and community.

The syllabi and textbook developed on the basis of above principle signify an attempt to implement it with a considerable change in the textbooks, teaching-learning methods, approaches etc. Such textbooks will provide the scope to the students to learn individually, in pair, in group and as a whole class and provide self-learning, improve the application and consolidation abilities of the children. In such a scenario, the teacher will be just an initiator, facilitator and guide and will create learner dominant classes.

During the process of designing and developing the textbooks, the core group personnel coordinators, writers and reviewers got a lot of inspiration and motivation from the Chief Secretary of Elementary Education.

Also, the guidance from IGNUS-erg and co-operation of UNICEF was easily and continuously available to the group during the entire process of developing the textbooks. After implementing the textbooks as a part of the pilot study, due efforts were done to make it faultless. Now, it is in the hands of the users and beneficiaries.

GCERT welcomes constructive and creative comments and suggestions which will be useful to undertake further revision and refinement.
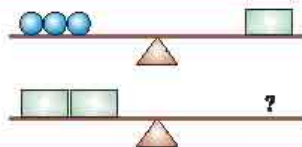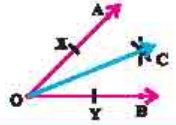
FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India :*

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage or our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild-life, and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement.

INDEX

1 Banking

❖ **Let's Learn New :**

The mischievous team of Std. 8 of Ekta Primary School, Rahul, Saijal, Nisar, Leela all decided to save their pocket-money.

Nisar : What shall we do with the saved pocket-money ?

Saijal : Whenever we need, we will withdraw.

Rahul : With whom the collected money be saved ?

Saijal : With sir.

Leela : Let's tell sir that there should be an arrangement in school for saved money.

Sir : Children, to save and return money of all students is a big task. For that proper system must be needed.

Children : Then, What should we do ?

Sir : Let's do one thing. Let's go to the bank and open an account for all the children.

 (All visit the bank)

Children : Namaste Madam, we want to open accounts in the bank.

Hemaben : Children, to open an account, you have to fill up a form.

Children : Which type of form ?

Hemaben : Here is the form. Take it and fill up your details in it.

Form

Date : _____

	First Name	Middle Name	Surname
First Account Holder	_____	_____	_____
Second Account Holder	_____	_____	_____
Third Account Holder	_____	_____	_____

	Date of Birth	PAN	Male/Female	Nationality	Relation with First A/c Holder
First Account Holder					
Second Account Holder					
Third Account Holder					

Class : ☐ Adult ☐ Senior Citizen ☐ Pensioner ☐ Staff ☐ Illiterate ☐ Blind
☐ Others

If you are minor then ....

Date of Birth of minor _____

Parents/Gaurdian Name _____

Address of Parents/Gaurdian _____

Relation with ☐ Father ☐ Mother ☐ Order by court ☐ Others

```
┌─────────────────────────────┐
│                             │
└─────────────────────────────┘
```
Signature of Parents/Gaurdian

If you are staff member then : ☐ In service ☐ Retired

Application type : ☐ Individual ☐ Partnership ☐ Pvt. Ltd. Company
☐ Ltd. Company ☐ Trust

Address :

First Account Holder _____

Second Account Holder _____

Third Account Holder _____

Signature in presence of Bank Officer :

Passport size Photo of First Account Holder	Passport size Photo of Second Account Holder	Passport size Photo of Third Account Holder
Signature	Signature	Signature

Identification Details :

For Photo ID : (1) Passport (2) PAN Card (3) Driving Licence (4) Government ID
(5) Voter ID (An attested copy of one out of these as given)

For Residence : (1) Electricity bill (2) Ration Card (3) Telephone Bill
(An attested copy of one out of these as given)

Saijal : All persons open their account as we do ?

Hemaben : No, each person has different requirements. According to that they do transactions and for that they open different accounts according to their requirements.

Rahul : What is the characterstics of each account ?

Hemaben :

Savings Account

- Maximum numbers of people open this account.

- On this account the bank provides interest at fixed rate.
- Main aim is that people save the money.

Current Account

- The people doing daily transactions in this account. For example : Merchants, Companies, Businessman, etc.
- On this account the bank does not provide interest.
- On this service the banks take necessary charges from the account holder.

Types of Account

Recurring Deposit Account

- It is monthly savings scheme.
- Time period and amount of saving is determined before opening the account.

Fixed Deposit Account

- Long term deposite.
- More interest is obtained.
- Time period is fixed at the time of opening the account.
- To withdraw money before fixed time period, less interest is obtained than fixed rate.

Leela : Madam, what is called the person opening the account ?

Hemaben : The person who opened the account is called 'Account Holder'.

Nisar : What to do to deposit amount ?

Hemaben : To deposit amount in the account "Deposit slip" is to be filled. You all fill one-one such slip.

(Children, you also fill your details in this slip)

Branch _____ Date _____	Branch _____ Date _____
Type of A/c _____	Type of A/c _____
Full Name _____	Full Name _____
A/c Number _____	A/c Number _____
	Telephone No. _____ Mobile No. _____

Details	Rs.	P.		Bank & Branch	Cheque No.	Cash notes	No.	Rs.	P.
						1000 ×			
						500 ×			
						100 ×			
						50 ×			
						20 ×			
						10 ×			
						5 ×			
Total				Total Amount (In words)		Coin			
				_____		Total			
				PAN _____					
						Sign. of Depositer			

(Cheque Deposit) (Cash Deposit)

Figure (1) : Deposit slip

Saijal : Madam, how will we come to know that the money has been deposited in our account ?

Hemaben : The bank notes all money transaction in a book and that book is called "Passbook".

Rahul : What does Account Holder do to withdraw money ?

Hemaben : The money can be withdrawn from the account by different methods. Let me give you details.

(1) **Withdrawl form :**

Date _____

Pay self the sum of Rupees _____

_____

Savings Bank Account No. _____ | Rs.

In the name/s of _____

Ledger folio No. _____

Initial of Ledger keeper _____ Signature of Account Holder

Figure (2) : Withdrawl form

The money is paid to the Account Holder only when withdrawn through withdrawl form.

(2) The bank provides cheque book to the Account Holder on a condition that according to the rule of the bank the fixed amount always be there in the account. The cheque book is provided to the Account Holder after writting the Account Number at every page of cheque book.

Date _____

Pay _____

_____ or Bearer

Rupees _____

Rs.

A/c No. [   ]

Bank of ×××××
Anand Branch
Anand 388001.
Branch code : 1000

|| 0001 || 38013003 : 003561 || 10

Figure (3) : Cheque

In a cheque, It is necessary to write date, amount in figure and word in proper columns and then account holder must sign at bottom of cheque. Below the cheque in white column cheque number is printed. It is instructed, nothing to be written in this column. Written cheque is accepted by cashier and after necessary verification, the written amount is paid to the account holder.

There are three types of cheque :

Bearer cheque   Order cheque   Crossed cheque
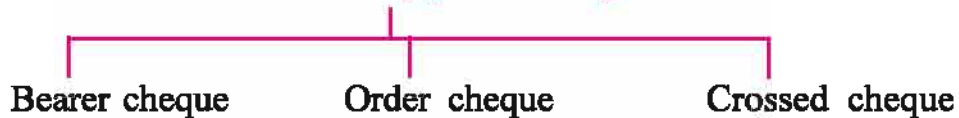
(1)   Bearer Cheque :

	Date 01/02/2013
P a y _____ Self _____	
_____	or Bearer
Rupees _____ Ten thousand only _____	
	Rs. 10,000/-
A/C No. 015062012	
Bank of ××××	
Anand Branch	*P. K. Sharma*
Anand 388001	
Branch Code : 1000	Parth K. Sharma

|| 0001 || 38013003 : 003561 || 10

Figure (4) : Bearer Cheque

The cheque, in which date has been written, at the place of name 'self' has been written, amount has been written in words and figure and signature of account holder has been done, is called bearer cheque.

If any person presents this cheque in the bank then that person can obtain the amount in cash. Therefore a person can get money instead of going himself he can send the other person. For this facility the big loss is that if the cheque is lost then unknown person can also get money from the bank. Therefore, this type of cheque is the most unsafe cheque.

(2)   Order Cheque :

	Date 28/12/2013
Pay ___Sajidbhai Saiyad_____	
_____	or Bearer
Rupees ___Two thousand & five hundred only___	
	Rs. 2,500 /-
A/C No. 203401307	
Bank of ××××× Anand Branch Anand 388001 Branch Code : 1000	Sofiya Rana
|| 0001 || 380014002 : 00132 || 15	

Figure (5) : Order Cheque

In the cheque after the space of name 'Or Bearer' is written. In the place of name, the name of any person is writen and 'Or Bearer' word is cancelled, then this cheque becomes order cheque. To whom this cheque belongs bank verifies the details of that person and the payment is made.

(3)   Crossed Cheque : (A/c Payee)

A/C Payee	Date 20-10-2013
Pay ___Arif Mansuri_____	
_____	or Bearer
Rupees ___Fifty thousand only___	
	Rs. 50,000 /-
A/C No. 0463002420	
Bank of ××××× Anand Branch Anand 388001 Branch Code : 1000	S.S. Patel Subhash S. Patel
|| 0001 || 38013003 : 005748 || 06	

Figure (6) : A/c Payee OR Crossed Cheque

In top left corner of cheque two parallel lines are drawn and between the lines or at the top of the cheque 'A/c Payee' is written, then this cheque in called 'A/c Payee' or cross-cheque. The amount of the cheque is deposited to the account of the person in whose favour the cheque is written. The amount written in chqeue can't be encashed. This is the safest cheque.

By ATM Card :

Figure (7) : ATM Card

In big cities and towns, different banks started their ATM centre. The full name of ATM is "Automated Teller Machine". To use this machine account holder must have ATM card. With the help of ATM card he can withdraw money from ATM machine related to that bank at any time and from any place. One can withdraw money from ATM in a day as limit of amount decided by the bank.

Hemaben : Children, understood ?

All : Yes.

Hemaben : Answer the following questions on the basis of figures (1) to (7).

(1) Where cheque number is printed in the cheque ?

(2) From the sample of Bearer Cheque, tell the account number.

(3) To whom Order Cheque is written ?

(4) How much amount is written in Crossed Cheque ?

(5) Who signed the Crossed Cheque ?

(6) Write full-form of 'ATM'.

Kishan : Yesterday my elder brother was telling to our father that he has come with Demand Draft from the bank. What is it ?

Hemaben : See, a person, companies or institutions use demand draft for safe transaction or to collect definite amount of fee.

Figure (8) : Demand Draft

- For Demand Draft an application form has been filled up in the bank with that amount of draft and bank charge is deposited at that time. Demand Draft is denoted by "DD" in shortly.

- There are two types of Demand Draft : (1) Order (2) A/c Payee

- To whom Demand Draft is written the person, institution or company can present to the bank with necessary identification and can get the money.

- Some times, for admission form of higher studies or examination fees or with application of job advertisement Demand Draft is expected.

Sir : Hemaben, does bank provide any other facility except transaction of money ?

Hemaben : Yes, many facilities are provided, such as .....

- The bank provides loan with definite guarantee to purchase house, for agriculture to farmers, for education to students, to purchase vehicle, for industries etc.

- To put personal valuable items the bank provides facility of safe deposit vault (Locker).

- Some banks accept insurance premium, electricity bill, telephone bill, student's fee etc.

- Through the bank, salary, pension, interest, dividend etc. are paid to the people.

- Some banks provide facility for transfer of foreign money exchange.

- Bank also provides the facilities of credit card to the people.

Saijal : Madam, you have provided us many useful details. We will do money transactions properly with the bank.

All : Yes Madam, thank you very much.

Words used : Bank

Account

Account Holder

Saving Account

Current Account

Bearer Cheque

Order Cheque

Crossed Cheque

Fixed Deposit Account

Demand Draft

Exercise

1. Give the answer by doing ✔ sign against correct alternative in the given alternatives :

(1) One who wants to open his / her account in the bank is known as .....

○ member   ○ account holder   ○ cashier

(2) The book having details of transaction of money is called.....

○ passbook   ○ note book   ○ cheque book

(3) How many types of cheques are there ?

○ One   ○ Two   ○ Three

(4) On which account interest is not obtained ?

○ Savings Account   ○ Current Account   ○ Fixed Deposit Account

(5) On which account the time period is fixed at the timing of opening ?

○ Savings Account   ○ Current Account   ○ Fixed Deposit Account

(6) On which cheque cash is not obtained, but deposited in the account ?

○ Crossed Cheque   ○ Order Cheque   ○ Bearer Cheque

(7) Which type of cheque is the most unsafe cheque ?

○ Crossed Cheque   ○ Order Cheque   ○ Bearer Cheque

(8) What is called Demand Draft in short ?

○ DB   ○ DD   ○ CD

(9) Which type of cheque is the safest cheque ?

⭘ Bearer Cheque   ⭘ Order Cheque   ⭘ Crossed Cheque

(10) How is the interest of fixed deposit account as compared to savings account ?

⭘ Less   ⭘ More   ⭘ Equal

2. Answer the following questions in short :

(1) Write the types of account.

(2) How many types of cheque are there ? What are they ?

(3) What is used to deposit money in the account ?

(4) Write the importance of bank.

(5) If there is no bank then ....

(6) For what purpose should one open the account in the bank ?

●

👉 **Answers** 👈

Exercise

1. (1) account holder   (2) passbook   (3) Three

(4) Current Account   (5) Fixed Deposit Account   (6) Crossed Cheque

(7) Bearer Cheque   (8) DD   (9) Crossed Cheque

(10) More

Compound Interest

Hardikbhai borrowed ₹ 50,000 at 10% rate of interest for 3 years from Niru Finance Company to get coloured his house. At the end of 3 years he got a notice to deposit ₹ 66,550 from finance company. But according to calculation done by Hardikbhai at the end of 3 years the amount should be ₹ 65,000. How did it happen ?

Let's see !

As the notice obtained on the same day Hardikbhai went to see the manager Smt. Vandanaben of Niru Finance Company.

Hardikbhai : Today I got notice to deposit ₹ 66,550 from you. But according to my calculation the amount with interest should be ₹ 65,000 then why did you give me notice to deposit ₹ 66,550 ?

Vandanaben : Tell me how did you do the calculation ?

Hardikbhai : See, by this method (Method of simple interest) I had taken ₹ 50,000 at 10 % rate for 3 years

i.e. Principal is ₹ 50,000, rate of interest is 10 % and time period is 3 years.

$$\therefore \text{Interest (I)} = \frac{PRN}{100}$$

$$= \frac{50000 \times 10 \times 3}{100}$$

$$= ₹ 15,000$$

And   Amount = Principal ₹ 50,000 + Interest ₹ 15,000

$\qquad$ = ₹ 65,000

Therefore,, I should deposit ₹ 65,000. It is true !

Vandanaben : You are true, but see the condition of loan. Here calculation of interest and interest on interest is done every year, it is written.

Hardikbhai : How ? tell me ?

Vandanaben : See, by this method.

Calculation of interest is done every year.

Principal (P) = 50,000, Rate of interest (R) = 10%, Time = 3 years

$\therefore$ Interest on 50,000 for first year.

$$I \ = \ \frac{PRN}{100}$$

$$\therefore \ I \ = \ \frac{50000 \times 10 \times 1}{100}$$

$$= ₹ \ 5000$$

$\therefore$ Interest of first year = ₹ 5000

Now principal for second year = Principal of first year + Interest of first year

$\qquad$ = ₹ 50,000 + ₹ 5000

$\qquad$ = ₹ 55,000

∴ Interest on 55,000 for 1 year (second year)

$$I = \frac{PRN}{100}$$

$$\therefore \quad I = \frac{55000 \times 10 \times 1}{100} = ₹ \ 5500$$

∴ Interest of second year = ₹ 5500

Now principal for third year = Principal of second year + Interest of second year

$$= ₹ \ 55,000 + ₹ \ 5500$$

$$= ₹ \ 60,500$$

∴ Interest on ₹ 60,500 for 1 year (third year)

$$I = \frac{PRN}{100}$$

$$\therefore \quad I = \frac{60500 \times 10 \times 1}{100}$$

$$= ₹ \ 6050$$

∴ Interest of third year = ₹ 6050

Total Interest = Interest of first year + Interest of second year + Interest of third year

$$= ₹ \ 5000 + ₹ \ 5500 + ₹ \ 6050$$

$$= ₹ \ 16,550$$

Now, Amount = Principal + Total Interest

$$= ₹ \ 50,000 + ₹ \ 16,550$$

$$= ₹ \ 66,550$$

Therefore, by doing calculation of interest every year and adding, the required amount = ₹ 66,550

Vandanaben : See, by doing calculation of interest every year you will have to deposit ₹ 66,550.

Hardikbhai : Thank you, Manager saheb !

Questioning

(1) If Hardikbhai borrowed ₹ 50,000 on interest from Niru Finance Company for 1 year, then what will be the calculation difference between Hardikbhai and the manager ?

(2) If Hardikbhai borrowed ₹ 50,000 on interest from Niru Finance Company for 2 years, then what will be the calculation difference between Hardikbhai and the manager ?

(3) On the basis of given figure which details do you get ?

XXXXX BANK
INTEREST RATES ON TERM DEPOSITS

Maturity Range		Interest Rates
180 Days to 365 Days 6 Months to 12 Months	®	7.00 %
1 Year to 2 years	®	9.00 %
2 Years to 4 Years	®	9.50 %

Interest of interest i.e. compound interest.

❖ **Learn :**

● For the first year the simple interest and compound interest both are same.

● If Interest is added every year then difference of simple interest and compound interest for first two years is equal to interest of interest of first year.

● Amount obtained at the end of any year is taken as a principal for the next year.

● Interest is calculated after every six months in nationalised banks.

Example 1 : Saijan deposits ₹ 1000 at 10 % rate of interest for 2 years in the Post-office at Compound Interest, then what amount will she get at the end of 2 years ?

Solution : Principal (P) = ₹ 1000, Rate of Interest (R) = 10 %,

Time (N) = 2 years

∴ Interest for 1 year on ₹ 1000 = (?)

$$I = \frac{PRN}{100}$$

$$\therefore \quad I = \frac{1000 \times 10 \times 1}{100}$$

$$= ₹ 100$$

Principal for second year = Principal of first year + Interest of first year

$$= ₹ 1000 + ₹ 100$$

$$= ₹ 1100$$

Interest for 1 year (second year) on ₹ 1100

$$I = \frac{PRN}{100}$$

$$\therefore \quad I = \frac{1100 \times 10 \times 1}{100}$$

$$= ₹ 110$$

Compound Interest = Interest of first year + Interest of second year

$$= ₹ 100 + ₹ 110$$

$$= ₹ 210$$

Amount = Principal + Compound Interest

$$= ₹\ 1000 + ₹\ 210$$

$$= ₹\ 1210$$

Therefore, Saijan will get ₹ 1210 at the end of 2 years.

Here we have done calculation of Compound Interest on the basis of formula of

Simple Interest $(I) = \dfrac{PRN}{100}$

Now, let's see the formula to calculate Compound Interest

$$A = P\left(1 + \dfrac{R}{100}\right)^{N}$$

Where,

A = Sum of Principal and Compound Interest = Amount

P = Principal capital

R = Rate of interest

N = Number of years

Example 2 : Neerav borrows loan of ₹ 20,000 at 8 % for 3 years at Compound Interest from a finance company, then at the end of 3 years how much total amount will Neerav have to return to the finance company ?

Solution : Here, P = _____

R = _____

N = _____

Method 1 :

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$= 20000\left(1 + \frac{8}{100}\right)^3$$

$$= 20000\left(\frac{108}{100}\right)^3$$

$$= 20000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100}$$

$$= \frac{2519424}{100}$$

$$= 25,194.24$$

Method 2 :

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$= 20000\left(1 + \frac{8}{100}\right)^3$$

$$= 20000\,(1.08)^3$$

$$= 20000\,(1.259712)$$

$$= 25,194.24$$

Therefore, Neerav will have to return ₹ 25,194.24 at the end of 3 years.

Example 3 : How much compound interest will Prathna get on ₹ 18,000 at 5% rate of interest for 2 years ?

Solution : Here P = ₹ 18000, R = 5 %, N = 2 years

Method 1 :

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$= 18000\left(1 + \frac{5}{100}\right)^2$$

$$= 18000\,(1+0.05)^2$$

$$= 18000\,(1.05)^2$$

$$= 18000\,(1.1025)$$

$$= 19,845$$

Method 2 :

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$= 18000\left(1 + \frac{5}{100}\right)^2$$

$$= 18000\left(\frac{105}{100}\right)^2$$

$$= 18000 \times \frac{105}{100} \times \frac{105}{100}$$

$$= \frac{198450}{10}$$

$$= 19,845$$

∴ Prathna will get ₹ 19,845 at the end of 2 years.

Compound Interest = Amount − Principal

$$= (A) - (P)$$

$$= ₹\ 19,845 - ₹\ 18,000$$

$$= ₹\ 1845$$

Therefore, Prathna will get ₹ 1845 as compound interest.

❖ **Remember :**

Generally the interest is added after every six months in the bank. In this situation, if this formula is used then rate of interest is halved and the time period is doubled.

Example 4 : Geeta borrows ₹ 10,000 at 12 % rate of interest for 1 year at compound interest. At the end of year how much amount will she have to return ? (Interest is added every six months.)

Solution : Here, Principal (P) = ₹ 10,000, Rate of Interest = 12 %,
Time (N) = 1 year

$$A = P\left(1 + \frac{R}{100 \times 2}\right)^{N \times 2}$$

$$= 10000\left(1 + \frac{12}{100 \times 2}\right)^{1 \times 2}$$

$$= 10000 \,(1 + 0.06)^2$$

$$= 10000 \,(1.06)^2$$

$$= 10000 \,(1.1236)$$

$$= 11,236$$

Therefore, Geeta will have to return ₹ 11,236 at the end of 1 year.

Practice 1

1. **By using formula, find compound interest and Amount :**

Sr. No.	Principal (₹)	Rate of Interest	Time Period (Year)
(i)	8000	10%	2
(ii)	6400	12%	2
(iii)	15,000	10%	2
(iv)	20,000	8%	3
(v)	50,000	5%	3

2. Samarth deposits ₹ 1,00,000 at 9 % rate of interest for 2 years in a nationalised bank at compound interest. So at the end of period how much amount will he get from the bank ?

3. Harsha borrows ₹ 5000 to purchase a sewing machine from 'Women Savings Bank' at 5 % rate of compound interest, so at the end of 3 years how much amount will she pay ?

4. Hiteshbhai borrows ₹ 25,000 at 10 % rate of compound interest to purchase a TV for 2 years, so at the end of period how much amount will Hiteshbhai pay to the bank ? (Interest is added after every six months.)

5. Tony deposits ₹ 35,000 at 10 % rate of compound interest for 4 years in a Co-operative society, so at the end of 4 years how much amount will Tony get ?

Activities : Get the details by visiting nearby Co-operative society, Post-office and Bank and answer the following questions :

(1) Your father wants to build a house. He wants to take ₹ 5,00,000 at interest for 5 years. So from which bank should he take loan that he will have to pay less interest ? how much ?

(2) Your brother wants to go foreign for higher education after passing Std. 12. From which bank can he avail the loan ? Which details will he have to submit ? How much amount of loan will be available for different type of education ? How much loan will be available to study in different countries ?

(3) Your mother wants to purchase a cow. Its value is ₹ 20,000. Your monthly income is ₹ 800, so how will you help your mother ? In which bank and in which account will you save your money ? How do you deposit to get more profit ?

Let's now see the difference between Simple interest and Compound interest :

Example 5 : Salim deposits ₹ 10,000 at simple interest and Devendra deposit ₹ 10,000 at compound interest. So every year how much interest will be obtained at 10% rate of interest ? (Calculate for time period of 4 years.)

Solution :

	Simple interest obtained by Salim	Compound interest obtained by Devendra	
First year interest	1000	1000	1000
Second year interest	1000	Last year interest + Interest = 1000 + 100	1100

Third year interest	1000	Last year interest + its interest = 1100 + 100	1210
Fourth year interest	1000	Last year interest + its interest = 1210 + 121	1331
Total interest	**4000**		**4641**

Therefore, at simple interest Salim gets ₹ 4000 and Devendra gets ₹ 4641.

Example 6 : Find the difference of simple interest and compound interest of ₹ 1250 at 10 % rate for 2 years.

Solution : Difference of simple interest and compound interest

$$P = ₹ 1250, R = 10 \%, N = 2 \text{ years}$$

Simple Interest	Compound Interest
$I = \dfrac{PRN}{100}$	$A = P\left(1 + \dfrac{R}{100}\right)^{N}$
$= \dfrac{1250 \times 10 \times 2}{100}$	$= 1250\left(1 + \dfrac{10}{100}\right)^{2}$
$= ₹ 250$	$= 1250 (1 + 0.1)^2$
	$= 1250 (1.1)^2$
	$= 1250 \times 1.21$
	$= 1512.50$
	Compound Interest
	$= \text{Amount} - \text{Principal}$
	$= ₹ 1512.50 - ₹ 1250$
	$= ₹ 262.50$

Difference of both Interests

Compound Interest − Simple Interest = ₹ 262.50 − ₹ 250

$$= ₹ 12.50$$

Example 7 : Dhruvkumar lends ₹ 7500 at 8% rate of interest for 1 year. His wife Purviben deposits the same amount at same rate of interest and for the same time period at compound interest in Mahila Co-operative Society. Who will get more amount ? (In Co-operative Society interest is added at every six months.)

Solution : Here, P = ₹ 7500, N = 1 year, R = 8%

∴ Interest obtained by Dhruvkumar

$$I = \frac{PRN}{100}$$

$$= \frac{7500 \times 8 \times 1}{100}$$

$$= 75 \times 8$$

$$= ₹ 600$$

Therefore, Dhruvkumar gets ₹ 600

∴ Compound interest obtained by Purviben

$$A = P\left(1+\frac{R}{100\times2}\right)^{N\times2}$$

$$= 7500\left(1+\frac{8}{100\times2}\right)^{1\times2}$$

$$= 7500\left(1+\frac{4}{100}\right)^{2}$$

$$= 7500\,(1 + 0.04)^2$$

$$= 7500\,(1.04)^2$$

$$= 7500 \times (1.0816)$$

$$= ₹\ 8112$$

Compound interest obtained by Purviben = Amount − Interest

= ₹ 8112 − ₹ 7500 = ₹ 612

∴ Compound interest obtained by Purviben = ₹ 612 and

Simple interest obtained by Dhruvkumar = ₹ 600

∴ The more interest which is obtained by Purviben = ₹ 612 − ₹ 600 = ₹ 12

Therefore, Purviben will get more amount.

Practice 2

1. **Write the suitable number and fill in the blanks given in the table :**

Sr. No.	Principal	Rate of interest	Time	Simple interest	Compound interest	Difference
(1)	5000	5 %	2	....	....	....
(2)	3000	10 %	2	....	....	....
(3)	25,000	12 %	3	....	....	....
(4)	64,000	6 %	3	....	....	....
(5)	10,000	8 %	2	....	....	....

2. Ratansingh borrowed ₹ 16,000 at 12.5 % rate of interest for 3 year to set up a motorpump on the well from 'Gurudev Co-operative Society' at compound interest. If he would have taken this loan at simple interest, then what would be the difference in the interest ?

3. Muhammadbhai took a loan of ₹ 60,000 at 9 % rate of interest for 3 years to repair his house. If Prafulchandra took the same amount at same rate of interest and for the same time period who will pay more interest and how much ?

4. Arvindbhai deposits ₹ 10,000 at 10 % rate of interest for 3 years in a Co-operative Society at simple interest and Pravinbhai deposits ₹ 10,000 at 10 % rate of interest for 3 years at compound interest in a bank. Find the difference between interests obtained by Arvindbhai and Pravinbhai.

5. Ranchodbhai took a loan of ₹ 50,000 at 9 % rate of interest for 2 years at simple interest to renovate his bungalow. If in place of simple interest the compound interest is paid then how much more more interest will he have to pay ?

Exercise

1. Ruchit borrowed ₹ 20,000 at 12% rate of interest for 3 years at compound interest. At the last how much amount will Ruchit pay as compound interest ?

2. Seema took a loan of ₹ 3,00,000 at 9% rate of interest for 2 years to purchase a motorcar from 'Shaikh Finance Company' at compound interest. At last how much total amount will she have to pay ?

3. Dharmakshi deposited ₹ 4096 in the post-office. Rate of interest is 6%. At the end of 3 years how much amount will the post-office pay to Dharmakshi ?

4. Poonamben took a loan of ₹ 3,20,000 at 5% rate of interest for 4 years at compound interest for the visit of her parents to 'Kailas Mansarovar' from a District Co-operative bank. At last how much amount will Poonamben return to the bank ?

5. Pooja took ₹ 20,000 at compound interest at 10% rate of interest for 3 years from 'Stuti Finance Company'. If Pooja took ₹ 20,000 at simple interest at 10% rate of interest for 3 years then how much less amount will she have to pay ?

6. Jigyaben gave ₹ 50,000 at 12% rate of interest for 2 years to Jitendrabhai at simple interest. If Jigyaben gave this amount at compound interest, how much more interest will be obtained by her ?

7. Jagrutiben deposits ₹ 44,000 at 9.5% rate of interest for 2 years. At last how much compound interest will she get ?

8. During the whole year the students of Standard VII of 'Sidsar Prathmik Shala' collected money from their pocketmoney and deposited the same in the commencement of Standard VIII to the class teacher. The class-teacher deposited the amount at compound interest at 10% rate of interest for 1 year. After 1 year when students passed standard VIII and promoted to standard IX then bank paid a total amount of ₹ 4410. How much amount will be obtained by the student ? (Interest is added half yearly in the bank.)

Sr. No.	Name of the students	Deposited Money	Interest	Total amount obtained by the student
1.	Ved	500		
2.	Mahir	400		
3.	Mosam	300		
4.	Aarjav	500		
5.	Khushi	400		
6.	Kunj	300		
7.	Harshi	300		
8.	Kashyap	400		
9.	Devanshi	500		
10.	Mantra	400		
	Total amount deposited to class teacher	4000	410	4410

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👉 **Answers** 👈

Practice 1

1. (1) ₹ 1680, ₹ 9680   (2) ₹ 1628.16, ₹ 8028.16   (3) ₹ 3150, ₹ 18150

(4) ₹ 5194.24, ₹ 25,194.24   (5) ₹ 7881.25, ₹ 57,881.25

2. ₹ 1,18,810   **3.** ₹ 5788.13   **4.** ₹ 30,387.66   **5.** ₹ 51,243.50

Practice 2

1.

Sr. No.	Simple interest (₹)	Compound interest (₹)	Difference (₹)
(1)	500	512.5	12.5
(2)	600	630	30
(3)	9000	10,123.20	1123.20
(4)	11520	12,225.02	705.02
(5)	1600	1664	64

2. ₹ 781.25   3. Muhammadbhai ₹ 16,200 and Prafulchandra ₹ 17,701.74

4. Pravinbhai will get ₹ 310 more interest   5. ₹ 405 will be more interest

Exercise

1. ₹ 8098.56   2. ₹ 3,56,430   3. ₹ 4878.40   4. ₹ 3,88,962   5. ₹ 620

6. ₹ 720   7. ₹ 8757.10

8.

Sr. No.	1	2	3	4	5	6	7	8	9	10
Interest (₹)	51.25	41	30.75	51.25	41	30.75	30.75	41	51.25	41
Amount (₹)	551.25	441	330.75	551.25	441	330.75	330.75	441	551.25	441

3   Work and Remuneration

❖ **Let's Learn new :**

     Anishabanu, Swituben, Josephbhai and Sunilbhai do the work to colour the saree. They work daily for five hours. After five days, they get remuneration as ₹ 3200, ₹ 2500, ₹ 2000 and ₹ 2500 respectively.

❖ **Answer the following questions :**

    1.   How much more rupees are obtained by Swituben as compared to josephbhai ?

        _____

    2.   How much less rupees are obtained by Sunilbhai as compared to Anishabanu ?

        _____

    3.   If remuneration paid for 1 saree is ₹ 10 then how many sari did they colour ?

        _____

    4.   How many average sarees did everyone colour daily ?

        _____

    5.   Why did everyone not get equal remuneration ?

        _____

    6.   If Sunilbhai wants to get more remuneration, what will he do ?

        _____

❖ **Think :**

What did you do to find, how much saree did they colour ?

Therefore to find work done in the given time by a person, a machine or a unit, divide the work done by time taken.

    "The work done in unit time is called rate of work."

Rate of work is expressed in 'per day', 'per hour', 'per minute' or 'per second'.

$$\text{Rate of work} = \frac{\text{Work done}}{\text{Time taken}}$$

Rate of work done by Sunilbhai $= \dfrac{\text{Work done}}{\text{Time taken}}$

$$= \frac{250 \text{ saree}}{5 \text{ days}}$$

$$= 50 \text{ sarees / day (read as fifty saree per day)}$$

- **Find and write the rate of work done for each :**

1. Swituben   _____

2. Josephbhai   _____

3. Anishabanu   _____

Example 1 : In a pencil making company Pestanji takes 6 days to make 480 packets of pencil and Manojbhai takes 12 days for the same. The owner of the company gives both to make 480 packets together. After completing the work he gives total ₹ 7200 between the two as remuneration. How much each will get ?

What is to be find out ?	**How ?**

- Rate of work done of Pestanji   → Rate of work done $= \dfrac{\text{Work done}}{\text{Time taken}}$

$$= \frac{480 \text{ Packets}}{6 \text{ days}}$$

$$= 80 \text{ Packets / day}$$

- Rate of work done of Manojbhai   → Rate of work done $= \dfrac{\text{Work done}}{\text{Time taken}}$

$$= \frac{480 \text{ Packets}}{12 \text{ days}} = 40 \text{ Packets / day}$$

- Combined rate of work done $\rightarrow$ (80 + 40) Packets / day

  (Work done by both in a day) = 120 Packets / day

- Time taken to complete the work = 1 day to make 120 packets

$\therefore$ Time taken to make 480 packets $= \dfrac{480}{120}$

$$= 4 \text{ days}$$

Remuneration per packet $= \dfrac{\text{Total Remuneration}}{\text{Total packets}}$

$$= \frac{7200}{480}$$

$$= ₹ 15$$

$\therefore$ Remuneration obtained by Pestanji $= 80 \times 4 \times 15$

$$= ₹ 4800$$

$\therefore$ Remuneration obtained by Manojbhai $= 40 \times 4 \times 15$

$$= ₹ 2400$$

Therefore, Pastanji will get ₹ 4800 and Manojbhai will get ₹ 2400.

Example 2 : Shardaben takes 10 hours and Kaminiben takes 15 hours to make 6000 pouch of water by a machine. If both jointly make total 6000 pouch and for that remuneration paid is ₹ 1200 then how much remuneration will each get ?

Rate of work done of Shardaben $= \dfrac{6000 \text{ pouch}}{10 \text{ hours}}$

$$= 600 \text{ pouch / hour}$$

$$\text{Rate of work done of Kaminiben} = \frac{6000 \, \text{pouch}}{15 \, \text{hours}}$$

$$= 400 \, \text{pouch / hour}$$

Combined rate of work done $\rightarrow$ (600 + 400) pouch / hour

(Work done by both in a day) = 1000 pouch / hour

Time taken to make 1000 pouch = 1 hour

$$\therefore \quad \text{Time taken to make 6000 pouch} = \frac{6000}{1000}$$

$$= 6 \, \text{hour}$$

$$\therefore \quad \text{Remuneration for each pouch} = \frac{1200}{6000}$$

$$= ₹ \, \frac{1}{5}$$

$$\text{Remuneration obtained by Shardaben} = 600 \times 6 \times \frac{1}{5}$$

$$= ₹ \, 720$$

$$\text{Remuneration obtained by Kaminiben} = 400 \times 6 \times \frac{1}{5}$$

$$= ₹ \, 480$$

Hence, Shardaben will get ₹ 720 and Kaminiben will get ₹ 480 at remuneration.

Example 3 : Manubhai, Sohilkhan and Kartikbhai take to prepare 18,000 boxes of soap in 90 days, 100 days and 150 days respectively for a soap manufacturing company. If all the three prepare 18,000 boxes jointly then they get 54,000 as remuneration. How much amount will each get ?

$$\text{Rate of work done of Manubhai} = \frac{18000 \, \text{boxes}}{90 \, \text{days}}$$

$$= 200 \, \text{boxes / day}$$

Rate of work done of Sohilkhan $= \dfrac{18000 \text{ boxes}}{100 \text{ days}} = 180$ boxes / day

Rate of work done of Kartikbhai $= \dfrac{18000 \text{ boxes}}{150 \text{ days}} = 120$ boxes / day

Combined rate of work done $\rightarrow (200 + 180 + 120)$ boxes / day

Work done by all three jointly in a day $= 500$ boxes / day

Time taken to manufacture 500 boxes $= 1$ day

$\therefore$ Time taken to manufacture 18000 boxes $= \dfrac{18000}{500} = 36$ days

$\therefore$ Remuneration for each box $= \dfrac{54000}{18000} = ₹ 3$

$\therefore$ Amount obtained by Manubhai $= 200 \times 36 \times 3$

$= ₹ 21,600$

$\therefore$ Amount obtained by Sohilkhan $= 180 \times 36 \times 3$

$= ₹ 19,440$

$\therefore$ Amount obtained by Kartikbhai $= 120 \times 36 \times 3$

$= ₹ 12,960$

Hence, Manubhai, Sokhilkhan and Kartikbhai will get ₹ 21,600, ₹ 19,440, ₹ 12,960 respectively.

Exercise

1. Nimesh completes his work in 20 minutes. Find his rate of work done.

2. Rupali takes 15 days to embroidery work in a sari. Find her rate of work done.

3. Nimesh completes a work in 8 days. Find his rate of work done.

4. Rambhai takes 3 hours to prepare 180 boxes of peda, but Sureshbhai prepares the same number of boxes in 6 hours. How much time will be taken if they do this work combinedly.

5. Salimbhai takes 30 hours to saw 9000 cubic feet work by machine in "Dharati Saw Mill" and Junaidbhai takes 45 hours. If both do the same work together and remuneration paid is ₹ 3000, how much remuneration will each get ?

6. To plough 30 acres of land by tractor, Shankarbhai takes 10 hours and Yakubbhai takes 15 hours. If both plough the same land together, they get ₹ 6000 as remuneration. How much amount will each get ?

7. Mariya takes 100 days and Sofiya takes 150 days to make 3000 cookers by machine in a pressure cooker manufacturing company. If both do this work together then they get ₹ 36,000 as remuneration. Find the amount obtained by each.

8. Ramjibhai takes 16 hours and Kanjibhai takes 20 hours to colour a 640 squarefeet wall. If both do this work together they get ₹ 5760 as remuneration. How much will each get ?

9. In a toy manufacturing company to make 2400 toys Dhara takes 20 days and Samira takes 30 days. If both make 2400 toys together, they get ₹ 12,000 as remuneration. How much remuneration will each get ?

10. Dipesh, Mustafa and Prakash take respectively 6 hours, 10 hours and 15 hours to make 360 measure scale in a company manufacturing measure scale of steel. If all the three do this work together, they get ₹ 720 as remuneration. How much amount will each get ?

●

☞ **Answers** ☜

Exercise

1. $\frac{1}{20}$ work / minute   2. $\frac{1}{15}$ work / day   3. $\frac{1}{8}$ work / day   4. 2 hours

5. Salimbhai ₹ 1800, Junaidbhai ₹ 1200 6. ₹ 3600 to Shankarbhai, ₹ 2400 to Yakubbhai

7. ₹ 21,600 to Mariya, ₹ 14,400 to Sofiya 8. ₹ 3200 to Ramjibhai, ₹ 2560 to Kanjibhai

9. ₹ 7200 to Dhara, ₹ 4800 to Samira.

10. Dipesh ₹ 360, Mustafa ₹ 216 and Prakash ₹ 144.

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Revision

1. Answer the following questions on the basis of the given cheque :

```
A/c payee

                                                    Date 20-10-2013
   Pay    Aarti Parmar
                                                    or Bearer
   Rupees    Twelve thousand and three hundred only
                                                    Rs. 12,300/-

   A/C No.    470120349
   Bank of    ×××××                                        D. N. Rohit
   Anand Branch
   Anand 388001
   Branch Code : 1111                                Divya N. Rohit

              || 0001 || 4000161358 : 01642 || 09
```

(1) Which type of cheque is this ?

(2) What is the date of cheque ?

(3) In whose account, the amount of cheque will be deposited ?

(4) Write the account number.

(5) How much amount is written in the cheque ?

(6) Whose account is this ?

(7) Write the cheque number.

2. Solve the sums :

(1) Chirag borrows ₹ 1,50,000 at 11% rate for 2 years from a 'Co-operative Society' to go foreign at compound interest. At last how much compound interest will Chirag have to pay ?

(2) Ketanbhai borrowed ₹ 2,50,000 at 9 % rate for 3 years to set up 'Harsh chemical factory' from Tina finance company at compound interest. If Ketanbhai borrowed this amount at simple interest instead of compound interest, how much less amount will he have to pay ?

(3) Rameshbhai took ₹ 1,50,000 at 10% rate for 1 year for a work at compound interest from his friend. At last how much will he have to pay ?

(4) Maheshbhai took ₹ 2,00,000 at 8% rate at compound interest from a 'Angadia Pedhi' to start a new shop. At the end of 2 years how much will he have to pay ?

3. Solve the following sums :

(1) Jaspal takes 50 minutes and Surjeet takes 70 minutes to pack 350 packets of medicine by machine in a pharmaceutical company. Find the combined rate of work of both.

(2) Jagrutiben takes 10 hours and Jaisangbhai takes 15 hours to prepare 30 kg papad by machine in a papad making home industry. If the same is done by both together, they get total ₹ 750 as remuneration. Find the amount obtained by each.

●

👉 **Answers** 👈

2. (1) ₹ 34,815   (2) ₹ 6257.25   (3) ₹ 1,65,000   (4) ₹ 2,33,280

3. (1) 12 packets / minute   (2) Jagrutiben ₹ 450 and Jaisangbhai ₹ 300

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4   Factorization-1

❖ **Remember :**

● Write all factors of 10.

  1, 2, 5 and 10

● Students, now find all factors of 12 as above.

  _____ ,   _____ , _____ , _____ , _____ , _____

● Find the factors of 10. (Factorize 10)

  $10 = 2 \times 5$

  Students, now do factors of 18 by other method.

  $18 =$ _____

● Find the factors of 28.

  $28 = 2 \times \boxed{14}$

  $\therefore \ 28 = 2 \times 2 \times \square$

● Find the factors of 48.

  $48 = 2 \times \boxed{24}$

  $\quad\ = 2 \times \square \times 12$

  $\quad\ = 2 \times \square \times \square \times 6$

  $48 = 2 \times \square \times \square \times \square \times 3$

We know about unknown numbers, on the basis of this we will get knowledge of factorization of polynomials.

Factorize the Monomial :

Here, $10 = 2 \times 5$

Similarly $2x = 2 \times x$

$6x = 2 \times 3 \times x$

$10x^2 = 2 \times 5 \times x \times x$

Therefore to factorize the monomial, write the prime factors of coefficients of monomial and unknown number in multiplication form on the basis of its exponent (power).

Example 1 : Factorize the given monomials :

(1) $15x^2y^2 = 3 \times 5 \times x \times x \times y \times y$

(2) $12xy^4 = 2 \times 2 \times 3 \times x \times y \times y \times y \times y$

Practice 1

1. **Fill in the blanks :**

(1) $2x^2y^2 = \underline{\hspace{1cm}} \times x \times x \times \underline{\hspace{1cm}} \times y$

(2) $10a^2b = 2 \times \underline{\hspace{1cm}} \times a \times \underline{\hspace{1cm}} \times b$

(3) $6xy = \underline{\hspace{1cm}} \times 3 \times x \times \underline{\hspace{1cm}}$

(4) $15mn^2 = 3 \times \underline{\hspace{1cm}} \times m \times \underline{\hspace{1cm}} \times n$

2. **Factorize the following monomials :**

(1) 25 (2) $6x^2y$ (3) $20x^2y^4$

(4) $24x^3y^2$ (5) $26xy$ (6) $18a^3b$

❖ **Factorization of Binomials :**

For unknown numbers, we know distributive law

$a \times (b + c) = (a \times b) + (a \times c)$ (distributive law)

$a (b + c) = ab + ac$

Therefore to factorize $ab + ac$, we get $(b + c)$ by dividing both terms by a. Therefore, a and $(b + c)$ are their factors.

$ab + ac = \underline{a} \times b + \underline{a} \times c$

$= a (b + c)$ (Taking 'a' common)

Example 2 : Factorize $4x + 6$

$4x + 6 = \underline{2 \times 2 \times x} + \underline{2 \times 3}$

$\qquad = 2(2x + 3)$ (Taking '2' as common)

Example 3 : Factorize $6a^2b - 3ab^2$

$6a^2b - 3ab^2 = \underline{2 \times 3 \times a \times a \times b} - \underline{3 \times a \times b \times b}$

$\qquad = 3ab\,(2a - b)$ (Taking '3ab' common)

Practice 2

1. **Fill in the blanks :**

(1) $x^2 - x = x \,(\underline{\qquad})$

(2) $8x^3 + 4x^2 = (\underline{\qquad})\,(2x + 1)$

(3) $3a^2 - 6 = 3\,(\underline{\qquad})$

(4) $xy - xz = x\,(\underline{\qquad})$

2. **Factorize the following binomials :**

(1) $10x + 5$ (2) $5x^2 + 15$ (3) $7a - 7b$ (4) $-3x + 6$

(5) $6x^3y^2 - 3x$ (6) $9xy^2 - 18x^2$ (7) $8 - 4xy$ (8) $9x - 27xyz$

(9) $12a^2b - 18ab^2$

❖ **Factorize the polynomials :**

Factorize $ax + ay + bx + by$.

$\quad = (ax + ay) + (bx + by)$ **OR** $= (ax + bx) + (ay + by)$

$\quad = a(x + y) + b\,(x + y)$ $\qquad = x(a + b) + y(a + b)$

$\quad = (x + y)\,(a + b)$ $\qquad\qquad = (x + y)\,(a + b)$

Example 4 : Factorize $4ab + 6ac + 2bx + 3cx$.

$$= \quad (4ab + 6ac) + (2bx + 3cx)$$

$$= \quad 2a\,(2b + 3c) + x(2b + 3c)$$

$$= \quad (2b + 3c)\,(2a + x)$$

Similarly form other pair of terms in example 4 and solve.

Example 5 : Factorize $8ax + 3by - 12bx - 2ay$.

$$= \quad 8ax - 2ay - 12bx + 3by \qquad \text{(by arranging the terms)}$$

$$= \quad 2a(4x - y) - 3b(4x - y)$$

$$= \quad (4x - y)(2a - 3b)$$

Similarly, solve again by making other pairs as in the example 5.

Practice 3

❖ **Factorize the following polynomials :**

(1) $xy + 2x + 4y + 8$

(2) $xy - 4x + 3y - 12$

(3) $x^2y + 5x^2 + y + 5$

(4) $6x^2 + 4xy - 3x - 2y$

(5) $15x - 4a + 6 - 10ax$

(6) $10m^2n + 9 + 6m + 15mn$

❖ **Factorization of Perfect Square Trinomial :**

Dear students, let's see the following expansions :

$(x + 2y)^2 = x^2 + 4xy + 4y^2$ ...(I)

$(2x + 3)^2 = 4x^2 + 12x + 9$ ...(II)

$(3x - 5y)^2 = 9x^2 - 30xy + 25y^2$ ...(III)

In the above polynomial (I), $x^2 + 4xy + 4y^2$ is a perfect square trinomial in which first term is x^2 and the last term is $4y^2$. Also both the terms are positive.

Similarly in polynomial (II), $4x^2 + 12x + 9$ is a perfect square trinomial and in polynomial (III), $9x^2 - 30xy + 25y^2$ is a perfect square trinomial. How is there first and last terms ? Verify yourself.

Therefore, in all the above three expressions it can be determined that in a perfect square trinomial, first term and the last term always have positive sign.

In polynomial (I) the middle term $4xy$ of perfect square trinomial is 2 times of multiplication of x and $2y$.

In polynomial (II) the middle term $12x$ of perfect square trinomial is 2 times of multiplication of $2x$ and 3.

In polynomial (III) the middle term $30xy$ of perfect square trinomial is 2 times of multiplication of $3x$ and $5y$.

Here in polynomial (I), x is square root of x^2 and $2y$ is square root of $4y^2$. And middle term is twice of multiplication of x and $2y$.

$$\therefore \text{ Middle term} = \pm 2 \times \sqrt{\text{First Term}} \times \sqrt{\text{Last Term}}$$

Now $(\text{Middle term})^2 = 4 \times \text{First term} \times \text{Last term}$

$$\therefore \text{ First term} = \frac{(\text{Middle Term})^2}{4 \times \text{Last Term}} \text{ and Last term} = \frac{(\text{Middle Term})^2}{4 \times \text{First Term}}$$

Example 6 : Determine whether the given trinomial is perfect square or not :

(1) $6x^2 - 12x + 4$

First term $= 6x^2$

Here in first term $6x^2$, 6 is not a perfect square number.

Therefore, the given trinomial is not a perfect square.

(2) $x^2 - 12x - 36$

In the given trinomial the last term has the negative sign, so it is not a perfect square.

(3) $x^2 + 12x - 36$

In the given trinomial the last term has the negative sign, so it is not a perfect square.

(4) $x^2 + 6x + 10$

In the given trinomial the last term is not perfect square, so it is not a perfect square trinomial.

(5) $16x^2 + 12x + 1$

First term $= 16x^2 = (4x)^2$ } Here first and the last terms are perfect

Last term $= 1 = (1)^2$ } squares and have the positive sign.

Middle term $= \pm 2 \times \sqrt{\text{First Term}} \times \sqrt{\text{Last Term}}$

$= \pm 2\sqrt{16x^2} \times \sqrt{1}$

$= \pm 2 \times 4x \times 1$

$= \pm 8x$ Here middle term must be $\pm 8x$

But the middle term of given trinomial is $+ 12x$, so given trinomial is not a perfect square.

Example 7 : Find the first term and last term of given polynomials such that they may becomes perfect square polynomial :

(1) $x^4 + 6x^2 +$ _____

Last term $= \dfrac{(\text{M.T.})^2}{4 \times \text{F.T.}} = \dfrac{(6x)^2}{4 \times x^4} = \dfrac{6x^2 \times 6x^2}{4x^4} = 9$

Here, First term $=$ F.T., Last term $=$ L.T., Middle term $=$ M.T.

(2) _____ $+ 10x + 25$

First term $= \dfrac{(\text{M.T.})^2}{4 \times \text{L.T.}} = \dfrac{(10x)^2}{4 \times 25} = \dfrac{10x^2 \times 10x^2}{4 \times 25} = x^2$

Example 8 : Factorize :

(1) $x^2 + 14x + 49$

$= (x)^2 + 2(x)(7) + (7)^2$ [Arranging terms as $a^2 \pm 2ab + b^2$]

$= (x + 7)^2$

(2) $25m^2 - 20mn + 4n^2$

$= (5m)^2 - 2(5m)(2n) + (2n)^2$ (Arranging terms as $a^2 \pm 2ab + b^2$)

$= (5m - 2n)^2$

(3) $4ax^2 + 12ax + 9a$

$= a(4x^2 + 12x + 9)$ (Taking 'a' as common)

$= a[(2x)^2 + 2(2x)(3) + (3)^2]$

$= a(2x + 3)^2$

Practice 4

1. Determine whether the given polynomials are perfect square or not :

(1) $x^2 + 2x + 4$ (2) $x^2 - 14x + 49$

(3) $a^2 + 10a + 25$ (4) $9x^2y^2 + 24xy + 8$

(5) $25x^2 - 35x + 49$ (6) $4x^2 + 4x + 1$

(7) $x^2 + 2 + \dfrac{1}{x^2}$ (8) $x^2 - 8x + 16$

2. Find the missing term such that given polynomial becomes a perfect square trinomial :

(1) $9a^2 +$ _____ $+ 16$

(2) _____ $- 12x + 9$

(3) $9x^2 + 30xy +$ _____

(4) _____ $+ 4xy + 4$

(5) $81x^2 +$ _____ $+ 4$

(6) $4a^2 +$ _____ $+ \dfrac{1}{4a^2}$

3. Factorize :

(1) $x^2 + 12x + 36$ (2) $4x^2 + 12xy + 9y^2$

(3) $9x^2 + 48x + 64$ (4) $x^2 - 8x + 16$

(5) $25x^2y^2 - 20xy + 4$ (6) $16x^2 + 40x + 25$

(7) $81 - 90xy + 25x^2y^2$ (8) $3x^3 - 30x^2 + 75x$

❖ **Factors of Six Terms :** $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

By reversing the expansion of square of trinomial

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

Here in six terms, the first three terms joined by addition are perfect square.

Fourth term	$= 2 \times \sqrt{F.T.} \times \sqrt{S.T.}$	Where,	F.T. = First term
Fifth term	$= 2 \times \sqrt{S.T.} \times \sqrt{T.T.}$		S.T. = Second term
Sixth term	$= 2 \times \sqrt{T.T.} \times \sqrt{F.T.}$		T.T. = Third term

(ii) $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

By reversing the expansion of square of binomial

$a^2 + b^2 + c^2 - 2ab + 2bc - 2ca = (a - b - c)^2$

Here in (i) and (ii), all the six terms are positive or in last three terms, any two terms are negative.

Example 9 : Factorize :

(1) $4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20zx$

$= (2x)^2 + (3y)^2 + (5z)^2 + 2(2x)(3y) + 2(3y)(5z) + 2(5z)(2x)$

$= (2x + 3y + 5z)^2$

(2) $16a^2 + 4b^2 + 36c^2 - 16ab + 24bc - 48ca$

$= (-4a)^2 + (2b)^2 + (6c)^2 + 2(-4a)(2b) + 2(2b)(6c) + 2(6c)(-4a)$

$= (-4a + 2b + 6c)^2$ or $(4a - 2b - 6c)^2$

Note : In the above example (2) out of six terms, the two terms which have negative sign and the variable which is common in them put negative sign of that variable.

Practice 5

❖ **Factorise :**

(1) $9x^2 + 4y^2 + 1 + 12xy + 4y + 6x$

(2) $16a^2 + 9b^2 + c^2 - 24ab + 6bc - 8ca$

(3) $a^4 + 4b^2 + 9 + 4a^2b - 12b - 6a^2$

(4) $9x^2 + 16y^2 + 25 + 24xy - 40y - 30x$

(5) $a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ca$

Exercise

1. **Fill in the blanks to correct the following statements :**

(1) $15x^3y = 3 \times$ _____ $\times$ _____ $\times x \times x \times y$

(2) $5x^4 - x^3 = x^3 ($ _____ $)$

(3) $-5a^2 + 10a =$ _____ $(a - 2)$

(4) $ab + a - 2b - 2 = (a - 2) ($ _____ $)$

(5) $16a^2 +$ _____ $+ 1 = (4a + 1)^2$

(6) _____ $+ 10x + 25 = (x + 5)^2$

(7) $4y^2 -$ _____ $+ 9 = (2y - 3)^2$

(8) $16x^2 - 72x + 81$ is perfect square of _____ .

(9) $a^2 -$ _____ $+ 0.04 = (a - 0.2)^2$

(10) Adding _____ to $9x^2 + 1$ it will become perfect square trinomial.

2. **Factorize :**

(1) $4ab + 8a - b - 2$

(2) $x^2y - 3x^2 + y - 3$

(3) $2x^2 - 5a - 5x + 2ax$

(4) $3ab + 12 - 4a - 9b$

(5) $x^2 + 49 + 14x$

(6) $16a^2 + 40ab + 25b^2$

(7) $m^4 - 16m^2 + 64$

(8) $4y^3 - 28y^2 + 49y$

(9) $25x^2 + 4y^2 + 9z^2 + 20xy + 12yz + 30zx$

(10) $4m^2 + 9n^2 + p^2 - 12mn + 6np - 4pm$

●

👉 **Answers** 👈

Practice 1

1. (1) 2, y (2)   5, a (3)   2, y (4)   5, n

2. (1) 5×5 (2) $2 \times 3 \times x \times x \times y$

(3) $2 \times 2 \times 5 \times x \times x \times y \times y \times y \times y$

(4) $2 \times 2 \times 2 \times 3 \times x \times x \times x \times y \times y$

(5) $2 \times 13 \times x \times y$ (6) $2 \times 3 \times 3 \times a \times a \times a \times b$

Practice 2

1. (1) $(x - 1)$ (2) $4x^2$ (3) $(a^2 - 2)$ (4) $y - z$

2. (1) $5(2x + 1)$ (2) $5(x^2 + 3)$ (3) $7(a - b)$ (4) $-3(x - 2)$ or $3(2 - x)$

(5) $3x(2x^2y^2 - 1)$ (6) $9x(y^2 - 2x)$ (7) $4(2 - xy)$ (8) $9x(1 - 3yz)$

(9) $6ab(2a - 3b)$

Practice 3

1. (1) $(x + 4)(y + 2)$ (2) $(x + 3)(y - 4)$ (3) $(x^2 + 1)(y + 5)$

(4) $(2x - 1)(3x + 2y)$ (5) $(5x + 2)(3 - 2a)$ (6) $(5mn + 3)(2m + 3)$

Practice 4

1. (1) not a perfect square   (2) perfect square   (3) perfect square

(4) not a perfect square   (5) not a perfect square   (6) perfect square

(7) perfect square   (8) perfect square

2. (1) $24a$ (2) $4x^2$ (3) $25y^2$ (4) x^2y^2 (5) $36x$ (6) 2

3. (1) $(x + 6)^2$ (2) $(2x + 3y)^2$ (3) $(3x + 8)^2$ (4) $(x - 4)^2$

(5) $(5xy - 2)^2$ (6) $(4x + 5)^2$ (7) $(9 - 5xy)^2$ (8) $3x(x - 5)^2$

Practice 5

1. (1) $(3x + 2y + 1)^2$ (2) $(4a - 3b - c)^2$ (3) $(a^2 + 2b - 3)^2$

    (4) $(3x + 4y - 5)^2$ (5) $(a - 2b + c)^2$

Exercise

1. (1) $5, x$ (2) $5x - 1$ (3) $-5a$ (4) $b + 1$ (5) $8a$

    (6) x^2 (7) $12y$ (8) $4x - 9$ (9) $0.4a$ (10) $\pm 6x$

2. (1) $(b + 2)(4a - 1)$ (2) $(x^2 + 1)(y - 3)$ (3) $(2x - 5)(x + a)$

    (4) $(a - 3)(3b - 4)$ (5) $(x + 7)^2$ (6) $(4a + 5b)^2$

    (7) $(m^2 - 8)^2$ (8) $y(2y - 7)^2$ (9) $(5x + 2y + 3z)^2$

    (10) $(2m - 3n - p)^2$

◆

Knowlegable

Pascal's Triangle

		Coefficient
$(x + y)^0$	1	1
$(x + y)^1$	$x + y$	1 1
$(x + y)^2$	$x^2 + 2xy + y^2$	1 2 1
$(x + y)^3$	$x^3 + 3x^2y + 3xy^2 + y^3$	1 3 3 1
$(x + y)^4$	$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$	1 4 6 4 1
$(x + y)^5$	$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$	1 5 10 10 5 1

Explanation : For example to write $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, 1, 3, 3, 1 are coefficients. Here the exponent (power) of x are written in descending (decreasing) order and power of y are written in ascending (increasing) order.

5 Factorization-2

❖ **Revision :**

Dear students, we have learnt factorization-1 on the basis of that fill up the blanks :

(1) $18x^2 = 2 \times 3 \times$ _____ $\times$ _____ $\times x$

(2) $4x^2 - x =$ _____

(3) $m^2y + 5m^2 + y + 5 =$ _____

(4) Place the correct term in the blanks such that polynomial become perfect square :

 (i) _____ $+ 10x + 25$

 (ii) $4y^2 -$ _____ $+ 1$

 (iii) $25x^2 + 10x +$ _____

● **The difference of two squares : factors of $a^2 - b^2$**

● **Expand :**

Example 1 : $(5x + 3y)(5x - 3y)$

$$= 5x(5x - 3y) + 3y(5x - 3y)$$

$$= 25x^2 - 15xy + 15xy - 9y^2$$

$$= 25x^2 - 9y^2$$

$$= (5x)^2 - (3y)^2$$

$$\therefore \quad (5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$$

Example 2 : $(2a + b)(2a - b)$

$$= 2a(2a - b) + b(2a - b)$$

$$= 4a^2 - 2ab + 2ab - b^2$$

$$= 4a^2 - b^2$$

$$= (2a)^2 - (b)^2$$

$$\therefore \quad (2a + b)(2a - b) = (2a)^2 - (b)^2$$

Example 3 : $(m + 2n)(m - 2n)$

$$= m(m - 2n) + 2n(m - 2n)$$

$$= m^2 - 2mn + 2mn - 4n^2$$

$$= m^2 - 4n^2$$

$$= (m)^2 - (2n)^2$$

$$\therefore \quad (m + 2n)(m - 2n) = (m)^2 - (2n)^2$$

From the above three examples :

$$(5x)^2 - (3y)^2 = (5x + 3y)(5x - 3y)$$

$$(2a)^2 - (b)^2 = (2a + b)(2a - b)$$

$$(m)^2 - (2n)^2 = (m + 2n)(m - 2n)$$

Therefore, the above examples can be denoted as :

$$a^2 - b^2 = (a + b)(a - b)$$

Similarly, $\quad p^2 - q^2 = $ _____

❖ **Factorize :**

Example 4 : $x^2 - 4$

$$= (x)^2 - (2)^2$$

$$= (x + 2)(x - 2)$$

Example 5 : $25x^2 - 9$

$$= (5x)^2 - (3)^2$$

$$= (5x + 3)(5x - 3)$$

Example 6 : $m^2 - 16n^2$

$$= (m)^2 - (4n)^2$$

$$= (m + 4n)(m - 4n)$$

Example 7 : $16a^2 - 25b^2$

$$= (4a)^2 - (5b)^2$$

$$= (4a + 5b)(4a - 5b)$$

Example 8 : $a^3b^3 - ab$

$$= ab \, (a^2b^2 - 1)$$

$$= ab \, [\, (ab)^2 - (1)^2]$$

$$= ab \, (ab + 1) \, (ab - 1)$$

Example 9 : $x^4 - y^4$

$$= (x^2)^2 - (y^2)^2$$

$$= (x^2 + y^2) \, (x^2 - y^2)$$

$$= (x^2 + y^2) \, (x + y) \, (x - y)$$

Practice 1

❖ **Factorize :**

(1) $m^2 - 16$

(2) $16x^2 - 49y^2$

(3) $4a^2b^2 - 1$

(4) $49 - 25x^2$

(5) $16x^4 - 81y^4$

(6) $a^4b^4 - 1$

(7) $x^3 - 49x$

(8) $18 \, x^3y^3 - 2xy$

❖ **Factors of polynomials like :** $(x \pm y)^2 - a^2, \ x^2 - (a \pm b)^2, \ (x \pm y)^2 - (a \pm b)^2$

❖ **Factorize :**

Example 10 : $(m + n)^2 - a^2$

$$= (x)^2 - (a)^2 \qquad \text{(Taking } m + n = x)$$

$$= (x + a) \, (x - a)$$

$$= [(m + n) + a] \, [(m + n) - a] \qquad \text{(Putting back } x = m + n)$$

$$= (m + n + a) \, (m + n - a)$$

Example 11 : $(2a - b)^2 - 4c^2$

$\qquad = (x)^2 - (2c)^2$ $\qquad$ (Taking $2a - b = x$)

$\qquad = (x + 2c)(x - 2c)$

$\qquad = [(2a - b) + 2c][(2a - b) - 2c)]$ $\qquad$ (Putting back $x = 2a - b$)

$\qquad = (2a - b + 2c)(2a - b - 2c)$

Example 12 : $x^2 - (a + b)^2$

$\qquad = (x)^2 - (m)^2$ $\qquad$ (Taking $a + b = m$)

$\qquad = (x + m)(x - m)$

$\qquad = [x + (a + b)][x - (a + b)]$ $\qquad$ (Putting back $m = a + b$)

$\qquad = (x + a + b)(x - a - b)$

Example 13 : $4x^2 - (5y - 1)^2$

$\qquad = (2x)^2 - (m)^2$ $\qquad$ (Taking $5y - 1 = m$)

$\qquad = (2x + m)(2x - m)$

$\qquad = [2x + (5y - 1)][2x - (5y - 1)]$ $\qquad$ (Putting back $m = 5y - 1$)

$\qquad = (2x + 5y - 1)(2x - 5y + 1)$

Example 14 : $(x + y)^2 - (a - b)^2$

$\qquad = (m)^2 - (n)^2$ $\qquad$ (Taking $x + y = m, a - b = n$)

$\qquad = (m + n)(m - n)$

$\qquad = [(x + y) + (a - b)][(x + y) - (a - b)]$

$\qquad\qquad\qquad\qquad$ (Putting back $m = x + y, n = a - b$)

$\qquad = (x + y + a - b)(x + y - a + b)$

Example 15 : $(2x - 1)^2 - (3y - 5)^2$

$\qquad = (m)^2 - (n)^2$ $\qquad$ (Taking $2x - 1 = m, 3y - 5 = n$)

$\qquad = (m + n)(m - n)$

$\qquad = [(2x - 1) + (3y - 5)][(2x - 1) - (3y - 5)]$

$\qquad\qquad\qquad\qquad$ (Putting back $m = 2x + 1, n = 3y - 5$)

$\qquad = (2x - 1 + 3y - 5)(2x - 1 - 3y + 5)$

$\qquad = (2x + 3y - 6)(2x - 3y + 4)$

Example 16 : $x^2 + 2xy + y^2 - z^2$

$$= (x^2 + 2xy + y^2) - (z)^2$$

$$= (x + y)^2 - (z)^2$$

$$= (m)^2 - (z)^2 \qquad \text{(Taking } x + y = m)$$

$$= (m + z)(m - z)$$

$$= [(x + y) + z][(x + y) - z] \qquad \text{(Putting back } m = x + y)$$

$$= (x + y + z)(x + y - z)$$

Example 17 : $x^2 - a^2 + 2ab - b^2$

$$= (x)^2 - (a^2 - 2ab + b^2)$$

$$= (x)^2 - (a - b)^2$$

$$= (x)^2 - (m)^2 \qquad \text{(Taking } a - b = m)$$

$$= (x + m)(x - m)$$

$$= [x + (a - b)][x - (a - b)] \qquad \text{(Putting back } m = a - b)$$

$$= (x + a - b)(x - a + b)$$

Example 18 : $x^2 + 4x + 4 - a^2 - 2ab - b^2$

$$= (x^2 + 4x + 4) - (a^2 + 2ab + b^2)$$

$$= (x + 2)^2 - (a + b)^2$$

$$= (m)^2 - (n)^2 \qquad \text{(Taking } x + 2 = m, a + b = n)$$

$$= (m + n)(m - n)$$

$$= [(x + 2) + (a + b)][(x + 2) - (a + b)]$$

$$\text{(Putting back } m = x + 2, n = a + b)$$

$$= (x + 2 + a + b)(x + 2 - a - b)$$

$$= (x + a + b + 2)(x - a - b + 2)$$

Practice 2

1. Factorize :

(1) $(4x + 3y)^2 - 49z^2$ (2) $(ab - 1)^2 - 64x^2$

(3) $81 - (5x - 3y)^2$ (4) $36z^2 - (x + 2y)^2$

2. Factorize :

(1) $(a + 8)^2 - (b - 3)^2$ (2) $(3x - 2y)^2 - (5a - 3b)^2$

(3) $(4x + 5)^2 - (2y + 3)^2$ (4) $(ab + 6)^2 - (mn - 7)^2$

3. Factorize :

(1) $36a^2 - 12a + 1 - 4b^2$ (2) $64 - x^2 - 10x - 25$

(3) $m^2n^2 - 4mn + 4 - x^2$ (4) $9m^2 - 25x^2 + 20xy - 4y^2$

4. Factorize :

(1) $9a^2 + 6a + 1 - x^2 - 2xy - y^2$

(2) $49x^2 - 14x + 1 - 64a^2 + 16ab - b^2$

(3) $4a^2 + 12a + 9 - m^2 + 2mn - n^2$

(4) $x^2 - 14x + 49 - a^2 - 12a - 36$

*

❖ **The factorization of the form $a^4 + b^4$:**

Example 19 : $x^4 + 4$

$$= \pm 2 \times \sqrt{x^4} \times \sqrt{4}$$

$$= \pm 2 \times x^2 \times 2$$

$$= \pm 4x^2$$

$x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2$ (Here adding and subtracting the middle term)

$$= (x^2 + 2)^2 - (2x)^2$$

$$= (m)^2 - (2x)^2 \qquad \text{(Taking } x^2 + 2 = m)$$

$$= (m + 2x)(m - 2x)$$

$$= [(x^2 + 2) + 2x][(x^2 + 2) - 2x] \qquad \text{(Putting back } m = x^2 + 2)$$

$$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$$

$$= (x^2 + 2x + 2)(x^2 - 2x + 2)$$

Practice 3

❖ **Factorize :**

(1) $4x^4 + y^4$ (2) $64a^4 + b^4$ (3) $81a^4 + 4$ (4) $4m^4 + 625$

*

❖ **The factorization of the form $x^4 + x^2y^2 + y^4$:**

❖ **Factorize :**

Example 20 : $x^4 + x^2 + 1$

$$\text{M.T.} = \pm\, 2 \times \sqrt{x^4} \times \sqrt{1}$$

$$= \pm\, 2 \times x^2 \times 1$$

$$= \pm\, 2x^2$$

Now, $x^4 + x^2 + 1$

$$= x^4 + 2x^2 + 1 - x^2 \qquad \text{(Putting in the difference of two squares)}$$

$$= (x^2 + 1)^2 - (x)^2$$

$$= (x^2 + 1 + x)\,(x^2 + 1 - x)$$

$$= (x^2 + x + 1)\,(x^2 - x + 1)$$

Example 21 : $4a^4 - 13a^2 + 9$

$$\text{M.T.} = \pm\, 2\sqrt{4a^4} \times \sqrt{9}$$

$$= \pm\, 2 \times 2a^2 \times 3$$

$$= \pm\, 12a^2$$

Now, $4a^4 - 13a^2 + 9$

$$= 4a^4 - 12a^2 + 9 - a^2$$

$$= (2a^2 - 3)^2 - (a)^2 \qquad \text{(Putting in the difference of two squares)}$$

$$= (m)^2 - (a)^2 \qquad\qquad \text{(Taking } 2a^2 - 3 = m)$$

$$= (m + a)\,(m - a)$$

$$= (2a^2 - 3 + a)\,(2a^2 - 3 - a) \qquad \text{(Putting back } m = 2a^2 - 3)$$

$$= (2a^3 + a - 3)\,(2a^2 - a - 3)$$

Practice 4

❖ **Factorize :**

(1) $a^4 + 6a^2 + 25$

(2) $m^4 - 7m^2 + 9$

(3) $4x^4 - 21x^2 + 25$

(4) $a^4 - 8a^2b^2 + 4b^4$

(5) $y^4 + 3y^2 + 4$

(6) $m^4 + 12m^2 + 64$

*

❖ **Factorization of $a^3 + b^3$ and $a^3 - b^3$:**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

❖ **Factorize :**

Example 22 : $a^3 + 64$

$$= (a)^3 + (4)^3$$

$$= (a + 4)[(a)^2 - (a)(4) + (4)^2]$$

$$= (a + 4)(a^2 - 4a + 16)$$

Example 23 : $8m^3 - 1$

$$= (2m)^3 - (1)^3$$

$$= (2m - 1)[(2m)^2 + (2m)(1) + (1)^2]$$

$$= (2m - 1)(4m^2 + 2m + 1)$$

Example 24 : If $a + b = 6$ and $ab = 8$ then find the value of $a^3 + b^3$.

$$a + b = 6$$

$$\therefore (a + b)^2 = (6)^2$$

$$\therefore a^2 + 2ab + b^2 = 36$$

$$\therefore a^2 + b^2 + 2(8) = 36 \quad \text{(Putting } ab = 8)$$

$$\therefore a^2 + b^2 + 16 = 36$$

$$\therefore a^2 + b^2 = 36 - 16$$

$$\boxed{\therefore a^2 + b^2 = 20}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$= (a + b)(a^2 + b^2 - ab)$$
$$= (6)(20 - 8) \qquad \text{(Putting } a^2 + b^2 = 20 \text{ and } ab = 8)$$
$$= (6)(12)$$

$$\boxed{a^3 + b^3 = 72}$$

Example 25 : If $a - b = -8$, $a^2 + b^2 = 40$ and $ab = -12$ then find the value of $a^3 - b^3$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$= (-8)(a^2 + b^2 + ab)$$
$$= (-8)[(40 + (-12)] \qquad \text{(Putting the given values)}$$
$$= (-8)(40 - 12)$$
$$= (-8)(28)$$

$$\boxed{a^3 - b^3 = (-224)}$$

Example 26 : Find the value using formula : $(12)^3 + (8)^3$

$$(12)^3 + (8)^3 = (12 + 8)[(12)^2 - (12)(8) + (8)^2]$$
$$= (20)(144 - 96 + 64)$$
$$= (20)(112)$$
$$= 2240$$

Practice 5

1. Factorize :

(1) $x^3 + 27$ (2) $a^3 + 125b^3$

(3) $8a^3b^3 - 27$ (4) $8x^3 - 125$

2. Find the values :

(1) If $a + b = 5$ and $ab = 6$ then find the value of $a^3 + b^3$.

(2) If $a + b = 8$, $ab = 15$ and $a^2 + b^2 = 34$ then find the value of $a^3 + b^3$.

(3) If $a - b = 2$, $ab = 24$ and $a^2 + b^2 = 52$ then find the value of $a^3 - b^3$.

3. **Find the value using formula :**

    (1) $(11)^3 + (9)^3$ (2) $(23)^3 + (7)^3$ (3) $(45)^3 - (25)^3$

<div align="center">*</div>

❖ **Factors of the type $ax^2 + bx + c$, $a \neq 0$:**

    (1) If multiplication of coefficients of first term and last term. $(a \times c)$ is positive, then find such two factors of $(a \times c)$ whose addition is equal to coefficient of middle term.

      ● If middle term is positive then both part will be positive and if middle term is negative then both part will be negative.

    (2) If multiplication of first term and last term $(a \times c)$ is negative, then find such two factors of $(a \times c)$ whose difference is equal to the coefficient of middle term.

      ● If middle term is positive, then bigger part will have positive sign and smaller part will have negative sign. If middle term is negative then bigger part will have negative sign and smaller part will have positive sign.

Example 27 : Factorize : $x^2 + 8x + 15$

Here, coefficient of x^2 is 1 and the last term is 15, so there multiplication will be 15. The factors of 15 are (15, 1) and (3, 5) in which $15 + 1 = 16$ and $3 + 5 = 8$. Therefore do two parts of middle term such that $3 + 5 = 8$.

$$x^2 + 8x + 15 = x^2 + (5 + 3)\, x + 15$$
$$= x^2 + 5x + 3x\ + 15$$
$$= x(x + 5) + 3(x + 5)$$
$$= (x + 3)\,(x + 5)$$

Example 28 : Factorize : $x^2 - 9x + 18$

Here, coefficient of x^2 is 1 and the last term is 8. So there multiplication will be 18. The factors of 18 are (18, 1), (2, 9) and (3, 6) in which $18 + 1 = 19$, $2 + 9 = 11$ and $3 + 6 = 9$. Therefore do two parts of middle term such that $-3 - 6 = -9$.

$$x^2 - 9x + 18 = x^2 - 3x - 6x + 18$$
$$= x\,(x - 3) - 6\,(x - 3)$$
$$= (x - 3)\,(x - 6)$$

Example 29 : Factorize : $x^2 + 5x - 24$

Here coefficient of x^2 is 1 and the last term is (-24), so there multiplication will be (-24). The factors of 24 are $(24, 1)$, $(12, 2)$, $(8, 3)$ and $(6, 4)$, in which $24 - 1 = 23$, $12 - 2 = 10$ and $8 - 3 = 5$. Therefore, do two parts of middle term such that $+8 - 3 = 5$.

$$x^2 + 5x - 24$$
$$= x^2 + 8x - 3x - 24$$
$$= x (x + 8) - 3 (x + 8)$$
$$= (x - 3) (x + 8)$$

Example 30 : Factorize : $x^2 - 3x - 18$

Here, coefficient of x^2 is 1 and the last term is (-18), so there multiplication will be (-18). The factors of 18 are $(1,18)$, $(2,9)$ and $(6,3)$, in which $18 - 1 = 17$, $9 - 2 = 7$ and $6 - 3 = 3$. Therefore do two parts of middle term such that $-6 + 3 = -3$.

$$x^2 - 3x - 18$$
$$= x^2 - 6x + 3x - 18$$
$$= x (x - 6) + 3 (x - 6)$$
$$= (x - 6) (x + 3)$$

Example 31 : Factorize : $3x^2 + 13x + 12$

Here, coefficient of x^2 is 3 and the last term is 12. So there multiplication will be 36. The factors of 36 are $(36, 1)$, $(12, 3)$, $(6, 6)$ and $(9, 4)$, in which $36 + 1 = 37$, $12 + 3 = 15$, $6 + 6 = 12$ and $9 + 4 = 13$. Therefore do two parts of middle term such that $9 + 4 = 13$.

$$3x^2 + 13x + 12$$
$$= 3x^2 + 9x + 4x + 12$$
$$= 3x (x + 3) + 4 (x + 3)$$
$$= (3x + 4) (x + 3)$$

Practice 6

❖ **Factorize :**

(1) $x^2 + 5x + 6$

(2) $x^2 + 15x + 50$

(3) $x^2 - 11x + 24$

(4) $x^2 - 7x + 12$

(5)　$x^2 + 6x - 27$

(6)　$a^2 + 4a - 21$

(7)　$m^2 - 2m - 8$

(8)　$n^2 - 4n - 45$

(9)　$4x^2 + 12x + 5$

(10)　$9y^4 - 13y^2 + 4$

(11)　$2a^2 - 19a - 21$

(12)　$6x^2 - 7x - 3$

Exercise

1.　Fill in the blanks :

(1)　$36 - x^2 = (6 + x)\ (\underline{\hspace{2cm}})$

(2)　$a^2 - b^2c^2 = \underline{\hspace{2cm}}\ (a + bc)$

(3)　$x^3 - 49x = \underline{\hspace{2cm}}\ (x + 7)\ (x - 7)$

(4)　$4x^2 - 25 = (2x + 5)\ (\underline{\hspace{2cm}})$

(5)　$x^2 + 5x + 6 = (x + 3)\ (\underline{\hspace{2cm}})$

(6)　$x^2 - x - 12 = \underline{\hspace{2cm}}\ (x - 4)$

(7)　$a^3 - 1 = \underline{\hspace{2cm}}\ (a^2 + a + 1)$

(8)　$m^3 + 125 = (m + 5)\ (\underline{\hspace{2cm}})$

2.　Factorize :

(1)　$16a^2b^2 - 36$

(2)　$625 - 64x^2$

(3)　$4x^5 - 64x$

(4)　$(4a - 5b)^2 - 16c^2$

(5)　$25 - (ab - 3x)^2$

(6)　$(x + 8)^2 - (2x - 3)^2$

(7)　$121x^2 - 22x + 1 - 9a^2 - 24ab - 16b^2$

(8)　$x^4 + 4y^4$

(9)　$x^4 - 3x^2 + 9$

(10)　$x^4 - 8x^2 - 65$

(11)　$x^6 - 27$

(12)　$64x^3 + 125y^3$

❖ **We learnt this much :**

● Factors of $a^2 - b^2 = (a + b)\ (a - b)$

● Factors of types : $(x \pm y)^2 - a^2$, $a^2 - (x \pm y)^2$, $(x \pm y)^2 - (a \pm b)^2$

● Factors of type : $a^4 + b^4$

● Factors of $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$

● Factors of $a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)$

● Factors of type : $ax^2 + bx + c$　$(a \neq 0)$

Answers

Practice 1

(1) $(m + 4)(m - 4)$

(2) $(4x + 7y)(4x - 7y)$

(3) $(2ab + 1)(2ab - 1)$

(4) $(7 + 5x)(7 - 5x)$

(5) $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$

(6) $(a^2b^2 + 1)(ab + 1)(ab - 1)$

(7) $x(x + 7)(x - 7)$

(8) $2xy(3xy + 1)(3xy - 1)$

Practice 2

1. (1) $(4x + 3y + 7z)(4x + 3y - 7z)$

(2) $(ab + 8x - 1)(ab - 8x - 1)$

(3) $(5x - 3y + 9)(-5x + 3y + 9)$

(4) $(6z + x + 2y)(6z - x - 2y)$

2. (1) $(a + b + 5)(a - b + 11)$

(2) $(3x - 2y + 5a - 3b)(3x - 2y - 5a + 3b)$

(3) $4(2x + y + 4)(2x - y + 1)$

(4) $(ab + mn - 1)(ab - mn + 13)$

3. (1) $(6a + 2b - 1)(6a - 2b - 1)$

(2) $(x + 13)(-x + 3)$

(3) $(mn + x - 2)(mn - x - 2)$

(4) $(3m + 5x - 2y)(3m - 5x + 2y)$

4. (1) $(3a + 1 + x + y)(3a + 1 - x - y)$

(2) $(7x + 8a - b - 1)(7x - 8a + b - 1)$

(3) $(2a + 3 + m - n)(2a + 3 - m + n)$

(4) $(x + a - 1)(x - a - 13)$

Practice 3

(1) $(2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$

(2) $(8a^2 + b^2 + 4ab)(8a^2 + b^2 - 4ab)$

(3) $(9a^2 - 6a + 2)(9a^2 + 6a + 2)$

(4) $(2m^2 + 10m + 25)(2m^2 - 10m + 25)$

Practice 4

(1) $(a^2 + 2a + 5)(a^2 - 2a + 5)$ (2) $(m^2 + m - 3)(m^2 - m - 3)$

(3) $(2x^2 + x - 5)(2x^2 - x - 5)$ (4) $(a^2 - 2b^2 + 2ab)(a^2 - 2b^2 - 2ab)$

(5) $(y^2 + 2 - y)(y^2 + 2 + y)$ (6) $(m^2 + 2m + 8)(m^2 - 2m + 8)$

Practice 5

1. (1) $(x + 3)(x^2 - 3x + 9)$ (2) $(a + 5b)(a^2 - 5ab + 25b^2)$

(3) $(2ab - 3)(4a^2b^2 + 6ab + 9)$ (4) $(2x - 5)(4x^2 + 10x + 25)$

2. (1) 35 (2) 152 (3) 152

3. (1) 2060 (2) 12,510 (3) 75,500

Practice 6

(1) $(x + 3)(x + 2)$ (2) $(x + 10)(x + 5)$

(3) $(x - 3)(x - 8)$ (4) $(x - 4)(x - 3)$

(5) $(x + 9)(x - 3)$ (6) $(a + 7)(a - 3)$

(7) $(m - 4)(m + 2)$ (8) $(n - 9)(n + 5)$

(9) $(2x + 5)(2x + 1)$ (10) $(3y + 2)(3y - 2)(y + 1)(y - 1)$

(11) $(a + 1)(2a - 21)$ (12) $(3x + 1)(2x - 3)$

Exercise

1. (1) $6 - x$ (2) $a - bc$ (3) x (4) $2x - 5$

(5) $x + 2$ (6) $x + 3$ (7) $a - 1$ (8) $(m^2 - 5m + 25)$

2. (1) $4(2ab + 3)(2ab - 3)$ (2) $(25 + 8x)(25 - 8x)$

(3) $4x(x^2 + 4)(x + 2)(x - 2)$ (4) $(4a - 5b + 4c)(4a - 5b - 4c)$

(5) $(5 + ab - 3x)(5 - ab + 3x)$ (6) $(3x + 5)(-x + 11)$

(7) $(11x + 3a + 4b - 1)(11x - 3a - 4b - 1)$

(8) $(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$ (9) $(x^2 + 3x + 3)(x^2 - 3x + 3)$

(10) $(x^2 + 5)(x^2 - 13)$ (11) $(x^2 - 3)(x^4 + 3x^2 + 9)$

(12) $(4x + 5y)(16x^2 - 20xy + 25y^2)$

◆

6 Equation

❖ **Magic Game :**

Manish tells a magic game in the class. He tells Mehul to assume a number. After that he asks to add 4 and do three times of that number. After doing this process he asks Mehul, "What is your result ?" Mehul says, '79'. After doing simple calculation. Manish says, "Your assumed number is 25". Mehul says, "Wow ! completely true'. By this way, Manish has surprised his friend by telling his assumed number. It is a magic !

You also want to learn such game ! Let's first understand the game played by Manish.

We don't know the assumed number of Mehul.

Suppose, the number assumed by Mehul is x.

Now adding 4 to three times of x, we get $3x + 4$ which is equal to 79.

∴ $3x + 4 = 79$

The solution of this type of equations you have learnt in Std. VII. Find the solution and verify it.

Manish also finds the solution by this way and tells the assumed number.

Now, you know any magic game ! You also make such a game.

Tell your friend to asssume a number. Tell him to subtract 7 from five times of that asssumed number. Ask his result and make an equation on that basis. On the basis of this equation tell him his supposed number.

(Write the equation in the following place by playing this game with your five friends. Write their solutions in your notebook.)

❖ **Let's Learn New :**

You have learnt to find the solution of the equation $2x + 5 = 21$ in Std. VII. Here the power of variable in equations is 1. These type of equations are called linear equations.

The equation having only one variable is called one variable equation.

The standard form of one variable equation is $ax + b = 0$, a, b are constant terms and $a \neq 0$.

❖ **Equations of type $ax + b = cx + d$:**

$2x + 5 = 3x - 2$, $4y - 3 = 5y - 2$ are also equations of one variable. The standard form of these type of equation is also $ax + b = 0$.

e.g. $2x + 5 = 3x - 2$

$5 = 3x - 2x - 2$

$5 + 2 = 3x - 2x$

$7 = x$

$x - 7 = 0$

Now, let's understand how to find the solutions of these type of equations through examples.

Example 1 : Solve the equation : $3x + 10 = 5x$

Solution : $3x + 10 = 5x$

∴ $3x = 5x - 10$ (Taking $5x$ on the other side of =)

∴ $3x - 5x = -10$

∴ $-2x = -10$

∴ $2x = 10$

$$\therefore \quad x = \frac{10}{2}$$

$$\therefore \quad x = 5$$

Therefore, solution of the given equation is $x = 5$.

Example 2 : Solve the equation : $2x + 5 = 5x - 2$

Solution : $2x + 5 = 5x - 2$

$$\therefore \quad 2x = 5x - 2 - 5$$

$$\therefore \quad 2x = 5x - 7$$

$$\therefore \quad 2x - 5x = -7$$

$$\therefore \quad -3x = -7$$

$$\therefore \quad 3x = 7$$

$$\therefore \quad x = \frac{7}{3} = 2\frac{1}{3}$$

Therefore, solution of the given equation is $x = 2\frac{1}{3}$.

Example 3 : Solve the equation : $\frac{2x}{5} + 1 = \frac{x}{3} + 3$

Solution : $\frac{2x}{5} + 1 = \frac{x}{3} + 3$

$$\therefore \quad \frac{2x}{5} = \frac{x}{3} + 3 - 1$$

$$\therefore \quad \frac{2x}{5} - \frac{x}{3} = 2$$

$$\therefore \quad \frac{6x - 5x}{15} = 2 \qquad \text{(Simplifying by taking L.C.M.)}$$

$$\therefore \quad \frac{x}{15} = 2$$

$\therefore \quad x = 2 \times 15$

$\therefore \quad x = 30$

Therefore, solution of the given equation is $x = 30$.

Example 4 : Solve the equation : $9 - 3 (x - 2) = 21 - x$

Solution : $9 - 3 (x - 2) = 21 - x$

$\therefore \quad 9 - 3x + 6 = 21 - x$

$\therefore \quad 15 - 3x = 21 - x$

$\therefore \quad -3x = 21 - x - 15$

$\therefore \quad -3x = 21 - 15 - x$

$\therefore \quad -3x = 6 - x$

$\therefore \quad -3x + x = 6$

$\therefore \quad -2x = 6$

$\therefore \quad 2x = -6$

$\therefore \quad x = \dfrac{-6}{2}$

$\therefore \quad x = -3$

Therefore, solution of the given equation is $x = -3$.

Example 5 : Solve the equation : $\dfrac{1}{3} (x + 1) - \dfrac{1}{2} (x - 1) = \dfrac{5}{6} (x + 1)$

Solution : $\dfrac{1}{3} (x + 1) - \dfrac{1}{2} (x - 1) = \dfrac{5}{6} (x + 1)$

$\therefore \quad \dfrac{2}{6}(x+1) - \dfrac{3}{6}(x-1) = \dfrac{5}{6}(x+1)$ (Making same denominator of $\dfrac{1}{3}, \dfrac{1}{2}, \dfrac{5}{6}$)

$\therefore \quad 2(x+1) - 3(x-1) = 5(x+1)$ \hspace{1cm} (Multiplying both sides by 6)

$\therefore \quad 2x + 2 - 3x + 3 = 5x + 5$

$\therefore \quad 2x - 3x + 2 + 3 = 5x - 5$

$\therefore \quad -x + 5 = 5x + 5$

$\therefore \quad -x = 5x + 5 - 5$

$\therefore \quad -x = 5x$

$\therefore \quad -x - 5x = 0$

$\therefore \quad -6x = 0$

$\therefore \quad 6x = 0$

$\therefore \quad x = 0$

Therefore, solution of the given equation is $x = 0$.

Practice 1

❖ **Solve the following equations :**

(1) $\quad 3x + 8 = -5x + 4$

(2) $\quad \dfrac{x}{2} - 2 = \dfrac{x}{3} + 1$

(3) $\quad 5x + \dfrac{7}{2} = \dfrac{3x}{2} - 14$

(4) $\quad x = \dfrac{4}{5}(x + 10)$

(5) $\quad 2y + \dfrac{5}{3} = \dfrac{26}{3} - y$

(6) $\quad \dfrac{2x}{3} + 1 = \dfrac{7x}{15} + 3$

(7) $3m = 5m - \dfrac{8}{5}$

(8) $8x + 4 = 3\,(x - 1) + 7$

(9) $\dfrac{1}{3}(2x - 1) - \dfrac{1}{4}(2x + 1) = \dfrac{1}{12}\,(2 - x)$

(10) $9x - 4 - 3\,(x - 4) = 4\,(x - 1)$

(11) $\dfrac{x}{2} + \dfrac{x+2}{3} + \dfrac{x+7}{4} = x$

(12) $2\,(x - 3) - 7 = 6 - 5\,(x + 1)$

*

❖ **Practical Sums (Problems) :**

Example 6 : The present age of a grandfather is ten times more than the present age of a grand daughter. If his present age is 54 years more than the present age of his grand daughter, then find the present ages of both.

Solution : Suppose the present age of grand daughter is x years. Now the present age of grandfather will be 10 times than the present age of grand daughter i.e. $10x$ years. Now the present age of grandfather is 54 than the present age of grand daughter. i.e. it will be $(x + 54)$ years.

Both are the present ages of grandfather.

∴ $10x = x + 54$

∴ $10x - x = 54$

∴ $9x = 54$

∴ $x = \dfrac{54}{9}$

∴ $x = 6$

Therefore, the present age of grand daughter is 6 years and the present age of grandfather is $10x = 10(6) = 60$ years.

Example 7 : There is a pond near a grass garden. From a herd of deers half of deers are grazing in the garden. The half of the remaining deers are playing with one another. Further, the half part of the remaining deer are taking rest in the garden and 9 deers are drinking water in the pond, then how many deers will be in the group ?

Solution : Suppose total number of deers in the groups is x.

The half part of deers i.e. $\dfrac{x}{2}$ are grazing in the garden.

$\therefore$ Remaining number of deers $= x - \dfrac{x}{2} = \dfrac{2x - x}{2} = \dfrac{x}{2}$

Now, half part of deers i.e. $\dfrac{x}{2}$ are playing with each other.

$\therefore$ The number of deers which are playing $= \dfrac{1}{2}\left(\dfrac{x}{2}\right) = \dfrac{x}{4}$

Now, remaining number of deers $= \dfrac{x}{2} - \dfrac{x}{4} = \dfrac{2x - x}{4} = \dfrac{x}{4}$

Here the half part of deers i.e. $\dfrac{x}{4}$ are resting.

$\therefore$ The number of deers taking rest $= \dfrac{1}{2}\left(\dfrac{x}{4}\right) = \dfrac{x}{8}$

Remaining deers are drinking water in the pond.

(No. of deers grazing in the garden) + (No. of deers playing with each other) + (No. of deers taking rest) + (No. of deers drinking water) = Total no. of deers

$\therefore \quad \dfrac{x}{2} + \dfrac{x}{4} + \dfrac{x}{8} + 9 = x$

$\therefore \quad \dfrac{4x}{8} + \dfrac{2x}{8} + \dfrac{x}{8} + 9 = x$

$\therefore \quad 4x + 2x + x + 72 = 8x$ $\qquad$ (Multiplying by 8)

$\therefore$ $7x + 72 = 8x$

$\therefore$ $72 = 8x - 7x$

$\therefore$ $72 = x$

$\therefore$ $x = 72$

$\therefore$ Therefore, the total number of deers are 72.

Example 8 : In two numbers, the larger number is five times the smaller number. If 21 is added to each, then larger number will be double of the smaller number, then find those numbers.

Solution : Suppose smaller number is x.

$\therefore$ Larger number will be 5.

Adding 21 to larger number, it will be $(5x + 21)$ and Adding 21 to smaller number, it will be $(x + 21)$.

Now $(5x + 21)$ is double than $(x + 21)$.

$\therefore$ $5x + 21 = 2 (x + 21)$

$\therefore$ $5x + 21 = 2x + 42$

$\therefore$ $5x = 2x + 42 - 21$

$\therefore$ $5x = 2x + 21$

$\therefore$ $5x - 2x = 21$

$\therefore$ $3x = 21$

$\therefore$ $x = \dfrac{21}{3}$

$\therefore$ $x = 7$

$\therefore$ Therefore, smaller number is $x = 7$ and the larger number is $5x = 5(7) = 35$.

Practice 2

1. The present age of Vinodbhai is three times more than the present age of his son. Before ten years his age was five times more than the age of his son. Find the present ages of both.

2. There are some children in the garden. In which one third of the children are playing games. The five more than the half of remaining children are taking breakfast. The remaining 4 children are drawing picture of garden. How many total children will be in the garden ?

3. By adding one third age of Salim's father before 5 years to the half of present age of his father we get 20, then find the present age of Salim's father.

4. The result obtained by subtracting 5 from eight times of a number is 4 more than the five times of that number. Find those numbers.

5. The result obtained by adding 5 to 3 times of a number is equal to the result obtained by subtracting 10 from 4 times of that number. Find those numbers.

6. Jiya has some chocolates. She distributed one-fifth part to Riya, one fourth part to Vansh and the the half part to Dhruv. Now if 5 chocolates are left, then find how many chocolates were with her ?

7. The present age of Ravi's father is four times the present age of Ravi. After 10 years the age of Ravi's father will be three times more than the age of Ravi. Find their present ages.

Practice 3

1. **Solve the following equations :**

   (1) $4x + 13 = 3x + 15$

   (2) $9x - 15 = 7x + 1$

   (3) $y + 18 = 2y - 7$

   (4) $17 - 3 (m - 5) = 2 - 13m$

(5) $\dfrac{4x+3}{3} = \dfrac{4x-1}{2} + \dfrac{1}{2}$

(6) $\dfrac{1}{2}(x-2) - \dfrac{2}{3}(x+3) = \dfrac{1}{2} - x$

(7) $5(3-2x) - 4(2-3x) = 3 - 5(x+2)$

2. Solve the following sums :

(1) The present age of Kiran's mother is six times the present age of Kiran. After 5 years, the age of Kiran will be one-third of her mother at that time. Find their present ages.

(2) The result obtained by adding 8 to seven times of a number is equal to the result obtained by subtracting 25 to ten times of that number. Find that number.

(3) There are some chocolates in a box. Each gets 7 chocolates when these chocolates are distributed equally among the students of a class. If there were 5 more students in the class, then each would get 1 chocolate less. Find the number of students in the class.

(4) If we subtract 8 from a number and then divide it by 5 or if we add 13 to the number and divide the result by 8, we get the same result. Find the number.

*

- **The solution of equation of the form $\dfrac{ax+b}{cx+d} = k$ ($cx + d \neq 0$, $a \neq ck$)**

The equation of the form $\dfrac{8x+4}{3x-2} = 5$, $\dfrac{7y+3}{4y+5} = \dfrac{2}{3}$ etc. are also the linear equations of one variable.

The generalized form of these type of equations is $\dfrac{ax+b}{cx+d} = k$ (a, b, c, d, k are constant numbers, $cx + d \neq 0$ and $a \neq ck$)

Earlier we have learnt $\dfrac{a}{b} = \dfrac{c}{d}$ then $ad = bc$.

Using this, we will find how to solve the equation of the form $\dfrac{ax+b}{cx+d} = k$.

Example 9 : Solve the equation : $\dfrac{4x-1}{3x+2} = \dfrac{3}{2}$

Solution : $\dfrac{4x-1}{3x+2} = \dfrac{3}{2}$

$\therefore \quad 2(4x - 1) = 3(3x + 2)$ (From $\dfrac{a}{b} = \dfrac{c}{d}$ we get $ad = bc$ by cross multiplication)

$\therefore \quad 8x - 2 = 9x + 6$

$\therefore \quad 8x = 9x + 6 + 2$

$\therefore \quad 8x = 9x + 8$

$\therefore \quad 8x - 9x = + 8$

$\therefore \quad -x = 8$

$\therefore \quad x = -8$

Therefore, solution of the given equation is $x = -8$.

Example 10 : Solve the equation : $\dfrac{2x-3}{5} = \dfrac{3x-4}{4}$

Solution : $\dfrac{2x-3}{5} = \dfrac{3x-4}{4}$

$\therefore \quad 4(2x - 3) = 5(3x - 4)$ $\hfill$ (By cross multiplication)

$\therefore \quad 8x - 12 = 15x - 20$

$\therefore \quad 8x = 15x - 20 + 12$

$\therefore \quad 8x - 15x = -8$

$\therefore \quad -7x = -8$

$\therefore \quad 7x = 8$

$\therefore \quad x = \dfrac{8}{7} = 1\dfrac{1}{7}$

Therefore, solution of the given equation is $x = 1\dfrac{1}{7}$.

Example 11 : Solve the equation : $\dfrac{3x+1}{2x}=\dfrac{3x+2}{2x+1}$

Solution : $\dfrac{3x+1}{2x}=\dfrac{3x+2}{2x+1}$

$\therefore \quad (3x+1)(2x+1)=2x(3x+2)$ (By cross multiplication)

$\therefore \quad 6x^2+3x+2x+1=6x^2+4x$

$\therefore \quad 6x^2+5x+1=6x^2+4x$

$\therefore \quad 6x^2+5x+1-6x^2-4x=0$

$\therefore \quad 5x-4x+1=0$

$\therefore \quad x+1=0$

$\therefore \quad x=0-1$

$\therefore \quad x=-1$

$\therefore \quad$ Therefore, solution of the given equation is $x=-1$

Example 12 : Solve the equation : $\dfrac{2x+7}{3x-1}+4=\dfrac{x+5}{3x-1}+\dfrac{15}{4}$

Solution : $\dfrac{(2x+7)}{3x-1}+4=\dfrac{x+5}{3x-1}+\dfrac{15}{4}$

$\therefore \quad \dfrac{2x+7}{3x-1}=\dfrac{x+5}{3x-1}+\dfrac{15}{4}-4$

$\therefore \quad \dfrac{2x+7}{3x-1}-\dfrac{x+5}{3x-1}=\dfrac{15}{4}-4$

$\therefore \quad \dfrac{(2x+7)-(x+5)}{3x-1}=\dfrac{15}{4}-\dfrac{16}{4}$

$\therefore \quad \dfrac{2x+7-x-5}{3x-1}=\dfrac{15-16}{4}$

$$\therefore \quad \frac{x+2}{3x-1} = \frac{-1}{4}$$

$\therefore \quad 4 (x + 2) = (-1) (3x - 1)$

$\therefore \quad 4x + 8 = -3x + 1$

$\therefore \quad 4x + 3x = 1 - 8$

$\therefore \quad 7x = -7$

$\therefore \quad x = \dfrac{-7}{7}$

$\therefore \quad x = -1$

$\therefore \quad$ Therefore, solution of the given equation is $x = -1$.

Verify : The solution you get of the equation, putting that in L.H.S. of equation and R.H.S. of equation, the results will be same).

In the above equation, put $x = -1$, then.

$$\textbf{L.H.S.} \quad = \quad \frac{2x+7}{3x-1} + 4$$

$$= \frac{2(-1)+7}{3(-1)-1} + 4$$

$$= \frac{-2+7}{-3-1} + 4$$

$$= \frac{5}{-4} + 4$$

$$= \frac{-5}{4} + 4$$

$$= \frac{-5}{4} + \frac{16}{4}$$

$$= \frac{-5+16}{4}$$

$$= \frac{11}{4}$$

R.H.S. $= \dfrac{x+5}{3x-1} + \dfrac{15}{4}$

$= \dfrac{(-1)+5}{3(-1)-1} + \dfrac{15}{4}$

$= \dfrac{-1+5}{-3-1} + \dfrac{15}{4}$

$= \dfrac{4}{-4} + \dfrac{15}{4}$

$= \dfrac{-4}{4} + \dfrac{15}{4}$

$= \dfrac{-4+15}{4}$

$= \dfrac{11}{4}$

Therefore, L.H.S. = R.H.S.. So the answer is correct.

Example 12 : Solve the equation : $2x\,(2x-1) - (2x+1)\,(2x-1) = 0$

Solution : $2x\,(2x-1) - (2x+1)\,(2x-1) = 0$

$\therefore \quad 4x^2 - 2x - (4x^2 - 1) = 0$

$\therefore \quad 4x^2 - 2x - 4x^2 + 1 = 0$

$\therefore \quad -2x + 1 = 0$

$\therefore \quad -2x = 0 - 1$

$\therefore \quad -2x = -1$

$\therefore \quad 2x = 1$

$\therefore \quad x = \dfrac{1}{2}$

Second Method :

$$2x\,(2x-1)-(2x+1)\,(2x-1)=0$$

∴ $(2x-1)\,[2x-(2x+1)\,]=0$ **(Taking common)**

∴ $(2x-1)\,[2x-2x-1]=0$

∴ $(2x-1)\,(-1)=0$ **(Multiplying bothsides by (−1)**

∴ $2x-1=0$

∴ $2x=1$

∴ $x=\dfrac{1}{2}$

Practice 4

1. **Solve the following equations :**

(1) $\dfrac{8x-3}{3x}=2$ (2) $\dfrac{30x}{7-6x}=30$

(3) $\dfrac{3a-4}{2-6a}=\dfrac{-2}{5}$ (4) $\dfrac{m}{m+15}=\dfrac{4}{9}$

(5) $\dfrac{7n+4}{n+2}=\dfrac{-4}{3}$ (6) $\dfrac{3}{2}\left(\dfrac{4x+1}{2x-5}\right)=\dfrac{3}{2}$

(7) $\dfrac{4x-1}{2x+1}=\dfrac{6x-5}{3x+2}$ (8) $\dfrac{4x-1}{4x+1}=\dfrac{2x+1}{2x-1}$

(9) $\dfrac{3x+1}{3x-4}=\dfrac{x-1}{x+1}$ (10) $\dfrac{5x-6}{8}=\dfrac{3x-8}{5}+1$

(11) $\dfrac{5x-4}{4}=\dfrac{3x-4}{5}+5$ (12) $\dfrac{x+10}{4}+\dfrac{x+3}{3}=\dfrac{x+4}{2}$

❖ **Practical Sums (Problems) :**

Example 14 : The ratio of present ages of Chintu and Jalu is 3 : 4. If the ratio of the age of Chintu after 3 years and the age of Jalu before 5 years is 3, then find the present age of each one.

Solution : The ratio of present ages of Chintu and Jalu is 3 : 4.

∴   Suppose the present age of Chintu is $3x$ years and that of Jalu is $4x$ years.

After 3 years the age of Chintu will be $(3x + 3)$ years and before 5 years, the age of Jalu is $(4x - 5)$ years.

Now, the ratio of $\dfrac{3x+3}{4x-5}$ is equal to 3.

∴ $\dfrac{3x+3}{4x-5} = 3$

∴ $3x + 3 = 3(4x - 5)$

∴ $3x + 3 = 12x - 15$

∴ $3x = 12x - 15 - 3$

∴ $3x = 12x - 18$

∴ $3x - 12x = -18$

∴ $-9x = -18$

∴ $9x = 18$

∴ $x = \dfrac{18}{9}$

∴ $x = 2$

∴   Therefore, the present age of Chintu is $3x = 3 \times 2 = 6$ years and the present age of Jalu is $4x = 4 \times 2 = 8$ years.

Example 15 : The sum of two numbers is 98. If the ratio of smaller number and larger number is 5 : 9, then find those numbers.

Solution : Suppose the samller number is x.

∴   The larger number will be $(98 - x)$

(because the sum of two numbers is 98.)

The ratio of smaller number and larger number will be $\frac{x}{98-x}$ which is given 5 : 9.

∴ $\frac{x}{98-x} = \frac{5}{9}$

∴ $9x = 5\,(98 - x)$

∴ $9x = 490 - 5x$

∴ $9x + 5x = 490$

∴ $14x = 490$

∴ $x = \frac{490}{14}$

∴ $x = 35$

∴   Smaller number $x = 35$ and larger number $98 - x = 98 - 35 = 63$

∴   Therefore, the larger number is 63 and the smaller number is 35.

Second Method :

Solution : The ratio of two numbers given is 5 : 9.

∴ Suppose those numbers are $5x$ and $9x$.

Now sum of both number is 98, which is given.

∴ $5x + 9x = 98$

$\therefore \quad 14x = 98$

$\therefore \quad x = \dfrac{98}{14}$

$\therefore \quad x = 7$

$\therefore$ First number $5x = 5(7) = 35$ and

Second number $9x = 9(7) = 63$

$\therefore$ Therefore, two numbers are 35 and 63.

Example 17 : The ratio of two natural numbers is 5 : 8, If we add 5 to smaller number and subtract 6 from larger number, then their ratio will be 4 : 5. Find those numbers.

Solution : The ratio of two numbers is 5 : 8.

$\therefore$ Suppose one number is $5x$ and second number is $8x$. It is clear that $5x$ is smaller number and $8x$ is larger number.

Now adding $5x$ to smaller number 5, it becomes $(5x + 5)$ and subtracting 6 from larger number it becomes $(8x - 6)$.

Their ratio 4 : 5 is given.

$\therefore \quad \dfrac{5x+5}{8x-6} = \dfrac{4}{5}$

$\therefore \quad 5(5x + 5) = 4(8x - 6)$

$\therefore \quad 25x + 25 = 32x - 24$

$\therefore \quad 25x - 32x = -24 - 25$

$\therefore \quad -7x = -49$

$\therefore \quad 7x = 49$

$\therefore \quad x = \dfrac{49}{7}$

$\therefore \quad x = 7$

$\therefore$ Smaller number is $5x = 5 \times 7 = 35$ and

Larger number is $8x = 8 \times 7 = 56$.

Example 17 : In a rational number, the denominator is 8 more than the numerator. If 17 is added to numerator and 1 is subtracted from denominator then we get $\frac{3}{2}$. Find that fraction.

Solution : Suppose numerator of rational number is x.

∴ Its denominator will be $x + 8$. (because denominator is 8 more than numerator)

∴ Rational number will be $\frac{x}{x+8}$.

By adding 17 to numerator we get $(x + 17)$ and subtracting 1 from denominator $(x + 8)$, we get $(x + 8) - 1 = (x + 7)$.

∴ We get new fraction $\frac{x+17}{x+7}$, which is equal to $\frac{3}{2}$.

∴ $\frac{x+17}{x+7} = \frac{3}{2}$

∴ $2(x + 17) = 3 (x + 7)$

∴ $2x + 34 = 3x + 21$

∴ $2x - 3x = 21 - 34$

∴ $-x = -13$

∴ $x = 13$

∴ $\frac{x}{x+8} = \frac{13}{13+8} = \frac{13}{21}$

Practice 5

1. The ratio of present ages of Veena and Kinjal is 2 : 3. The ratio of ages of Veena after 4 years and the age of Kinjal 4 years before is 4 : 1. Find their present ages.

2. The ratio of present ages of Ajay and Vijay is 4 : 5. After 8 years the ratio of their ages will be 5 : 6, then find their present ages.

3. The ratio of two numbers is 1 : 2. If their sum is 45, then find those numbers.

4. The sum of two numbers is 30 and their ratio is 3 : 2. Find those numbers.

5. In a rational number, the numerator is 5 less than its denominator. If 1 is subtracted from the numerator and 2 is added to the denominator, the new rational number will be $\frac{1}{5}$. Find original rational number.

6. The numerator of a rational number is 3 less than its denominator. If we take 3 times of numerator and increase 20 to the denominator, the new rational number will be $\frac{1}{8}$. Find original rational number.

Exercise

1. Solve the following equations :

(1) $\dfrac{2x+3}{x-2} = \dfrac{1}{4}$

(2) $\dfrac{22x-7}{11x+3} = 5$

(3) $\dfrac{3}{4}\left(\dfrac{4x+1}{2x+5}\right) = \dfrac{1}{2}$

(4) $\dfrac{3x+2}{x-3} - \dfrac{1}{3} = \dfrac{2x+3}{x-3} - \dfrac{1}{2}$

(5) $\dfrac{2x-3}{3x-1} = \dfrac{2x+5}{3x+1}$

2. Solve the following sums :

(1)   The ratio of present ages of Alpa and Jalpa is 5 : 7. After 5 years the ratio of their ages will be 3 : 4. Find their present ages.

(2)   The sum of two numbers is 108. If their ratio is 5 : 7, then find those numbers.

(3)   The numerator of a rational number is 2 less than 3 times of its denominator. If 8 is added to its numerator and 6 is added to its denominator then new fraction will be $\frac{5}{3}$. Find that fraction.

(4)   The ratio of rupees of Tinu and Minu is 2 : 5. If both have ₹ 50 more, then ratio is 4 : 7. How much rupees will both have ?

Answers

Practice 1

(1) $\dfrac{1}{2}$ (2) 18   (3) (−5)   (4) 40   (5) $2\dfrac{1}{3}$ (6) 10   (7) $\dfrac{4}{5}$

(8) 0   (9) 3   (10) (−6)   (11) (−29)   (12) 2

Practice 2

(1) The age of son is 20 years, The age of father is 60 years. (2) 27

(3) 26 years   (4) 3   (5) 15   (6) 100 chocolates

(7) Age of Ravi is 20 years and age of his father is 80 years.

Practice 3

1. (1) 2   (2) 8   (3) 25   (4) (−3)   (5) $1\dfrac{1}{2}$ (6) $4\dfrac{1}{5}$ (7) (−2)

2. (1) Age of Kiran is 5 years and age of her mother is 30 years.

(2) 11   (3) 30 students   (4) 43

Practice 4

1. (1) $1\dfrac{1}{2}$ (2) 1   (3) $5\dfrac{1}{3}$ (4) 12   (5) $\dfrac{-4}{5}$ (6) −3   (7) $-\dfrac{1}{3}$

(8) 0   (9) $\dfrac{3}{11}$ (10) 6   (11) 8   (12) − 18

Practice 5

(1) Age of Veena is 4 years, Age of Kinjal is 6 years

(2) Age of Ajay is 32 years, Age of Vijay is 40 years

(3) 15 and 30   (4) 18 and 12

(5) $\dfrac{3}{8}$ (6) $\dfrac{1}{4}$

Exercise

1. (1) -2 (2) $\dfrac{-2}{3}$ (3) $\dfrac{7}{8}$ (4) $1\dfrac{2}{7}$ (5) $\dfrac{1}{10}$

2. (1) Age of Alpa is 25 years, Age of Jalpa is 35 years   (2) 45 and 63   (3) $\dfrac{7}{3}$

(4) Tinu has ₹ 50 and Minu has ₹ 125

◆

Inventor of basic elements of Geometry - Euclid

- He was born in 330 A.D. in Ethens.

- He gave a place by correcting geometrical concepts and giving recognition as a subject in a well disciplined syllabus.

- He made the easy arrangement by briefing their truths and works done by earlier geometers.

- He made theorems and proof more logical.

- He tried to give clear understanding (knowledge) of basic structure of geometry like point, line and plane.

- The main work of Euclid and his book are known as 'Elements'. It is translated in all the languages of the world.

- He began with the conceptual terms like line, point, plane, etc. After that he determines their postulates.

7 Constructions

❖ **Let's Remember :**

(1) Construct $\overline{AB}$ such that AB = 4.5 cm.

(2) Construct ΔXYZ such that XY = 5 cm, YZ = 5 cm and XZ = 5 cm.

(3) Construct ΔPQR such that PQ = 4.5 cm, QR = 6.5 cm and PR = 8 cm.

(4) Construct a right angled ΔFDE such that DE = 4 cm, Hypotenuse FE = 5 cm.

● **Activity 1 :**

● Construct an angle of measure 179° with the help of protector.

● **Activity 2 :**

● Draw opposite rays and measure their angle.

● What will be the degree of measure of angle ?

● Friends, think and discuss with your teachers.

● **Activity 3 :**

● Draw a line *l* and take a point R on it.

● Draw a semicircle taking R as centre and 3 cm radius. Where it intersects the line *l*, name them A and B.
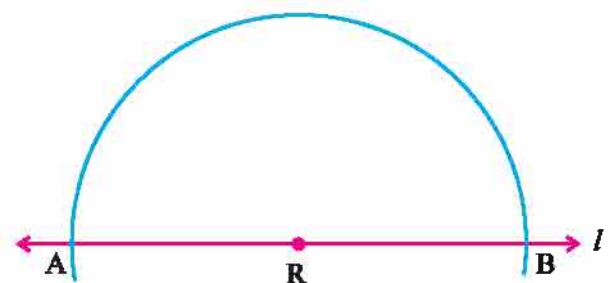
- Taking A and B as centre respectively, draw an arc of radius 3 cm on the semicircle. Where it intersects the arc, name them P and Q respectively.
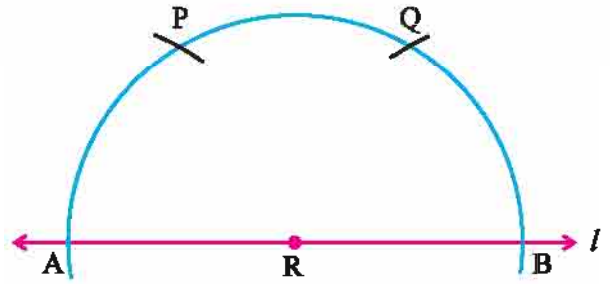
- Construct $\overrightarrow{RP}$ and $\overrightarrow{RQ}$

- With the help of protector measure $\angle QRB$, $\angle PRA$ and $\angle PRQ$. Write their measurements.
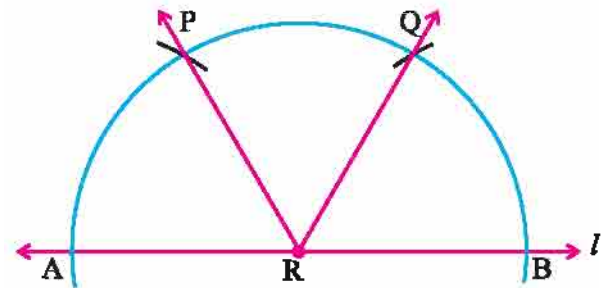
We have learnt to construct angle with the help of protector. Without the help of protector we can draw angles with the help of scale and rounder.

Construction 1 : Given a line and a point outside it, to construct a line perpendicular to the given line and passing through the given point.

Data : A line l and a point P are given such that $P \notin l$.

To construct : $\overleftrightarrow{PQ} \perp l$

Steps of construction :

(1) A line and a point P outside it is given.

(2) Taking P as a centre and with arbitrary radius draw an arc of circle that intersects l in two points, say them A and B respectively.

• P

(3) Taking A and B as centres one by one and keeping the same radius, draw arcs in the half plane not containing P. Where these two arcs intersect, name that point as Q.

(4) Draw $\overleftrightarrow{PQ}$.

Thus, $\overleftrightarrow{PQ} \perp l$

($\overleftrightarrow{PQ}$ will be perpendicular to l.)
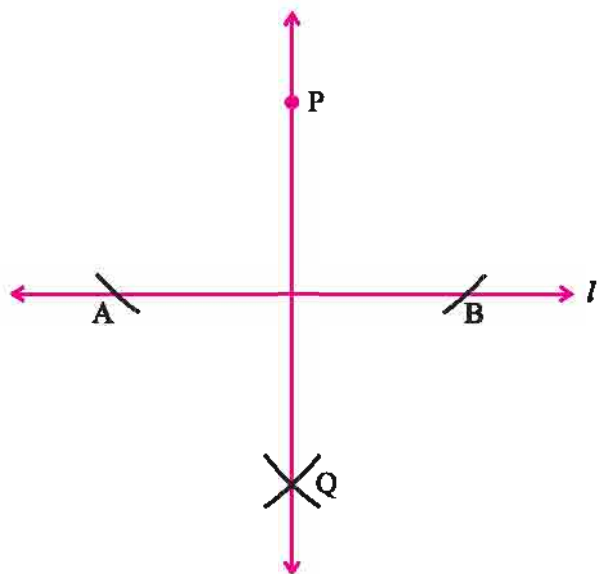
Note : Here in each step, to draw different figures are not necessary, we have to draw the figure which is obtained after step (4). The steps of construction are necessary.

Construction 2 : Given a line and a point on it, to draw a perpendicular line passing through the given point.

Data : A line l and a point P on it. P ∈ l .

To construct : $\overleftrightarrow{PQ} \perp l$

Steps of construction :

(1) A line l and a point P on it, is given.

(2) Taking P as centre, draw an arc of circle with an arbitrary radius which intersects them on both sides of P. Name these points as A and B respectively.

(3) Taking A and B as centres one by one and radius more than half of $\overline{AB}$ draw arcs of circle on one side of P. Where these both arcs intersect name that point Q.
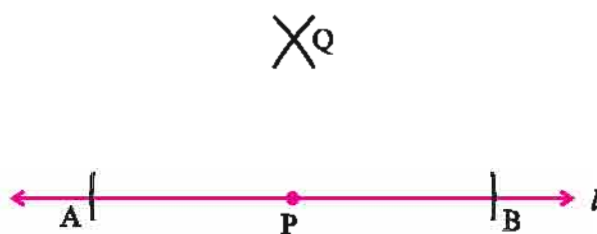
(4) Draw $\overleftrightarrow{PQ}$.

Thus, $\overleftrightarrow{PQ} \perp l$ can be obtained.
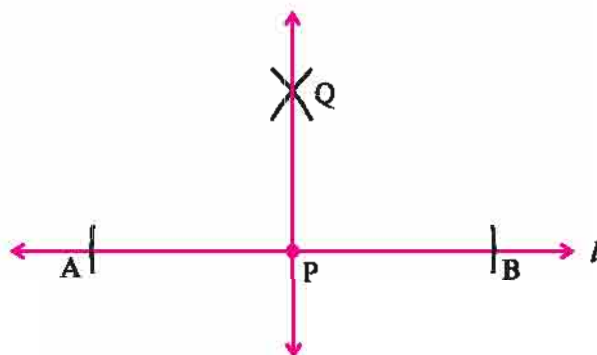
Construction 3 : Construct perpendicular bisector $\overleftrightarrow{AB}$ of $\overline{PQ}$ such that PQ = 5.7 cm.

Data : PQ = 5.7 cm is given.

To construct : To draw perpendicular bisector $\overleftrightarrow{AB}$ of $\overline{PQ}$.

Steps of construction :

(1) PQ = 5.7 cm is given.

(2) Taking radius more than half the measure of $\overline{PQ}$ and P and Q centres one by one, draw arcs in both half planes of $\overline{PQ}$. Where these two arcs intersect name them as A and B respectively.

(4) Draw $\overleftrightarrow{AB}$.

Thus, $\overleftrightarrow{AB}$ is perpendicular bisector of $\overline{PQ}$.
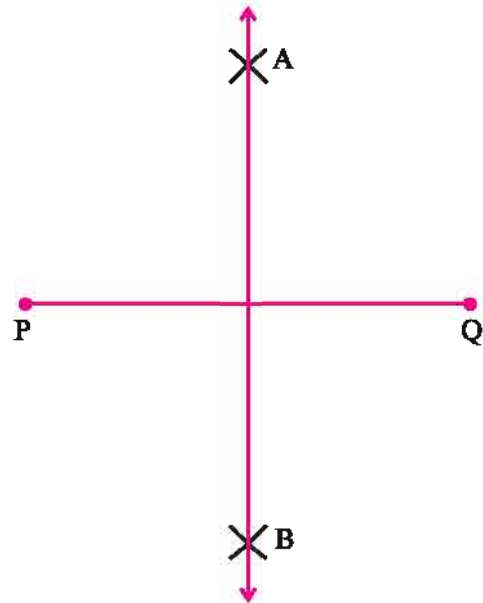
We have learnt to construct angle with the help of protector. Without help of protector we can draw angles with the help of scale and compass.

Construction 4 : To construct an angle of measure 60°.

Data : $\overrightarrow{BX}$ is given.

To construct : $m\angle ABX = 60°$

Steps of construction :

(1) Draw $\overrightarrow{BX}$.

(2) Taking B as centre and an arbitrary radius, draw an arc of semicircle which intersects $\overrightarrow{BX}$. Let this arc interest $\overrightarrow{BX}$ in C.
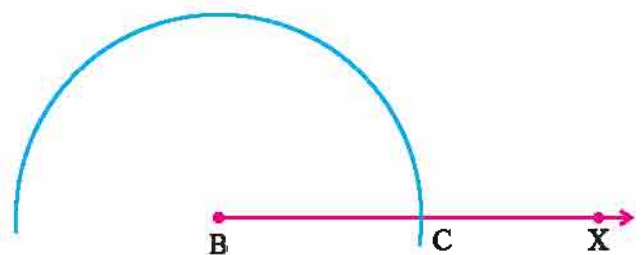
(3) Taking C as centre and radius equal to $\overline{BC}$ draw an arc of the circle which intersects the semicircular arc at A.
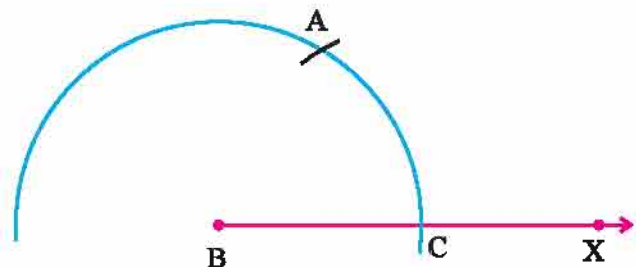
(4) Draw $\overrightarrow{BA}$.

Thus, $m\angle ABX = 60°$

(Verify with the help of protector.)
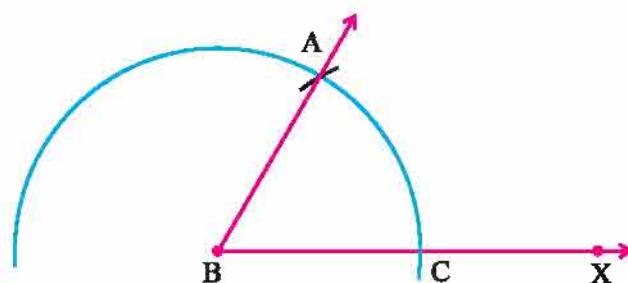
Construction 5 : To construct an angle of measure 120°.

Data : $\overrightarrow{QY}$ is given.

To construct : $m\angle PQY = 120°$

Steps of construction :

(1) Draw $\overrightarrow{QY}$.

(2) Taking an arbitrary radius and Q as centre. Draw an arc of circle which intersects $\overrightarrow{QY}$ at R.
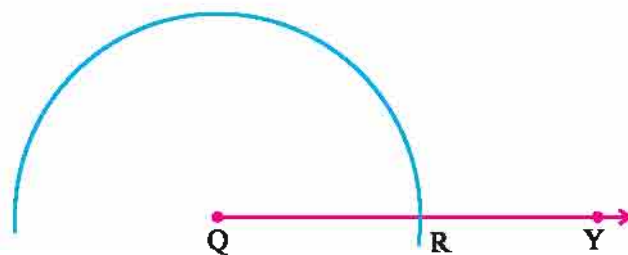
(3) Now, taking R as centre and the same radius draw an arc of circle which intersects the semicircle at point S.
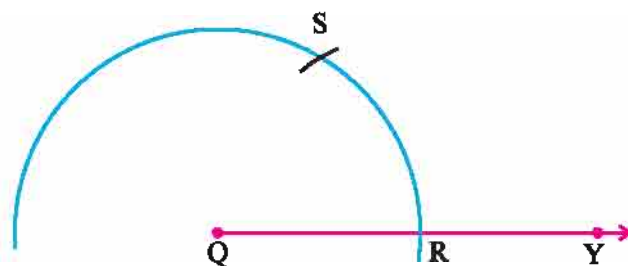
(4) Taking S as centre and the same radius draw an arc of circle in opposite direction of R which intersects the semicircle at P.
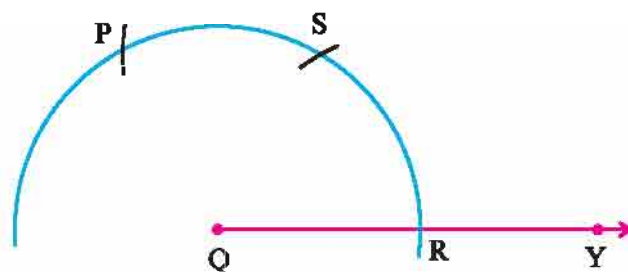
(5) Draw $\overrightarrow{QP}$.

Thus, $m\angle PQY = 120°$
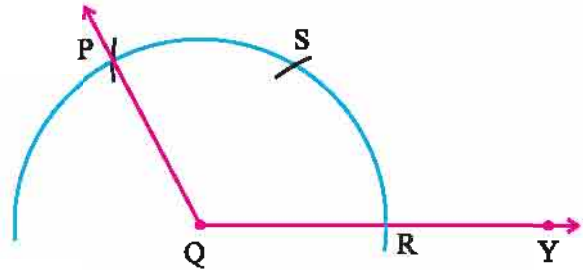
Construction 5 : To construct bisector of $\angle AOB$.

Data : $\angle AOB$ is given.

To construct : To construct bisector of $\angle AOB$.

Steps of construction :

(1) Draw $\angle AOB$.
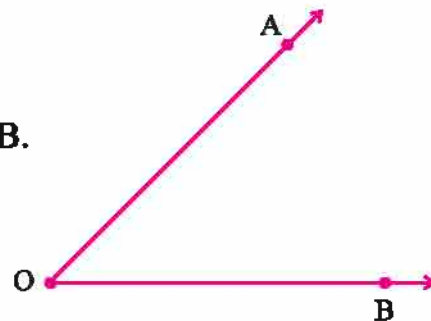
(2) Taking O as centre and an arbitrary radius draw an arc of circle which intersects $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
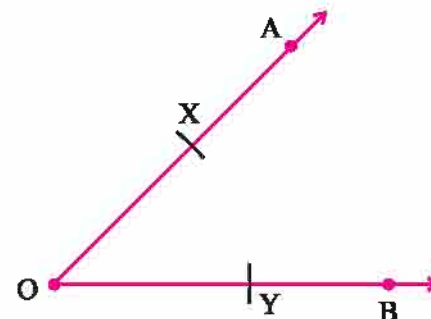
(3) Taking the same radius and X and Y as centres one by one, draw arcs in the interior of angle, where these arcs intersect each other, name that point as C.
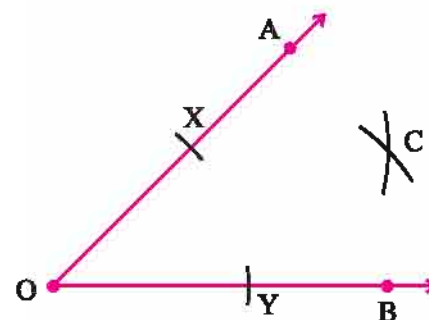
(4) Draw $\overrightarrow{OC}$.

(5) $\overrightarrow{OC}$ bisects $\angle AOB$.

Thus, $\overrightarrow{OC}$ is the bisector of $\angle AOB$.
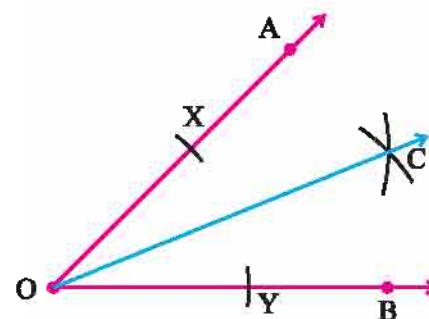
Construction 7 : To construct an angle congruent to the given angle.

Data : ∠ABC is given.

To construct : ∠PQR such that ∠PQR ≅ ∠ABC.

Steps of construction :

(1) ∠ABC is given.

(2) Draw $\overrightarrow{QR}$.

(3) Taking B as centre and an arbitrary radius, draw an arc of the circle which intersects $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at point E and D respectively.

(4) Without changing the above radius and taking O as centre, draw an arc of the circle which intersects $\overrightarrow{QR}$ at a point. Name that point as S.

(5) Taking S as centre and ED as radius, draw an arc of the circle which intersects the earlier arc at a point P.
Draw $\overrightarrow{QP}$.
Thus, ∠PQR ≅ ∠ABC
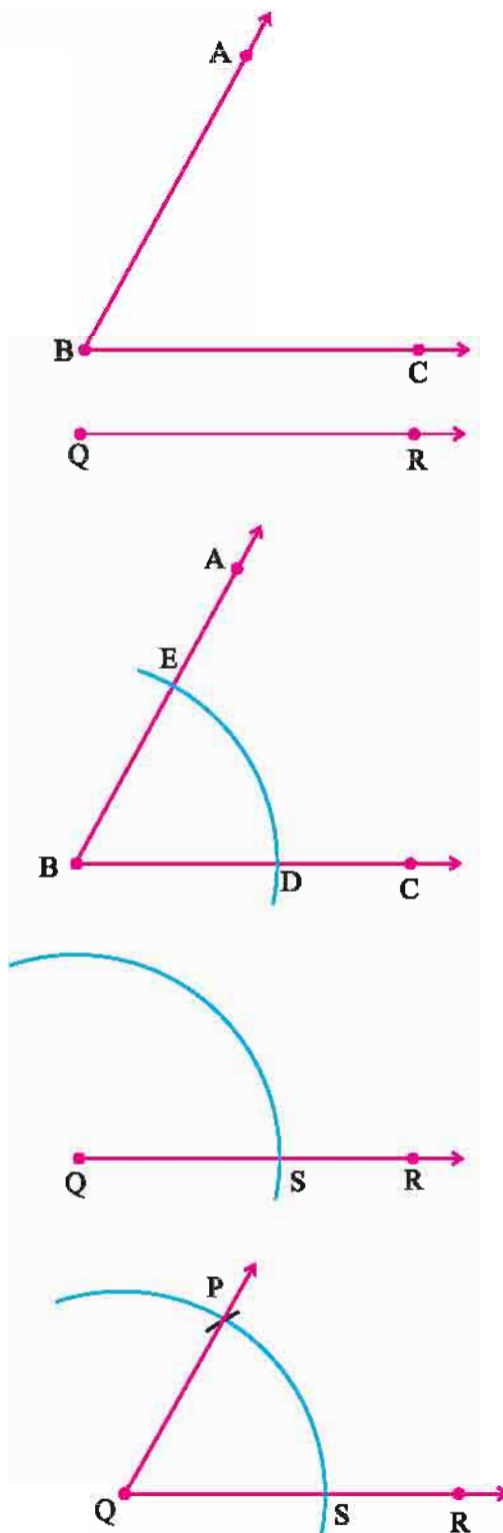
Construction 8 : To construct an angle of measure 90°.

Data : $\overrightarrow{BX}$ is given.

To construct : To construct $m\angle ABC = 90°$ $(C \in \overrightarrow{BX})$

Steps of construction :

(1) Draw $\overrightarrow{BX}$.

(2) Taking B as centre and an arbitrary radius, draw a semicircle. Name the point of intersection of $\overrightarrow{BX}$ and semicircle as C.

(3) Taking C as centre and the same radius, draw an arc on the semicircle which intersect the semicircle at a point D.

(4) Taking D as centre and the same radius, draw an other arc in the opposite direction of C which intersects the semicircle at a point. Name that point as E.

(5) Taking E and D as centre one by one and the same radius draw arcs which intersects each other at a point. Name that point A.

(6) Draw $\overrightarrow{BX}$.

Thus, $m\angle ABC = 90°$.
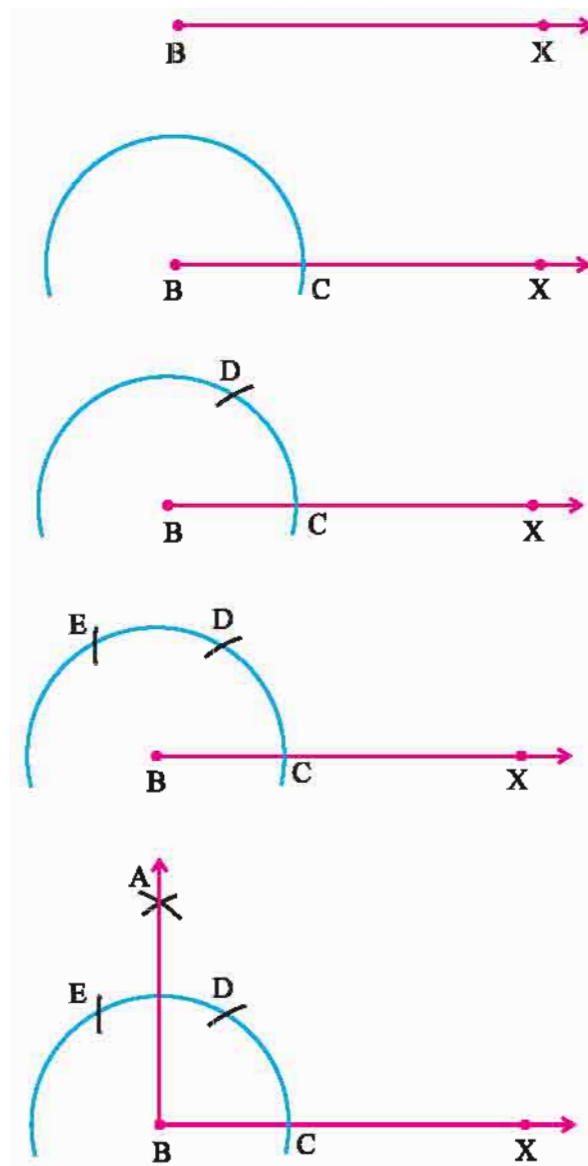
Example 1 : To construct an angle of measure with the help of compass and scale and write steps of construction.

Data : $\overrightarrow{BX}$ is given.

To construct : $m\angle GBC = 75°$

Steps of construction :

(1) Construct an angle of measure 90° according to construction 7. Where $\overrightarrow{BA}$ intersect the semicircle, Name that point as F.

(2) Now, taking F and D as centre one by one and radius equal to $\overline{FD}$, draw arcs of the circle which intersect each other at point G. Draw $\overrightarrow{BG}$.

Thus, $m\angle GBC = 75°$
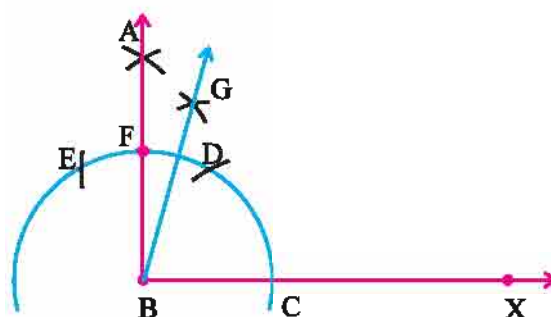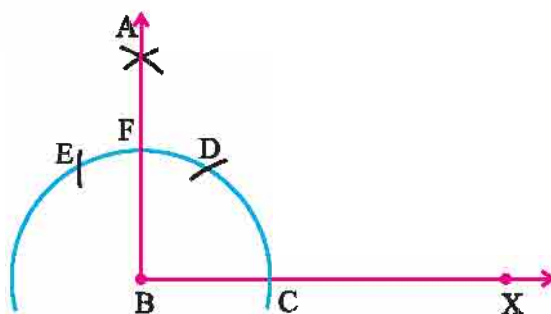
Exercise

(1) To construct perpendicular bisector of $\overline{XY}$ such that XY = 5 cm. Write steps of construction.

(2) Draw an obtuse angle without the help of protector. Construct an angle congruent to this angle. Write steps of construction.

(3) Without use of protector, construct an angle of measure 150°. Construct the bisector of this angle and write steps of construction.

(4) Construct an angle such that $m\angle PQR = 45°$. Construct $\angle ABC \cong \angle PQR$ and write steps of construction.

(5) Without use of protector construct an angle of measure 105°. Draw its bisector. Write steps of construction.

(6) With the help of rounder and scale, construct the following measure of angles and write their steps of construction :

(i) 15°   (ii) 135°   (iii) 30°   (iv) 165°

Revision

1. Write the suitable answer in the given table :

Sr. No.	Details	1	2	3
1.	Types of Account			
2.	Methods of withdrawing money			
3.	Types of cheques			
4.	Name three banks which you have seen			

2(A)   Fill in the blanks :

(1) $a^2 - 4a^3 = a^2$ (_____).

(2) _____ $+ 14m + 49 = (m + 7)^2$

(3) $4n^2 -$ _____ $+ 25 = (2n - 5)^2$

(4) $25a^2 - 70ab + 49b^2$ is square of _____

(5) Adding _____ to $x^2 + 100$, it will become a perfect square trinomial.

(6) $m^2n^2 - 1 = (mn + 1)$ (_____)

(7) $12a^2 - 75 = 3$ (_____) $(2a + 5)$

(8) $x^2 - 2x - 8 = (x - 4)$ (_____)

(9) $1 - y^3 = ($_____$) (1 + y + y^2)$

(B)   Factorize :

(1) $ab^3 + ab$

(2) $x^3 - 5x^2 + 2x - 10$

(3) $4a^2b^2 + 4ab + 1$

(4) $4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20zx$

(5) $m^4 - 8m^2 + 16$

(6) $64m^3 - mn^2$

(7) $4x^2 + 4x + 1 - y^2 + 10y - 25$

(8) $4x^4 - 12x^2 + 9$

3(A) Solve the following :

(1) Munnabhai takes 5 days and Sultansingh takes 7.5 days to make 75 water tanks by a machine in a factory. If both do this work together they get ₹ 3750 as remuneration to make 75 tanks. How much will each get ?

(2) Nileshbhai borrowed ₹ 35,000 from Jahnavi at 10% rate of interest for 1 year at simple interest. If he took same amount from a nationalised bank at compound interest instead of simple interest, then how much more amount would Nileshbhai have to pay ? (Interest is added half yearly in a bank.)

3(B) Solve the equations :

(1) $\dfrac{4x+13}{x+5} = 3$

(2) $\dfrac{2}{3}\dfrac{(-3x+1)}{(5-x)} = \dfrac{1}{2}$

(3) $\dfrac{2x+3}{x} + 3 = \dfrac{5}{x}$

(4) $\dfrac{4x+1}{2} = \dfrac{5}{2}$

4. Draw the following constructions and write their steps of construction :

(1) Draw the perpendicular bisector of $\overline{XY}$ such that XY = 6.6 cm.

(2) Without use of protector draw acute angle. Construct an angle congruent to the given angle.

(3) Construct the following angles with the help of compass and scale :

(i) 22.5°   (ii) 45°   (iii) 112.5°

5. Solve the following problems :

(1) The ground floor of a room is of rectangular shape. Its length is two and half times of its breadth. If the perimeter of this ground floor is 70 meter, then find its length.

(2) The result obtained by subtracting 5 from eight times of a number is 4 more than five times the same number, then find that number.

(3) The sum of two number is 45. If their ratio is 7 : 8, then find these numbers.

(4) The denominator of a rational number is 1 more than three times of its numerator. If its numerator is doubled and 2 is added to denominator, then the rational number becomes $\frac{1}{2}$. Find that rational number.

●

☞ **Answers** ☜

2.(A) (1) $1 - 4a$ (2) m^2 (3) $20n$ (4) $5a - 7b$

(5) $\pm 20x$ (6) $mn - 1$ (7) $2a - 5$ (8) $x + 2$ (9) $1 - y$

(B) (1) $ab\,(b^2 + 1)$ (2) $(x - 5)\,(x^2 + 2)$ (3) $(2ab + 1)^2$

(4) $(2x + 3y + 5z)^2$ (5) $(m^2 - 4)^2$ (6) $m\,(8m + n)\,(8m - n)$

(7) $(2x - y + 6)\,(2x + y - 4)$ (8) $(2x^2 - 3)^2$

3.(A) (1) ₹ 2250 to Munnabhai and ₹ 1500 to Sultansingh.

(2) Nileshbhai would have to pay ₹ 87.5 more.

(B) (1) 2 (2) $-1\frac{2}{9}$ (3) $\frac{2}{5}$ (4) 1

5. (1) Length 25 meter   (2) 3 (3) $21, 24$ (4) $\frac{3}{10}$

◆

8 Introduction to Computers-3

● **openoffice.org – Presentation (Impress) :**

We know the primary information of presentation in standard 7. We got information to create a slide, change a slide and change different layout of a slide.
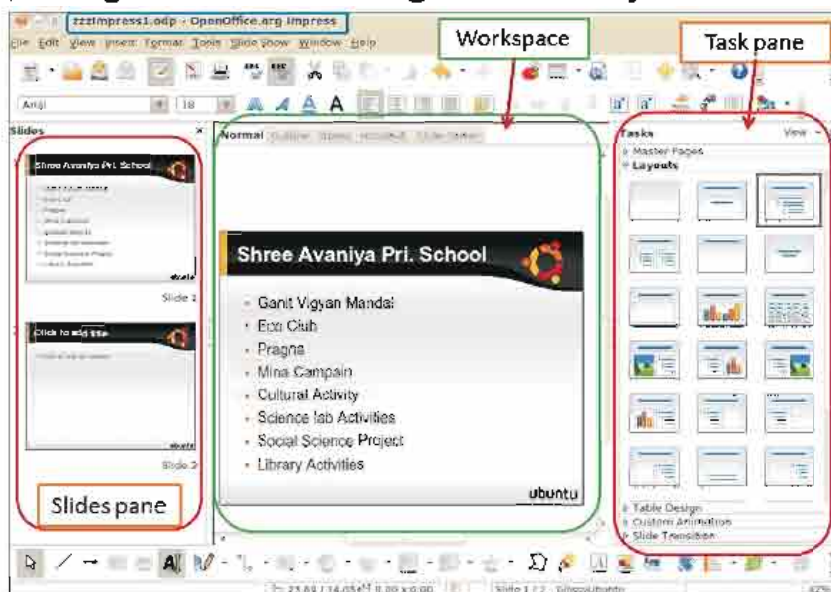
8.1 Presentation

> **Activity :** Under to your teacher's guidance prepare a presentation which is running in your school (As shown in figure).

So, dear students, you have prepared a simple presentation with the help of your teacher. Let's now know about some other tools to make this presentation more effective.

In std. VII, we knew about two options of master pages and slide layouts present in Task-Pane. Let's now make this slide more dynamic and attractive with the help of third option of the same Task-Pane.

Custom animation can be opened by two ways :

(1) Select this option from Slide Show Menu as shown in figure.

(2) Select this option from Task-Pane.
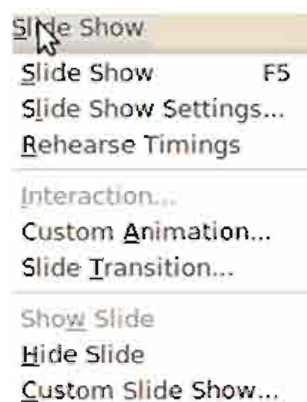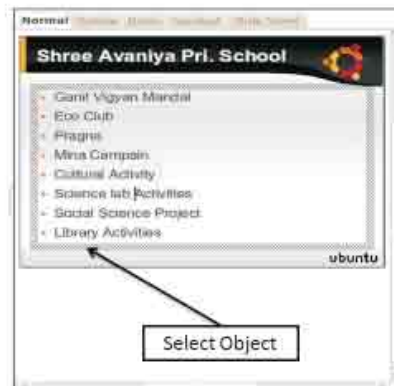
Select Object

8.2 Select Object for Animation

Select any one object from the selected slide. After that click on "Add" button present in Custom Animation Tab as shown in figure.
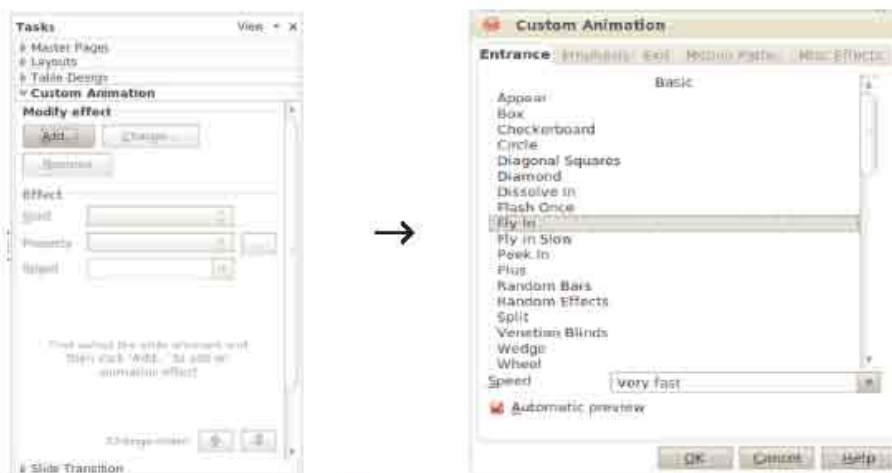
8.3 Custom Animation

Now dialogue box will be opened showing the different effects of Custom Animation according to the figure. By selecting any one effect, speed of effect can be set. Therefore, animation effects can be given to the selected object. Similarly to all objects of all slides, Animation effects of different types can be given.

The details and sequence of animation according to figure can be seen after setting the animation of object. If animation is to 'Remove' or 'Change' then by selecting this effect, both process can be done. By the given option of Change Order, the sequence of animation can be done up and down. By 'Play' the animation preview of slide can be seen.
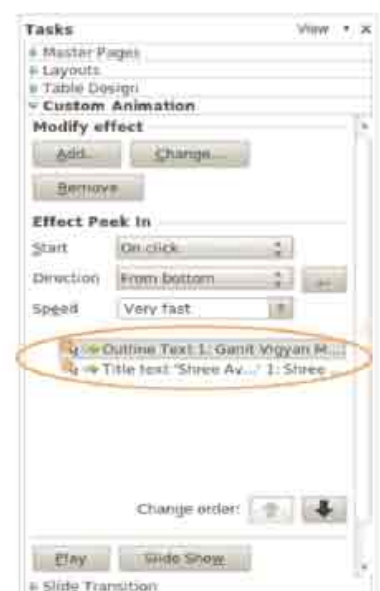
8.4 Modify Effect

● **Slide Transition :**

After completing one slide, where to put second slide by which graphical effect, the speed of effect and trasition is shown in figure.

By selecting any one effect as shown in slide transition tab of Task-Pane and "Apply" button, it can be done on whole presentation. Here you can set the time also.

● **Slide Show :**

Any one option can be selected out of the following to start Slide Show :

(1) Select option of Slide Show → Slide Show

(2) 'Click' F5 key.

(3) 'Click' on the Slide Show button.

8.5 Slide Transition

Activity : Make the slides of content of any one lesson and give effect to it.

● **Openoffice.org (Spreadsheet) calc. :**

Spreadsheet is a simple electronic form which has horizontal rows and vertical columns. With the help of this form the data can be shown properly and stored. For example, attendance sheet of school, Salary sheet, result sheet etc. are formed in this format. Spreadsheet includes workbook, worksheet, row, column and shell.

According to figure 8.6, In software of openoffice Menubar, Toolbar and Status bar can be seen. In work-sheet there are 1024 columns and 1048576 rows.
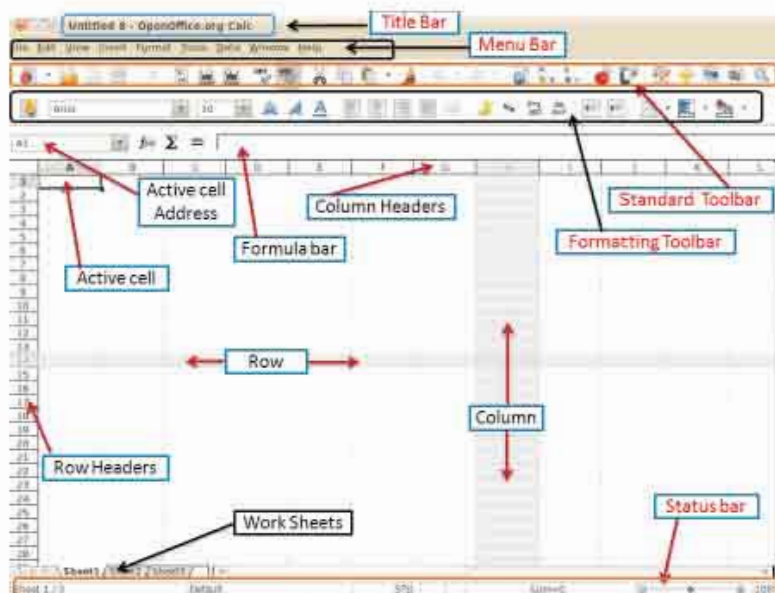
8.6 Untitled Worksheet

Think : How many squares are there in a worksheet made with 1024 columns and 1048576 rows.

● In one workbook of spreadsheet there are three worksheet by default. You can increase more.

● Here name of columns are like A, B, C, D, ... and names of rows are as 1, 2, 3, 4, ... can be seen in the spreadsheet.

● Here each cell has its own cell address. The address of selected cell is seen in active cell address of Formula bar. As shown in figure active cell with black border will be seen Highlighted.
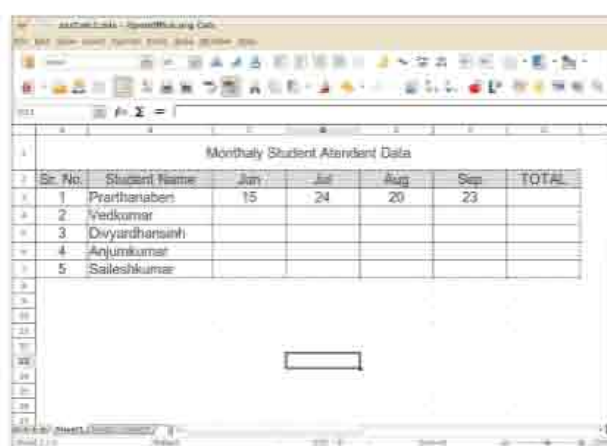
Standard Toolbar

Formatting Toolbar
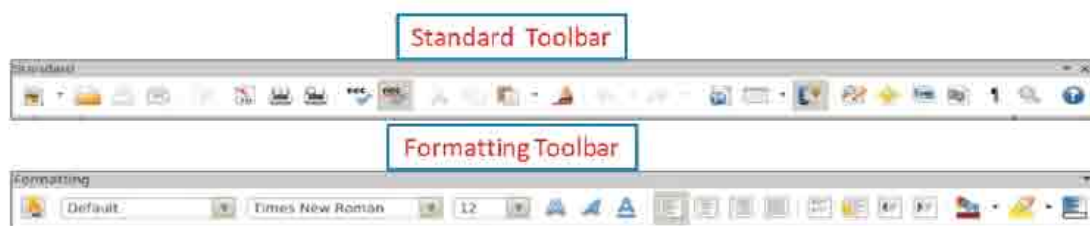
8.7 Toolbar

● **To prepare attendance sheet with the help of spreadsheet :**

In the following table, fill the data of attendance of yours and your friends in your notebook under the guidance of your teacher.

Monthly Attendance Report

Sr. No.	Student Name	Jun	Jul	Aug	Sep	TOTAL
1	Prarthanaben	15	24	20	23	
2	Vedkumar					
3	Divyardhansinh					
4	Anjumkumar					
5	Saileshkumar					

Make this tab in spreadsheet software.

8.8 Inserting Data

Above figure will be seen as data entry and completed in Calc. program.
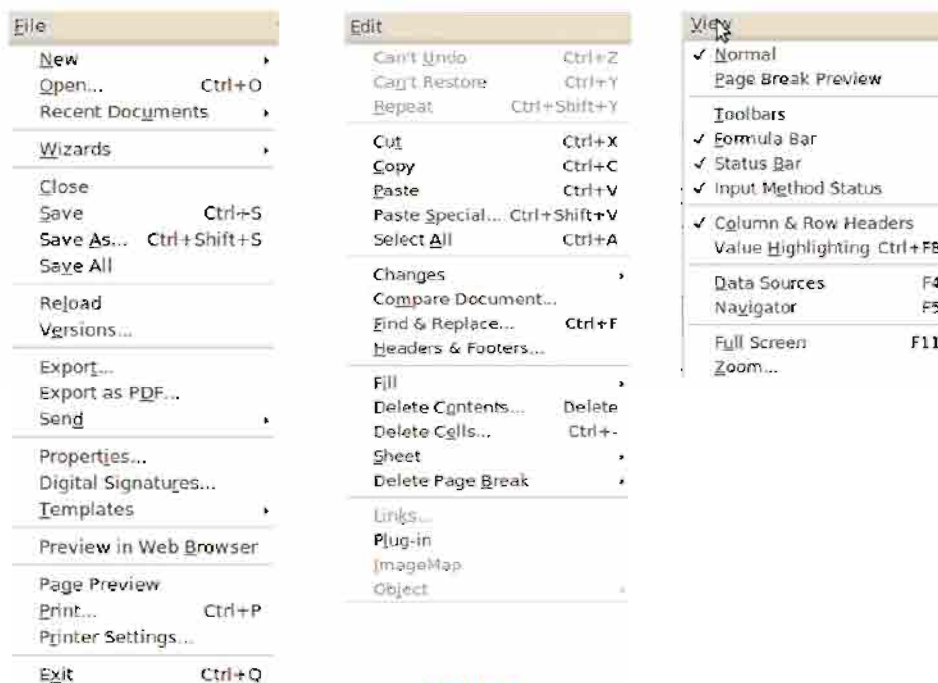
● **Save / Save as :**

With the help of save option, save the calc file in File → Save as or Standard toolbar. Save this file by '.ods' extension.
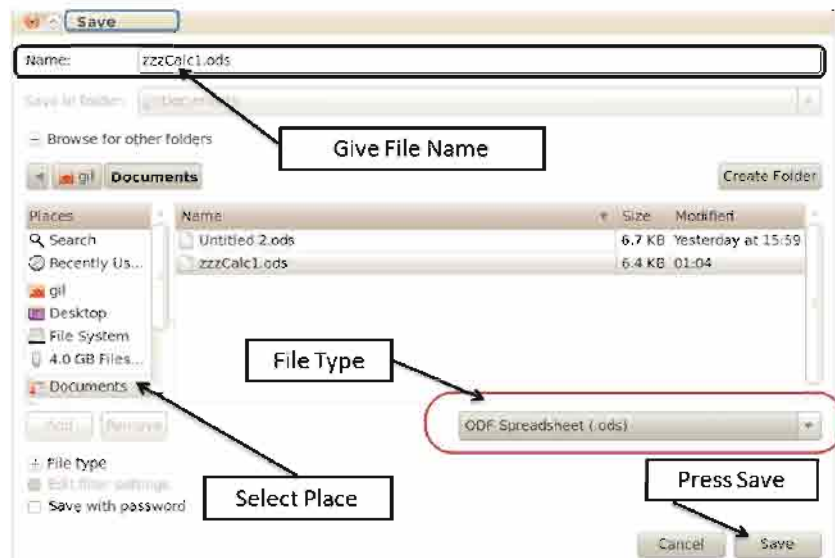
8.9 Save Spreadsheet

● **File / Edit / View :**

Similarly as shown in figure, there are File, Edit and View Menu as in other programme of Openoffice.org.

8.10 Menu

● **Insert Menu :**

   Cell : To add new cell in existing worksheet.

   Row / Column : To add new plane row or column in existing worksheet.

   Sheet : For adding new worksheet.

● **Applying Formula :**

   8.11 Insert Menu

   Dear students, you will have to use formula to get total attendance of four months in the square of total attendance in table of attendance sheet. The simple meaning of formula is mathematical formulae. As to solve a sum of mathematics the values are substituted in different formulae. Similarly we will have to make formula by using address of different cells in this program.

8.12 Applying Formula

   Observe the figure 8.12. "Click" in the square of total of students of student number 5. To apply formula first of all put = (equal to) sign, then cell address 'C7', of attendance of June month of this child, after that + (addition) sign, then cell address 'D7' of attendance of July month. Similarly putting cell address E7 and F7 and taking (+) sign between them we will get the formula. As "Enterkey" is pressed, automatically sum will be done. Similarly the sum of attendance of other student can be done.

   Note : Generally as well as cell is selected, data present in it are seen in formula bar. But in the cell where formula is inserted, as we select the cell formula will be seen in the formula bar.

Activity : Prepare table for the calculation for simple interest.

Sr. No.	Name	Principle (Rs.)	Rate of Intrest (%)	Number of Year	Simple Intrest I =(PRN)/100
1	Nilesh	12000	5	2	
2					
3					
4					
5					

According to formula of simple interest $I = \dfrac{PRN}{100}$ apply it to each column.

Note : Prepare formulae with the use of equations of Area, Volume, Perimeter etc.

● **Functions :**

With the option of Insert → Function wizard will be open for different functions and we can see different functions of different categories in calc. For example, in the category of mathematics many functions related to Sum, Round, Average are given.

8.13 Applying Formula

According to figure 8.13 click and insert G5 cell = SUM (C5, D5, E5, F5) and apply enter key. This sum function will give the details of addition (sum).

8.14 Formula bar

The above calculation addition can also be applied as shown in figure 8.14.

Teacher Activity : Give the details to the students from the Function Wizard. Give the practical knowledge.
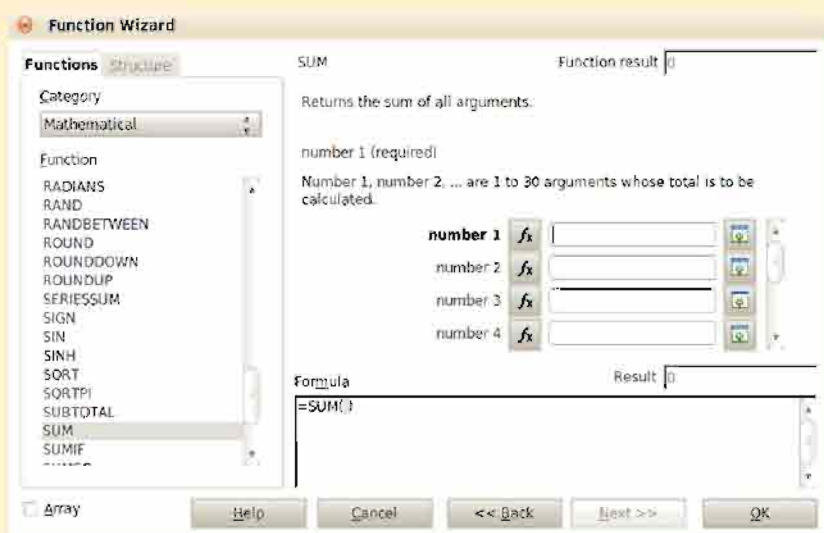
8.15 Function Wizard

● **Charts :**

Graphs are used to present the complicated matters in a simple way in the form of figure. There are many options available in calc program to construct and to make changes in graph. With this we can draw various types of graphs such as Column, Bar, Pie, Area and Line. Let's prepare column chart from the sample of attendance sheet.

Selection for Chart (B2 to F7)

|	A	B	C	D	E	F
1		Monthaly Student Atendent Data				
2	Sr. No.	Student Name	Jun	Jul	Aug	Sep
3	1	Prarthanaben	15	24	20	23
4	2	Vedkumar	14	22	19	24
5	3	Divyardhansinh	15	23	17	24
6	4	Anjumkumar	13	20	15	20
7	5	Saileshkumar	10	21	16	22

8.16 Select Data

Follow the following steps to prepare the Graph :

● We have to prepare a chart giving details about the student's name and attendance of four months. Select region from student name cell B2 to student-5 attendance of september cell F7 (B2 : F7).

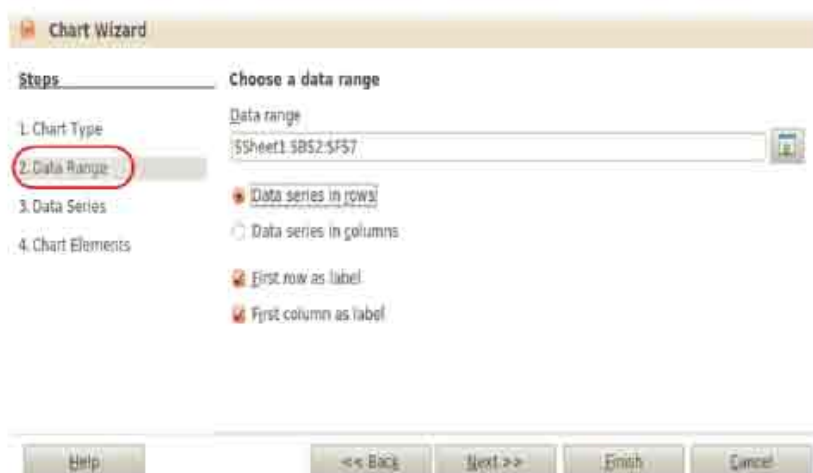● Select options chart from Insert → Chart or Toolbar.

● **Step-1 :** Select column from the various options of chart type.
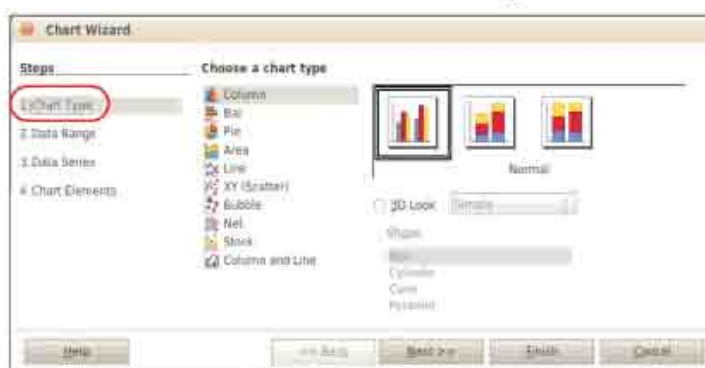
8.17 Chart Wizard Step-1

● **Step-2 :** This option is for to select data range for the chart. But initially we have already selected this range, so it will be printed on screen.

8.18 Chart Wizard Step-2

● **Step-3 :** Now in this option range for name and categories options are already selected. Therefore, go for the next step.
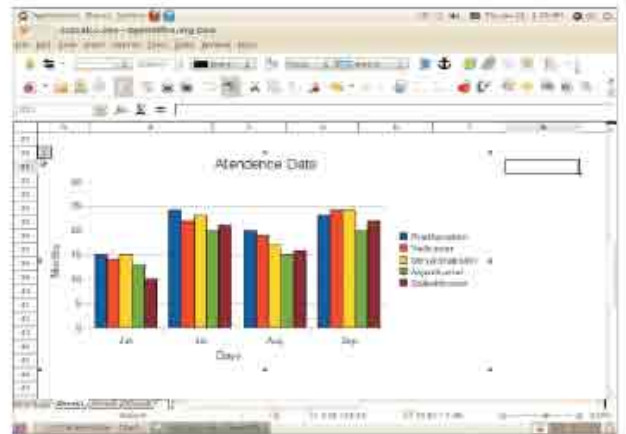

8.19 Chart Wizard Step-3

- **Step-4 :** In this option click 'Finish' after filling simple details like title of chart, text in X-axis and text in Y-axis.

8.20 Chart Wizard Step-4

8.21 Chart

You will see the final chart as shown in figure 8.21, whose size can be modified.
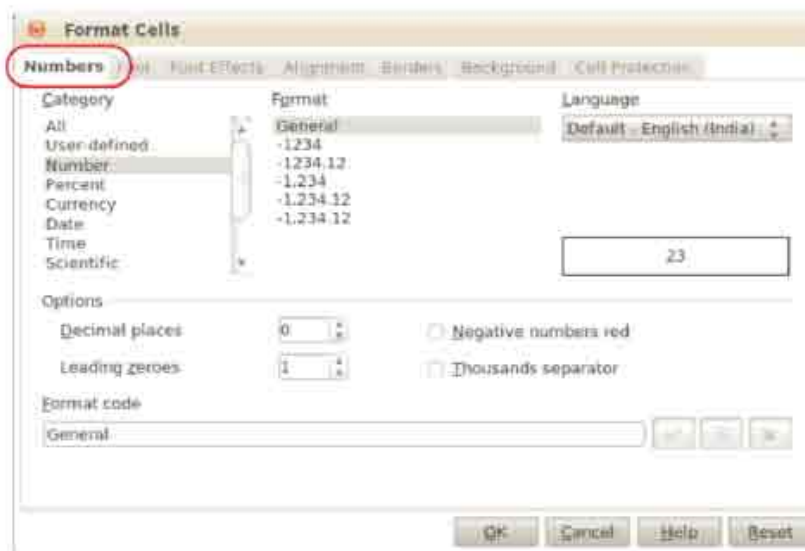
- ### Format Cells :

Select the Rows, Columns or Cells which are to be formatted, click on cells of format menu, then dialogue box as shown in figure will open.

- ### Numbers :

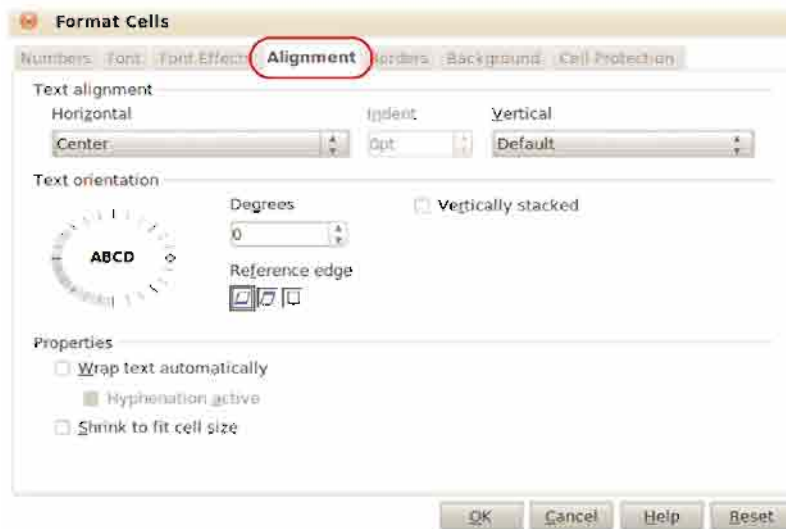You can set the format and information of the selected cell with this Tab.

8.22 Format Menu

8.23 Format Cells
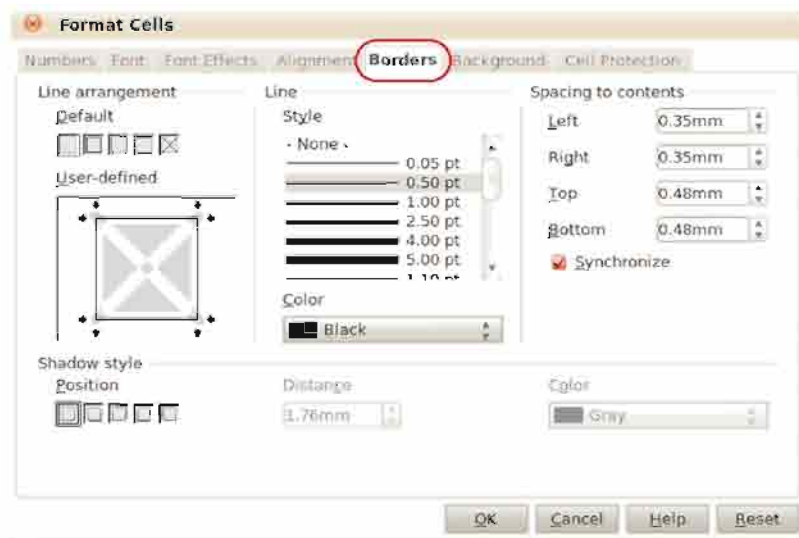
● **Alignment :**

The alignment and orientation of the text can be set with this tab.

8.24 Allignment

● **Borders :**

The style of border, colour, by default and user defined of the cell can be set with this tab.

8.25 Borders

● **Data Menu :**

Sort : The sort option is used to arrange the data in ascending or descending order. As we click any cell in attendance sheet prepared in calc, click 'Data → Sort', the dialogue box as shown in figure will open. By selecting the proper options, the informations will be sorted as per columnwise.
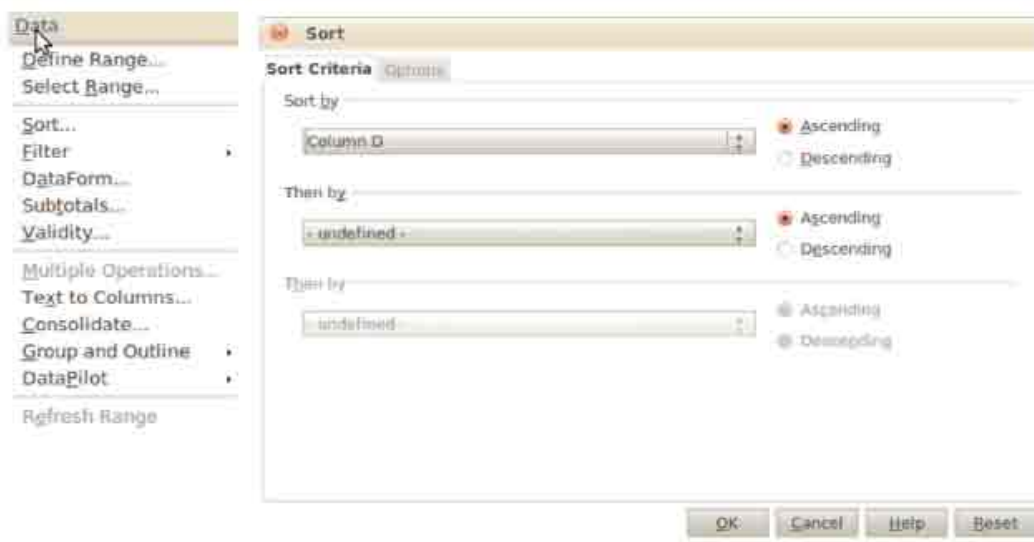
(8.26 Data Sort)

● **Filter :**

This facility is used to sort the required data from the given data. The data column is displayed with ▼ button by clicking 'Data → Filter' after clicking heading column. By clicking this button the necessary options for column of filter will be seen.

Sr. No	Student Name	Jun	Jul	Aug	Sep
1	Prarthanaben	15	24	20	23
2	Vedkumar	14	22	19	24
3	Divyardhansinh	15	23	17	24
4	Anjumkumar	13	20	15	20
5	Saileshkumar	10	21	16	22

Monthly Student Attendance Data

(8.27 Data Filter)

Activity : To use formula and various functions of calc to prepare worksheet based on the result data.

♦ ♦ ♦