A challenge!

On a centimeter squared paper, make as many rectangles as you can, such that the area of the rectangle is 16 sq cm (consider only natural number lengths).

(a) Which rectangle has the greatest perimeter?

(b) Which rectangle has the least perimeter?

If you take a rectangle of area 24 sq cm, what will be your answers?

Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter? With the least perimeter? Give example and reason.

What have we discussed?

1. Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.
2. (a) Perimeter of a rectangle = 2 × (length + breadth)

   (b) Perimeter of a square = 4 × length of its side

   (c) Perimeter of an equilateral triangle=3 × length of a side
3. Figures in which all sides and angles are equal are called regular closed figures.
4. The amount of surface enclosed by a closed figure is called its area.
5. To calculate the area of a figure using a squared paper, the following conventions are adopted:

   a. Ignore portions of the area that are less than half a square.

   b. If more than half a square is in a region. Count it as one square.

   c. If exactly half the square is counted, take its area as $\frac{1}{2}$ sq. units
6. (a) Area of a rectangle = length × breadth

   (b) Area of a square = side × side

9 Algebra

9.1 Introduction

Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

9.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 9.1 (a). Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 9.1 (b)].

Fig. 9.1

Then Ameena adds one more L and this goes on as shown by the dots in Fig 9.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.

While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = $2 \times$ number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$.

Note that $2n$ is same as $2 \times n$.

Ameena says to her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required $= 2 \times 1 = 2$

For $n = 2$, the number of matchsticks required $= 2 \times 2 = 4$

For $n = 3$, the number of matchsticks required $= 2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".

Do you agree with Sarita?

9.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was:

Number of matchsticks required = 2*n*

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4,....

Let us look at Table 1 once again. In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4,... . We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

9.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 9.2(a).

(a) (b) (c) (d)

Fig. 9.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....

Can you complete the entries left blank in the table?

Sarita comes up with the rule:

Number of matchsticks required = 3*n*

She has used the letter n for the number of Cs; n is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita?

Remember $3n$ is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 9.3(a).

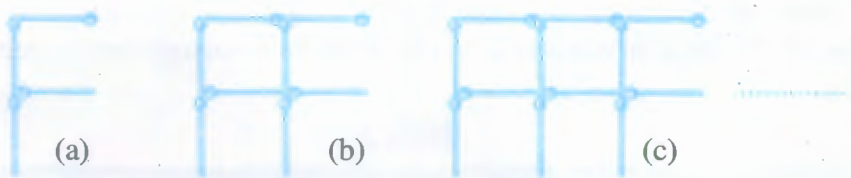


Fig. 9.3

Can you now write the rule for making patterns of F?

In the examples on matchstick patterns, we use the variable n to give us the rule for the number of sticks required to make a pattern. An important use of variables in mathematics is in writing general rules like those we have found.

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U ( ), V ( ), triangle ( ), square ( ) etc. Choose any five and write the rules for making matchstick patterns with them.

9.5 More Examples of Variables

We have used the letter n to show a variable. Raju asks, "Why not m"? There is nothing special about n, any letter can be used.

Let us now consider variables in a more familiar situation.

One may use any letter as m, l, p, x, y, z etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But n in the examples we have looked at above is a variable. It takes on various values 1, 2, 3, 4,.....

Students went to buy notebooks from the school bookstore. Price of one notebook is Rs 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	....	m	....
Total cost in rupees	5	10	15	20	25	....	$5m$	....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule:

The total cost in rupees $= 5 \times$ number of note books required $= 5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry Rs 5×5 or Rs 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 9.4). How many children can there be in the drill?

Fig. 9.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2×10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here r is a variable which stands for the number of rows and so take on values on 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

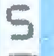
Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles. But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,.... ,10,..... ,20,..... ,30,..... . Using x, we write Sarita's marbles $= x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

> The expression $(x + 10)$ cannot be simplified further.Do not confuse $x + 10$ with $10x$, they are different.In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.We may check this for some values of x. For example,If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x - 3)$. The expression $(x - 3)$ is read as x minus three. As you would expect, when x is 12, $(x - 3)$ is 9 and when x is 15, $(x - 3)$ is 12.

EXERCISE 9.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.
   - a) A matchstick pattern of letter **T** as 
   - b) A matchstick pattern of letter **Z** as 
   - c) A matchstick pattern of letter **U** as 
   - d) A matchstick pattern of letter **V** as 
   - e) A matchstick pattern of letter **E** as 
   - f) A matchstick pattern of letter **S** as 
   - g) A matchstick pattern of letter **A** as 
2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)
4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)
5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)
7. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

9.6 Use of Variables in Common Rules

Let us now see how certain common rules in mathematics that we have already learnt are expressed using variables.

Rules from geometry

We have already learnt about the perimeter of a square and of a rectangle in the chapter on Mensuration. Here, we go back to them to write them in the form of a rule.

1. Perimeter of a square We know that perimeter of any polygon (a closed figure made up of 3 or more line segments) is the sum of the lengths of its sides. A square has 4 sides and they are equal in length (Fig 9.6). Therefore,

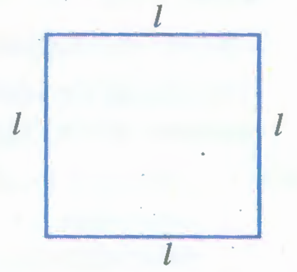


Fig 9.6

The perimeter of a square = Sum of the lengths of the sides of the square = $l \times l \times l \times l = 4l$.

Thus, we get the rule for the perimeter of a square. The use of the variable

l allows us to write the general rule in a way that is concise and easy to remember.

We may take the perimeter also to be represented by a variable, say p. Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, $p = 4l$

2. Perimeter of a rectangle We know that a rectangle has four sides. For example, the rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length. Thus, in the rectangle ABCD, let us denote by l, the length of the sides AB or CD and, by b, the length of the sides AD or BC. Therefore,

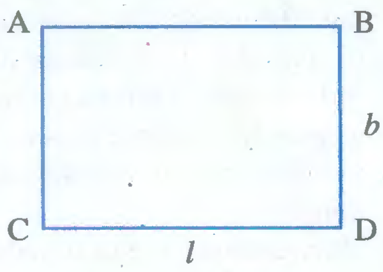


Fig 9.7

Perimeter of a rectangle = length of AB + length of BC

$$\begin{aligned} &+ \text{length of CD} + \text{length of AD} \\ &= l + b + l + b \\ &= (l + l) + (b + b) \\ &= 2l + 2b \end{aligned}$$

The rule, therefore, is

The perimeter of a rectangle = $2l + 2b$

where, l and b are respectively the length and breadth of the rectangle.

Discuss what happens if $l = b$.

If we denote the perimeter of the rectangle by the variable p, the rule for perimeter of a rectangle becomes $p = 2l + 2b$

Note : Here, both l and b are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

In your studies of geometry you will come across several rules and formulas dealing with perimeters and areas of plane figures, and surface areas and volumes of three-dimensional figures. Also, you may obtain formulas for the sum of internal angles of a polygon, the number of diagonals of a polygon and so on. The concept of variables which you have learnt will prove very useful in writing all such general rules and formulas.

Rules from arithmetic

3. Commutativity of addition of two numbers:

We know that

$4 + 3 = 7$ and $3 + 4 = 7$

i.e. $4 + 3 = 3 + 4$

As we have seen in the chapter on whole numbers, this is true for any two numbers. This property of numbers is known as the **commutativity of addition of numbers.** Commuting means interchanging. Commuting the order of numbers in addition does not change the sum. The use of variables allows us to express the generality of this property in a concise way. Let a and b be two variables which can take any number value.

Then, $\boldsymbol{a + b = b + a}$

Once we write the rule this way, all special cases are included in it. If $a = 4$ and $b = 3$, we get $4 + 3 = 3 + 4$. If $a = 37$ and $b = 73$, we get $37 + 73 = 73 + 37$ and so on.

4. Commutativity of multiplication of two numbers

We have seen in the chapter on whole numbers that for multiplication of two numbers, the order of the two numbers being multiplied does not matter. For example,

$4 \times 3 = 12$, $3 \times 4 = 12$

Hence, $4 \times 3 = 3 \times 4$

This property of numbers is known as **commutativity of multiplication of numbers.** Commuting (interchanging) the order of numbers in multiplication does not change the product. Using variables a and b as in the case of addition, we can express the commutativity of multiplication of two numbers as $\boldsymbol{a \times b = b \times a}$

Note that a and b can take any number value. They are variables. All the special cases like $4 \times 3 = 3 \times 4$ or $37 \times 73 = 73 \times 37$ follow from the general rule.

5. **Distributivity of numbers:**

Suppose we are asked to calculate 7×38. We obviously do not know the table of 38. So, we do the following:

$7 \times 38 = 7 \times (30 + 8)$
$= 7 \times 30 + 7 \times 8$
$= 210 + 56$
$= 266$

Here we assumed $7 \times 38 = 7 \times 30 + 7 \times 8$, i.e. we assumed that multiplication by 7 can be distributed over the addition of 30 and 8. This is always true for any three numbers like 7, 30 and 8. This property is known as distributivity of multiplication over addition of numbers. Let us take another example:
We know $9 \times 13 = 117$, $9 \times 11 = 99$ and $9 \times 2 = 18$;

Also $99 + 18 = 117$ and $11 + 2 = 13$.

We may therefore write

$9 \times (11 + 2) = 9 \times 13$
$= 117$
$= 99 + 18$
$= 9 \times 11 + 9 \times 2$

or $9 \times (11 + 2) = 9 \times 11 + 9 \times 2$

Thus, the distributivity property is verified for the numbers 9, 11 and 2. By using variables, we can write this property of numbers also in a general and concise way. Let *a*, *b* and *c* be three variables, each of which can take any number value. Then,

$$\boldsymbol{a \times (b + c) = a \times b + a \times c}$$

Properties of numbers are fascinating. You will learn many of them in your study of numbers this year and in your later study of mathematics. Use of variables allows us to express these properties in a very general and concise way. One more property of numbers is given in question 5 of Exercise 9.2. Try to find more such properties of numbers and learn to express them using variables.

EXERCISE 9.2

1. The side of an equilateral triangle is shown by l. Express the perimeter of the equilateral triangle using l.
2. The side of a regular hexagon (Fig 9.8) is denoted by l. Express the perimeter of the hexagon using l. (**Hint:** A regular hexagon has all its six sides equal in length.)

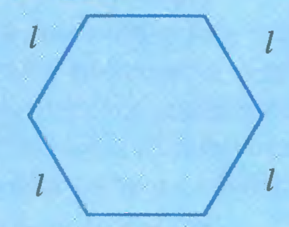


Fig. 9.8

3. Consider the sum of three numbers 14, 27 and 13, we may do the sum in two ways:
   a. We may first add 14 and 27 to get 41 and then add 13 to it to get the total sum 54 or
   b. We may add 27 and 13 to get 40 and then add it to14 to get the total sum 54.Thus,

   $(14 + 27) + 13 = 14 + (27 + 13)$

   This can be done for any three numbers. This property is known as the **associativity of addition of numbers**. Express this property which we have already studied in the chapter on Whole Numbers, in a general way, by using variables a, b and c.

9.7 Expressions with Variables

Recall that in arithmetic we have come across expressions like $(2 \times 10) + 3$, $3 \times 100 + (2 \times 10) + 4$ etc. These expressions are formed from numbers like 2, 3, 4, 10, 100 and so on. To form expressions we use all the four number operations of addition, subtraction, multiplication and division. For example, to form $(2 \times 10) + 3$, we have multiplied 2 by 10 and then added 3 to the product. Examples of some of the other arithmetic expressions are:

$3 + (4 \times 5)$,	$-3 \times 4 + 5$,
$8 - 7 \times 2$,	$14 - (5 - 2)$,
$6 \times 2 - 5$,	$5 \times 7 - 3 \times 4$,
$7 + 8 \times 2$	$5 \times 7 - (3 \times 4 - 7)$ etc.

Expressions can be formed from variables too. In fact, we already have seen expressions with variables, for example: $2n$, $5m$, $x + 10$, $x - 3$ etc. These expressions with variables are obtained by operations of addition, subtraction, multiplication and division on variables. For example, the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

We know that variables can take different values; they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication and division can be done on them.

One important point must be noted regarding the expressions containing variables. A number expression like $(4 \times 3) + 5$ can be immediately evaluated as

$(4 \times 3) + 5 = 12 + 5 = 17$

But an expression like $(4x + 5)$, which contains the variable x, cannot be evaluated. Only if x is given some value, an expression like $(4x + 5)$ can be evaluated. For example,

when $x = 3$, $4x + 5 = (4 \times 3) + 5 = 17$ as found above.

In the following exercise we shall look at how few simple expressions have been found.

Expression	How formed?
(a) $y + 5$	5 added to y
(b) $t - 7$	7 subtracted from t
(c) $10\,a$	a multiplied by 10
(d) $\frac{x}{3}$	x divided by 3
(e) $-5\,q$	q multiplied by -5
(f) $3\,x + 2$	first x multiplied by 3, then 2 added to the product
(g) $2\,y - 5$	first y multiplied by 2, then 5 subtracted from the product

Write 10 other such simple expressions and tell how they have been formed.

We should also be able to write an expression through given instruction about how to form it. Look at the following example :

Give expressions for the following :

(a)	12 subtracted from z	$z - 12$
(b)	25 added to r	$r + 25$
(c)	p multiplied by 16	$16p$
(d)	y divided by 8	$\frac{y}{8}$
(e)	m multiplied by –9	$-9m$
(f)	y multiplied by 10 and then 7 added to the product	$10y + 7$
(g)	n multiplied by 2 and subtracted from the product	$2n - 1$

Sarita and Ameena decide to play a game of expressions. They take the variable x and the number 3 and see how many expressions they can make. The condition is that they should use not more than one out of the four number operations and every expression must have x in it. Can you help them?

Is $(3x + 5)$ allowed?
Is $(3x + 3)$ allowed?

Sarita thinks of $(x + 3)$.

Then, Ameena comes up with $(x - 3)$.

Next she suggests $3x$. Sarita then immediately makes $\frac{x}{3}$.

Under the given condition are these the only four expressions that they can get?

Next they try combinations of y, 3 and 5. The condition is that they should use not more than one operation of addition or subtraction and one operation of multiplication or division. Every expression must have y in it. Check, if their answers are right.

$y + 5$, $y + 3$, $y - 5$, $y - 3$,
$3y$, $5y$, $\frac{y}{3}$, $\frac{y}{5}$, $3y + 5$, $3y - 5$, $5y + 3$, $5y - 3$

Is $\left(\frac{y}{3} + 5\right)$ allowed ?

Is $(y + 8)$ allowed?
Is $15y$ is allowed

Can you make some more expressions?

EXERCISE 9.3

1. Make up as many expressions with numbers (no variables) as you can from three numbers 5, 7 and 8. Every number should be used not more than once. Use only addition, subtraction and multiplication.

   (**Hint:** Three possible expressions are $5 + (8 - 7)$, $5 - (8 - 7)$, $(5 \times 8) + 7$; make the other expressions.)

2. Which out of the following are expressions with numbers only?
   (a) $y + 3$
   (b) $(7 \times 20) - 8z$
   (c) $5\ (21 - 7) + 7 \times 2$
   (d) 5
   (e) $3x$
   (f) $5 - 5n$
   (g) $(7 \times 20) - (5 \times 10) - 45 + p$

3. Identify the operations (addition, subtraction, division, multiplication) in forming the following expressions and tell how the expressions have been formed.
   (a) $z + 1$, $z - 1$, $y + 17$, $y - 17$
   (b) $17y$, $\frac{y}{17}$, $5z$
   (c) $2y + 17$, $2y - 17$
   (d) $7m$, $-7m + 3$, $-7m - 3$

4. Give expressions in the following cases.
   (a) 7 added to p
   (b) 7 subtracted from p
   (c) p multiplied by 7
   (d) p divided by 7
   (e) 7 subtracted from $-m$
   (f) $-p$ multiplied by 5
   (g) $-p$ divided by 5
   (h) p multiplied by -5

5. Give expressions in the following cases.
   (a) 11 added to $2m$
   (b) 11 subtracted from $2m$
   (c) 5 times y to which 3 is added
   (d) 5 times y from which 3 is subtracted
   (e) y is multiplied by -8
   (f) y is multiplied by -8 and then 5 is added to the result
   (g) y is multiplied by 5 and the result is subtracted from 16
   (h) y is multiplied by -5 and the result is added to 16.

9.8 Using Expressions Practically

We have already come across practical situations in which expressions are useful. Let us remember some of them.

	Situation (described in ordinary language)	Variable	Statements using expressions
1.	Sarita has 10 more marbles than Ameena.	Let Ameena have x marbles.	Sarita has $(x + 10)$ marbles).
2.	Balu is 3 years younger than Raju	Let Raju's age be x years	Balu's age is $(x - 3)$ years)
3.	Bikash is twice as old as Raju.	Let Raju's age be x years	Bikash's age is $2x$ years.

Let us look at some other such situations.

	Situation (described in ordinary language)	Variable	Statements using expressions
5.	How old will Susan be 5 years from now	Let y be Susan's present age in years.	Five years from now Susan will be (y + 5) years old.
6.	How old was Susan 4 years ago?	Let y be Susan's present age in years.	Four years ago Susan was (y – 4) years old.
7.	Price of wheat per kg is Rs less than price of rice per kg.	Let price of rice per kg be Rs p.	Price of wheat per kg is Rs (p – 5)
8.	Price of oil per litre is 5 times the price of rice per kg.	Let price of rice per kg be Rs p.	Price of oil per litre is Rs 5p.
9.	The speed of a bus is 10km/hour more than the speed of a truck going on the same road.	Let the speed of the truck be y km/hour.	The speed of the bus is (y + 5) km/ hour.

Try to find more such situations. You will realise that there are many statements in ordinary language, which you will be able to change to statements using expressions with variables. In the next section, we shall see how we use these statements using expressions for our purpose.

EXERCISE 9.4

1. Answer the following:

(a) Take Sarita's present age to be y years

a. What will be her age 5 years from now?

b. What was her age 3 years back?

c. Sarita's grandfather is 6 times her age. What is the age of her grandfather?

d. Grandmother is 2 years younger than grandfather. What is grandmother's age?

e. Sarita's father's age is 5 years more than 3 times Sarita's age. What is her father's age?

(b) The length of a rectangular hall is 4 meters less than 3 times the breadth of the hall. What is the length, if the breadth is b meters?

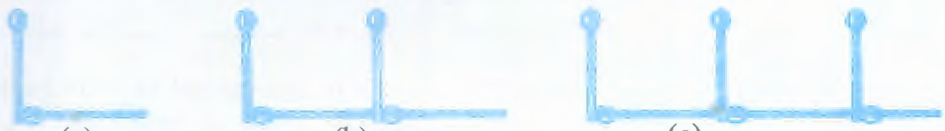
(c) A rectangular box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and the breadth of the box in terms of the height.

(d) Meena, Beena and Leena are climbing the steps to the hill top. Meena is at step s, Beena is 8 steps ahead and Leena 7 steps behind. Where are Beena and Meena? The total number of steps to the hill top is 10 less than 4 times what Meena has reached. Express the total number of steps using s.

(e) A bus travels at v km per hour. It is going from Daspur to Beespur. After the bus has travelled 5 hours, Beespur is still 20 km away. What is the distance from Daspur to Beespur? Express it using v.

9.9 What is an Equation?

Let us recall the matchstick pattern of the letter L given in Fig 9.1. For our convenience, we have the Fig . redrawn here.



(a) (b) (c)

The number of matchsticks required for different number of Ls formed was given in Table 1. We repeat the table here.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

We know that the number of matchsticks required is given by the rule, $2n$, if n is taken to be the number of Ls formed.

Appu always thinks differently. He asks, "We know how to find the number of matchsticks required for a given number of Ls. What about the other way round? How does one find the number of Ls formed, given the number of matchsticks"?

We ask ourselves a definite question.

How many Ls are formed if the number of matchsticks given is 10?

This means we have to find the number of Ls (i.e. n), given the number of matchsticks

$2n = 10$ (1)

Here, we have a condition to be satisfied by the variable n. This condition is an example of an equation.

Our question can be answered by looking at Table 1. Look at various values of n. If $n = 1$, the number of matchsticks is 2. Clearly, the condition is not satisfied, because 2 is not 10. We go on checking.

n	$2n$	Condition satisfied? Yes/No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

We find that only if $n = 5$, the condition, i.e. the equation $2n = 10$ is satisfied. For any value of n other than 5, the equation is not satisfied.

Let us look at another equation.

Balu is 3 years younger than Raju. Taking Raju's age to be x years, Balu's age is $(x - 3)$ years. Suppose, Balu is 11 years old. Then, let us see how our method gives Raju's age.

We have Balu's age, $x - 3 = 11$ (2)

This is an equation in the variable x. We shall prepare a table of values of $(x - 3)$ for various values of x.

x	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$x - 3$	0	1	–	–	–	–	–	–	–	9	10	11	12	13	–	–

Complete the entries which are left blank. From the table, we find that only for $x = 14$, the condition $x - 3 = 11$ is satisfied. For other values, for example for $x = 16$ or for $x = 12$, the condition is not satisfied. Raju's age, therefore, is 14 years.

To summarise, **an equation is a condition on a variable. It is satisfied only for a definite value of the variable.** For example, the equation $2n = 10$ is satisfied only by the value 5 of the variable n. Similarly, the equation $x - 3 = 11$ is satisfied only by the value 14 of the variable x.

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example: The statement $2n$ is greater than 10, i.e. $2n > 10$ is not an equation. Similarly, the statement $2n$ is smaller than 10 i.e. $2n < 10$ is not an equation. Also, the statements

$(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Now, let us consider $8 - 3 = 5$

There is an equal sign between the LHS and RHS. Neither of the two sides contains a variable. Both contain numbers. We may call this a numerical equation. Usually, the word equation is used only for equations with one or more variables.

Let us do an exercise.

State which of the following are equations with a variable. In the case of equations with a variable, identify the variable.

(a) $x + 20 = 70$	(Yes, x)
(b) $8 \times 3 = 24$	(No, this a numerical equation)
(c) $2p > 30$	(No)
(d) $n - 4 = 100$	(Yes, n)
(e) $20b = 80$	(Yes, b)
(f) $\frac{y}{8} < 50$	(No)

Following are some examples of an equation. (The variable in the equation is also identified).

Fill in the blanks as required :

$x + 10 = 30$	(variable x)	(3)
$p - 3 = 7$	(variable p)	(4)
$3n = 21$	(variable______)	(5)
$\frac{t}{5} = 4$	(variable ______)	(6)
$2l + 3 = 7$	(variable ______)	(7)
$2m - 3 = 5$	(variable ______)	(8)

9.10 Solution of an Equation

We saw in the earlier section that the equation

$$2n = 10 \qquad (1)$$

was satisfied by $n = 5$. No other value of n satisfies the equation. **The value**

of the variable in an equation which satisfies the equation is called a solution to the equation. Thus, $n = 5$ is a solution to the equation $2\ n = 10$.

Note, $n = 6$ is not a solution to the equation $2n = 10$; because for $n = 6$, $2n = 2 \times 6 = 12$ and not 10.

Also, $n = 4$ is not a solution. Tell, why not?

Let us take the equation $x - 3 = 11$ (2)

This equation is satisfied by $x = 14$, because for $x = 14$,

LHS of the equation = 14 – 3 = 11 = RHS

It is not satisfied by $x = 16$, because for $x = 16$,

LHS of the equation = 16 – 3 = 13, which is not equal to RHS.

Thus, $x = 14$ is a solution to the equation $x - 3 = 11$ and $x = 16$ is not a solution to the equation. Also, $x = 12$ is not a solution to the equation. Explain, why not?

Now complete the entries in the following table and explain why your answer is Yes/No.

Equation	Value of the variable	Solution (Yes/No)
1. $x + 10 = 30$	$x = 10$	No
2. $x + 10 = 30$	$x = 30$	No
3. $x + 10 = 30$	$x = 20$	Yes
4. $p - 3 = 7$	$p = 5$	No
5. $p - 3 = 7$	$p = 15$	–
6. $p - 3 = 7$	$p = 10$	–
7. $3n = 21$	$n = 9$	–
8. $3n = 21$	$n = 7$	–
9. $\frac{t}{5} = 4$ 10.	$t = 25$	–
10. $\frac{t}{5} = 4$	$t = 20$	–
11. $2l + 3 = 7$	$l = 5$	–
12. $2l + 3 = 7$	$l = 1$	–
13. $2l + 3 = 7$	$l = 2$	–

9.11 Getting a solution to the Equation

In finding the solution to the equation $2n = 10$, we prepared a table for various values of n and from the table picked up the value of n which was the solution to the equation (i.e. satisfies the equation). What we used is **a trial and error method**. It is not a **direct** and **practical** way of finding a solution. We shall now look for a direct way of solving an equation, i.e., finding the solution of the equation. We shall learn a more systematic method of solving equations only next year. For the present, we shall learn how to handle simple equations like.

(a) $x + 10 = 30$ (b) $x - 3 = 10$

(c) $2n = 10$ (d) $\frac{m}{5} = 4$

Solving $x + 10 = 30$

From earlier classes, we know how to find the number in the box in the statement $\square + 10 = 30$.

Compare the equation in x $\quad x + 10 = 30$ (a)

and $\quad \square + 10 = 30$ (b)

If we replace $\square$ by x in (b), we get the equation. It means that finding the number in the box is the same as finding the value of x which satisfies the equation.

The number in the box is such that 10 added to it gives 30. In other words, it is 10 taken away from 30, i.e., 20. Thus the solution to the equation is $x = 20$.

We may verify the solution:

LHS $= x + 10 = 20 + 10 = 30$ or LHS = RHS

Now, solve this way $\quad y + 5 = 12$

Compare the equation with $\quad \square + 5 = 12$

We know $\quad \square = 12 - 5 = 7$

Therefore, the required solution is $\quad y = \square = 7$

Solve by the above method $\quad p + 7 = 10$

Solving $x - 3 = 10$

Compare $\quad x - 3 = 10.$

with $\quad \square - 3 = 10$

This means that finding the solution to the equation in x is the same as finding the number in the box. Now, the number in the box is given by addition,

$\boxed{} = 10 + 3 = 13$

Therefore, the solution to the equation $x - 3 = 10$ is $x = 13$, which we already know.

We may verify the solution:

$$\text{LHS} = x - 3 = 13 - 3 = 10 = \text{RHS}$$

Solve this way $y - 7 = 5$

Solving $2n = 10$

We note $2n = 2 \times n$

The equation we want to solve is, therefore

$$2 \times n = 10$$

Compare this with $2 \times \boxed{} = 10$

Solving for n is the same as finding the number in the box. We know that the number in the box can be found by division, $\boxed{} = \frac{10}{2} = 5$

The solution to the equation $2n = 10$ is, therefore, $n = 5$, which we know already.

We may verify the solution. LHS $= 2 \times n = 2 \times 5 = 10 =$ RHS

Solving $\frac{m}{5} = 4$

We compare $\frac{m}{5} = 4$ with $\frac{\boxed{}}{5}$

Solving for m is the same as finding the number in the box. We know from earlier classes that this can be done by multiplication,

$\boxed{} = 5 \times 4 = 20$

Hence the solution to the equation is $m = 20$.

We may verify the solution: LHS $= \frac{m}{5} = \frac{20}{5} = 4 =$ RHS

9.12 Using an Equation

Appu, Sarita and Ameena are very excited. They tell the class that they have

found a method to solve puzzles. They want to explain it to the whole class.

First, they ask Sara to hold a number in her mind. Then they ask her to multiply the number by 5 and tell the result. She says 60. Appu immediately says that Sara had 12 in her mind. Sara agrees. The class is surprised.

Ameena explains:

Sara held some number in mind. It could have been anything. So we took it to be x. Now multiplying x by 5 gives $5x$. Sara said that she got 60. Thus, we have the condition $5x = 60$

This condition is just a simple equation of the kind we have learnt. We solved this equation by our simple method. We replaced the equation *in x by*

$$5 \times \boxed{} = 60$$

We knew $\boxed{} = \frac{60}{5} = 12$

Thus $x = 12$ is the required solution, i.e., it is the number Sara held in her mind.

The whole class clapped. They had learnt how useful equations were. The mathematics teacher complemented Appu, Sarita and Ameena. She added, "Puzzles and problems from everyday life which are much more challenging than that presented by the trio can be solved by using equations. However, for doing that we have to learn a systematic and general method of solving equations. We shall learn such a method next year".

While at the end of the chapter let us closely look at the process of solving an equation. We took an equation to be a condition on the variable in the equation. For example, the equation

$5x = 60$ is a condition on x.

Only one value of x, i.e. $x = 12$, satisfies the equation. To begin with we do not know this value. Solving the equation means finding this unknown value. We can look *at x* as this unknown. This is what Appu, Sarita and Ameena did. They did not know the number Sara had in her mind. They took it as unknown, called it x and got the condition that it satisfies. This condition was an equation. By solving the equation, they found the 'unknown'.

Forming and solving an equation, therefore, is a powerful method of finding unknown values and therefore of solving puzzles and problems.

Beginning of Algebra

It is said that algebra as a branch of Mathematics began about 1550 BC, i.e. more than 3500 years ago, when people in Egypt started using symbols to denote unknown numbers.

Around 300 BC, use of letters to denote unknowns and forming expressions from them was quite common in India. Many great Indian mathematicians, **Aryabhatt** (born AD 476), **Brahmagupta** (born AD 598), **Mahavira** (who lived around AD 850) and **Bhaskara II** (born AD 1114) and others, contributed a lot to the study of algebra. They gave names such as *Beeja*, *Varna* etc. to unknowns and used first letters of colour names [e.g., ka from *kala* (black), nee from *neela* (blue)] to denote them.

The Indian name for algebra, *Beejaganit*, dates back to these ancient Indian mathematicians.

The word 'algebra' is derived from the title of the book, **'Aljebar w'al almugabalah'**, written about AD 825 by an Arab mathematician, Mohammed Ibn Al Khowarizmi of Baghdad.

EXERCISE 9.5

1. State which of the following are equations (with a variable). Give reason for your answer. Identify the variable from the equations with a variable.

(a) $17 = x + 7$	(b) $(t - 7) > 5$	(c) $\frac{4}{2} = 2$
(d) $5 \times 4 - 8 = 2x$	(e) $x - 2 = 0$	(f) $2m < 30$
(g) $2n + 1 = 11$	(h) $7 = (11 \times 2) + p$	(i) $\frac{3q}{2} < 5$
(j) $20 - (10 - 5) = 3 \times 5$	(k) $7 - x = 5$	

2. Complete the entries in the third column of the table.

S.No.	Equation	Value of variable Equation satisfied Yes/No
(a)	$10y = 80$	$y = 10$
(b)	$10y = 80$	$y = 8$
(c)	$4l = 20$	$l = 20$
(d)	$4l = 20$	$l = 5$
(e)	$b + 5 = 9$	$b = 5$
(f)	$b + 5 = 9$	$b = 4$
(g)	$h - 8 = 5$	$h = 13$
(h)	$h - 8 = 5$	$h = 0$
(i)	$p + 3 = 1$	$p = 3$
(j)	$p + 3 = 1$	$p = 0$
(k)	$p + 3 = 1$	$p = -2$

3. Pick out the solution from the values given in the bracket next to each equation. Show that the other values do not satisfy the equation.

(a) $5m = 60$ (10, 5, 12, 15)

(b) $n + 12 = 20$ (12, 8, 20, 0)

(c) $p - 5 = 5$ (0, 10, 5 – 5)

(d) $\frac{q}{2} = 7$ (7, 2, 10, 14)

(e) $r - 4 = 0$ (4, – 4, 8, 0)

(f) $x + 4 = 2$ (– 2, 0, 2, 4)

4. (a) Complete the table and by inspection of the table find the solution to the equation m + 10 = 16

m	1	2	3	4	5	6	7	8	9	10	—	—	—
$m + 10$	—	—	—	—	—	—	—	—	—	—	—	—	—

(b) Complete the table and by inspection of the table, find the solution to the equation $5t = 35$.

t	3	4	5	6	7	8	9	10	11	—	—	—	—
$5t$	—	—	—	—	—	—	—	—	—	—	—	—	—

5. Solve

(a) $x + 5 = 12$	(b) $y - 2 = 10$
(c) $7p = 210$	(d) $\frac{q}{2} = 5$
(e) $t + 100 = 125$	(f) $1 - 20 = 30$

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.
2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter n, l, m, p, x, y, z, etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.
6. Variables allow us to express many common rules in both geometry and arithmetic in a general way. For example, the rule that the sum of two numbers remains the same if the order in which the numbers are taken is reversed can be expressed as $a + b = b + a$. Here, the variables a and b stand for any number, 1, 32, 1000, –7, –20, etc.
7. An equation is a condition on a variable. It is expressed by saying that an expression with a variable is equal to a fixed number, e.g. $x - 3 = 10$.

8. An equation has two sides, LHS and RHS, between them is the equal (=) sign.
9. The LHS of an equation is equal to its RHS only for a definite value of the variable in the equation. We say that this definite value of the variable satisfies the equation. This value itself is called the solution of the equation.
10. For getting the solution of an equation, one method is the trial and error method. In this method, we give some value to the variable and check whether it satisfies the equation. We go on giving this way different values to the variable' until we find the right value which satisfies the equation.
11. We need a more systematic way of getting a solution of the equation than the trial and error method. In case of very simple equations, the variable can be replaced by a place holder $\boxed{}$. From earlier classes we know how to get the value of $\boxed{}$. This is the value of the variable which is the solution to the equation.
12. When we are given an equation, its solution, i.e., the value of the variable which satisfies the equation; is unknown to us. Solving the equation means finding the unknown value, we can look at the variable in the equation as an unknown. Starting from the unknown we can see up the equation. Solving the equation is thus a method of finding the unknown. It is therefore a powerful method of solving puzzles and problems.

10 Ratio and Proportion

10.1 Introduction

In our daily life many a times we compare two quantities of the same type. For example, Avnee and Shari collected flowers for scrap notebook. Avnee collected 30 flowers and Shari collected 45 flowers.

So, we may say that Shari collected 45 – 30 = 15 flowers more than Avnee.

This is one way of comparison by taking difference. Height of Rahim is 150 cm and that of Avnee is 140 cm. so we may say that the height of Rahim is 150 cm – 140 cm = 10 cm more than Avnee.

If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

Consider another example.

Cost of a car is Rs 2,50,000 and that of a motorbike is Rs 50,000. If we calculate the difference between the costs, it is Rs 2,00,000 and if we compare by division; that is $\frac{2,50,000}{50,000} = \frac{5}{1}$

We can say that the cost of the car is five times the cost of the motorbike. Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about 'Ratios'.

10.2 Ratio

Consider the following:

Isha's weight is 25 kg and her father's weight is 75 kg. How many times Father's weight is of Isha's weight? It is three times.

Cost of a pen is Rs 10 and cost of a pencil is Rs 2. How many times the cost of a pencil is the cost of a pen? Obviously it is five times.

In the above examples, we compared the two **quantities in** terms of 'how many times'. This comparison is known as **the Ratio. We denote** ratio using symbol ':'

Consider the earlier examples again. We can say,

The ratio of father's weight to Isha's weight = $\frac{75}{25} = \frac{3}{1} = 3:1$

The ratio of the cost of a pen to the cost of a pencil = $\frac{10}{2} = \frac{5}{1} = 5:1$

Try These

1. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?
2. Ravi walks 6 km in an hour while Roshan walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Roshan?

Let us look at this problem.

In a class there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.

(b) Number of boys to the total number of students.

First we need to find the total number of students, which is,

Number of girls + Number of boys = 20 + 40 = 60.

Then, the ratio of number of girls to the total number of students is $\frac{40}{60} = \frac{3}{3}$.

Find the answer of part (b) in the similar manner.

Now consider the following example.

Length of a house lizard is 20 cm and the length of a crocodile is 4 m. "I am 5 times bigger than you", says the lizard. As we can see this is really absurd. A lizard's length cannot be 5 times of the length of a crocodile. So what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in meters. So we have to convert their lengths into the same unit.

Length of the crocodile = 4 m = 4 × 100 = 400 cm.

Therefore, ratio of the length of the crocodile to the length of the lizard

$$= \frac{400}{20} = \frac{20}{1} = 20:1.$$

Two quantities can be compared only if they are in the same unit.

Now what is the ratio of the length of the lizard to the length of the crocodile? It is $\frac{20}{400} = \frac{1}{20} = 1:20$.

Observe that the two ratios 1 : 20 and 20 : 1 are different from each other. The ratio 1 : 20 is the ratio of the length of the lizard to the length of the crocodile whereas, 20 : 1 is the ratio of the length of the crocodile to the length of the lizard.

Now consider another example.

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length? Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm = 18 × 10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil $= \frac{8}{180} = \frac{2}{45} = 2:45$.

Try These

1. Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.

2. Cost of a toffee is 50 paise and cost of a chocolate is Rs 10. Find the ratio of the cost of a toffee to the cost of a chocolate.
3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

A B

Think of some more such type of situations when you compare two quantities of same type in different units.

We use the concept of ratio in many situations of our daily life without realising that we do so.

Compare the drawings A and B. B looks more natural A B than A. Why?

The legs in the picture A are too long in comparison to the other body parts. This is because we normally expect a certain ratio of the length of legs to the length of whole body.

Compare the two pictures of a pencil. Is the first one looking like a full pencil? No.

Why not? The reason is that the thickness and the length of the pencil are not in the correct ratio.

Hey! We have the same ratio in different situations:

Consider the following:

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room = $\frac{30}{20} = \frac{3}{2} = 3:2$
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys = $\frac{24}{16} = \frac{3}{2} = 3:2$

The ratio in both the examples is 3 : 2.

- Note the ratios 30: 20 and 24: 16 in lowest form are same as 3: 2. These are equivalent ratios.

Can you think of some more examples having the ratio 3: 2?

It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2: 3.

Ratio of the breadth of a table to the length of the table is 2: 3.

Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Then, the ratio of marbles that Sheena and Shabnam have is 2: 3.

Can you write some more situations for this ratio? Give any ratio to your friends and ask them to frame situations.

Ravi and Rani started a business and invested money in the ratio 2: 3. After one year the total profit was Rs 40,000.

Ravi said "we would divide it equally", Rani said "I should get more as I have invested more".

It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 2 : 3 are 2 and 3.

Sum of these terms = 2 + 3 = 5

What does this mean?

This means if the profit is Rs 5 then Ravi should get Rs 2 and Rani should get Rs 3.

Or, we can say that Ravi gets 2 parts and Rani gets 3 parts out of the 5 parts.

That is Ravi should get $\frac{2}{5}$ of the total profit and Rani should get $\frac{3}{5}$ of the total profit.

If the total profit were Rs 500

Ravi would get Rs × 500 = Rs 200

and Rani would get × 500 = Rs 300

Now, if the profit were Rs 40,000, could you find the share of each?

Ravi's share = Rs × 40000 = Rs 16,000.

And Rani's share = Rs × 40000 = Rs 24,000.

Can you think of some more examples where you have to divide a number of things in some ratio? Frame three such examples and ask your friends to solve them.

Try These

1. Find the ratio of number of notebooks to the number of books in your bag.
2. Find the ratio of number of desks and chairs in your classroom.
3. Find the number of students above twelve years of age in your class. Then, find the ratio of number of students with age above twelve years and the remaining students.
4. Find the ratio of number of doors and the number of windows in your classroom.
5. Draw any rectangle and find the ratio of its length to its breadth.

Let us look at the kind of problems we have solved so far.

Example 1: Length and breadth of a rectangular field are 50 m and 15 m respectively. Find the ratio of the length to the breadth of the field.

Solution: Length of the rectangular field = 50 m

Breadth of the rectangular field = 15 m

The ratio of the length to the breadth is 50 : 15

The ratio can be written as $\frac{50}{15} = \frac{50 \div 5}{15 \div 5} = \frac{10}{3} = 10 : 3$

Thus, the required ratio is 10 : 3.

Example 2: Find the ratio of 90 cm to 1.5 m.

Solution: The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.

Therefore, the required ratio is 90 : 150.

$= \frac{90}{150} = \frac{90 \div 3}{150 \div 30} = \frac{3}{5}$

Required ratio is 3 : 5.

Example 3: There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of :

(a) The number of females to number of males.

(b) The number of males to number of females.

Solution: Number of females = 25

Total number of workers = 45

Number of males = 45 – 25 = 20

Therefore, the ratio of number of females to the number of males = 25 : 20 = 5 : 4

And the ratio of number of males to the number of females

= 20: 25 = 4: 5.

(Notice that there is a difference between the two ratios 5: 4 and 4: 5).

Example 4: Give two equivalent ratios of 6: 4.

Solution: Ratio 6: 4 = $\frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{12}{8}$

Therefore, 12: 8 is an equivalent ratio of 6: 4

Similarly, the ratio 6: 4 = $\frac{4}{6} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$

So, 3:2 is another equivalent ratio of 6: 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Write two more equivalent ratios of 6: 4.

Example 5: Fill in the missing numbers:

$$\frac{14}{21} = \frac{\square}{3} = \frac{6}{\square}$$

Solution: In order to get the first missing number, we consider the fact that 21 = 3 × 7. i.e. when we divide 21 by 7 we get 3. This indicates that to get the missing number of second ratio, 14 must also be divided by 7.

When we divide, we have, 14 ÷ 7 = 2

Hence, the second ratio is $\frac{2}{3}$

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\frac{6}{9}$

Therefore, $\frac{14}{21} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$ [These are all equivalent ratios.]

Example 6: Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km.)	10	$\boxed{}$	4	$\boxed{}$	$\boxed{}$
Distance from John's home to school (in km.)	5	4	$\boxed{}$	3	1

(c) If the ratio of distance of Mary's home to the distance of Kalam's home from school is 1 : 2, then who lives nearer to the school?

Solution: (a) John lives nearer to the school (As the ratio is 2 : 1).

(b)

Distance from Mary's home to school (in km.)	10	$\boxed{8}$	4	$\boxed{6}$	$\boxed{2}$
Distance from John's home to school (in km.)	5	4	$\boxed{2}$	3	1

(c) Since the ratio is 1 : 2, so Mary lives nearer to the school.

Example 7: Divide Rs 60 in the ratio 1 : 2 between Kriti and Kiran.

Solution: The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

This means if there are Rs 3, Kriti will get Re 1 and Kiran will get Rs 2. Or, we can say that Kriti gets 1 part and Kiran gets 2 parts out of every 3 parts.

Therefore, Kriti's share = Rs $\frac{1}{3} \times 60$ = Rs 20

And Kiran's share = Rs $\frac{2}{3} \times 60$ = Rs 40

EXERCISE 10.1

1. There are 20 girls and 15 boys in a class.
   a. What is the ratio of number of girls to the number of boys?
   b. What is the ratio of number of girls to the total number of students in the class?
2. Out of 30 students in a class, 6 like football, 12 like cricket and remaining like tennis. Find the ratio of
   (a) Number of students liking football to number of students liking tennis.
   (b) Number of students liking cricket to total number of students.

3. See the figure and find the ratio of
   a. Number of triangles to the number of circles inside the rectangle.
   b. Number of squares to all the figures inside the rectangle.
   c. Number of circles to all the figures inside the rectangle.

4. Distances travelled by Hamid and Akhtar in an hour are 9 km and 12 km. Find the ratio of speed of Hamid to the speed of Akhtar.
5. Find the ratio of the following:
   (a) 81 to 108 (b) 98 to 63
   (c) 33 km to 121 km (d) 30 minutes to 45 minutes
6. Find the ratio of the following:
   (a) 30 minutes to 1.5 hours (b) 40 cm to 1.5 m
   (c) 55 paise to Re 1 (d) 500 ml to 2 litres
7. In a year, Seema earns Rs 1,50,000 and saves Rs 50,000. Find the ratio of
   (a) Money that Seema earns to the money she saves.
   (b) Money that she saves to the money she spends.

8. There are 102 teachers in a school of 3300 students. Find the ratio of the number of teachers to the number of students.
9. Out of 1800 students in a school, 750 opted basketball, 800 opted cricket and remaining opted table tennis. If a student can opt only one game, find the ratio of
   a. Number of students who opted basketball to the number of students who opted table tennis.
   b. Number of students who opted cricket to the number of students opting basketball.
   c. Number of students who opted basketball to the total number of students.
10. Cost of a dozen pens is Rs 180 and cost of 8 ball pens is Rs 56. Find the ratio of the cost of a pen to the cost of a ball pen.
11. Present age of father is 42 years and that of his son is 14 years. Find the ratio of
   (a) Present age of father to the present age of son.
   (b) Age of the father to the age of son, when son was 12 years old.
   (c) Age of father after 10 years to the age of son after 10 years.
   (d) Age of father to the age of son when father was 30 years old.

10.3 Proportion

Consider this situation:

Raju went to the market to purchase tomatoes. One shopkeeper tells him that the cost of tomatoes is Rs 40 for 5 kg. Another shopkeeper gives the cost as 6 kg for Rs 42. Now, what should Raju do? Should he purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help him decide? No. Why not?

Think of some way to help him. Discuss with your friends.

Consider another example.

Bhavika has 28 marbles and Vini has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vini and Vini gave 90 flowers to Bhavika. But Vini was not satisfied. She felt that she has given more flowers to Bhavika than the marbles given by Bhavika to her.

What do you think? Is Vini correct?

To solve this problem both went to Vini's mother Pooja.

Pooja explained that out of 28 marbles, Bhavika gave 14 marbles to Vini.

Therefore, ratio is 14 : 28 = 1 : 2.

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is 90 : 180 = 1 : 2.

Since both the ratios are the same, so the distribution is fair.

Two friends Ashma and Pankhuri went to market to purchase hair clips. They purchased 20 hair clips for Rs 30. Ashma gave Rs 12 and Pankhuri gave Rs 18. After they came back home, Ashma asked Pankhuri to give 10 hair clips to her. But Pankhuri said, "since she has given more money so she should get more clips. And according to her, Ashma should get 8 hair clips and she should get 12 hair clips.

Can you tell who is correct, Ashma or Pankhuri? Why?

Ratio of money given by Ashma to the money given by Pankhuri

= Rs 12 : Rs 18 = 2 : 3

According to Ashma's suggestion, ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 10 : 10 = 1 : 1

According to Pankhuri's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 8 : 12 = 2 : 3

Now, notice that according to Ashma's distribution, ratio of hair clips and the ratio of money given by them is not the same. But according to the Pankhuri's distribution the two ratios are the same.

Hence, we can say that Pankhuri's distribution is correct.

Sharing a ratio means something!

Consider the following examples:

- Raj purchased 3 pens for Rs 15 and Anu purchased 10 pens for Rs 50, whose pens are more expensive?

  Ratio of number of pens purchased by Raj to the number of pens purchased by Anu = 3 : 10.

Ratio of their costs = 15 : 50 = 3 : 10

Both the ratios 3 : 10 and 15 : 50 are equal. Therefore, the pens were purchased for the same price by both.

- Rahim sells 2 kg of apples for Rs 60 and Roshan sells 4 kg of apples for Rs 120. Whose apples are more expensive?

Ratio of the weight of apples = 2 kg : 4 kg = 1 : 2

Ratio of their cost = 60 : 120 = 6 : 12 = 1 : 2

So, the ratio of weight of apples = ratio of their cost.

Since both the ratios are equal, hence, we say that they are in proportion. They are selling apples at the same rate.

If two ratios are equal, we say that they are in proportion and use the symbol '::' to equate the two ratios.

For the first example, we can say 3, 10, 15 and 50 are in proportion which is written as 3 : 10:: 15 : 50 and is read as 3 is to 10 as 15 is to 50.

For the second example, we can say 2, 4, 60 and 120 are in proportion which is written as 2 : 4 :: 60 : 120 and is read as 2 is to 4 as 60 is to 120.

Let us consider another example.

A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

Now, ratio of the two distances travelled by the man is 35 to 70 = 1 : 2 and the ratio of the time taken to cover these distances is 2 to 4 = 1 : 2 .

Hence, the two ratios are equal i.e. 35 : 70 = 2 : 4.

Therefore, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

Hence, we can write it as 35 : 70 :: 2 : 4 and read it as 35 is to 70 as 2 is to 4. Hence, he can travel 70 km in 4 hours with that speed.

Now, consider this example. Cost of 2 kg of apples is Rs 60 and a 5 kg watermelon costs Rs 15.

Now, ratio of the weight of apples to the weight of watermelon is 2 : 5.

And ratio of the cost of apples to the cost of the watermelon is 60 : 15 = 4 : 1.

Here, the two ratios 2 : 5 and 60 : 15 are not equal,

i.e. 2 : 5 $\neq$ 60 : 15

Therefore, the four quantities 2, 5, 60 and 15 are not in proportion.

If two ratios are not equal then we say that they are not in proportion.

Check whether the given ratios are equal, i.e. they are in proportion.
If yes, then write them in the proper form.

1. 1 : 5 and 3 : 15
2. 2 : 9 and 18 : 81
3. 15 : 45 and 5 : 25
4. 4 : 12 and 9 : 27
5. Rs 10 to Rs 15 and 4 to 6

In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as extreme terms. Second and third terms are known as middle terms.

For example, in 35 : 70 : : 2 : 4;

35, 70, 2, 4 are the four terms. 35 and 4 are the extreme terms. 70 and 2 are the middle terms.

Example 8 : Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution : 25 g : 30 g $= \frac{25}{30} = 5 : 6$

40 kg : 48 kg $= \frac{40}{48} = 5 : 6$

So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion, i.e. 25 : 30 :: 40 : 48

The middle terms in this are 30, 40 and the extreme terms are 25, 48.

Example 9 : Are 30, 40, 45 and 60 in proportion?

Solution : Ratio of 30 to 40 = $\frac{30}{40}$ = 3 : 4

Ratio of 45 to 60 = $\frac{45}{60}$ = 3 : 4

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example 10 : Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution : Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)

= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)

= 1 : 18

Since, 3 : 40 $\neq$ 1 : 18, therefore, the given ratios do not form a proportion.

EXERCISE 10.2

1. Determine if the following are in proportion.
   (a) 15, 45, 40, 120 (b) 33, 121, 9,96
   (e) 4, 6, 8, 12 (f) 33, 44, 75, 100
2. Write True (T) or False (F) against each of the following statements :
   (a) 16 : 24 :: 20 : 30 (b) 21: 6 :: 35 : 10
   (c) 12 : 18 :: 28 : 12 (d) 8 : 9 :: 24 : 27
3. Are the following statements true?
   (a) 40 persons : 200 persons = Rs 15 : Rs 75
   (b) 7.5 litres : 15 litres = 5 kg : 10 kg
   (c) 99 kg : 45 kg = Rs 44 : Rs 20
   (d) 32 m : 64 m = 6 sec : 12 sec
   (e) 45 km : 60 km = 12 hours : 15 hours
4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (a) 25 cm : 1 m and Rs 40 : Rs 160
   (b) 39 litres : 65 litres and 6 bottles : 10 bottles
   (c) 2 kg : 80 kg and 25 g : 625 g
   (d) 200 ml : 2.5 litre and Rs 4 : Rs 50

10.4 Unitary Method

Consider the following situations:

- Two friends Reshma and Seema went to market to purchase notebooks. Reshma purchased 2 notebooks for Rs 24. What is the price of one notebook?
- A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

These are examples of the kind of situations that we face in our daily life. How would you solve these?

Reconsider the first example:

Cost of 2 notebooks is Rs 24.

Therefore, cost of 1 notebook = Rs 24 ÷ 2 = Rs 12.

Now, if you were asked to find cost of 5 such notebooks. It would be

Rs 12 + Rs 12 + Rs 12 + Rs 12 + Rs 12 = Rs 12 × 5 = Rs 60

Reconsider the second example:

We want to know how many litres are needed to travel 1 km.

To go, 80 km, petrol needed = 2 litres.

Therefore, to travel 1 km, petrol needed = $\frac{2}{80} = \frac{1}{40}$ litres.

Now, if you are asked to find how many litres of petrol are required to cover 120 km?

Then petrol needed = $\frac{1}{40} \times 120$ litres = 3 litres.

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Try These

1. Prepare five similar problems and ask your friends to solve them.
2. Read the table and fill in the blanks.

Time	Distance travelled by Karan	Distance travelled by Kriti
2 hours	8 km	6 km
1 hour	4 km	
4 hours		

We see that,

Distance travelled by Karan in 2 hours = 8 km

Distance travelled by Karan in 1 hour = $\frac{8}{2}$ km = 4 km

Therefore, distance travelled by Karan in 4 hours = 4×4 = 16 km

Similarly, to find the distance travelled by Kriti in 4 hours, first find the distance travelled by her in 1 hour.

Example 11 : If the cost of 6 cans of juice is Rs 210, then what will be the cost of 4 cans of juice?

Solution : Cost of 6 cans of juice= Rs 210

Therefore, cost of one can of juice $= \frac{210}{6}$ = Rs 35

Therefore, cost of 4 cans of juice = Rs 35×4 = Rs 140.

Thus, cost of 4 cans of juice is Rs 140.

Example 12 : A motorbike travels 220 km in 5 litres of petrol. How much distance will it cover in 1.5 litres of petrol?

Solution : In 5 litres of petrol, motorbike can travel 220 km.

Therefore, in 1 litre of petrol, motor bike travels $\frac{220}{5}$ km

Therefore, in 1.5 litres, motorbike travels $\frac{220}{5} \times 1.5$ km

$\frac{220}{5} \times \frac{15}{10}$ km = 66 km.

Thus, the motorbike can travel 66 km in 1.5 litres of petrol.

Example 13 : If the cost of a dozen soaps is Rs 153.60, what will be the cost of 15 such soaps?

Solution : We know that 1 dozen = 12

Since, cost of 12 soaps = Rs 153.60

Therefore, cost of 1 soap = $\frac{153.60}{12}$ = Rs 12.80

Therefore, cost of 15 soaps = Rs 12.80×15 = Rs 192

Thus, cost of 15 soaps is Rs 192.

Example 14 : Cost of 105 envelopes is Rs 35. How many envelopes can be purchased for Rs 10?

Solution : In Rs 35, the number of envelopes that can be purchased = 105

Therefore, in Re 1, number of envelopes that can be purchased

$$= \frac{105}{35}$$

Therefore, in Rs 10, the number of envelopes that can be purchased $= \frac{105}{35} \times 10 = 30$

Thus, 30 envelopes can be purchased for Rs 10.

Example 15 : A car travels 90 km in $2\frac{1}{2}$ hours.

(a) How much time is required to cover 30 km with the same speed?

(b) Find the distance covered in 2 hours with the same speed.

Solution : (a) In this case, time is unknown and distance is known. Therefore, we proceed as follows :

$$2\frac{1}{2} = \frac{5}{2} \text{ hours} = \frac{5}{2} \times 60 \text{ minutes} = 150 \text{ minutes.}$$

90 km is covered in 150 minutes

Therefore, 1 km can be covered in $\frac{1}{2}$ minutes

Therefore, 30 km can be covered in $\frac{1}{2} \times 30$ minutes

i.e. 50 minutes

Thus, 30 km can be covered in 50 minutes.

(b) In this case, distance is unknown and time is known.

Therefore, we proceed as follows :

Distance covered in $2\frac{1}{2}$ hours (i.e. $\frac{1}{2}$ hours) = 90 km

Therefore, distance covered in 1 hour $= 90 \div \frac{5}{2}$ km

$$= 90 \times \frac{2}{5} = 36 \text{ km}$$

Therefore, distance covered in 2 hours = 36 × 2 = 72 km.

Thus, in 2 hours, distance covered is 72 km.

EXERCISE 10.3

1. If the cost of 7 m of cloth is Rs 294, find the cost of 5 m of cloth.
2. Ekta earns Rs 1500 in 10 days. How much will she earn in 30 days?
3. If it has rained 276 mm in the last 3 days, how many cm of rain will fall in one full week (7 days)? Assume that the rain continues to fall at the same rate.
4. Cost of 5 kg of wheat is Rs 30.50.
   (a) What will be the cost of 8 kg of wheat?
   (b) What quantity of wheat can be purchased in Rs 61?
5. The temperature dropped 15 degree celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?

What have we discussed?

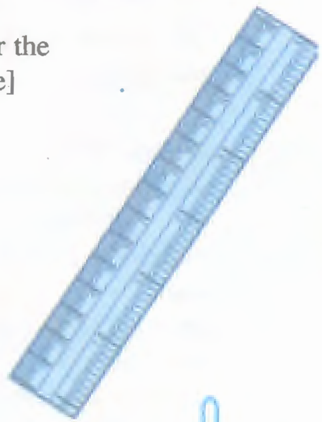
1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.
2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio. For example, Isha's weight is 25 kg and her father's weight is 75 kg. We say that Isha's father's weight and Isha's weight are in the ratio 3 : 1.
3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.
4. The same ratio may occur in different situations.
5. Note that the ratio 3 : 2 is different from 2 : 3. Thus, the order in which quantities are taken to express their ratio is important.
6. A ratio may be treated as a fraction, thus the ratio 10 : 3 may be treated as $\frac{10}{3}$.

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent. Thus, 3 : 2 is equivalent to 6 : 4 or 12 : 8.
8. A ratio can be expressed in its lowest form. For example, ratio 50 : 15 is treated as $\frac{50}{15}$; in its lowest form $\frac{50}{15} = \frac{10}{3}$
   Hence, the lowest form of the ratio 50 : 15 is 10 : 3.

   Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities. Thus, 3, 10, 15, 50 are in proportion, since $\frac{3}{10} = \frac{15}{50}$. We indicate the proportion by 3 : 10 :: 15 : 50, it is read as 3 is to 10 as 15 is to 50. In the above proportion, 3 and 50 are the extreme terms and 10 and 15 are the middle terms.
9. The order of terms in the proportion is important. 3, 10, 15 and 50 are in proportion, but 3, 10, 50 and 15 are not, since is $\frac{10}{3}$ not equal to $\frac{50}{15}$.
10. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method. Suppose the cost of 6 cans is Rs 210. To find the cost of 4 cans, using the unitary method, we first find the cost of 1 can. It is Rs $\frac{210}{3}$ or Rs 35. From this, we find the price of 4 cans as Rs 35 × 4 or Rs 140.

11 Practical Geometry

We see a number of shapes with which we are familiar. We also make a lot of pictures. These pictures include different shapes. We have learnt about some of these shapes in earlier chapters as well. Why don't you list those shapes that you know about alongwith how they appear? We enjoy drawing things and putting down our ideas. To do this effectively we have to learn to draw the shapes and also learn to analyse things around us, and from these identify the known shapes. While we draw pictures we keep in mind the proportions. Some of it we have learnt in the earlier chapters and also learnt to talk about some shapes that are common and are discussed in geometry. A few of these we have dealt with in some detail and have looked at their properties.

S. No.	Name	Figure	Description	Use
1.	The ruler [or the straight edge]		A ruler ideally has no marking on it. However, the ruler in your instruments box is graduated into centimetres along one edge (and sometimes into inches along the other edge).	To draw line segment and to measure their lengths.
2.	The Compasses		A pair – a pointer on one end and a pencil on the other	To mark off equal lengths but not to measure them. To draw arcs and circles

3.	The Divider		A pair of pointers	To compare lengths
4.	Set-Squares		Two triangular pieces – one of them has 45°, 45°, 90° angles at the vertices and the other has 30°, 60°, 90° angles at the vertices.	To draw perpendiculars and parallels.
5.	The Protractor		A semi-circular devicegraduated into 180 degree – parts. The measure starts from 0° on the right hand side and ends with 180° on the left hand side and vice versa	To draw and measure angles.

We are going to consider **"Ruler and compasses constructions"**, using ruler, only to draw lines, and compasses, only to draw arcs.

Be careful while doing these constructions.

Here are some tips to help you.

(a) Draw thin lines and mark points lightly.

(b) Maintain instruments with sharp tips and fine edges.

(c) Have two pencils in the box, one for insertion into the compasses and the other to draw lines or curves and mark points.

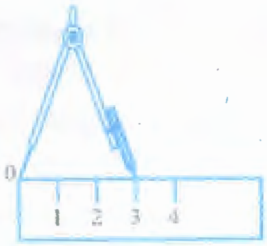
11.2 The Circle

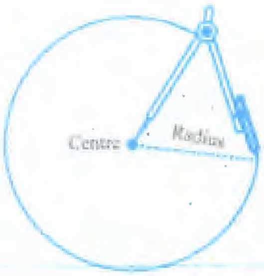
Look at the wheel shown here. Every point on its boundary is at an equal distance from its centre. Can you mention a few such objects and draw them? Think about five such objects which have this shape.

11.2.1 Construction of a circle when its radius is known

Suppose we want to draw a circle of radius 3 cm. We need to use our compasses.

Here are the steps to follow.





Step 1 Open the compasses for the required radius of 3cm.

Step 2 Mark a point with a sharp pencil where we want the centre of the circle to be. Name it as O.

Step 3 Place the pointer of the compasses on O.

Step 4 Turn the compasses slowly to draw the circle. Be careful to complete the movement around in one instant.

Think, discuss and write

How many circles can you draw passing through

(a) **one given point, say, P?**

(b) **two given points, say P?**

(c) **two given points, A and B?**

(d) **four scattered points?**

EXERCISE 11.1

1. Draw a circle of radius 3.2 cm.
2. With the same centre O, draw two circles of radii 4 cm and 2.5 cm.
3. Draw a circle and any two of its diameters. If you join the ends of these diameters, what is the figure obtained? What figure is

obtained if the diameters are perpendicular to each other? How do you check your answer?

4. Draw any circle and mark points A, B and C such that
   (a) A is on the circle.
   (b) B is in the interior of the circle.
   (c) C is in the exterior of the circle.

11.3 A Line Segment

Remember that a line segment is bounded by two end-points. This makes it possible to measure its length with a ruler.

If we know the length of a line segment, it becomes possible to represent it by a diagram. Let us see how we do this.

11.3.1 Construction of a line segment of a given length

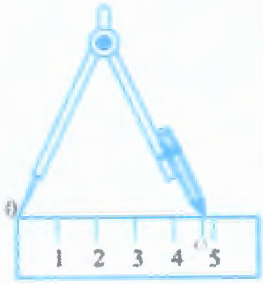
Suppose we want to draw a line segment of length 4.7 cm. We can use our ruler and mark two points A and B which are 4.7 cm apart. Join A and B and get $\overline{AB}$. While marking the points A and B, we should look straight down at the measuring device. Otherwise we will get an incorrect value.

Use of ruler and compasses

A better method would be to use compasses to construct a line segment of a given length.

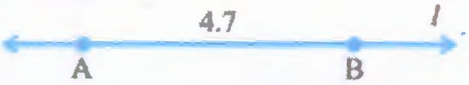
Step 1 Draw a line l. Mark a point A on a line l.

Step 2 Place the compasses pointer on the zero mark of the ruler. Open it to place the pencil point upto the 4.7cm mark.



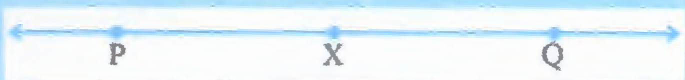
Step 3 Taking caution that the opening of the compasses has not changed, place the pointer on A and swing an arc to cut l at B.

Step 4 $\overline{AB}$ is a line segment of required length.



EXERCISE 11.2

1. Draw a line segment of length 7.3 cm using a ruler.
2. Construct a line segment of length 5.6 cm using ruler and compasses.
3. Construct $\overline{AB}$ of length 7.8 cm. From this, cut off $\overline{AC}$ of length 4.7 cm. Measure BC.
4. Given $\overline{AB}$ of length 3.9 cm, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$. Verify by measurement.



(**Hint:** Construct PX such that length of PX = length of AB; then cut off XQ such that XQ also has the length of AB.)

5. Given AB of length 7.3 cm and CD of length 3.4 cm, construct a line segment XY such that the length of XY is equal to the difference between the lengths of AB and CD. Verify by measurement.

11.3.2 Constructing a copy of a given line segment

Suppose you want to draw a line segment whose length is equal to that of a given line segment $\overline{AB}$.

A quick and natural approach is to use your ruler (which is marked with centimetres and millimeters) to measure the length of $\overline{AB}$ and then use the same length to draw another line segment CD.

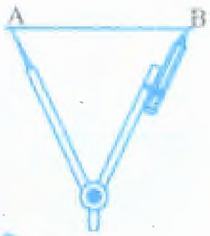
A second approach would be to use a transparent sheet and trace AB onto another portion of the paper. But these methods may not always give accurate results.

A better approach would be to use ruler and compasses for making this construction.

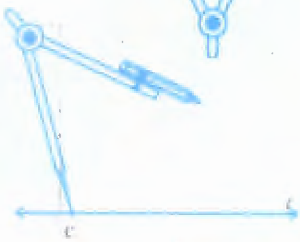
To make a copy of $\overline{AB}$.

Step 1 Given AB whose length is not known.

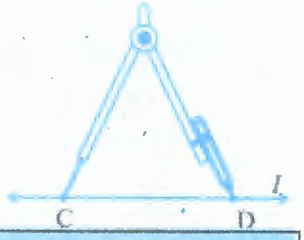
Step 2 Fix the compasses pointer on A and the pencil end on B. The opening of the instrument now gives the length of $\overline{AB}$.



Step 3 Draw any line l. Choose a point C on l. Without changing the compasses setting, place the pointer on C.



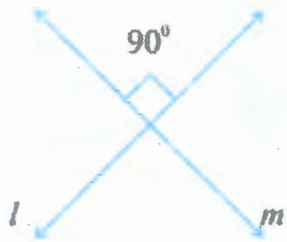
Step 4 Swing an arc that cuts l at a point, say, D. Now $\overline{CD}$ is a copy of $\overline{AB}$.



EXERCISE 11.3

1. Draw any line segment $\overline{PQ}$. Without measuring , construct a copy of $\overline{PQ}$.
2. Given some line segment $\overline{AB}$, whose length you do not know, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$.

11.4 Perpendiculars



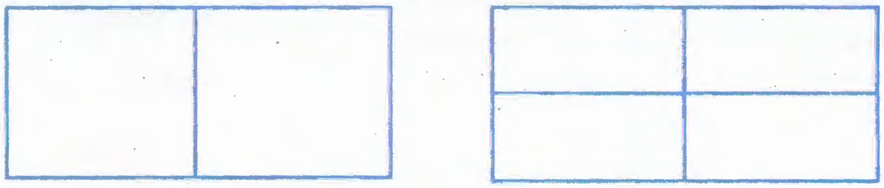
You know that two lines (or rays or segments) are said to be perpendicular if they intersect such that the angles formed between them are right angles. In the figure, the lines l and m are perpendicular.

Actually no paper folding is needed to demonstrate perpendicular lines; the corners of a foolscap paper or your notebook indicate lines meeting at right angles.

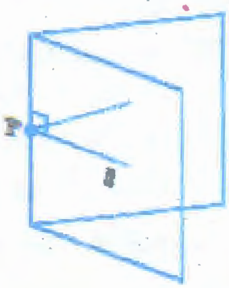
Do This

Where else do you see perpendicular lines around you?

Take a piece of paper. Fold it down the middle and make the crease. Fold the paper once again down the middle in the other direction. Make the crease and open out the page. The two creases are perpendicular to each other.



11.4.1 Perpendicular to a line through a point on it



Given a line l drawn on a paper sheet and a point P lying on the line. It is easy to have a perpendicular to l through P.

We can simply fold the paper such that the lines on both sides of the fold overlap each other.

Tracing paper or any transparent paper could be better for this activity. Let us take such a paper and draw any line l on it. Let us mark a point P anywhere on l.

Fold the sheet such that l is reflected on itself; adjust the fold so that the crease passes through the marked point P. Open out; the crease is perpendicular to l.

Think, discuss and write

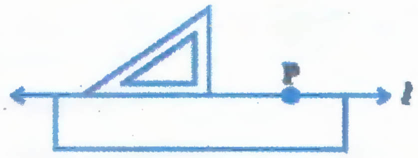
How would you check if it is perpendicular? Note that it passes through P as required.

A challenge: Drawing perpendicular using ruler and a set-square (An optional activity).

Step 1 A line l and a point P are given. Note that P is on the line l.

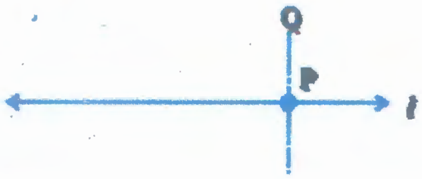
Step 2 Place a ruler with one of its edges along l. Hold this firmly.

Step 3 Place a set-square with one of its edges along the already aligned edge of the ruler such that the right angled corner is in contact with the ruler.



Step 4 Slide the set-square along the edge of ruler until its right angled corner coincides with P.

Step 5 Hold the set-square firmly in this position. Draw $\overline{PQ}$ along the edge of the set-square.



$\overline{PQ}$ is perpendicular to l. (How do you use the $\perp$ symbol to say this?).

Verify this by measuring the angle at P.

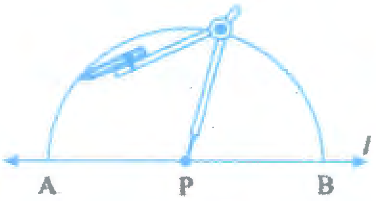
Can we use another set-square in the place of the 'ruler'? Think about it.

Method of ruler and compasses

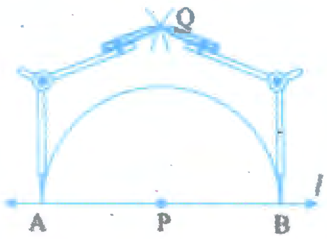
As is the preferred practice in Geometry, the dropping of a perpendicular can be achieved through the "ruler-compasses" construction as follows :

Step 1 Given a point P on a line l.

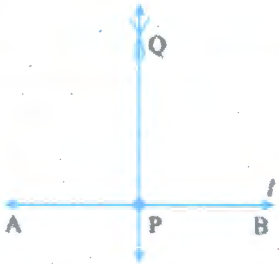
Step 2 With P as centre and a convenient radius, construct an arc intersecting the line l at two points A and B.



Step 3 With A and B as centres and a radius greater than AP construct two arcs, which cut each other at Q.

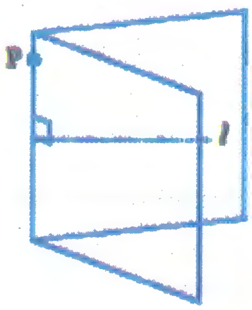


Step 4 Join PQ. Then $\overline{PQ}$ is perpendicular to l. We write $\overline{PQ} \perp l$.



11.4.2 Perpendicular to a line through a point not on it

(Paper folding)



If we are given a line l and a point P not lying on it and we want to draw a perpendicular to l through P, we can again do it by a simple paper folding as before.

Take a sheet of paper (preferably transparent). Draw any line l on it.

Mark a point P away from l.

Fold the sheet such that the crease passes through P.

The parts of the line l on both sides of the fold should overlap each other.

Open out. The crease is perpendicular to l and passes through P.

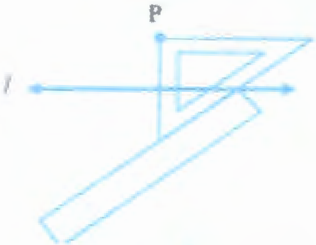
Method using ruler and a set-square (An optional activity)

Step 1 Let l be the given line and P be a point outside l.

Step 2 Place a set-square on l such that one arm of its right angle aligns along l.

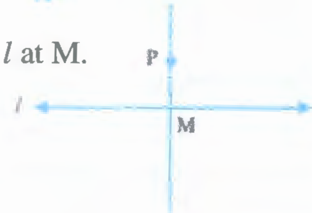
Step 3 Place a ruler along the edge opposite to the right angle of the set-square.

Step 4 Hold the ruler fixed. Slide the set-square along the ruler till the point P touches the other arm of the set-square.



Step 5 Join PM along the edge through P, meeting l at M.

Now $\overline{PM} \perp l$.

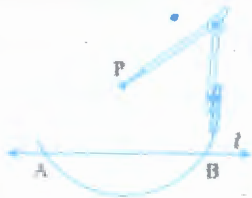


Method using ruler and compasses

A more convenient and accurate method, of course, is the ruler-compasses method.

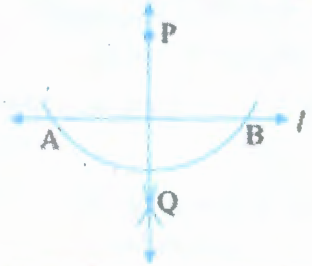
Step 1 Given a line l and a point P not on it.

Step 2 With P as centre, draw an arc which intersects line l at two points A and B.



Step 3 Using the same radius and with A and B as centres, construct two arcs that intersect at a point, say Q, on the other side.

Step 4 Join PQ. Thus, $\overline{PQ}$ is perpendicular to *l*.



EXERCISE 11.4

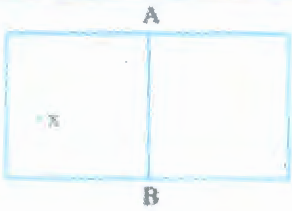
1. Draw any line segment $\overline{AB}$. Mark any point M on it. Through M, draw a perpendicular to $\overline{AB}$. (use ruler and compasses)
2. Draw any line segment $\overline{PQ}$. Take any point R not on it. Through R, draw a perpendicular to $\overline{PQ}$. (use ruler and set-square)
3. Draw a line *l* and a point X on it. Through X, draw a line segment XY perpendicular to *l*.

   Now draw a perpendicular to XY at Y. (use ruler and compasses)

11.4.3 The perpendicular bisector of a line segment

Do This

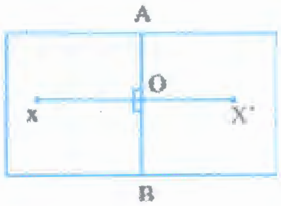
Fold a sheet of paper. Let be the fold. Place an ink-dot X, as shown, anywhere. Find the image X' of X, with AB as the mirror line.



Let $\overline{AB}$ and $\overline{XX'}$ intersect at O. Is OX = OX' ? Why?

This means that $\overline{AB}$ divides $\overline{XX'}$ into two parts of equal length. $\overline{AB}$ bisects $\overline{XX'}$ or $\overline{AB}$ is a bisector of $\overline{XX'}$. Note also that $\angle$ AOX and $\angle$ BOX are right angles. (Why?).

Hence, $\overline{AB}$ is the perpendicular bisector of $\overline{XX'}$. We see only a part of $\overline{AB}$ in the figure. Is the perpendicular bisector of a line joining two points the same as the axis of symmetry?

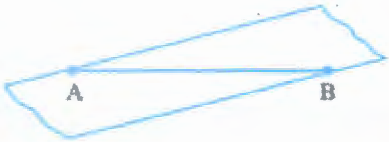


Do This

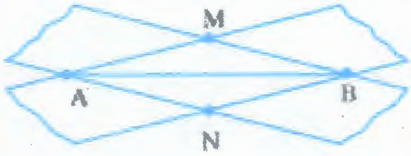
(Transparent tapes)

Step 1 Draw a line segment $\overline{AB}$.

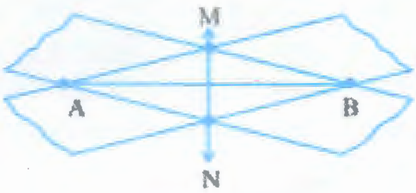
Step 2 Place a strip of a transparent rectangular tape diagonally across with the edges of the tape on the end points A and B, as shown in the figure.



Step 3 Repeat the process by placing another tape over A and B just diagonally across the previous one. The two strips cross at M and N.



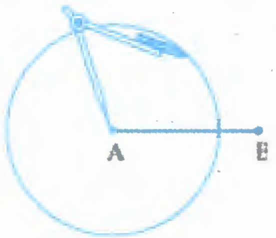
Step 4 Join M and N. Is $\overline{MN}$ a bisector of $\overline{AB}$? Measure and verify. Is it also the perpendicular bisector of $\overline{AB}$? Where is the mid point of $\overline{AB}$?



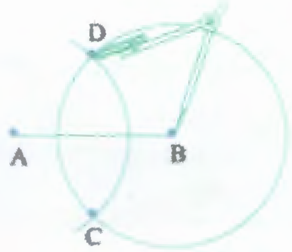
Construction using ruler and compasses

Step 1 Draw a line segment $\overline{AB}$ of any length.

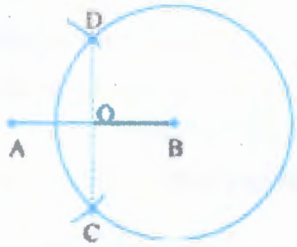
Step 2 With A as centre, using compasses, draw a circle. The radius of your circle should be more than half the length of $\overline{AB}$.



Step 3 With the same radius and with B as centre, draw another circle using compasses. Let it cut the previous circle at C and D.



Step 4 Join $\overline{CD}$. It cuts AB at O. Use your divider to verify that O is the midpoint of $\overline{AB}$. Also verify that $\angle$COA and $\angle$COB are right angles. Therefore, $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.



In the above construction, we needed the two points C and D to determine $\overline{CD}$. Is it necessary to draw the whole circle to find them? Is it enough if we draw merely small arcs to locate them? In fact, that is what we do in practice?

Try These

In Step 2 of the construction using ruler and compasses, what would happen if we take the length of radius to be smaller than half the length of $\overline{AB}$?

EXERCISE 11.5

1. Draw $\overline{AB}$ of length 7.3 cm and find its axis of symmetry.
2. Draw a line segment of length 9.5 cm and construct its perpendicular bisector.
3. Draw the perpendicular bisector of $\overline{XY}$ whose length is 10.3 cm.
   (a) Take any point P on the bisector drawn. Examine whether PX = PY.
   (b) If M is the mid point of $\overline{XY}$, what can you say about the lengths MX and XY?
4. Draw a line segment of length 12.8 cm. Using compasses, divide it into four equal parts. Verify by actual measurement.

5. With $\overline{PQ}$ of length 6.1 cm as diameter, draw a circle.
6. Draw a circle with centre C and radius 3.4 cm. Draw any chord $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$ and examine if it passes through C.

11.5 Angles

11.5.1 Constructing an angle of a given measure

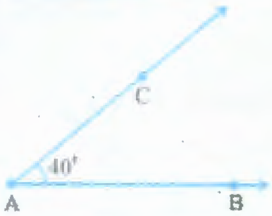
Suppose we want an angle of measure 40°. Here are the steps to follow:

Step 1 Draw $\overline{AB}$ of any length.

Step 2 Place the centre of the protractor at A and the and the zero edge along $\overline{AB}$.

Step 3 Start with zero near B. Mark point C at 40°.

Step 4 Join AC. $\angle$BAC is the required angle.



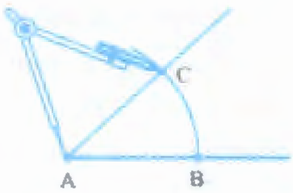
11.5.2 Constructing a copy of an angle of unknown measure

Suppose an angle (whose measure we do not know) is given and we want to make a copy of this angle. As usual, we will have to use only a straight edge and the compasses.

Given $\angle$A, whose measure is not known.

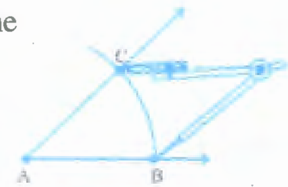
Step 1 Draw a line *l* and choose a point P on it.

Step 2 Place the compasses at A and draw an arc to cut the rays of ∠A at B and C.



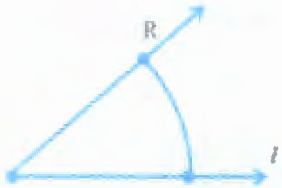
Step 3 Use the same compasses setting to draw an arc with P as centre, cutting *l* in Q.

Step 4 Set your compasses to the length BC with the same radius.



Step 5 Place the compasses pointer at Q and draw the arc to cut the arc drawn earlier in R.

Step 6 Join PR. This gives us ∠ P. It has the same measure as ∠ A.



This means ∠ QPR has same measure as ∠ BAC.

11.5.3 Bisector of an angle

Do This

Take a sheet of paper. Mark a point O on it. With O as initial point, draw two rays $\overline{OA}$ and $\overline{OB}$. You get ∠AOB. Fold the sheet through

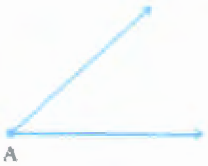
O such that the rays $\overline{OA}$ and $\overline{OB}$ coincide. Let OC be the crease of paper which is obtained after unfolding the paper.

OC is clearly a line of symmetry for $\angle$AOB.

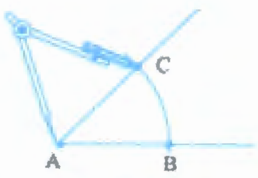
Measure $\angle$AOC and $\angle$COB. Are they equal? OC the line of symmetry, is therefore known as the angle bisector of $\angle$ AOB.

Construction with ruler and compasses

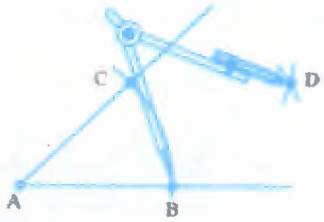
Let an angle, say, $\angle$A be given.



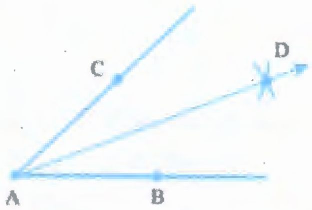
Step 1 With A as centre and using compasses, draw an arc that cuts both rays of $\angle$A . Label the points intersection as B and C.



Step 2 With B as centre, draw (in the interior of $\angle$A) an arc whose radius is more than half the length BC.



Step 3 With the same radius and with C as centre, draw another arc in the interior of $\angle$ A . Let the two arcs intersect at D. Then $\overline{AD}$ is the required bisector of $\angle$A .



> **Try These**
>
> In step 2 above, what would happen if we take radius to be smaller than half the length BC?

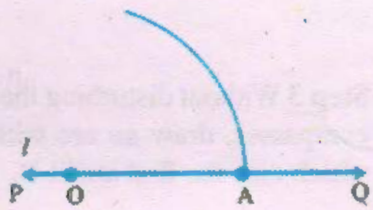
11.5.4 Angles of special measures

There are some elegant and accurate methods to construct some angles of special sizes which do not require the use of the protractor. We discuss a few here.

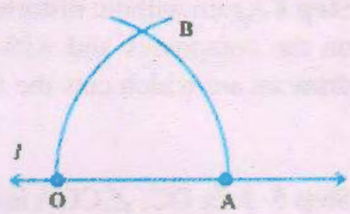
Constructing a 60° angle

Step 1 Draw a line l and mark a point O on it.

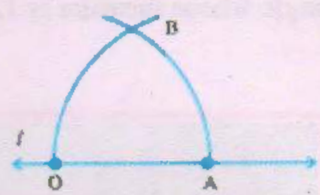
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line $\overline{PQ}$ at a point say, A.



Step 3 With the pointer at A (as centre), now draw an arc that passes through O.



Step 4 Let the two arcs intersect at B. Join OB. We get $\angle$BOA whose measure is 60°.



Constructing a 30° angle

Construct an angle of 60° as shown earlier. Now, bisect this angle. Each angle is 30°, verify by using a protractor.

> **Try these**
>
> How will you construct a 15° angle?

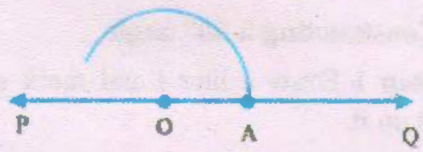
Constructing a 120° angle

An angle of 120° is nothing but twice of an angle of 60°.

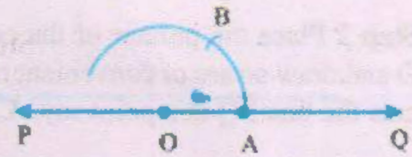
Therefore, it can be constructed as follows :

Step 1 Draw any line PQ and take a point O on it.

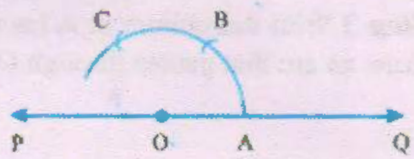
Step 2 Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line at A.



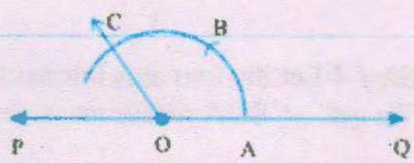
Step 3 Without disturbing the radius on the compasses, draw an arc with A as centre which cuts the first arc at B.



Step 4 Again without disturbing the radius on the compasses and with B as centre, draw an arc which cuts the first arc at C.



Step 5 Join OC, $\angle$COA is the required angle whose measure is 120°.



Try these

How will you construct a 150° angle?

Constructing a 90° angle

Construct a perpendicular to a line from a point lying on it, as discussed earlier. This is the required 90° angle.

Try These

How will you construct a 45° angle

1. Draw $\angle$ POQ of measure 75° and find its line of symmetry.
2. Draw an angle of measure 147° and construct its bisector.
3. Draw a right angle and construct its bisector.

4. Draw an angle of measure 153° and divide it into four equal parts.
5. Construct with ruler and compasses, angles of following measures:
   (a) 60° (b) 30° (c) 90° (d) 120° (e) 45° (f) 135°
6. Draw an angle of measure 45° and bisect it.
7. Draw an angle of measure 135° and bisect it.

What have we discussed?

This chapter deals with methods of drawing geometrical shapes.

1. We use the following mathematical instruments to construct shapes:
   (i) A graduated ruler (ii) The compasses
   (iii) The divider (iv) Set-squares
   (v) The protractor
2. Using the ruler and compasses, the following constructions can be made:
   (i) A circle, when the length of its radius is known.
   (ii) A line segment, if its length is given.
   (iii) A copy of a line segment.
   (iv) A perpendicular to a line through a point
   (a) on the line (b) not on the line.
   (v) The perpendicular bisector of a line segment of given length.
   (vi) An angle of a given measure.
   (vii) A copy of an angle.
   (viii) The bisector of a given angle.
   (ix) Some angles of special measures such as
   (a) 90° (b) 45° (c) 60° (d) 30° (e) 120°
   (f) 135°

ANSWERS

EXERCISE 1.1

1. 11,000; 11,001; 11,002

2. 10,000; 9,999; 9,998

3. 0

4. 20

5. (a) 24,40,702 (b) 1,00,200
(c) 11, 000, 00 (d) 23, 45,671

6. (a) 93 (b) 9,999
(c) 2, 08,089 (e) 76, 54,320

7. (a) 503 is on the left of 530 ; 530 > 503
(b) 307 is on the left of 370; 370 > 307
(c) 56,789 is on the left of 98,765; 98,765 > 56,789
(d) 98,30,415 is on the left of 1,00,23,001 ; 98,30,415 < 1,00,23,001

8. (a) F (b) F (c) T (d) T (e) T
(f) F (g) F (h) F (i) T (j) F
(k) F (l) T (m) F

EXERCISE 1.2

1. (a) 1,408 (b) 4,600

2. (a) 1,76,800 (b) 16,600 (c) 2,91,000 (d) 27,90,000
(e) 85,500 (f) 10, 00,000

3. (a) 5,940 (b) 54,27,900 (c) 81,26,500 (d) 1,92,25,000

4. (a) 76,014 (b) 87,108 (c) 2,60,064 (d) 1,68,840

EXERCISE 1.3

1. (a) ____ **2.** Yes

3. Both of them will be ‘l’

4. (a) 73,528 (b) 54,42,437 (c) 20,600 (d) 5,34,375
(e) 17,640

5. $123456 \times 8 + 6 = 987654$

$1234567 \times 8 + 7 = 9876543$

EXERCISE 2.1

1. (a) 1, 2, 3, 4, 6, 8, 12, 24 (b) 1, 3, 5, 15

(c) 1, 3, 7, 21 (d) 1, 3, 9, 27

(e) 1, 2, 3, 4, 6, 12 (f) 1, 2, 4, 5, 10, 20

(g) 1, 2, 3, 6, 9, 18 (h) 1, 23

(i) 1, 2, 3, 4, 6, 9, 12, 18, 36

2. (a) 5, 10, 15, 20, 25 (b) 8, 16, 24, 32, 40 (c) 9, 18, 27, 36, 45

3. (i) → (b) (ii) → (d) (iii) → (a)

(iv) → (f) (v) → (e)

4. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99

EXERCISE 2.2

1. (a) even number (b) even number

2. (a) F (b) T (c) T (d) F

(e) F (f) F (g) F (h) T

(i) F (j) T

3. 17 and 71, 37 and 73, 79 and 97

4. Prime numbers : 2, 3, 5, 7, 11, 13, 17, 19

Composite numbers : 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

5. 7

6. (a) 3 + 41 (b) 5 + 31 (c) 5 + 19 (d) 5 + 13

(This could be one of the ways. There can be other ways also.)

7. 3, 5; 5, 7 ; 11, 13

8. (a) prime number (b) composite number

(c) prime number, composite number

(d) 2 (e) 4 (f) 2

EXERCISE 2.3

1.

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
429714	Yes	Yes	No	No	Yes	No	Yes	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	No	No	No

2. (a), (f), (g), (i) **3.** (a) 2 and 8 (b) 0 and 9 **4.** (a) 8 (b) 6

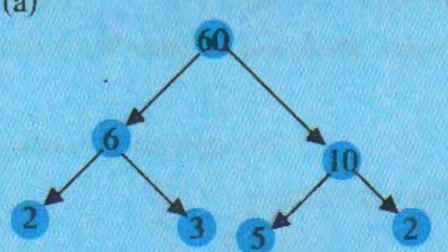
EXERCISE 2.4

1. (a) 1, 2, 4 (b) 1, 5 (c) 1, 5 (d) 1, 2, 4, 8
2. (a) 1, 2, 4 (b) 1, 5
3. (a) 24, 48, 72 (b) 36, 72, 108
4. 12, 24, 36, 48, 60, 72, 84, 96
5. (a), (b), (e), (f)

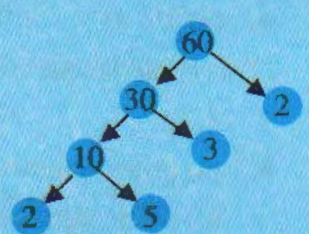
EXERCISE 2.5

1. (a) F (b) T (c) F (d) T
   (e) F (f) F (g) T (h) T
   (i) T (j) F

2. (a)



(b)



3. 1 and the number itself
4. 9999, $9999 = 3 \times 3 \times 11 \times 101$
5. 10000, $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
6. (i) $2 \times 3 \times 4 = 24$ is divisible by 6.
   (ii) $5 \times 6 \times 7 = 210$ is divisible by 6.
7. (c)
8. Yes
9. $2 \times 3 \times 5 \times 7 = 210$

EXERCISE 2.6

1. (a) 6 (b) 6 (c) 6 (d) 9
   (e) 12 (f) 34 (g) 35 (h) 7
   (i) 9 (j) 3
2. (a) 1 (b) 2 (c) 1
3. No; 1

EXERCISE 2.7

1. 3 kg **2.** 75 cm **3.** 120 **4.** 960
5. 7 minutes 12 seconds past 7 a.m. **6.** 95 **7.** 1152
8. (a) 36 (b) 60 (c) 30 (d) 60

EXERCISE 3.1

1. (a) O, B, C, D, E.
   (b) Many answers are possible. Some are $\overleftrightarrow{DE}, \overleftrightarrow{DO}, \overleftrightarrow{DB}, \overleftrightarrow{EO}$ etc.
   (c) Many answers are possible. Some are: $\overrightarrow{DB}, \overrightarrow{DE}, \overrightarrow{OB}, \overrightarrow{OE}, \overrightarrow{EB}$ etc.
   (d) Many answers are possible. Some are: $\overline{DE}, \overline{DO}, \overline{EO}, \overline{OB}, \overline{EB}$ etc.
2. $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{AD}, \overleftrightarrow{BA}, \overleftrightarrow{BC}, \overleftrightarrow{BD}, \overleftrightarrow{CA}, \overleftrightarrow{CB}, \overleftrightarrow{CD}, \overleftrightarrow{DA}, \overleftrightarrow{DB}, \overleftrightarrow{DC}$
3. (a) Many answers. One answer is (b) Many answers. One answer is $\overleftrightarrow{AE}$ (c) $\overleftrightarrow{CO}$ or $\overleftrightarrow{OC}$
   (d) Many answers are possible. Some are, $\overleftrightarrow{CO}, \overleftrightarrow{AE}$ and $\overleftrightarrow{AE}, \overleftrightarrow{EF}$
4. (a) Countless (b) Only one.

5. (a) T (b) T (c) T (d) F (e) F
(f) F (g) T (h) F (i) F (j) F
(k) T

EXERCISE 3.2

1. Open: (a), (c); Closed: (b), (d), (e). **4.** (a) Yes (b) Yes

EXERCISE 3.3

1. $\angle A$ or $\angle DAB$; $\angle B$ or $\angle ABC$; $\angle C$ or $\angle BCD$; $\angle D$ or $\angle CDA$
2. (a) A (b) A, C, D. (c) E, B, O, F.

EXERCISE 3.4

2. (a) $\triangle ABC$, $\triangle ABD$, $\triangle ADC$.
(b) Angles: $\angle B$, $\angle C$, $\angle BAC$, $\angle BAD$, $\angle CAD$, $\angle ADB$, $\angle ADC$
(c) Line segments: $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, $\overline{AD}$, $\overline{BD}$, $\overline{DC}$
(d) $\triangle ABC$, $\triangle ABD$

EXERCISE 3.5

1. The diagonals will meet in the interior of the quadrilateral.
2. (a) $\overline{KL}$, $\overline{NM}$ *and* $\overline{KN}$, $\overline{ML}$ (b) $\angle K$, $\angle M$ and $\angle N$, $\angle L$
(c) $\overline{KL}$, $\overline{KN}$ *and* $\overline{NM}$, $\overline{ML}$, *or* $\overline{KL}$, $\overline{LM}$ *and* $\overline{NM}$, $\overline{NK}$,
(d) $\angle K$, $\angle L$ and $\angle M$, $\angle N$ or $\angle K$, $\angle L$ and $\angle L$, $\angle M$ etc.

EXERCISE 3.6

1. (a) O (b) $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, (c) $\overline{AC}$, (d) $\overline{ED}$,
(e) O, P (f) Q (g) OAB (Shaded portion)
(h) Segment ED (Shaded portion)
2. (a) Yes (b) No
4. (a) True (b) True

EXERCISE 4.1

1. Accurate measurement will be possible.
2. Yes. (because C is 'between' A and B).
3. B lies between A and C.

4. D is the mid point of $\overline{AG}$ (because, AD = DG = 3 units).
5. AB = BC and BC = CD, therefore, AB = CD

EXERCISE 4.2

1. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$
   (e) $\frac{3}{4}$ (f) $\frac{3}{4}$
2. (a) 6 (b) 8 (c) 8 (d) 2
3. (a) 1 (b) 2 (c) 2 (d) 1
   (e) 3 (f) 2

EXERCISE 4.3

1. (i) → (c); (ii) → (d); (iii) → (a); (iv) → (e); (v) → (b).
2. Acute : (a) and(f); Obtuse : (b); Right: (c); Straight: (e); Reflex : (d).

EXERCISE 4.4

1. (i) 90°; (ii) 180°.
2. (a) T (b) F (c) T (d) T (e) T
3. (a) Acute: 23°, 89°; (b) Obtuse: 91°, 179°.
7. (a) acute (b) obtuse (if the angle is less than 180°)
   (c) straight (d) acute (e) an obtuse angle
8. 90°, 30°, 180°

EXERCISE 4.5

1. (a) and (c) **2.** 90°
3. One is a 30° – 60° – 90° set square; the other is a 45° – 45° – 90° set square.
   The angle of measure 90° (i.e. a right angle) is common between them.
4. (a) Yes (b) Yes (c) BH, DF (d) All are true.

EXERCISE 4.6

1. (a) Scalene triangle (b) Scalene triangle (c) Equilateral triangle
   (d) Right triangle (e) Isosceles right triangle
   (f) Acute-angled triangle

2. (i) →(e); (ii) →(g); (iii) → (a); (iv)→ (f);
(v)→ (d); (vi) → (c); (vii)→ (b).

3. (a) Acute-angled and isosceles. (b) Right-angled and scalene.
(c) Obtuse-angled and isosceles. (d) Right-angled and isosceles.
(e) Equilateral and acute angled. (f) Obtuse-angled and scalene.

4. (b) is not possible. (Remember: The sum of the lengths of any two sides of a triangle has to be greater than the third side.)

EXERCISE 4.7

1. (a) T (b) T (c) T (d) T (e) F
(f) F

2. (a) A rectangle with all sides equal becomes a square.
(b) A parallelogram with each angle a right angle becomes a rectangle.
(c) A rhombus with each angle a right angle becomes a square.
(d) All these are four-sided polygons made of line segments.
(e) The opposite sides of a square are parallel, so it is a parallelogram.

3. A square is a 'regular' quadrilateral

EXERCISE 4.8

1. (a) is not a closed figure and hence is not a polygon.
(b) is a polygon of six sides.
(c) and (d) are not polygons since they are not made of line segments.

2. (a) A Quadrilateral (b) A Triangle (c) A Pentagon (5-sided)
(d) An Octagon

EXERCISE 4.9

1. (a) →(ii); (b)→ (iv); (c)→(v); (d) →(iii); (e)→ (i).

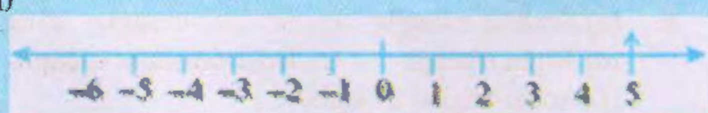
2. (a), (b) and (c) are cuboids; (d) is a cylinder; (e) is a sphere.

EXERCISE 5.1

1. (a) Decrease in weight (b) 30 km south (c) 326 A.D.
(d) Gain of Rs 700 (e) 100 m below sea level

2. (a) + 2000 (b) – 800 (c) + 200
(d) – 700

3. (a) + 5

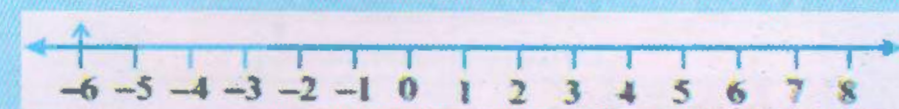


(b) – 10

(c) + 8

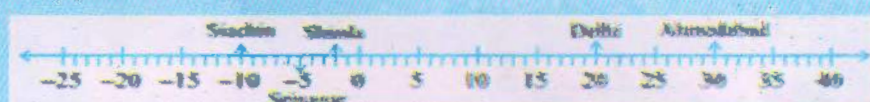
(d) – 1

(e) – 6



4. (a) F (b) negative integer
(c) B → + 4, E → – 10 (d) E (e) D, C, B, A, O, H, G, F, E

5. (a) – 10°C, – 2°C, + 30°C, + 20°C, – 5°C
(b)



(c) Siachin (d) Ahmedabad and Delhi

6. (a) 9 (b) – 3 (c) 0 (d) 10
(e) 6 (f) 1

7. (a) – 6, – 5, – 4, – 3, – 2, – 1 (b) – 3,– 2, – 1, 0, 1, 2, 3
(c) – 14, – 13, – 12, – 11, – 10, – 9
(d) – 29, – 28, – 27, – 26, – 25, – 24

8. (a) – 19, – 18, – 17, – 16 (b) – 11, – 12, – 13, – 14

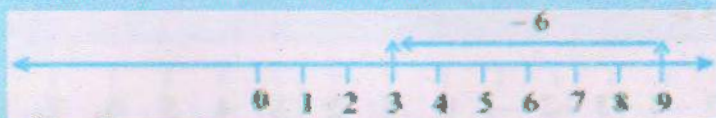
9. (a) T (b) F; – 100 is to the left of – 10 on number line
(c) F; greatest negative integer is – 1 (d) F; – 26 is smaller than – 25

10. (a) 2 (b) – 4 (c) to the left (d) to the right

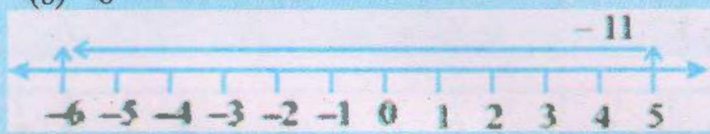
EXERCISE 5.2

1. (a) 8 (b) 0 (c) – 4 (d) – 5

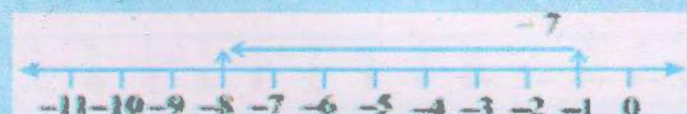
2. (a) 3



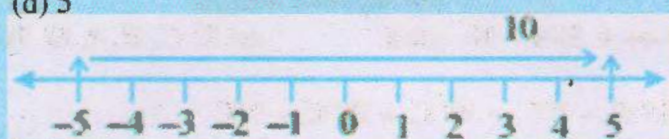
(b) – 6



(c) – 8

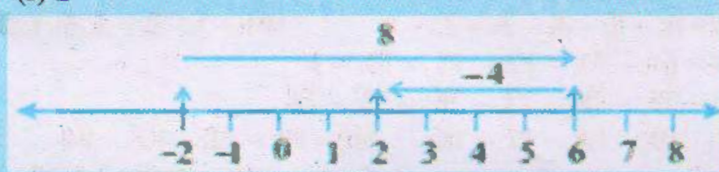


(d) 5



(e) – 6

(f) 2



3. (a) 4 (b) 5 (c) 9 (d) –100
(e) – 650 (f) – 317

4.	(a) – 217	(b) 0	(c) – 81	(d) 50	
5.	(a) 4	(b) –38			

EXERCISE 5.3

1.	(a) 15	(b) – 18	(c) 3	(d) – 33	(e) 35
	(f) 8				
2.	(a) <	(b) >	(c) >	(d) >	
3.	(a) 8	(b) – 13	(c) 0	(d) – 8	(e) 5
4.	(a) 10	(b) 10	(c) – 105	(d) 92	

EXERCISE 6.1

1.	(i) $\frac{1}{2}$	(ii) $\frac{2}{4}$	(iii) $\frac{8}{9}$	(iv) $\frac{4}{8}$
	(v) $\frac{1}{6}$	(vi) $\frac{1}{4}$	(vii) $\frac{3}{7}$	(viii) $\frac{3}{12}$
	(ix) No part	(x) $\frac{4}{9}$	(xi) $\frac{4}{8}$	(xii) $\frac{1}{2}$

3. Shaded portions do not represent the given fractions.

4. $\frac{8}{24}$ **5.** $\frac{40}{60}$

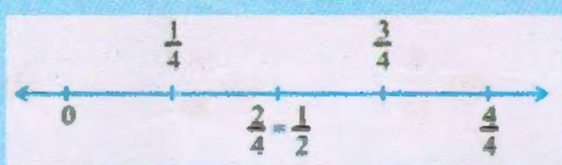
6. (a) Arya will divide each sandwich into three equal parts, and give one part of each sandwich to each one of them.

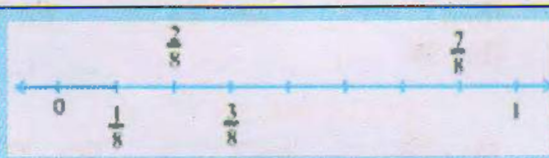
(b) $\frac{1}{3}$

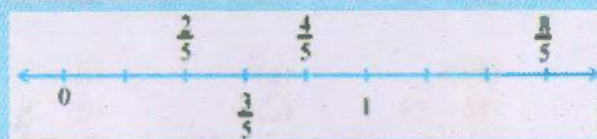
7. $\frac{5}{10}$

EXERCISE 6.2

1. (a)



(b) 

(c) 

2. (a) $6\frac{2}{3}$ (b) $2\frac{1}{5}$ (c) $2\frac{6}{7}$ (d) $5\frac{3}{5}$
(e) $3\frac{1}{6}$ (f) $3\frac{8}{9}$

3. (a) $\frac{31}{4}$ (b) $\frac{41}{7}$ (c) $\frac{17}{6}$ (d) $\frac{53}{5}$
(e) $\frac{66}{7}$ (f) $\frac{76}{9}$

EXERCISE 6.3

1. (a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; Yes (b) $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}, \frac{6}{15}$; No

2. (a) $\frac{1}{2}$ (b) $\frac{4}{6}$ (c) $\frac{3}{9}$ (d) $\frac{2}{8}$
(e) $\frac{3}{4}$ (i) $\frac{6}{18}$ (ii) $\frac{4}{8}$ (iii) $\frac{12}{16}$
(iv) $\frac{8}{12}$ (v) $\frac{4}{16}$
(a), (ii); (b), (iv); (c), (i); (d), (v); e), (iii)

3. (a) 28 (b) 16 (c) 12 (d) 20
(e) 3

4. (a) $\frac{12}{20}$ (b) $\frac{9}{15}$ (c) $\frac{18}{30}$ (d) $\frac{27}{45}$

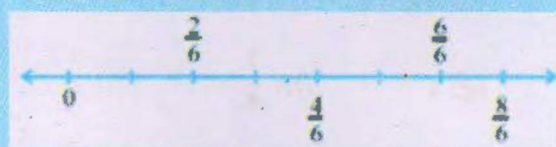
5. (a) equivalent (b) not equivalent (c) not equivalent

6. (a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{13}$ (e) $\frac{1}{4}$

EXERCISE 6.4

1. (a) $\frac{1}{8} < \frac{3}{8} < \frac{4}{8} < \frac{6}{8}$ (b) $\frac{3}{9} < \frac{4}{9} < \frac{6}{9} < \frac{8}{9}$

(c)



$\frac{5}{6} > \frac{2}{6}$, $\frac{3}{6} > \frac{2}{6}$, $\frac{1}{6} < \frac{6}{6}$, $\frac{8}{6} > \frac{5}{6}$

2. (a) $\frac{3}{6} < \frac{5}{6}$ (b) $\frac{1}{7} < \frac{1}{4}$ (c) $\frac{4}{5} > \frac{0}{5}$ (d) $\frac{3}{20} < \frac{4}{20}$

4. (a) $\frac{2}{6} (= \frac{1}{3}), \frac{3}{6} (= \frac{1}{2}), \frac{1}{6}, \frac{5}{6}, \frac{4}{6}, \frac{0}{6}, \frac{6}{6}$; $\frac{0}{6} < \frac{1}{6} < \frac{2}{6} < \frac{3}{6} < \frac{4}{6} < \frac{5}{6} < \frac{6}{6}$

5. (a) $\frac{1}{6} < \frac{1}{3}$ (b) $\frac{3}{4} > \frac{2}{6}$ (c) $\frac{2}{3} > \frac{2}{4}$ (d) $\frac{6}{6} = \frac{3}{3}$

(e) $\frac{5}{6} < \frac{5}{5}$

6. (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{25}$

(e) $\frac{1}{6}$ (f) $\frac{1}{5}$ (g) $\frac{1}{5}$ (h) $\frac{1}{6}$

(i) $\frac{4}{25}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $\frac{4}{25}$

(a), (e), (h), (j), (k), ; (b), (f), (g), (c), (d), (i), (l)

7. Ila reads less 8. Rohit

EXERCISE 6.5

1. (a) + (b) − (c) +

2. (a) $\frac{3}{5}+\frac{3}{5}=\frac{6}{5}$ (b) $\frac{8}{8}+\frac{8}{8}+\frac{2}{8}=\frac{18}{8}=\frac{9}{4}$

3. (a) $\frac{1}{6}$ (b) $\frac{11}{15}$ (c) $\frac{2}{7}$ (d) 1 (e) $\frac{11}{9}$

(f) $\frac{1}{3}$ (g) 1 (h) $\frac{7}{5}$ (i) $\frac{1}{3}$ (j) $\frac{1}{4}$

(k) $\frac{0}{2}$ (=0) (l) $\frac{9}{5}$ (m) $\frac{2}{3}$ (n) $\frac{3}{5}$

4. The complete wall. 5. 6 kg

6. (a) $\frac{4}{10}$ $(=\frac{2}{5})$ (b) $\frac{8}{21}$ (c) $\frac{6}{6}$ (=1) (d) $\frac{7}{27}$

7. $\frac{1}{5}$ 8. $\frac{2}{7}$

EXERCISE 6.6

1. (a) $\frac{17}{21}$ (b) $\frac{23}{30}$ (c) $\frac{46}{63}$ (d) $\frac{22}{21}$

(e) $\frac{17}{30}$ (f) $\frac{22}{15}$ (g) $\frac{5}{12}$ (h) $\frac{3}{6}$ $(=\frac{1}{2})$

(i) $\frac{3}{10}$ (j) $\frac{1}{6}$ (k) $\frac{2}{6}$ $(=\frac{1}{3})$ (l) $\frac{10}{24}$ $(=\frac{5}{12})$

(m) $\frac{23}{12}$ (n) $\frac{6}{6}$ (=1) (o) 5 (p) $\frac{95}{12}$

(q) $\frac{9}{5}$ (r) $\frac{5}{6}$ (s) $\frac{2}{3}$ (t) $\frac{6}{3}$ (=2)

2. $\frac{23}{20}$ metre **3.** $\frac{17}{6}$

4. (a) $\frac{7}{8}$ (b) $\frac{7}{10}$ (c) $\frac{1}{3}$

5. Length of the other piece = $\frac{5}{8}$ metre

6. The distance walked by Nandini = $\frac{4}{10}$ $(=\frac{2}{5})$ km

EXERCISE 7.1

1.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	3	1	2
(b)	1	1	0	4

2.

	Hundreds (100)	Tens (10)	Ones (1)	Tenths $(\frac{1}{10})$
(a)	0	1	9	4
(b)	0	0	0	3
(c)	0	1	0	6
(d)	2	0	5	9

3. (a) 0.7 (b) 20.9 (c) 14.6 (d) 102.0 (e) 600.8

4. (a) 0.5 (b) 3.7 (c) 265.1 (d) 3.6 (e) 70.8
(f) 8.8 (g) 4.2 (h) 1.5 (i) 0.4 (j) 2.4
(k) 3.6 (l) 4.5

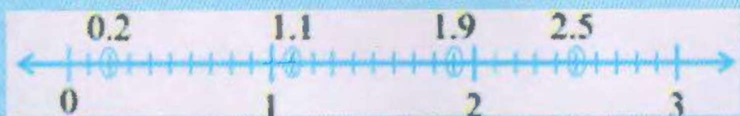
5. (a) $\frac{6}{10}, \frac{3}{5}$ (b) $\frac{25}{10}, \frac{5}{2}$ (c) 1, 1 (d) $\frac{38}{10}, \frac{19}{5}$ (e) $\frac{137}{10}, \frac{137}{10}$

(f) $\frac{212}{10}, \frac{106}{5}$ (g) $\frac{64}{10}, \frac{32}{5}$

6. (a) 0.2cm (b) 3.0 cm (c) 11.6 cm (d) 4.2 cm
(e) 16.2 cm (f) 8.3 cm

7. (a) 0 and 1; 1 (b) 5 and 6; 5 (c) 2 and 3; 3
(d) 6 and 7; 6 (e) 9.0 itself is a whole number (f) 4 and 5; 5

8.



9. A, 0.8 cm; B, 1.3 cm; C, 2.2 cm; D, 2.9 cm

10. (a) 9.5 cm (b) 6.5 cm

EXERCISE 7.2

1.

	Ones	Tenths	Hundredth	Number
(a)	0	2	6	0.26
(b)	1	3	8	1.38
(c)	1	2	8	1.28

2. (a) 3.25 (b) 102.63 (c) 30.025 (d) 211.902
(e) 12.241

3.

	Hundredths	Tens	Ones	Tenths	Hundredth	Thousands
(a)	0	0	0	2	9	0
(b)	0	0	2	0	8	0
(c)	0	1	9	6	0	0
(d)	1	4	8	3	2	0
(e)	2	0	0	8	1	2

4. (a) 29.41 (b) 30.483 (c) 137.05 (d) 0.764
(e) 23.206 (f) 725.09

5. (a) Zero point zero three
(b) One point two zero
(c) Seventeen point three eight
(d) One hundred eight point five six
(e) Ten point zero seven
(f) Zero point zero three two
(g) Two hundred ten points one zero nine
(h) Five point zero zero eight

EXERCISE 7.3

1. (a) 0.4 (b) 0.07 (c) 3 (d) 0.5
(e) 0.11 (f) 2.012 (g) 1 (h) 1.23
(i) 0.19 (j) both and same (k) 1.490 (l) both are same
(m) 5.64 (n) 1.8000 (o) 2.05

EXERCISE 7.4

1. (a) Rs. 0.05 (b) Rs 0.75 (c) Rs 3.60 (d) Rs 4.50
(e) Rs. 0.20 (f) Rs. 50.90 (g) Rs 7.25
2. (a) 0.15 m (b) 0.06 m (c) 1.36 m (d) 2.45 m
(e) 9.07 m (f) 4.19 m
3. (a) 0.5 cm (b) 6.0 cm (c) 16.4 cm (d) 9.8 cm
(e) 16.7 cm (f) 9.3 cm
4. (a) 0.008 km (b) 0.088 km (c) 0.888 km (d) 8.888 km
(e) 70.005 km (f) 29.037 km
5. (a) 0.002 kg (b) 0.1 kg (c) 3.750 kg (d) 2.7 kg
(e) 5.008 kg (f) 26.05 kg
6. (a) 230 paisa (b) 9240 g (c) 35 mm (d) 3050 m
(e) 881 cm (f) 1305 paisa (g) 15038 m (h) 14007 g
(i) 1106 cm (j) 2 mm

EXERCISE 7.5

1. (a) 38.587 (b) 29.432 (c) 27.63 (d) 38.355
(e) 13.175 (f) 343.89
2. Rs 68.35 3. Rs 26.30 4. 5.25 m 5. 3.75 m

EXERCISE 7.6

1. (a) Rs 2.50 (b) 47.46 m (c) Rs 3.04 (d) 3.155 km
(e) 1.793 kg

2. (a) 3.476 (b) 5.78 (c) 11.71 (d) 1.753

3. Rs 14.35 **4.** Rs 6.75

5. 15.55 m **6.** 9.850 km

7. 4.425 kg **8.** 3.042 km

9. 22.775 km **10.** 18.270 kg

EXERCISE 8.1

1. (a) 12 cm (b) 133 cm (c) 60 cm (d) 15 cm
(e) 15 cm (f) 52 cm

2. 100 cm or 1 m **3.** 7.5 m **4.** 106 cm

5. 9.6 km **6.** (a) 12 cm (b) 27 cm (c) 22 cm

7. 39 cm **8.** 5 m **9.** 20 cm

10. (a) 7.5 cm (b) 10 cm (c) 5 cm

11. 10 cm

12. Rs 20,000

EXERCISE 8.2

1. (a) 9 sq units (b) 5 sq units (c) 4 sq units (d) 8 sq units
(e) 10 sq units (f) 4 sq units (g) 6 sq units (h) 5 sq units
(i) 9 sq units (j) 4 sq units (k) 5 sq units (l) 8 sq units
(m) 14 sq units (n) 18 sq units

EXERCISE 8.3

1. (a) 12 sq cm (b) 252 sq cm (c) 6 sq km (d) 7.29 sq m

2. (a) 100 sq cm (b) 196 sq cm (c) 25 sq m

3. 6 m **4.** Rs 8000 **5.** 3.375 sq m **6.** 15.33 sq m

7. 12.96 sq m **8.** (a) 240 (b) 42

EXERCISE 9.1

1. (a) $2n$ (b) $3n$ (c) $3n$ (d) $2n$
(e) $5n$ (f) $5n$ (g) $6n$

2. (a) and (d); The number of matchsticks required in each of them is 2

3. $5n$ **4.** $50b$ **5.** $5s$

6. t km **7.** $2x + 10$

EXERCISE 9.2

1. $3l$ **2.** $6l$ **3.** $(a + b) + c = a + (b + c)$

EXERCISE 9.3

2. (c), (d)

3. (a) Addition, subtraction, addition, subtraction
(b) Multiplication, division, multiplication
(c) Multiplication and addition, multiplication and subtraction
(d) Multiplication, multiplication and addition, multiplication and subtraction

4. (a) $p + 7$ (b) $p - 7$ (c) $7p$ (d) $\frac{p}{7}$
(e) $-m - 7$ (f) $-5p$ (g) $\frac{-p}{5}$ (h) $-5p$

5. (a) $2m + 11$ (b) $2m - 11$ (c) $5y + 3$ (d) $5y - 3$
(e) $-8y$ (f) $-8y + 5$ (g) $16 - 5y$ (h) $-5y + 16$

EXERCISE 9.4

1. (a) (i) $y + 5$ (ii) $y - 3$ (iii) $6y$ (iv) $6y - 2$
(v) $3y + 5$
(b) $(3b - 4)$ metres (c) length = $5h$ cm,
breadth = $5h - 10$ cm
(d) $s + 8$, $s - 7$, $4s - 10$
(e) $(5v + 20)$ km

EXERCISE 9.5

1. (a) an equation with variable x
(d) an equation with variable x
(e) an equation with variable x
(g) an equation with variable n
(h) an equation with variable p
(k) an equation with variable x

2.	(a) No	(b) Yes	(c) No	(d) Yes	(e) No
		(f) Yes	(g) Yes	(h) No	(i) No(j) No
		(k) Yes			
3.	(a) 12	(b) 8	(c) 10	(d) 14	(e) 4
	(f) – 2				
4.	(a) 6	(b) 7			
5.	(a) 7	(b) 12	(c) 30	(d) 10	
	(e) 25	(f) 50			

EXERCISE 10.1

1.	(a) 4 : 3	(b) 4 : 7			
2.	(a) 1 : 2	(b) 2 : 5			
3.	(a) 3 : 2	(b) 2 : 7	(c) 2 : 7		
4.	3 : 4	**5.** (a) 3 : 4	(b) 14 : 9	(c) 3 : 11	(d) 2 : 3
6.	(a) 1 : 3	(b) 4 : 15	(c) 11 : 20	(d) 1 : 4	
7.	(a) 3 : 1	(b) 1 : 2			
8.	17 : 550				
9.	(a) 3 : 1	(b) 16 : 15	(c) 5 : 12		
10.	15 : 7				
11.	(a) 3 : 1	(b) 10 : 3	(c) 13 : 6	(d) 15 : 1	

EXERCISE 10.2

1.	(a) Yes	(b) No	(c) Yes	(d) Yes	
2.	(a) T	(b) T	(c) F	(d) T	
3.	(a) T	(b) T	(c) T	(d) T	(e) F

4. (a) Yes, Middle Terms – 1 m, Rs 40; Extreme Terms – 25 cm, Rs 160

(b) Yes, Middle Terms – 65 litres, 6 bottles; Extreme Terms – 39 litres, 10 bottles

(c) No.

(d) Yes, Middle Terms – 2.5 litres, Rs 4 ; Extreme Terms – 200 ml, Rs 50

EXERCISE 10.3

1. Rs 210 **2.** Rs 4500 **3.** 644 mm

4. (a) Rs 48.80 (b) 10 kg

5. 5 degrees