

#### P445(H/E) HIGHER MATHEMATICS 2015

Time: 3 Hours | Class: 12th [M. M.: 100

[astructions-(i) All questions are compulsory. (ii) Read instructions carefully of the question paper and then answers of the questions. (iii) Question paper has two sections - Section - 'A' and Section - 'B'. (iv) In the Section - 'A' Question Nos. 1 to 5 are objective type, which contain the - choose the correct option, answer in one word/sentence, fill in the blanks, True/False and match the columns. Each question carries 5 marks. (v) In the Section - 'B' question Nos. 6 to 24 has Internal option. (vi) Q.Nos. 6 to 10 carry 2 marks each. (vii) Q.Nos. 11 to 17 carry 4 marks each. (viii) Q.Nos. 18 to 22 carry 5 marks each. (ix) Q.Nos. 23 and 24 carry 6 marks each.

Section'A

Q.1. Choose the correct options-

 $(5 \times 1 = 5)$ 

(a) Fraction form of  $\frac{1}{(x+3)(x+4)}$  is-

(i) 
$$\frac{1}{(x+3)} + \frac{1}{(x+4)}$$

(ii) 
$$\frac{1}{(x+3)} - \frac{1}{(x+4)}$$

(iii) 
$$\frac{1}{(x+4)} - \frac{1}{(x+3)}$$

(iv) 
$$\frac{1}{2} \left[ \frac{1}{x+3} + \frac{1}{x+4} \right]$$

- (b) The perpendicular distance of the plane 3x 6y + 5z = 12 from origin is be-
  - (i)  $\frac{-\sqrt{70}}{12}$

(ii)  $\frac{-12}{\sqrt{70}}$ 

(iii)  $\frac{12}{\sqrt{70}}$ 

(iv)  $\frac{\sqrt{70}}{12}$ 

(c) The unit vector in the direction of " $\hat{i} + \hat{j} + \hat{k}$ " is be-

(i) 
$$\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

(ii) 
$$\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$$

(iii) 
$$\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$$

(iv) 
$$\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$$

- (d) Differential coefficient of "log (sin x)" with respect to 'x' is-
  - (i) cot x

(ii) cosec x

(iii) tan x

- (iv) sec x
- (e) By Newton-Raphson's method the formula for finding the square root of any number "y" is-

(i) 
$$x_{n+1} = \frac{1}{2} \left[ x_n + \frac{y}{x_n} \right]$$

(ii) 
$$x_{n+1} = \frac{1}{2} \left[ x_0 + \frac{y}{x_0} \right]$$

((ii) 
$$x_{n+1} = \frac{1}{3} \left[ 2x_n + \frac{y}{x_n^2} \right]$$

(iv) 
$$x_{p+1} = \frac{1}{3} \left[ 2x_0 + \frac{y}{x_0^2} \right]$$

Q.2. Answers in one word/sentences- 
$$(5 \times 1 = 5)$$

(i) Write the equation of a straight line which passes through the point (2, 1, 3) and has direction-rations (1, 3, 2)

- (ii) If a, b, c are the position vectors of the vertises of the triangle ABC, then write the formula of the area of ΔABC.
- (iii) Write the value of  $\int \frac{dx}{ax+1}$ .
- (iv) Define the positive co-relation.
- (v) What is the value of √12 by Newton-Raphson's method after first iteration?
- 03. Fill in the blanks-

$$(5 \times 1 = 5)$$

- (i) Is be  $\sin^{-1} x + \cos^{-1} x = \dots$
- (ii) Sphere  $3x^2 + 3y^2 + 3z^2 6x 12y + 6z + 2 = 0$  has centre ......
- (iii) If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are coplanar then  $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$  will be ..........

- QA. Write the True/False-

$$(5 \times 1 = 5)$$

- (i) Distance the point P (x, y, z) from the plane- X-Y is be  $\sqrt{x^2 + y^2 + z^2}$ .
- (ii) Differential coefficient of  $e^x$  with respect to  $\sqrt{x}$  is  $\sqrt{x} \cdot e^x$ .
- (iii)  $f(x) = 2x^3 21x^2 + 36x 30$  is maximum at x = 1.

- (iv) According to the Newton-Raphson's method the approximate root of the equation f(x) = 0 is  $x_n$  then be  $x_n = x_{n+1} - \frac{f(x)}{f'(x_n)}$ .
- By the method of Newton-Raphson, the cube root of 10, after first iteration is 2.167.
- Q5. Match the correct pair-

$$(5 \times 1 = 5)$$

'A'

(a) 
$$\int \frac{dx}{x^2 + a^2}$$

(i) 
$$\log \left[ x - \sqrt{x^2 - a^2} \right]$$

(b) 
$$\int \frac{dx}{\sqrt{a^2-x^2}}$$

(b) 
$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$
 (ii)  $\frac{1}{a} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a}$ 

(c) 
$$\int \sqrt{a^2 - x^2} \, dx$$

(iii) 
$$\frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

(d) 
$$\int \frac{dx}{\sqrt{x^2 - a^2}}$$
 (iv) a.  $\tan^{-1}$ 

(e) 
$$\int \sqrt{a^2 + x^2} dx$$
 (v)  $\sin^{-1}\left(\frac{x}{a}\right)$ 

(v) 
$$\sin^{-1}\left(\frac{x}{a}\right)$$

(vi) 
$$\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left[x+\sqrt{x^2+a^2}\right]$$

(vii) 
$$\log \left[ x + \sqrt{x^2 - a^2} \right]$$

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#### Section 'B'

- Q.6. Prove that- $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$
- (Or) If  $\overrightarrow{OP} = \hat{i} + 4\hat{j} 3\hat{k}$  and  $\overrightarrow{OQ} = 2\hat{i} 2\hat{j} k$  then find the modulus of  $\overrightarrow{PQ}$ .
- Q.7. Prove that vectors  $2\hat{i} 3\hat{j} + 5\hat{k}$  and  $-2\hat{i} + 2\hat{j} + 2\hat{k}$  are mutually perpendicular.
- (Or) If  $a = 2\hat{i} 3\hat{j} + \hat{k}$  and  $b = 3\hat{i} + 2\hat{j}$ , then find  $a \times b$ .
- Q.8. Find the vector equation of sphere whose centre is (2, -3, 4) and radius is 5.
- (Or) Find the distance of point (2, -1, 3) from the plane  $\vec{r} \cdot (3\hat{i} + 2\hat{j} 6\hat{k}) + 15 = 0$ .



Q.9. Evaluate 
$$-\int \frac{dx}{1+\cos 2x}$$

(Or) Evaluate 
$$-\int \frac{1}{1-4x} dx$$

Q.10. Evaluate 
$$\int_{0}^{x/4} \sin 2x \, dx$$
.

(Or) Evaluate 
$$-\int \frac{\sec x}{(\sec x - \tan x)} dx$$
.

Q.11. Resolve the following fraction into partial fractions 
$$-\frac{16}{(x+2)(x^2-4)}$$

(Or) Resolve the following fraction into partial fractions 
$$-\frac{2x+1}{(x-1)(x^2+1)}$$



Q.12. Prove that-  

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} \left[ x \sqrt{1 - y^2} + y \sqrt{1 - x^2} \right].$$

- (Or) If  $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ , then prove that, xy + yz + zx = 1.
- 0.13. Find the differential coefficient of sinx by first principle.
- (Or) If  $y = \log (\log \sin x)$ , then evaluate  $\frac{dy}{dx}$ .

Q.14. Differentiate, 
$$tan^{-1} \left[ \frac{\cos x + \sin x}{\cos x - \sin x} \right]$$
 with respect to x.

- (Or ) Find the differential coefficient with respect to x of  $\frac{e^{2x} + e^{-2x}}{e^{2x} e^{-2x}}$ .
- Q.15. If the edge of a cube is increasing at the rate of 5cm/sec., find the rate of increasing of its volume when its edge is 8cm long?



Simplifying Test P

- (Or) Prove that,  $f(x) = x^3 3x^2 + 3x 100$  is an increasing function in R.
- Q.16. If "r" is a coefficient of conetation of two variables x and y, then prove that-

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_{x-}\sigma_y}$$
 Where  $\sigma_x^2$ ,  $\sigma_y^2$  and  $\sigma_{x-y}^2$  are the variance of x, y and x - y respectively.

(Or) If n = 10, 
$$\sum x = 50$$
,  $\sum y = 30$ ,  $\sum x^2 = 290$ ,  $\sum y^2 = 300$ ,  $\sum xy = -115$ , then find the coefficient of correlation.

- Q.17. If " $\theta$ " be the angle between two regression lines and regression coefficients are  $b_{yx} = 1.6$  and  $b_{xy} = A$ , then find the value of tan $\theta$ . 4
- (Or) Prove that coefficient of correlation is the Geometric mean of regression co-efficients.
- Q.18. If  $\cos \alpha$ ,  $\cos \beta$ ,  $\cos \gamma$  are direction-cosines of any straight line, then prove that  $-\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
- (Or) Equation of the sphere is,  $2x^2 + 2y^2 + 2z^2 - 8x + 12y - 16z + 8 = 0$  find its centre and ratios:

Q.19. Prove that 
$$-\lim_{x\to 0} \left(\frac{e^x-1}{x}\right) = 1$$

(Or) Evaluate 
$$\lim_{x\to 0} \left(\frac{1-\cos 2x}{x}\right)$$
.

Q.20. Find the area of circle, 
$$x^2 + y^2 = a^2$$
.

(Or) Prove that 
$$-\int_{0}^{1} \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$
.

Q21. Solve the Differential Equation,  

$$(1+x) y dx + (1-y) x dy = 0$$
.

(Or) Solve the differential equation, 
$$(x^2 + xy) dy = (x^2 + y^2) dx$$
.

(Or) A bag contains 8 black and 5 white balls 2 balls are drawn. Find the probability that both the balls are white.

Q23. Prove that the lines.

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$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5} \text{ are coplanar. Find also}$$
the point of intersection.

- (Or) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and whose centre lies on the plane 3x y + z = 2.
- Q.24. Prove by vector method. cos(A - B) = cos A. cos B + sin A. sin B.
- (Or) Find the shortest distance between the lines.

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + t\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$
 and  $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + s$ 

$$\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right).$$