

E-445(H/E) HIGHER MATHEMATICS 2016

Time: 3 Hours | Class: 12th | M. M.: 100

Instructions-

- All questions are compulsory.
- (ii) Read instructions carefully of the question paper and then answers of the questions.
- (iii) Question paper has two sections-Section 'A' and Section 'B'.
- (iv) In the section- A' Question Nos. 1 to 5 are objective type.
 Each question carries 5 marks.
- V) In the section 'B' Question Nos. 6 to 24 has Internal option.
- vi) Q. Nos. 6 to 10 carry 2 marks each.
- /ii) Q. Nos. 11 to 17 carry 4 marks each.
- /iii) Q. Nos. 18 to 22 carry 5 marks each.
- x) Q. Nos. 23 to 24 carry 6 marks each.

Section 'A'

Q. 1. Choose the correct options:

 $5 \times 1 = 5$

- (i) The value of 2cos⁻¹x is:
 - (a) $\cos^{-1}(2x^2+1)$

(b)
$$\cos^{-1} \frac{2x}{1+x^2}$$

(c)
$$\cos^{-1}(2x^2-1)$$

(d)
$$\tan^{-1} \frac{2x}{1-x^2}$$

The partial fraction form of $\frac{1}{x^2 + 2x}$ is: (ii)

(a)
$$\frac{1}{2} \left[\frac{1}{x} - \frac{1}{x+2} \right]$$
 (b) $\frac{1}{2} \left[\frac{1}{x} + \frac{1}{x+2} \right]$

(b)
$$\frac{1}{2} \left[\frac{1}{x} + \frac{1}{x+2} \right]$$

(c)
$$\frac{1}{2} \left[\frac{1}{x+2} - \frac{1}{x} \right]$$

(d)
$$\frac{1}{x} - \frac{1}{x+2}$$

The value of $\int_{1}^{1} \cos x \, dx$ is: (iii)

(b) sin b + sin a

(d) sin b - sin a

Projection of vector \vec{a} on \vec{b} (iv)

(a)
$$\frac{\overline{a.b}}{|\overline{b}|}$$

(b)
$$\frac{1}{|\vec{a}|}$$

(c)
$$\frac{\bar{a}}{|\bar{a}|}$$

(d)
$$\frac{\mathbf{a} \times \mathbf{b}}{|\vec{\mathbf{b}}|}$$

The value of $\frac{d}{dx} \sin x^2$ is: (v)

(a) cos x²

(b) x sin x2

(c) 2x cos x2

(d) x cos x2

Q.2. Answar in one word/sentence:

5 × 1

- (i) Write Simpson's one-third rule formula in numerical meth
- In Newton-Raphson's method write the formula for fine cube root of the number N.
- (iii) In which interval does the root of equation $x^3 2x 5 = 0$
- (iv) If x_n is a nearer root of equation f(x) = 0, then write the value x_{n+1} by Newton-Raphson's method.
- (v) On which principle is Simpson's rule based?
- Q.3. Match the correct pair:

 $5 \times 1 = 5$

'A'

(a) \int \tan xdx

1. $\frac{1}{2a}\log\frac{x-a}{x+a}, x>a$

(b) ∫cot x dx

 $2. \ \frac{1}{2a} \log \frac{a+x}{a-x}, x < a$

(c) ∫sec xdx

3. log sin x

(d) $\int \frac{1}{x^2 - a^2} dx$

4. -log cos x

 $(5) \int \frac{1}{a^2-x^2} dx$

5. log (sec x + tan x)

| | | | •pjg | | | |
|-------|--|--|------------------|--|--|--|
| Q.4. | Fill in | the blanks: | $5 \times 1 = 5$ | | | |
| | (1) | Equation of the plane which intercepts unit ler dinate axes is | igth from coor- | | | |
| | (ii) | The acute angle between the planes $2x - y + z$ 2z = 3 is | = 6 and x + y + | | | |
| | (iii) | The equation of plane parallel to x - axis is | | | | |
| | (iv) | The perpendicular distance of point (18, 5, 12 | | | | |
| | · (v) | Differential coefficient of sin x3 with respect to | o x³ is | | | |
| Q. 5. | Write | True/False in the following statements: | 5 × 1 = 5 | | | |
| | (i) | The value of coefficient of correlation is great | iter than 1. | | | |
| | (ii) | The coefficient of correlation is the geometric mean of the regression co-efficients | | | | |
| | (iii) | When a body is thrown up, the sign of g is positive when it goes up. | | | | |
| | (iv) | In vector Algebra displacement is a vecto | r quantity. | | | |
| | (v) | Square of a vector is equal to square of its Section 'B' | | | | |
| Q.6. | Find the direction cosiness of the vector $6\hat{i} + 2\hat{j} - 3\hat{k}$ | | | | | |
| (OR) | - | If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then find the value of $\vec{a}\times\vec{b}$ | | | | |
| | | | | | | |
| Q.7. | If $\vec{a} = 2\hat{i} + \lambda \hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 3\hat{j} - 2\hat{k}$ are perpendicular to each oth then find the value of scalar λ | | | | | |
| | | | | | | |

- (OR) Find the angle between two vectors $\vec{a} = 2\hat{i} + \hat{j} 3\hat{k}$ and $\vec{b} = 3\hat{i} 2\hat{j} 2\hat{k}$
- Q. 8. Find the vector equation of the sphere with centre $(\hat{i} + 2\hat{j} 3\hat{k})$ and radius 5 units.
- (OR) Write the condition for intersecting the two lines $r = a_1 + \lambda b_1$ and $r = a_2 + \lambda b_2$
- Q.9. Find the value of $\int \sec^2 x \tan^3 x dx$.
- (OR) Find the value of $\int \frac{1}{e^2 + 1} dx$
- Q. 10. Find the value of $\int x \sin x dx$ 2
- (OR) Find the value of $\int \frac{2x+1}{\sqrt{x^2+x+1}} dx$
- Q. 11. Prove that:

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$$

(OR) Prove that:

$$\sin^{-1} 3/5 + \sin^{-1} 8/17 = \sin^{-1} \frac{77}{85}$$

Q. 12. Resolve
$$\frac{11-2x}{x^2+9x+14}$$
 into partial fractions.

(OR) Resolve $\frac{x}{1+x^3}$ into partial fractions.

Q. 13. Differentiate
$$\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}$$
 with respect to x. 4

(OR) If
$$y = \frac{1 - \cos x}{1 + \cos x}$$
, then find the value of $\frac{dy}{dx}$

Q. 14. If
$$y = \tan x + \sec x$$
, then prove that
$$\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$$

(OR) Find the differential coefficient of
$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
 with respect to $\tan^{-1}x$.

- Q.15. The edge of a cube is increasing at the rate of 7 cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm long? 4
- (OR) Find two positive numbers whose product is 64 and the sum is minimum.
- Q. 16. Calculate the correlation coefficient between x and y for the following data:

| x | 5 | 9 20 | 13 25 | 17 33 | 21 35 |
|---|----|------|----------|----------|----------|
| у | 12 | | | | |

(OR) Find correlation coefficient f(x, y), where

$$cov(x, y) = -2.25,$$

$$var(x) = 6.25$$
,

and
$$var(y) = 20.25$$

Q. 17. An article costs Rs. 75 at Gwalior. Find the corresponding most appropriate value at Bhopal using the following data:
4

| | Gwalior | Bhopal | |
|--------------------|---------|--------|--|
| Mean value | 65 | 67 | |
| Standard Deviation | 2.5 | 3.5 | |

Three Telation coefficient between the values of the two cities is 0.8

- (OR) If the regression line of y on x is ax + by + c = 0 and that of x on y is $a_1x + b_1y + c_1 = 0$, then prove that $ab_1 \le a_1b$.
- Q. 18. Find the equation to the plane through the point (-1, 3, 2) and perpendicular to the planes x + 2y + 2z = 11 and 3x + 3y + 2z = 15.
- (OR) Prove that the angle between any two diagonals of a cube is $\tan^{-1}(2\sqrt{2})$
- Q. 19. Find the value of $\lim_{x\to 0} \frac{x^3 \cot x}{1-\cos x}$

(OR)
$$f(x) = \begin{cases} \frac{\sin ax}{\sin bx}, & x \neq 0 \\ \frac{a}{b}, & x = 0 \end{cases}$$
 Test the continuity of function at $x = 0$.

Q. 20. Find the value of
$$\int \frac{dx}{3+2\cos^2 x}$$

- (OR) Find the area included between the two curves $y^2 = 9x$ and $x^2 = 9y$.
- Q. 21. Solve the differential equation $x + y \frac{dy}{dx} = 2y$.
- (OR) Solve the differential equation $\frac{dy}{dx} + 2y = \sin x$.
- Q. 22. Tickets are marked from 1 to 16 and mixed up. One ticket is taken out at random. Find the probability of its being a multiple of 2 or 3. 5
- (OR) A dice is thrown twice. A success is an even number on each throw.
 Find the probability distribution of the number of successes.
- Q.23. Prove that the points (1, 2, 3), (3, 0, 3), (-2, -3, -3) and (3, 4, 6) are coplanar.
- (OR) The coordinates of ends of one diameter of a sphere are (1, 0, 1) and (5, 4, 5). Find the equation, centre and diameter of sphere.
- Q.24. If $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ then find the value of $\vec{a} \times (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \times \vec{c})$
- (OR) Prove that the triangle, whose position vectors of the vertices are $2\hat{i}+4\hat{j}-\hat{k}$, $4\hat{i}+5\hat{j}+\hat{k}$ and $3\hat{i}+6\hat{j}-3\hat{k}$ respectively, is an isosceles right angled triangle.