

A-436 (E/H) HIGHER MATHEMATICS 2017

Time: 3 Hours | Class: 12th | M. M.: 100

Instructions:

- (i) All questions are compulsory.
- (ii) Read instructions carefully of the question paper and then write answers of the question
- (iii) Question paper has two sections-Section 'A' and Section 'B'.
- (iv) In the Section 'A' Question Nos.1 to 5 are objective type. Each question carries 5 marks.
- (v) In the Section 'B' Question Nos. 6 to 26 has Internal option.
- (vi) Q.Nos 6 to 10 carry 2 marks each.
- (vii) Q. Nos. 11 to 14 carry 3 marks each.
- (viii) Q. Nos. 15 to 21 carry 4 marks each.
- (ix) Q. Nos. 22 to 26 carry 5 marks each.

Section 'A'

Q. 1. Choose the correct options:

 $5 \times 1 = 5$



(i) if
$$\frac{2x+3}{x^2+5c+6} = \frac{A}{x+2} + \frac{1}{x+3}$$
 then the value of A + B is:

(a) 3

(b)2

(c) 5

(d) 4



(ii) The value of $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$ is:

(a) cos x

(b) cot-1 x

(c) $\sin^{-1} \frac{1}{x}$

(d) sin-1 x

(iii) The value of $\int_0^3 x^2 dx$ is:

(a) $\frac{26}{3}$

(b) $\frac{28}{3}$

(c) $\frac{25}{3}$

(d) $\frac{8}{3}$

(iv) The unit vector in the direction of a is:

(a)
$$\frac{\overrightarrow{a}}{|\overrightarrow{a}|}$$

(b)
$$\overrightarrow{a} | \overrightarrow{a} |$$

 $(c) a^2$

(d) i

(v) Differential coeffcient of log sin x is:

(a) cos x

(b) tan x



- Q. 2. Write True/ False in the following statements: $5 \times 1 = 5$
 - (i) The value of correlation coefficient lies between 2 to + 2
 - If regression coefficient are 0.8 and 0.2 then the value of correlation coeffcient is + 0.4
 - (iii) The function f(x) = 5x + 2 is increasing on set of real numbers.
 - (iv) If the position vector of centre of sphere is \overrightarrow{c} and radius is a then the vector equartion of sphere is $|\overrightarrow{r} + \overrightarrow{c}| = a$.
 - (v) Two vectros \overrightarrow{a} and \overrightarrow{b} are perpendicular if $\overrightarrow{a} \cdot \overrightarrow{b} = 0$
- Q. 3. Fill in the blanks:

 $5 \times 1 = 5$

- The perpendicular distance of a point (5, 12, 13) from Y-axis is.......
- (ii) The equation of the plane passing through the origin (0, 0, 0) is
- (iii) The intercept cut by the plane 2x + y z = 5 on X-axis is
- (iv) The centre of the sphere $x^2 + y^2 + z^2 + 3x + 5y + 2z = 0$ is
- (v) The differential coefficient of sin 3x with respect to 3x is.........

Q.4. Give answer in one word/sentence:

 $5 \times 1 = 5$

- In Newton-Raphson's method write formula for finding square root of the number N.
- (ii) Write trapezoidal rule formula in numerical methods.
- (iii) Write cube root of 10by Newton-Raphson's method after first iteration.
- (iv) In which interval does the root of equation $x^3 + x 3 = 0$ lie.
- (v) Write the coefficient of y with odd subscripts in Simpson's Rule.
- Q.5. Match the correct pair.

 $5 \times 1 = 5$

'A'

(i) sec x + c

'B'

(b)
$$\int \frac{dx}{x\sqrt{x^2-1}}$$

(ii)
$$\frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right] + c$$

(c)
$$\int \sqrt{a^2 - x^2} dx$$

(iii)
$$\log \left[x + \sqrt{x^2 - a^2} \right] + c$$

(d)
$$\int \frac{dx}{\sqrt{a^2 - x^2}}$$

(iv)
$$\log \tan \frac{x}{a} + c$$

(e)
$$\int \frac{dx}{a^2 + x^2}$$

(v)
$$\sin^{-1} \frac{x}{a} + c$$

(vi)
$$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$$



Section B

- Q.6. Prove that the vectors $\vec{a} = 5\hat{i} + 15\hat{j}$ and $\vec{b} = 3\hat{i} + 9\hat{j}$ are parallel. 2
- (OR) If the position vectors of the points A and B are $7\hat{i} + 3\hat{j} + \hat{k}$ and $2\hat{i} + 5\hat{j} + 4\hat{k}$ respectively, then find the magnitude of \overrightarrow{AB} .
- Q.7. The displacement of a particle by the force $\vec{F} = 2\hat{i} \hat{j} \hat{k}$ is $\vec{d} = 3\hat{i} + 2\hat{j} 5\hat{k}$, then find the work done by the force.

- (OR) If $\overrightarrow{a} = 2\hat{i} 3\hat{j} + \hat{k}$ and $\overrightarrow{b} = 3\hat{i} + 2\hat{j} + \hat{k}$, then find the value of $\overrightarrow{a} \times \overrightarrow{b}$.
- Q.8. For what value of λ the planes $\vec{r} \cdot (2\hat{i} + \lambda \hat{j} 3\hat{k}) = 2$ and $\vec{r} \cdot (\lambda \hat{i} 3\hat{j} + \hat{k}) = 5$ are perpendicular to each other.
- (OR) Write the formula to tind the shortest distance between two straight lines whose vector equation are

$$r = a_1 + \lambda b_1$$

and
$$\overrightarrow{r} = \overrightarrow{a}_2 + \mu \overrightarrow{b}_2$$

where λ and μ are scalars.

Q.9. Find the value of
$$\int \frac{\cos(\log x)}{x} dx$$
.



- (OR) Prove that $\int \sec x \, dx = \log (\sec x + \tan x) + c$.
- Q. 10. find the value of $\int xe^x dx$.

2

- (OR) find the value of $\int x \tan^2 x dx$.
- Q. 11. Find the distance between the parallel planes 2x 2y + z + 3 = 0 and 4x 4y + 2z + 5 = 0.
- (OR) In which radio does the YZ plane divide the line joining the points (-2, 4, 7) and (3, -5, 8).
- Q. 12. Prove that the symmetrical form of equation x = ay + b, z = cy + d is

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}.$$

- (OR) Find the equation of the sphere whose centre is (3, 2, 1) and radius is 5.
- Q. 13. Prove by vector method that the angle in a semicircle is right angle. 3
- (OR) Prove that $\overrightarrow{a} \times (\overrightarrow{b} + \overrightarrow{c}) + \overrightarrow{b} \times (\overrightarrow{c} + \overrightarrow{a}) + \overrightarrow{c} \times (\overrightarrow{a} + \overrightarrow{b}) = \overrightarrow{0}$
- Q. 14. Find the vector equation of the spher where the points (2, 3, 5) and (4, 9, -3) are extremities of its diameter.
- (OR) Find the angle between the planes $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} + \hat{j}) = 4$.
- Q. 15. Resolve $\frac{13x+18}{2x^2+5x+3}$ into partial fractions.
- (OR) Resolve $\frac{8x^2-9}{(3x+2)(x^2+5)}$ into partial fractions.
- Q. 16. Prove that $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$.
- (OR) Express $\sin \left[2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right]$ in simplest form.
- Q. 17. Find differential coefficient of $\sqrt{\tan \sqrt{x}}$
- (OR) If $y = \cot^{-1} \left[\frac{\sqrt{1 + x^2} + 1}{x} \right]$, then find the value of $\frac{dy}{dx}$.

Q. 18. Differentiate (cosx)cosx with respect to x.

4

(OR) if
$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots + \infty}}}$$
 then prove that

$$1-2$$
 $\frac{dy}{dx} = \sin x$

Q. 19. The side of a square sheet of metal is increasing at the rate of 5cm/ minute. At what are its area is increasing when the side is 20cm long?

4

- (OR) Find the maximum value of $2x^3 24x + 107$ in the interval [1, 3].
- Q. 20. Calculate the correlation coefficient between x and y for the following data:

Y	65	66	67	67	68	69	70	72
У	67	68	65	68	72	72	69	71

(OR) Calculate cov (X,Y) between two variables x and y where:

$$\sum x_i = 15$$
, $\sum y_i = 36$, $\sum x_i y_i = 110$, $n = 5$

Q.21. Prove that correlation coefficient is the geometric mean of the regression coefficiens.

(OR) Calculate regression coefficient byx and i for variables x and y for the following data:

$$\sum x = 24$$
, $\sum y = 44$, $xy = 306$, $\sum x^2 = 164$, $\sum y' = 574$, $n = 4$

- Q. 22. Find the equation of the plane passing through the point (4, 5, 1), (0, -1, -1) and (-4, 4, 4).
- (OR) Prove that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Find the point of intersection of these lines.
- Q.23. If $f(x) = \log_e \left(\frac{1-x}{1+x}\right)$ then prove that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$.
- (OR) Find the value of $\lim_{X \to 0} \frac{\tan x \sin x}{x^3}$
- Q. 24. Find the value of $\int \frac{dx}{5+4\sin x}$
- (OR) Prove that $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx = \frac{\pi}{4}.$
- 25. Solve the differenatial equation $(x-1)\frac{dy}{dx} = 2x^3y$.
- (OR) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + 5xy + 4y^2}{x^2}$.
- Q. 26. A card is drawn at random from a well shuffled pack of of 52 cards. Find the probability thaty it is neither an ace nor a king.
- (OR) A fair coin is tossed six times. What is the probability of getting at least three heads.?