Answers & Explanations

Q.1 **(i)** Ans. Here, $a = 55, d = 50 - 55 = -5$.

Let the n^{th} term of the given AP be the first negative term.

Then, $a_n < 0$

$\Rightarrow \{a + (n-1)d\} < 0$

$\Rightarrow \{55 + (n-1) \times (-5)\} < 0$

$\Rightarrow (55 - 5n + 5) < 0$

$\Rightarrow (60 - 5n) < 0$

$\Rightarrow 60 < 5n$

$\Rightarrow 5n > 60$

$\Rightarrow n > 12$

$\therefore n = 13$

Hence, the 13^{th} term is the first negative term of the given AP.

(ii) Ans. The given systems of linear equations are $x + 3y = 6$ and $3x + 9y = 24$

$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{6}{24} = \frac{1}{4}$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and the given system of linear equations has no solution.

(iii) Ans. Here, $a = 20, d = 24 - 20 = 4$ and $a_n = 360$

Let n be the number of terms.

Now, $a_n = 360 \Leftrightarrow a + (n-1)d = 360$

$\Leftrightarrow 20 + (n-1)(4) = 360$ [$\because a = 20, d = 4$]

$\Leftrightarrow 20 + 4n - 4 = 360$

$\Leftrightarrow 4n + 16 = 360 \Leftrightarrow 4n = 360 - 16 = 344$

$\Leftrightarrow n = \frac{344}{4} = 86$

Hence, the given AP contains 86 terms.

(iv) Ans. The given systems of linear equations are $x + 5y = 9$ and $3x + ky = 8$

$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{5}{k}$ and $\frac{c_1}{c_2} = \frac{9}{8}$

For no solution, we must have: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \frac{1}{3} = \frac{5}{k} \neq \frac{9}{8}$

$\Rightarrow \frac{1}{3} = \frac{5}{k}$

$\therefore k = 15$

Hence, $k = 15$.

$$\Rightarrow \frac{1}{3} = \frac{5}{15} \neq \frac{9}{8}$$

$$\Rightarrow \frac{1}{3} = \frac{1}{3} \neq \frac{9}{8}$$

Hence there is no solution for the system of linear equations given.

(v) Ans. Here, $a = 1, b = -4$ and $c = 9$

$\therefore \alpha + \beta = -\frac{b}{a} = 4$ and $\alpha \cdot \beta = \frac{c}{a} = 9$

$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha \cdot \beta} = \frac{4}{9}$

Hence, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{4}{9}$.

(vi) Ans. If $\alpha = 9 - 4\sqrt{5}$, then $\beta = 9 + 4\sqrt{5}$

$\therefore \alpha + \beta = (9 - 4\sqrt{5}) + (9 + 4\sqrt{5}) = 18$

and $\alpha \cdot \beta = (9 - 4\sqrt{5})(9 + 4\sqrt{5}) = 81 - 80 = 1$

Quadratic equation is $x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$

$\therefore x^2 - 18x + 1 = 0$

Hence, the required quadratic equation is $x^2 - 18x + 1 = 0$.

Q.2 (i) Ans. Let a and d be the first term and common difference in AP respectively.

Given, $a_7 = 35, a_{13} = 77$

7^{th} term of an AP

$a_7 = a + (7-1)d = 35$

$\Rightarrow a + 6d = 35 \quad \ldots\ldots(1)$

13^{th} term of the AP

$a_{13} = a + (13-1)d = 77$

$\Rightarrow a + 12d = 77 \quad \ldots\ldots(2)$

Subtracting (1) from (2), we get

$6d = 42 \quad \Rightarrow d = 7$

Hence, the common difference of the AP$= 7$.

(ii) Ans. The given system of linear equations are $x + 4y = 7$ and $5ax + (a + b)y = 35$

The condition of infinite many solutions $= \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{1}{5a} = \frac{4}{(a+b)} = \frac{7}{35}$

$\Rightarrow \frac{1}{5a} = \frac{4}{(a+b)} = \frac{1}{5}$

$\therefore \frac{1}{5a} = \frac{1}{5}$

$\therefore 5a = 5 \ \Rightarrow a = \frac{5}{5} = 1$

Hence, $a = 1$.

(iii) Ans. Let $\alpha = \frac{7}{5}$ and $\beta = -\frac{5}{3}$

$\therefore$ Sum of the zeros $= \alpha + \beta = \frac{7}{5} - \frac{5}{3} = \frac{21-25}{15} = -\frac{4}{15}$

And product of the zeros $= \alpha \cdot \beta = \frac{7}{5} \times \left(-\frac{5}{3}\right) = -\frac{7}{3}$

$\therefore$ Quadratic equation is $x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$

$= x^2 - \left(-\frac{4}{15}\right)x + \left(-\frac{7}{3}\right) = 0$

$= x^2 + \frac{4}{15}x - \frac{7}{3} = 0$

(iv) Ans. Extra salary will be $=$ $Rs. 25{,}000 -$ $Rs. 20{,}000$

$=$ $Rs. 5{,}000$

Which will be divided among 25 members = $\frac{5000}{25}$ = 200

Then, the average salary of the group $= 20{,}000 + 200 = Rs. 20{,}200$

Hence, the average salary of the group is $Rs. 20{,}200$.

(v) Ans. All possible outcomes are $2, 3, 4, 5, 6, 7, \ldots\ldots, 50$.

Total number of all possible outcomes $= 49$

Favourable outcomes are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$.

Number of all favourable outcomes $= 15$

Let E be the event of getting a prime number.

$\therefore P(E) = \frac{\text{Number of all favourable outcomes}}{\text{Total number of all possible outcomes}} = \frac{15}{49}$

Hence, the probability of the prime number $= \frac{15}{49}$.

(vi) Ans. Given: median $= 24$, mode $= 16$ and mean=?

We know that

$Mode\ = (3 \times Median) - (2 \times Mean)$

$\Rightarrow 16\ = (3 \times 24) - (2 \times Mean)$

$\Rightarrow 16\ = 72 - (2 \times Mean)$

$\Rightarrow (2 \times Mean)\ = 72 - 16 = 56$

$\Rightarrow Mean = \frac{56}{2} = 28$

Hence, $Mean = 28$.

Q.3 (i) Ans. Let the present age of Mohan's father $= x$ years and the present age of Mohan $= y$ years

According to questions,

$x = 5y$

$x - 5y = 0 \ldots\ldots (1)$

5 years ago, the age of Mohan's father $= (x - 5)$ years and the age of Mohan $= (y - 5)$ years

$(x - 5) - (y - 5) = 24$

$\Rightarrow x - 5 - y + 5 = 24$

$\Rightarrow x - y = 24 \ldots\ldots (2)$

On subtracting (1) from (2), we get

$4y = 24 \quad \Rightarrow y = 6$

Putting $y = 6$ in equation (1), we get

$x - 5 \times 6 = 0 \ \Rightarrow x = 30$

$\therefore x = 30, y = 6$

Hence, the present age of Mohan and Mohan's father are 6 years and 30 years respectively.

(ii) Ans. The given equation is $(p - q)x^2 + (q - r)x + (r - p) = 0$

Here, $a = (p - q), b = (q - r)\ and\ c = (r - p)$

$\therefore D = b^2 - 4ac$

$= (q - r)^2 - 4(p - q)(r - p)$

$= q^2 + r^2 - 2qr - 4(pr - p^2 - qr + pq)$

$= q^2 + r^2 - 2qr - 4pr + 4p^2 + 4qr - 4pq$

$= (-2p)^2 + q^2 + r^2 + 2(-2p)q + 2qr + 2r(-2p)$

$= (-2p + q + r)^2 \qquad [\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

For real and equal roots, we must have: $D = 0$

$\Rightarrow (-2p + q + r)^2 = 0$

$\Rightarrow -2p + q + r = 0$

$\Rightarrow 2p = q + r$

Hence, it is proved.

(iii) Ans. Since, Loss will be divided according to their investment ratio

A : B

2000 : 1500

4 : 3

$\therefore$ Loss of B = $\frac{3}{7} \times 700$ = Rs. 300

(iv) Ans. Total number of student = 100

(a) Let the chosen student be a boy.

Number of outcomes favorable for a boy = 60

$\therefore P(choosing\ a\ boy) = \frac{60}{100} = \frac{3}{5}$

(b) Let the chosen student be a girl.

Number of outcomes favorable for a girl = 40

$\therefore P(choosing\ a\ girl) = \frac{40}{100} = \frac{2}{5}$

(v) Ans. We have,

Class	$0-4$	$4-8$	$8-12$	$12-16$	$16-20$	$20-24$
Frequency	8	21	15	34	26	12

As the class 12-16 has maximum frequency, so it is modal class.

$\therefore x_k = 12, h = 4, f_k = 34, f_{k-1} = 15\ and\ f_{k+1} = 26$

Mode, $M_o = x_k + \frac{f_k - f_{k-1}}{2f_k - f_{k-1} - f_{k+1}} \times h$

$= 12 + \frac{34-15}{2\times34-15-26} \times 4$

$= 12 + \frac{19}{68-41} \times 4$

$= 12 + \frac{76}{27}$

$= 12 + 2.814 = 14.814$

Hence, mode = 14.814.

Q.4 (i) Ans: We have, $3^{x+1} + 3^{2-x} = 28$

$\Rightarrow 3^x \cdot 3 + 3^2 \cdot 3^{-x} = 28$

$\Rightarrow 3^x \cdot 3 + \frac{9}{3^x} = 28$

Put $3^x = y$

$\Rightarrow 3y + \frac{9}{y} = 28$

$\Rightarrow 3y^2 + 9 = 28y$

$\Rightarrow 3y^2 - 28y + 9 = 0$

$\Rightarrow 3y^2 - 27y - y + 9 = 0$

$\Rightarrow 3y(y-9) - 1(y-9) = 0$

$\Rightarrow (y-9)(3y-1) = 0$

$\therefore y = 9 \ and \ y = \frac{1}{3}$

$y = 3^x = 9$

$3^2 = 9 \Rightarrow x = 2$

And y = $3^x = \frac{1}{3}$

$3^x = 3^{-1} \Rightarrow x = -1$

Hence, $x = 2 \ and -1.$

(ii) Ans. When two dice are thrown simultaneously.

Total number of all possible outcomes $= 36$

a. Let E be the event of getting that the sum of two numbers is 8.

The favorable outcomes are

$(2,6), (3,5), (4,4), (5,3), and \ (6,2).$

Number of favorable outcomes$= 5$

$$\therefore P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of all possible outcomes}} = \frac{5}{36}$$

b. Let E' be the event of getting that the sum of two numbers is not 8.

$$\therefore P(E') = 1 - P(E)$$

$$= 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$$

Hence, the probability of getting the sum of two numbers is 8 = $\frac{5}{36}$ and the probability of getting the sum of two numbers is not 8 $= \frac{31}{36}$.

(iii) Ans. We prepare the cumulative frequency, as given below:

Class	Frequency (f_i)	Cumulative Frequency
$0-4$	10	10
$4-8$	12	22
$8-12$	14	36
$12-16$	14	50
$16-20$	22	72
$20-24$	11	83
	$\sum f_i = 83$	

$\therefore N = 83 \Rightarrow \frac{N}{2} = \frac{83}{2} = 41.5$

The cumulative frequency just greater than 41.5 is 50 and its corresponding class is $12-16$.

The median class is $12-16$.

$l = 12, h = 4, f = 14, cf = 36$ and $\frac{N}{2} = \frac{83}{2} = 41.5$

Median, $M_e = l + \left\{\frac{\left(\frac{N}{2} - cf\right)}{f}\right\} \cdot h$

$= 12 + \left\{\frac{(41.5 - 36)}{14}\right\} \cdot 4$

$= 12 + \frac{5.5}{14} \cdot 4$

$= 12 + 1.57 = 13.57$

Hence, median$= 13.57$.

Q.5 (i) Ans. Let a and d be the first term and common difference in AP respectively.
Given, $a_9 = 49$, $a_{17} = 105$

9^{th} term of an AP

$a_9 = a + (9 - 1)d = 49$

$\Rightarrow a + 8d = 49 \quad \ldots\ldots(1)$

17^{th} term of the AP

$a_{17} = a + (17 - 1)d = 105$

$\Rightarrow a + 16d = 105 \quad \ldots\ldots(2)$

Subtracting (1) from (2), we get

$8d = 56 \quad \Rightarrow d = 7$

Putting $d = 7$, in equation (1), we get

$a = 49 - 56 = -7$

We know that, the sum of first n terms $S_n = \left\{\frac{n}{2}[2a + (n-1)d]\right\}$

∴ The sum of first 12 terms of an AP

$S_{12} = \left\{\frac{12}{2}[2 \times (-7) + (12 - 1) \times 7]\right\}$

$= \{6[-14 + 77]\} = 6 \times 63 = 378$

Hence, the sum of first 12 terms of the AP $= 378$.

(ii) Ans. According to the question,

Work done by P and Q together in one day $= \frac{1}{12}\ parts$

Work done by Q and R together in one day $= \frac{1}{18}\ parts$

Work done by R and P together in one day $= \frac{1}{15}\ parts$

$\therefore P + Q = \frac{1}{12}$ .....(1)

$Q + R = \frac{1}{18}$ .....(2)

and $R + P = \frac{1}{15}$ .....(3)

Adding equations (1), (2) and (3), we get

$2(P + Q + R) = \frac{1}{12} + \frac{1}{18} + \frac{1}{15} = \frac{15+10+12}{180} = \frac{37}{180}$

$\Rightarrow P + Q + R = \frac{37}{360}$ ....(4)

Putting the value of equation (1) in equation (4), we get

$\frac{1}{12} + R = \frac{37}{360} \Rightarrow R = \frac{37}{360} - \frac{1}{12} = \frac{37-30}{360} = \frac{7}{360}$

$\therefore$ Work done in 1 day by R is $\frac{7}{360}$ part

Hence, R can finish the whole work in $\frac{360}{7}$ day's $= 51\frac{3}{7}$ days.

(iii) Ans. We have,

Class	Mid-value (x_i)	Frequency(f_i)	Cumulative frequency	$f_i \times x_i$
$0 - 10$	5	8	8	40
$10 - 20$	15	16	24	240
$20 - 30$	25	10	34	250
$30 - 40$	35	5	39	175
$40 - 50$	45	11	50	495
		$\sum f_i = 50$		$\sum f_i \times x_i = 1200$

Mean, $\bar{x} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \frac{1200}{50} = 24$

Here, $N = 50 \Rightarrow \frac{N}{2} = \frac{50}{2} = 25$

Cumulative frequency is just greater than 25 is 34 and the corresponding class is $20 - 30$.

Thus, the median class is $20 - 30$.

$\therefore l = 20, h = 10, f = 10, c = 24$ and $\frac{N}{2} = 25$

Median, $M_e = l + \frac{\left(\frac{N}{2} - c\right)}{f} \times h$

$= 20 + \frac{(25-24)}{10} \times 10 = 20 + \frac{1}{10} \times 10 = 20 + 1 = 21$

$\therefore$ Median $= 21$

Mode= $3(Median) - 2(Mean)$
$= 3(21) - 2(24) = 63 - 48 = 15$
Hence, $Mean = 24$, Mode= 15 and Median= 21.