MAHARASHTRA BOARD CLASS 10
MATHS PART 2 ANSWERS

Answers & Explanations

Q.1 (i) Ans. Let a, b and c be the the sides of the triangle.

$\therefore a = 10, b = 14, c = 20$

$\because$ c is the largest side of the triangle

$a^2 + b^2 = 10^2 + 14^2$

$= 100 + 196 = 296$

and $c^2 = 20^2 = 400$

$\because c^2 > a^2 + b^2$

Hence, the given triangle is an obtuse angled triangle.

(ii) Ans. Area of the circle $= 1296\pi\ cm^2$

Let r be the radius of the circle.

$\therefore$ Area of the circle $= \pi r^2$

$\Rightarrow \pi r^2 = 1296\pi$

$\Rightarrow r^2 = 1296 \Rightarrow r^2 = 36^2$

$\Rightarrow r = 36\ cm$

We know that, the length of the longest chord of the circle is diameter.

$\therefore$ The length of the longest chord of the circle = Diameter

$= 2r = 2 \times 36 = 72\ cm$

(iii) Ans. We have,

$\cos^2 5° - \sin^2 85°$

$= \cos^2 5° - \sin^2(90 - 5°)$

$= \cos^2 5° - \cos^2 5°$ $(\because \sin(90° - \theta) = \cos\theta\ )$

$= 0$

Hence, $\cos^2 5° - \sin^2 85° = 0.$

(iv) Ans. Let the required points be $P(0, y)$.

$AP^2 = BP^2$

$A(-2, -3)$ and $B(3, -4)$

$\Rightarrow (0 + 2)^2 + (y + 3)^2 = (0 - 3)^2 + (y + 4)^2$

$\Rightarrow 4 + y^2 + 6y + 9 = 9 + y^2 + 8y + 16$

$\Rightarrow 2y = 13 - 25 = -12$

$\Rightarrow y = -6$

Hence, the required point is $P(0, -6)$.

(v) Ans. Let a be the side of the cube.

Volume of the cube $= a^3$

$\Rightarrow a^3 = 1728 = 12^3$

$\Rightarrow a = 12\ cm$

Total surface area of the cube $= 6a^2$

$= 6 \cdot 12^2 = 6 \cdot 144 = 864\ cm^2$

(vi) Ans. Here, volume of the pyramid$= 210\ cm^3$, height $= 15\ cm$

$\therefore$ Volume of the pyramid $= \frac{1}{3} \times Height \times \text{Area of the base}$

$$\text{Area of the base} = \frac{\text{Volume of the pyramid } \times 3}{\text{Height}}$$

$$= \frac{210\ \times 3}{15} = \frac{210}{5} = 42\text{ cm}$$

Hence, the area of the base of the pyramid is 42 cm.

Q.2 (i) Ans. Let $\Delta ABC \sim \Delta DEF$ such that ar(ΔABC) = 196 cm^2 and ar(ΔDEF) = 144 cm^2

Let AP and DQ be the corresponding medians of ΔABC and ΔDEF respectively

Given median of the first triangle is 14.0 cm.

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians.

$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AP^2}{DQ^2}$

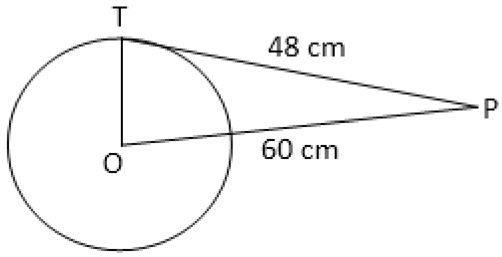
$\Rightarrow \left(\frac{AP}{DQ}\right)^2 = \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{196}{144} = \left(\frac{14}{12}\right)^2$

$\Rightarrow \frac{AP}{DQ} = \frac{14}{12} \quad \Rightarrow \frac{14.0}{DQ} = \frac{14}{12}$

$\Rightarrow DQ = \frac{14.0 \times 12}{14} = 12\text{ cm}$

Hence, the corresponding median is 12 cm.

(ii) Ans. Let O be the centre of the given circle and P be a point such that $OP = 60\ cm$



Let PT be tangent such that $PT = 48\ cm$

Join OT.

Now, PT is a tangent at T and OT is the radius through T.

$\therefore PT \perp OT$

In the right ΔOTP, we have

$OP^2 = OT^2 + PT^2$ [By Pythagoras Theorem]
$OT = \sqrt{OP^2 - PT^2}$
$= \sqrt{60^2 - 48^2} = \sqrt{3600 - 2304} = \sqrt{1296} = 36\ cm$
Hence, radius of the circle is 36 cm.

(iii) Ans. We have, $21\sec^2\theta - 18\tan^2\theta = 24$

$\Rightarrow 21\,(1+\tan^2\theta) - 18\tan^2\theta = 24$ $(\because 1+\tan^2\theta = \sec^2\theta)$

$\Rightarrow 21 + 21\tan^2\theta - 18\tan^2\theta = 24$

$\Rightarrow 3\tan^2\theta = 3$

$\Rightarrow \tan\theta = 1 = \frac{\text{opposite}}{\text{adjacent}}$

$\therefore \sin\theta = \frac{opposite}{hypotenuse} = \frac{1}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$

Hence, $\sin\theta = \frac{1}{\sqrt{2}}$.

(iv) Ans. Given, $\sin 5A = \cos(A - 30°)$
$\Rightarrow \cos(90° - 5A) = \cos(A - 30°)$ $(\because \cos(90 - A) = sinA)$
$\Rightarrow 90° - 5A = A - 30°$
$\Rightarrow 6A = 120°$
$\Rightarrow A = 20°$
Hence, $\angle A = 20°$.

(v) Ans. The given points are $A(5,2), B(1,-1)$ and $C(k,2)$.
The given points A, B and C are collinear.
$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
$\Rightarrow 5(-1-2) + 1(2-2) + k(2+1) = 0$
$\Rightarrow -15 + 0 + 3k = 0$
$\Rightarrow 3k = 15$
$\Rightarrow k = \frac{15}{3}$
Hence, $k = 5$.

(vi) Ans. Let l and b be the length and breadth of the rectangle respectively.

$\therefore$ Area of the rectangle $= lb$ square units

Length of new rectangle $= 1.5l$, breadth of new rectangle $= 0.7b$

$\therefore$ Area of the new rectangle $= 1.5l \times 0.7b$ square units

$= 1.05lb$ square units

$\therefore$ % increase in area of rectangle

$= \frac{\text{Area of the new rectangle} - \text{Area of the rectangle}}{\text{Area of the rectangle}} \times 100$

$= \frac{1.05lb - lb}{lb} \times 100$

$= \frac{(1.05 - 1)lb}{lb} \times 100 = 5\%$

Q.3 (i) Ans. Given, radius of the circle, $r = 6\ cm$

$\therefore$ Diameter of the circle , $d = 2r = 12\ cm$

Length of the plastic band $= 3d + 2\pi r = 3d + \pi d$

$= 3 \times 12 + 12\pi = 12(3 + \pi)$ cm

Hence, Length of the plastic band $= 12(3 + \pi)$ cm.

(ii) Ans. We have, $\frac{\sin^2 46° + \sin^2 44°}{\cos^2 46° + \cos^2 44°} + 5(\cos^2 27° + \cos^2 63°)$

$= \frac{\sin^2 46° + \sin^2(90° - 46°)}{\cos^2 46° + \cos^2(90° - 46°)} + 5(\cos^2 27° + \cos^2(90° - 27°))$

$(\because \sin(90° - \theta) = \cos\theta \ \ and \ \ \cos(90° - \theta) = \sin\theta)$

$= \frac{\sin^2 46° + \cos^2 46°}{\cos^2 46° + \sin^2 46°} + 5(\cos^2 27° + \sin^2 27°)$

$= \frac{1}{1} + 5(1) \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 6$

(iii) Ans.We have,

$11 \sin^2\theta + 7\cos^2\theta = 8$

$\Rightarrow 11\sin^2\theta + 7(1 - \sin^2\theta) = 8 \qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow 11\sin^2\theta + 7 - 7\sin^2\theta = 8$

$\Rightarrow 4\sin^2\theta = 1 \quad \Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30°$

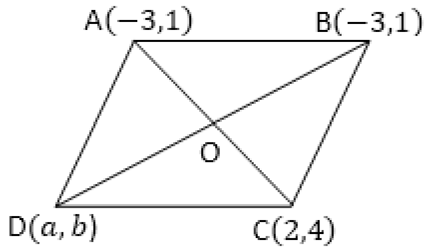
$\Rightarrow \theta = \frac{\pi}{6}$

Hence, $\theta = \frac{\pi}{6}$.

(iv) Ans.Let $A(-3,1), B(-3,1), C(2,4)$ be the three vertices of a parallelogram $ABCD$.

Join AC and BD, intersecting each other at O.

Let its fourth co-ordinate $D(a,b)$.



We know that the diagonals of a parallelogram bisect each other.

Therefore, O is the midpoint of AC as well as that of BD.

Midpoint of AC = $\left(\frac{-3+2}{2}, \frac{1+4}{2}\right)$ i.e, $\left(\frac{-1}{2}, \frac{5}{2}\right)$

Midpoint of BD = $\left(\frac{-3+a}{2}, \frac{1+b}{2}\right)$

$\therefore \frac{-1}{2} = \frac{-3+a}{2}$

$\Rightarrow -2 = -6 + 2a$

$\Rightarrow 2a = -2 + 6 = 4$

$\Rightarrow a = 2$

and $\frac{5}{2} = \frac{1+b}{2}$

$\Rightarrow 10 = 2 + 2b$

$\Rightarrow 2b = 10 - 2 = 8$

$b = 4$

$\Rightarrow a = 2$ and $b = 4$

Hence, the fourth vertex is D$(2,4)$.

(v) Ans.Let h be the height of the trapezium

Given, area of the trapezium is $320\ cm^2$,

Sum of the parallel sides of the trapezium $= 64\ cm$

Area of the trapezium $= \frac{1}{2} \times (sum\ of parallel\ sides) \times height$

$\Rightarrow \frac{1}{2} \times 64 \times h = 320$

$\Rightarrow 32h = 320$

$\Rightarrow h = \frac{320}{32}\ cm$

$\Rightarrow h = 10\ cm$

Hence, the height of the trapezium $= 10\ cm$.

Q.4 (i) Ans. Given $\frac{25 \sin\theta + 5\cos\theta}{25\sin\theta - 5\cos\theta}$

Dividing numerator and denominator by $\cos\theta$, we get

$$\frac{25\frac{\sin\theta}{\cos\theta} + 5\frac{\cos\theta}{\cos\theta}}{25\frac{\sin\theta}{\cos\theta} - 5\frac{\cos\theta}{\cos\theta}}$$

$$= \frac{25\tan\theta + 5}{25\tan\theta - 5} = \frac{25\times\frac{15}{25} + 5}{25\times\frac{15}{25} - 5}$$

$$= \frac{15 + 5}{15 - 5} = \frac{20}{10} = 2$$

Hence, $\frac{25\sin\theta + 5\cos\theta}{25\sin\theta - 5\cos\theta} = 2$.

(ii) Ans. Steps of construction:

Step 1: Draw a circle of radius 5.5 cm.

Step 2: Mark a point P on it.

Step 3: Draw any chord PQ.

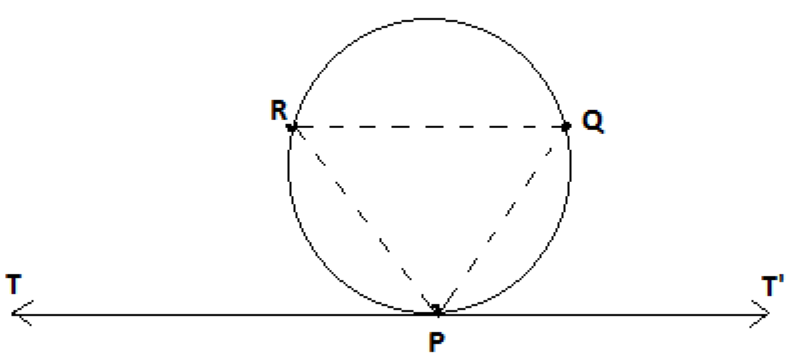
Step 4: Take a point R in the major arc QP.

Step 5: Join PR and RQ.

Step 6: Make $\angle QPT = \angle PQR$.

Step 7: Extend the line from TP to T', as shown in the figure.

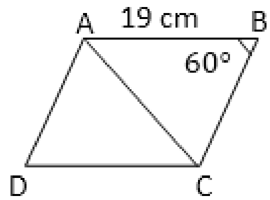
Then, TT' is the required tangent at P.



(iii) Ans. Let a be the side of the rhombus.

Given, perimeter of a rhombus is 76 cm

∴ Side of the rhombus $(a) = \frac{76}{4} = 19\ cm$



∴ Area of rhombus $ABCD\ = ar(\Delta ABC) + ar(\Delta ADC)$

Area of equilateral triangle $ABC = \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 19 \times 19 = \frac{361}{4}\sqrt{3}\ cm^2$

Area of equilateral triangle $ADC = \frac{361}{4}\sqrt{3}\ cm^2$

∴ Area of rhombus $ABCD\ = ar(\Delta ABC) + ar(\Delta ADC)$

$= \left(\frac{361}{4}\sqrt{3} + \frac{361}{4}\sqrt{3}\right) cm^2$

$= \frac{361}{2}\sqrt{3}\ cm^2$

Q.5 (i) Ans. We have,

$\frac{\sin^2 54° + \sin^2 36°}{\cos^2 54° + \cos^2 36°} + 11(\sin^2 37° + \sin^2 53°) - 11(\cos^2 35° + \cos^2 55°)$

$= \frac{\sin^2 54° + \sin^2(90° - 54°)}{\cos^2 54° + \cos^2(90° - 54°)} + 11(\sin^2 37° + \sin^2(90° - 37°)) - 11(\cos^2 35° + \cos^2(90 - 35°)$

$(\because \sin(90° - \theta) = \cos\theta\ \ and\ \ \cos(90° - \theta) = \sin\theta)$

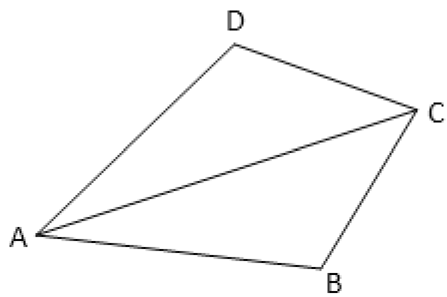
$= \frac{\sin^2 54° + \cos^2 54°}{\cos^2 54° + \sin^2 54°} + 11(\sin^2 37° + \cos^2 37°) - 11(\cos^2 35° + \sin^2 35°)$

$= \frac{1}{1} + 11(1) - 11(1)$ $\qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 1$

Hence, $\frac{\sin^2 54° + \sin^2 36°}{\cos^2 54° + \cos^2 36°} + 11(\sin^2 37° + \sin^2 53°) - 11(\cos^2 35° + \cos^2 55°) = 1.$

(ii) Ans. Join A and C. Then,



Area of quadrilateral $ABCD = ar(\Delta ABC) + ar(\Delta ACD)$
Given, $A(3,2), B(-3,1), C(2,0)$and $D(a,-1)$
Area of $\Delta ABC = \frac{1}{2}[3(1-0) - 3(0-2) + 2(2-1)]$
$= \frac{1}{2}[3+6+2]$
$= \frac{11}{2}$ Sq. units
$\therefore\ ar(\Delta ACD) =$ Area of quadrilateral $ABCD - ar(\Delta ABC)$
$= \left(6 - \frac{11}{2}\right) = \frac{1}{2}$ square units
Area of $\Delta\ ACD = \frac{1}{2}[3(0+1) + 2(-1-2) + a(2-0)]$
$\Rightarrow \frac{1}{2}[3 - 6 + 2a] = \frac{1}{2}$ Sq. units
$\Rightarrow [-3 + 2a] = 1$
$\Rightarrow 2a = 4$
$\Rightarrow a = 2$
Hence, $a = 2$.

(iii) Ans. Let R and r be the larger and smaller radius of the bucket respectively and h be the height of the bucket.

Given, $R = 28\ cm, h = 45\ cm\ and\ r = ?$

Volume of the conical bucket $= 48510\ cm^3$

$\therefore$ Volume of the frustum of the cone $= \frac{\pi h}{3}[R^2 + r^2 + Rr]$ cubic units

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 45 \times [28^2 + r^2 + 28r] = 48510$

$\Rightarrow \frac{22}{7} \times 15 \times [28^2 + r^2 + 28r] = 48510$

$\Rightarrow 784 + r^2 + 28r = \frac{339570}{330} = 1029$

$\Rightarrow r^2 + 28r - 245 = 0$

$\Rightarrow r^2 + 35r - 7r - 245 = 0$

$\Rightarrow (r + 35)(r - 7) = 0$

$\Rightarrow r = 7\ cm\ \ (\because r \neq 35)$

Hence, the smaller radius of the conical bucket is $7\ cm$.