MAHARASHTRA BOARD CLASS 12
MATHS QUESTIONS

MATHEMATICS – XII

Time: 3 Hrs **Max. Marks: 80**

Note:

(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.
(iii) Graph of L.P.P. should be drawn on graph paper only.
(iv) Use of logarithmic table is allowed.
(v) Answers to the questions of Section – I and Section – II should be written in only one answer book.
(vi) Answer to every new question must be written on a new page.

SECTION – I

Q1. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions:

(i) If $[3x^2 + 10xy + 5y^2] = [x \quad y]A\begin{bmatrix} x \\ y \end{bmatrix}$ and $A - A^T = 0$, then find the value of A. [2]

(a) $\begin{bmatrix} 3 & 10 \\ 10 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 3 \\ 5 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -5 \\ -5 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 5 \\ 5 & 5 \end{bmatrix}$

(ii) A straight line perpendicular to the line 2x + y = 3 is passing through (1, 1). Its y -intercept is [2]

(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(iii) Cartesian equation of the plane $\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (3 - 2s + 2t)\hat{k}$ is [2]

(a) $2x + y = 5$ (b) $2x - y = 5$
(c) $2x + z = 5$ (d) $2x - z = 5$

(B) Attempt any three of the following:

(i) Let p = "Poornima comes to the theatre"
q = "Lalitha comes to the theatre"
r = "Sandya comes to the theatre"
s = "Kalai comes to the theatre"
Write the following statements in the symbolic form:

a) Kalai comes to the theatre if and only if Sandhya comes and Poornima doesn't come"
b) Sandhya comes to the theatre provided that Kalai doesn't come, but, if Kalai comes, then Lalitha doesn't come. [2]

(ii) What is the value of $2\sin X + 4\sin Y$ if the below three conditions are satisfied:

(a) $\frac{\cos X}{3} = \frac{\cos Y}{4} = \frac{1}{5}$

(b) $\frac{\pi}{2} < X < 0$

(c) $-\frac{\pi}{2} < Y < 0$? [2]

(iii) Find the length of the median PT of the triangle PQR whose sides PQ and PR are given by the vectors $3\hat{\imath} + 4\hat{k}$ and $5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ [2]

(iv) If the vectors $-17\hat{\imath} - 12\hat{\jmath} + \hat{k}, 5\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $-6\hat{\imath} - 11\hat{\jmath} + \hat{k}$ are coplanar, find the value of λ. [2]

(v) If a plane $x + y + 2z - 2 = 0$ is parallel to the plane $x + y + 2z - 5 = 0$ then what is the distance between two parallel planes? [2]

Q2. (A) Attempt any two of the following: **(6)**

(i) What is the range of values that p could be if

(a) $\sqrt{2cot\beta + \frac{1}{\sin^2\beta}} = p - cot\beta$

(b) $3\pi < 4\beta < 4\pi$? [3]

(ii) Find the direction ratio of the line passing through the origin if the line makes 60 degrees with the line $x - 2y - z + 3 = 0$. [3]

(iii) Show that the equation $x^2 - 4xy + 4y^2 - 3x + 6y - 10$ represents a pair of lines. Find the angle between the two lines. [3]

(B) Attempt any two of the following: **(8)**

(i) (Using matrices, find the value of p if the points $A(p, 2-2p), B(-p+1, 2p), C(-4-p, 6-2p)$ are collinear and also find the coordinates of A, B and C such that none of the points overlap? [4]

(ii) What is the value of $\cot\left(\sum_{k=1}^{29} cot^{-1}(1 + \sum_{m=1}^{k} 2m)\right)$? [4]

(iii) The tangent to a circle with centre O is x-axis. This circle also touches another circle with equation $x^2 + y^2 - 6y + 5 = 0$. What is the equation of the locus of the point O? [4]

Q3. (A) Attempt any two of the following: **(6)**

(i) $y = 2 - \frac{x}{2}$ and $y = 2 - \frac{3z}{2}$ are the equation of lines forming a tetrahedron which is bounded by four different planes. The four planes include the three coordinate planes (XY plane, YZ plane and ZX plane) and the plane $2x + 4y + 6z = 8$. Find the volume of the tetrahedron so formed. [3]

(ii) A line segment PQ, formed by two points in space, P$(\vec{p})$ and Q $(\vec{q})$, has another point $A(\vec{a})$ dividing it internally such that $\frac{PA}{AQ} = \frac{m}{n}$.

a. Show that $\vec{a} = \frac{m\vec{q} + n\vec{p}}{m+n}$

b. If the coordinates of P and Q are (1,-2,1) and (1,4,-2) and $PA : AQ = 2 : 1$, then find the position vector of point A. [3]

(iii) A textile company makes two different models A and B of a product. Each piece of model A requires 6 labour hours for fabricating and 2 labour hours for finishing. Each piece of model B requires 8 hours for fabricating and 6 hours for finishing. For fabricating and finishing, the maximum labour hours available

are 120 and 60 respectively. The company makes a profit of Rs.4000 and Rs.6000 on each piece of model A and model B respectively. How many pieces of Model A and Model B should be manufactured per week to make a maximum profit? What is the maximum profit per week?

[3]

(B) Attempt any two of the following: **(8)**

(i) The farthest planet X moves in a highly elliptical orbit with the Sun situated at one of the two foci. At the most distant point of its orbit, X gets to within 7.4 billion kilometres from the Sun. At the closest point of its orbit, X gets to within 4.4 billion kilometres. Find the equation of the orbit of the planet X assuming centre is at (0, 0). [4]

(ii) Show that the points P, Q, R with position vectors $\bar{p} = 3\hat{\imath} + 2\hat{k}, \bar{q} = 4\hat{\imath} + 3\hat{\jmath}$ and $\bar{r} = 8\hat{\imath} + \hat{\jmath} - \hat{k}$ respectively form the vertices of a right triangle. [4]

(iii) If $A(1,8,4); B(0,-11,4); C(2,-3,1)$ are three points in space and D is the point on BC which dropped along normal to BC from A. Determine the coordinates of D. [4]

SECTION – II

Q4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions:

(i) The order and degree of the differential equation $v = u\frac{dv}{du} + \frac{2}{\frac{dv}{du}}$ are [2]

(a) 1,2
(b) 1,-1
(c) 1,1
(d) 0,1

(ii) For $f(\theta) = \theta^4 + |\theta|$, let $I_a = \int_0^{\pi} f(\cos\theta)d\theta$, $I_b = \int_0^{\pi/2} f(\sin\theta)d\theta$, then $\frac{I_a}{I_b} = ?$ [2]

(a) 1
(b) 2
(c) ½
(d) 4

(iii) Joe is a mechanic working in a car service company. The time Joe takes to run a maintenance check for one car and to change its engine oil is based on the recommendation of that car's manufacturer follows exponential distribution with a mean of 20 minutes. What is the probability that the time taken for Joe to finish the work on a car would be between 15 and 25 minutes? [2]

(a) 0.1859
(b) 0.4724
(c) 0.7135
(d) 0.4001

(B) Attempt any THREE of the following:

(i) If $v = g(t^2 + 2)$ and $g'(3) = 5$, then what is the value of $\frac{dv}{dt}$ at $t = 1$? [2]

(ii) An apartment complex has 125 apartments to rent. If x is the number of apartments they rent, then their monthly profit is given by

$P(x) = -4x^2 + 800x - 20000$

For maximizing their profit, how many apartments should they rent? [2]

(iii) Evaluate $\int \sqrt{(1+cos\alpha)}d\alpha$ [2]

(iv) (iv) In an exhibition, there is a game where they can win many cute prizes. The game is defined as follows:

A box is placed which contains many rods of different lengths. The player is asked to select some rods randomly from the box. The length of any one of the randomly picked rod has a continuous uniform distribution. This distribution is over the interval [7, 10]. The lengths are measured in centimetres.

For winning a clock, the player must select six rods such that more than four rods which selected are shorter than 0.076 m. What is the probability of the player winning a clock?

[2]

(v) If $\sqrt{p} = qe^{\beta \cot\theta}$ where q and θ are real numbers, then what is the value of $\frac{d^2p}{d\beta^2} - 4pcot^2\theta$? [2]

Q5. (A) Attempt any TWO of the following:

(i) If f(t) $= \begin{cases} me^t + n + 1, & if\ t \leq 0 \\ mt^2 + n(t+3), & if\ 0 < t \leq 1 \\ mcos(\pi t) + 7nt, & if\ t > 1 \end{cases}$ so that f(t) is continuous for $t \in R$, then what are the values of m and n? [3]

(ii) What is the value of $\frac{dt}{d\theta}$ $if\ t = e^{\theta + e^{\theta + e^{\theta + \cdots}}}$? [3]

(iii) What is the value of the integral $\int_0^1 \left(\frac{dt}{1+t+t^2}\right)$? [3]

(A) Attempt any TWO of the following:

(i) In which quadrant, the normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1,1)$ will cut the curve again. [4]

(ii) Evaluate $\int \frac{x^2-1}{x^4+1}\, dx.$ [4]

(iii) Evaluate $\lim_{n\to\infty} \left[\frac{(n+1)(n+2)\ldots\ldots(4n)}{n^{3n}}\right]^{1/n}$ [4]

Q6. (A) Attempt any TWO of the following:

(i) If the area between X-axis, $y = f(x)$ curve and the lines $x = 1$ and $x = b$ is equal to $\log\left(b + \sqrt{b^2+1}\right)$, then determine $f(x)$ for all $b > 1$. [3]

(ii) Maria came out from railway station and started walking towards the main road with her luggage. After a certain distance on the footpath near the road where there was heavy movement of vehicles, she thought to hire a taxi to reach her home quicker. The time between any two taxis arriving at the main road for giving a lift, follows exponential distribution with a mean of 600 seconds. The probability that Maria waits more than 'x' minutes is 10%. What is the value of 'k'? [3]

(iii) In a binomial variate, X with $n = 6$ if $P(X = 2) = 9\,P(X = 4)$, find variance. [3]

(B) Attempt any TWO of the following:

(i) Determine the solution of differential equation $\frac{dy}{dx} + \frac{2y}{x} = \sin x$. [4]

(ii) Compute the Karl Pearson's correlation coefficient for the given data

A	B
9	6
8	9
7	8

6	7
11	10
10	11
5	5

[4]

(iii) Anil is having two bags with two different colored balls in which bag-1 contains 3 black and 4 white balls; and bag-2 contains 4 black and 3 white balls. To select a bag he throws a die and if it shows 1 or 3, bag-1 is selected or bag-2 for the rest numbers on the die. Determine the probability of picking a black ball by Anil from the selected bag. [4]