CHAPTER EIGHT

GRAVITATION

8.1 INTRODUCTION

Early in our lives, we become aware of the tendency of all material objects to be attracted towards the earth. Anything thrown up falls down towards the earth, going uphill is lot more tiring than going downhill, raindrops from the clouds above fall towards the earth and there are many other such phenomena. Historically it was the Italian Physicist Galileo (1564-1642) who recognised the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration. It is said that he made a public demonstration of this fact. To find the truth, he certainly did experiments with bodies rolling down inclined planes and arrived at a value of the acceleration due to gravity which is close to the more accurate value obtained later.

A seemingly unrelated phenomenon, observation of stars, planets and their motion has been the subject of attention in many countries since the earliest of times. Observations since early times recognised stars which appeared in the sky with positions unchanged year after year. The more interesting objects are the planets which seem to have regular motions against the background of stars. The earliest recorded model for planetary motions proposed by Ptolemy about 2000 years ago was a 'geocentric' model in which all celestial objects, stars, the sun and the planets, all revolved around the earth. The only motion that was thought to be possible for celestial objects was motion in a circle. Complicated schemes of motion were put forward by Ptolemy in order to describe the observed motion of the planets. The planets were described as moving in circles with the centre of the circles themselves moving in larger circles. Similar theories were also advanced by Indian astronomers some 400 years later. However a more elegant model in which the Sun was the centre around which the planets revolved – the 'heliocentric' model – was already mentioned by Aryabhatta (5th century A.D.) in his treatise. A thousand years later, a Polish monk named Nicolas

Copernicus (1473-1543) proposed a definitive model in which the planets moved in circles around a fixed central sun. His theory was discredited by the church, but notable amongst its supporters was Galileo who had to face prosecution from the state for his beliefs.

It was around the same time as Galileo, a nobleman called Tycho Brahe (1546-1601) hailing from Denmark, spent his entire lifetime recording observations of the planets with the naked eye. His compiled data were analysed later by his assistant Johannes Kepler (1571-1640). He could extract from the data three elegant laws that now go by the name of Kepler's laws. These laws were known to Newton and enabled him to make a great scientific leap in proposing his universal law of gravitation.

8.2 KEPLER'S LAWS

The three laws of Kepler can be stated as follows:

1. Law of orbits : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse (Fig. 8.1a). This law was a deviation from the Copernican model which allowed only circular orbits. The ellipse, of which the circle is a special case, is a closed curve which can be drawn very simply as follows.

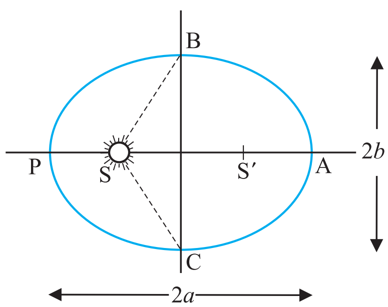


Fig. 8.1(a) *An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.*

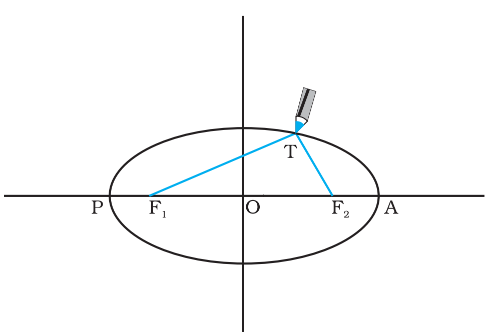


Fig. 8.1(b) *Drawing an ellipse. A string has its ends fixed at F_1 and F_2. The tip of a pencil holds the string taut and is moved around.*

Select two points F_1 and F_2. Take a length of a string and fix its ends at F_1 and F_2 by pins. With the tip of a pencil stretch the string taut and then draw a curve by moving the pencil keeping the string taut throughout.(Fig. 8.1(b)) The closed curve you get is called an ellipse. Clearly for any point T on the ellipse, the sum of the distances from F_1 and F_2 is a constant. F_1, F_2 are called the focii. Join the points F_1 and F_2 and extend the line to intersect the ellipse at points P and A as shown in Fig. 8.1(b). The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two focii merge into one and the semi-major axis becomes the radius of the circle.

2. Law of areas : The line that joins any planet to the sun sweeps equal areas in equal intervals of time (Fig. 8.2). This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.

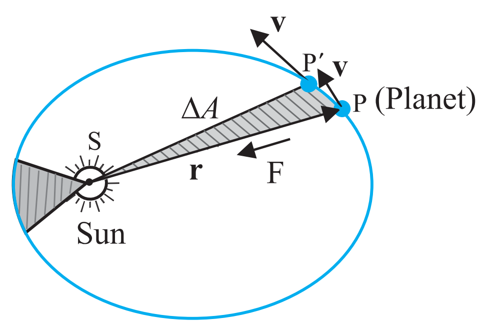


Fig. 8.2 *The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.*

3. Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

Table 8.1 gives the approximate time periods of revolution of eight* planets around the sun along with values of their semi-major axes.

* Refer to information given in the Box on Page 182

Table 8.1 **Data from measurement of planetary motions given below confirm Kepler's Law of Periods**

(a ≡ Semi-major axis in units of 10^{10} m.
T ≡ Time period of revolution of the planet in years(y).
Q ≡ The quotient (T^2/a^3) in units of 10^{-34} y^2 m^{-3}.)

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99
Pluto*	590	248	2.99

The law of areas can be understood as a consequence of conservation of angular momentum whch is valid for any central force . A central force is such that the force on the planet is along the vector joining the sun and the planet. Let the sun be at the origin and let the position and momentum of the planet be denoted by **r** and **p** respectively. Then the area swept out by the planet of mass m in time interval Δt is (Fig. 8.2) $\Delta\mathbf{A}$ given by

$$\Delta\mathbf{A} = \tfrac{1}{2}\ (\mathbf{r} \times \mathbf{v}\Delta t) \tag{8.1}$$

Hence

$$\Delta\mathbf{A}/\Delta t = \tfrac{1}{2}\ (\mathbf{r} \times \mathbf{p})/m, \text{ (since } \mathbf{v} = \mathbf{p}/m)$$
$$= \mathbf{L} / (2\ m) \tag{8.2}$$

where **v** is the velocity, **L** is the angular momentum equal to (**r** × **p**). For a central force, which is directed along **r**, **L** is a constant as the planet goes around. Hence, $\Delta\mathbf{A}/\Delta t$ is a constant according to the last equation. This is the law of areas. Gravitation is a central force and hence the law of areas follows.

Johannes Kepler (1571–1630) was a scientist of German origin. He formulated the three laws of planetary motion based on the painstaking observations of Tycho Brahe and coworkers. Kepler himself was an assistant to Brahe and it took him sixteen long years to arrive at the three planetary laws. He is also known as the founder of geometrical optics, being the first to describe what happens to light after it enters a telescope.

* *Refer to information given in the Box on Page 182*

▶ ***Example 8.1*** Let the speed of the planet at the perihelion *P* in Fig. 8.1(a) be v_P and the Sun-planet distance SP be r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at the aphelion $\{r_A, v_A\}$. Will the planet take equal times to traverse *BAC* and *CPB* ?

Answer The magnitude of the angular momentum at *P* is $L_p = m_p r_p v_p$, since inspection tells us that $\mathbf{r}_p$ and $\mathbf{v}_p$ are mutually perpendicular. Similarly, $L_A = m_p r_A v_A$. From angular momentum conservation

$$m_p r_p v_p = m_p r_A v_A$$

or
$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$ ◀

Since $r_A > r_p$, $v_p > v_A$.

The area *SBAC* bounded by the ellipse and the radius vectors *SB* and *SC* is larger than SBPC in Fig. 8.1. From Kepler's second law, equal areas are swept in equal times. Hence the planet will take a longer time to traverse *BAC* than *CPB*.

8.3 UNIVERSAL LAW OF GRAVITATION

Legend has it that observing an apple falling from a tree, Newton was inspired to arrive at an universal law of gravitation that led to an explanation of terrestrial gravitation as well as of Kepler's laws. Newton's reasoning was that the moon revolving in an orbit of radius R_m was subject to a centripetal acceleration due to earth's gravity of magnitude

$$a_m = \frac{V^2}{R_m} = \frac{4\pi^2 R_m}{T^2} \tag{8.3}$$

where *V* is the speed of the moon related to the time period *T* by the relation $V = 2\pi R_m / T$. The time period *T* is about 27.3 days and R_m was already known then to be about 3.84×10^8m. If we substitute these numbers in Eq. (8.3), we get a value of a_m much smaller than the value of acceleration due to gravity g on the surface of the earth, arising also due to earth's gravitational attraction.

Central Forces

We know the time rate of change of the angular momentum of a single particle about the origin is

$$\frac{d\mathbf{l}}{dt} = \mathbf{r} \times \mathbf{F}$$

The angular momentum of the particle is conserved, if the torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ due to the force **F** on it vanishes. This happens either when **F** is zero or when **F** is along **r**. We are interested in forces which satisfy the latter condition. Central forces satisfy this condition. A 'central' force is always directed towards or away from a fixed point, i.e., along the position vector of the point of application of the force with respect to the fixed point. (See Figure below.) Further, the magnitude of a central force F depends on r, the distance of the point of application of the force from the fixed point; $F = F(r)$.

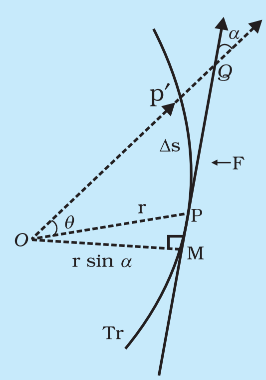
In the motion under a central force the angular momentum is always conserved. Two important results follow from this:

(1) The motion of a particle under the central force is always confined to a plane.

(2) The position vector of the particle with respect to the centre of the force (i.e. the fixed point) has a constant areal velocity. In other words the position vector sweeps out equal areas in equal times as the particle moves under the influence of the central force.

Try to prove both these results. You may need to know that the areal velocity is given by : $dA/dt = ½\, r\, v \sin \alpha$.

An immediate application of the above discussion can be made to the motion of a planet under the gravitational force of the sun. For convenience the sun may be taken to be so heavy that it is at rest. The gravitational force of the sun on the planet is directed towards the sun. This force also satisfies the requirement $F = F(r)$, since $F = G\, m_1 m_2/r^2$ where m_1 and m_2 are respectively the masses of the planet and the sun and G is the universal constant of gravitation. The two results (1) and (2) described above, therefore, apply to the motion of the planet. In fact, the result (2) is the well-known second law of Kepler.



Tr is the trajectory of the particle under the central force. At a position P, the force is directed along **OP**, *O is the centre of the force taken as the origin. In time Δt, the particle moves from P to P′, arc PP′ = $\Delta s = v\,\Delta t$. The tangent PQ at P to the trajectory gives the direction of the velocity at P. The area swept in Δt is the area of sector POP′ $\approx (r \sin\alpha)$ PP′/2 = $(r\, v \sin \alpha)\, \Delta t/2$.)*

This clearly shows that the force due to earth's gravity decreases with distance. If one assumes that the gravitational force due to the earth decreases in proportion to the inverse square of the distance from the centre of the earth, we will have $a_m \,\alpha\, R_m^{-2}$; $g \,\alpha\, R_E^{-2}$ and we get

$$\frac{g}{a_m} = \frac{R_m^2}{R_E^2} \simeq 3600 \qquad (8.4)$$

in agreement with a value of $g \simeq 9.8$ m s^{-2} and the value of a_m from Eq. (8.3). These observations led Newton to propose the following Universal Law of Gravitation :

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

The quotation is essentially from Newton's famous treatise called 'Mathematical Principles of Natural Philosophy' (Principia for short).

Stated Mathematically, Newton's gravitation law reads : The force **F** on a point mass m_2 due to another point mass m_1 has the magnitude

$$|\mathbf{F}| = G\,\frac{m_1\, m_2}{r^2} \qquad (8.5)$$

Equation (8.5) can be expressed in vector form as

$$\mathbf{F} = G\,\frac{m_1\, m_2}{r^2}\left(-\hat{\mathbf{r}}\right) = -G\,\frac{m_1\, m_2}{r^2}\,\hat{\mathbf{r}}$$

$$= -G\,\frac{m_1\, m_2}{|\mathbf{r}|^3}\,\hat{\mathbf{r}}$$

where G is the universal gravitational constant, $\hat{\mathbf{r}}$ is the unit vector from m_1 to m_2 and $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ as shown in Fig. 8.3.

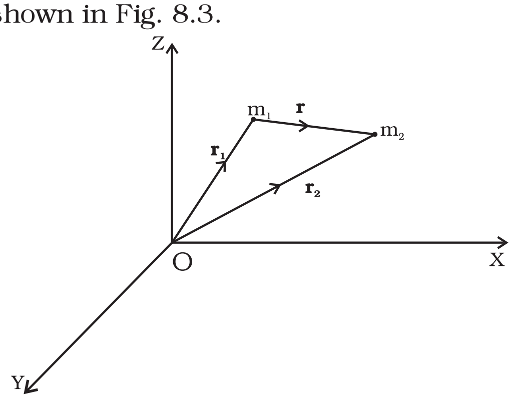


Fig. 8.3 *Gravitational force on m_1 due to m_2 is along* **r** *where the vector* **r** *is* ($\mathbf{r}_2 - \mathbf{r}_1$).

The gravitational force is attractive, i.e., the force **F** is along – **r**. The force on point mass m_1 due to m_2 is of course – **F** by Newton's third law. Thus, the gravitational force $\mathbf{F}_{12}$ on the body 1 due to 2 and $\mathbf{F}_{21}$ on the body 2 due to 1 are related as $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

Before we can apply Eq. (8.5) to objects under consideration, we have to be careful since the law refers to **point** masses whereas we deal with extended objects which have finite size. . If we have a collection of point masses,the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses as shown in Fig 8.4.

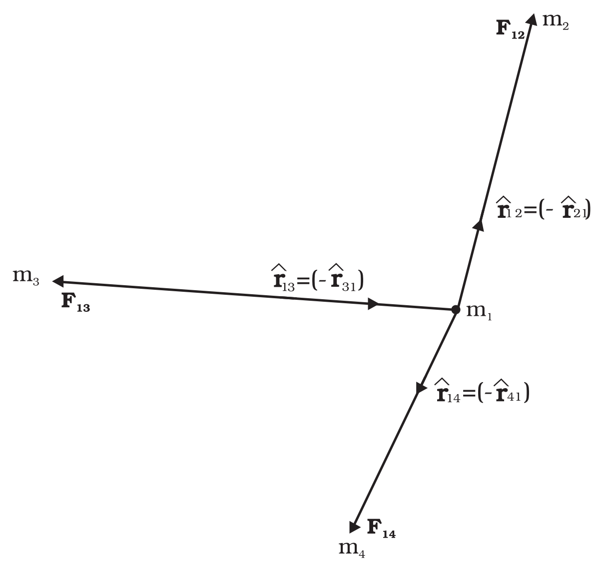


Fig. 8.4 *Gravitational force on point mass m_1 is the vector sum of the gravitational forces exerted by m_2, m_3 and m_4.*

The total force on m_1 is

$$\mathbf{F}_1 = \frac{Gm_2\, m_1}{r_{21}^2}\,\hat{\mathbf{r}}_{21} + \frac{Gm_3\, m_1}{r_{31}^2}\,\hat{\mathbf{r}}_{31} + \frac{Gm_4\, m_1}{r_{41}^2}\,\hat{\mathbf{r}}_{41}$$

▶ ***Example 8.2*** Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC.
(a) What is the force acting on a mass 2*m* placed at the centroid G of the triangle?
(b) What is the force if the mass at the vertex A is doubled ?
Take AG = BG = CG = 1m (see Fig. 8.5)

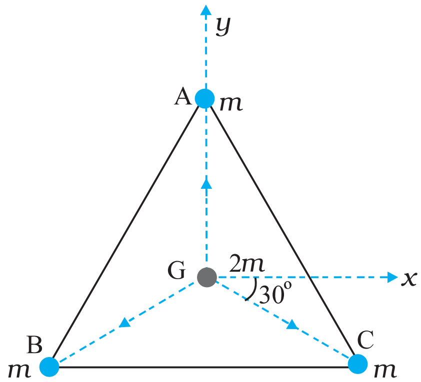


Fig. 8.5 *Three equal masses are placed at the three vertices of the Δ ABC. A mass 2m is placed at the centroid G.*

Answer (a) The angle between GC and the positive x-axis is 30° and so is the angle between GB and the negative x-axis. The individual forces in vector notation are

$$\mathbf{F}_{GA} = \frac{Gm(2m)}{1}\hat{\mathbf{j}}$$

$$\mathbf{F}_{GB} = \frac{Gm(2m)}{1}\left(-\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right)$$

$$\mathbf{F}_{GC} = \frac{Gm(2m)}{1}\left(+\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right)$$

From the principle of superposition and the law of vector addition, the resultant gravitational force $\mathbf{F}_R$ on $(2m)$ is

$$\mathbf{F}_R = \mathbf{F}_{GA} + \mathbf{F}_{GB} + \mathbf{F}_{GC}$$

$$\mathbf{F}_R = 2Gm^2\,\hat{\mathbf{j}} + 2Gm^2\left(-\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right)$$

$$+ 2Gm^2\left(\hat{\mathbf{i}}\cos 30^o - \hat{\mathbf{j}}\sin 30^o\right) = 0$$

Alternatively, one expects on the basis of symmetry that the resultant force ought to be zero.

(b) By symmetry the x-component of the force cancels out. The y-component survives.

$$\mathbf{F}_R = 4Gm^2\hat{\mathbf{j}} - 2Gm^2\hat{\mathbf{j}} = 2Gm^2\hat{\mathbf{j}}$$ ◀

For the gravitational force between an extended object (like the earth) and a point mass, Eq. (8.5) is not directly applicable. Each point mass in the extended object will exert a force on the given point mass and these force will not all be in the same direction. We have to add up these forces vectorially for all the point masses in the extended object to get the total force. This is easily done using calculus. For two special cases, a simple law results when you do that :

(1) **The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.** Qualitatively this can be understood as follows: Gravitational forces caused by the various regions of the shell have components along the line joining the point mass to the centre as well as along a direction prependicular to this line. The components prependicular to this line cancel out when summing over all regions of the shell leaving only a resultant force along the line joining the point to the centre. The magnitude of this force works out to be as stated above.

Newton's Principia

Kepler had formulated his third law by 1619. The announcement of the underlying universal law of gravitation came about seventy years later with the publication in 1687 of Newton's masterpiece **Philosophiae Naturalis Principia Mathematica**, often simply called the **Principia**.

Around 1685, Edmund Halley (after whom the famous Halley's comet is named), came to visit Newton at Cambridge and asked him about the nature of the trajectory of a body moving under the influence of an inverse square law. Without hesitation Newton replied that it had to be an ellipse, and further that he had worked it out long ago around 1665 when he was forced to retire to his farm house from Cambridge on account of a plague outbreak. Unfortunately, Newton had lost his papers. Halley prevailed upon Newton to produce his work in book form and agreed to bear the cost of publication. Newton accomplished this feat in eighteen months of superhuman effort. The **Principia** is a singular scientific masterpiece and in the words of Lagrange it is "the greatest production of the human mind." The Indian born astrophysicist and Nobel laureate S. Chandrasekhar spent ten years writing a treatise on the **Principia**. His book, Newton's **Principia for the Common Reader** brings into sharp focus the beauty, clarity and breath taking economy of Newton's methods.

(2) **The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.**
Qualitatively, we can again understand this result. Various regions of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

8.4 THE GRAVITATIONAL CONSTANT

The value of the gravitational constant G entering the Universal law of gravitation can be determined experimentally and this was first done by English scientist Henry Cavendish in 1798. The apparatus used by him is schematically shown in figure.8.6

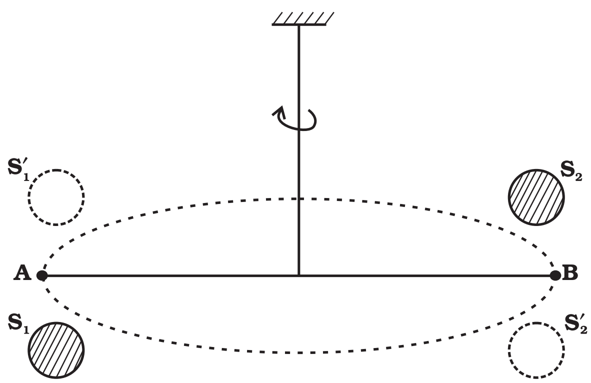


Fig. 8.6 *Schematic drawing of Cavendish's experiment. S_1 and S_2 are large spheres which are kept on either side (shown shades) of the masses at A and B. When the big spheres are taken to the other side of the masses (shown by dotted circles), the bar AB rotates a little since the torque reverses direction. The angle of rotation can be measured experimentally.*

The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown. The big spheres attract the nearby small ones by equal and opposite force as shown. There is no net force on the bar but only a torque which is clearly equal to F times the length of the bar,where F is the force of attraction between a big sphere and its neighbouring small sphere. Due to this torque, the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque . If θ is the angle of twist of the suspended wire, the restoring torque is proportional to θ, equal to $\tau\theta$. Where τ is the restoring couple per unit angle of twist. τ can be measured independently e.g. by applying a known torque and measuring the angle of twist. The gravitational force between the spherical balls is the same as if their masses are concentrated at their centres. Thus if d is the separation between the centres of the big and its neighbouring small ball, M and m their masses, the gravitational force between the big sphere and its neighouring small ball is.

$$F = G\frac{Mm}{d^2} \tag{8.6}$$

If L is the length of the bar AB , then the torque arising out of F is F multiplied by L. At equilibrium, this is equal to the restoring torque and hence

$$G\frac{Mm}{d^2}L = \tau\,\theta \tag{8.7}$$

Observation of θ thus enables one to calculate G from this equation.

Since Cavendish's experiment, the measurement of G has been refined and the currently accepted value is

$$G = 6.67\times10^{-11}\ \mathrm{N\ m^2/kg^2} \tag{8.8}$$

8.5 ACCELERATION DUE TO GRAVITY OF THE EARTH

The earth can be imagined to be a sphere made of a large number of concentric spherical shells with the smallest one at the centre and the largest one at its surface. A point outside the earth is obviously outside all the shells. Thus, all the shells exert a gravitational force at the point outside just as if their masses are concentrated at their common centre according to the result stated in section 8.3. The total mass of all the shells combined is just the mass of the earth. Hence, at a point outside the earth, the gravitational force is just as if its entire mass of the earth is concentrated at its centre.

For a point inside the earth, the situation is different. This is illustrated in Fig. 8.7.

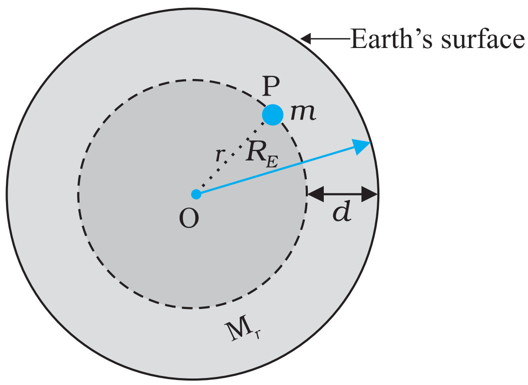


Fig. 8.7 *The mass m is in a mine located at a depth d below the surface of the Earth of mass M_E and radius R_E. We treat the Earth to be spherically symmetric.*

Again consider the earth to be made up of concentric shells as before and a point mass m situated at a distance r from the centre. The point P lies outside the sphere of radius r. For the shells of radius greater than r, the point P lies inside. Hence according to result stated in the last section, they exert no gravitational force on mass m kept at P. The shells with radius $\leq r$ make up a sphere of radius r for which the point P lies on the surface. This smaller sphere therefore exerts a force on a mass m at P as if its mass M_r is concentrated at the centre. Thus the force on the mass m at P has a magnitude

$$F = \frac{Gm\,(M_r)}{r^2} \tag{8.9}$$

We assume that the entire earth is of uniform density and hence its mass is $M_E = \frac{4\pi}{3} R_E^3\,\rho$ where M_E is the mass of the earth R_E is its radius and ρ is the density. On the other hand the mass of the sphere M_r of radius r is $\frac{4\pi}{3}\rho r^3$ and hence

$$F = Gm\left(\frac{4\pi}{3}\rho\right)\frac{r^3}{r^2} = Gm\left(\frac{M_E}{R_E^3}\right)\frac{r^3}{r^2}$$

$$= \frac{Gm\,M_E}{R_E^{\ 3}}\,r \tag{8.10}$$

If the mass m is situated on the surface of earth, then $r = R_E$ and the gravitational force on it is, from Eq. (8.10)

$$F = G\,\frac{M_E m}{R_E^2} \tag{8.11}$$

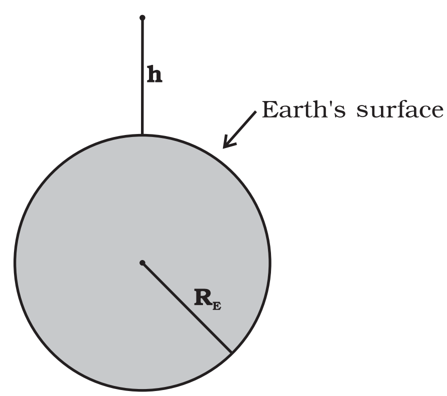
The acceleration experienced by the mass m, which is usually denoted by the symbol g is related to F by Newton's 2nd law by relation F = mg. Thus

$$g = \frac{F}{m} = \frac{GM_E}{R_E^2} \tag{8.12}$$

Acceleration g is readily measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment (or otherwise), combined with knowledge of g and R_E enables one to estimate M_E from Eq. (8.12). This is the reason why there is a popular statement regarding Cavendish : "Cavendish weighed the earth".

8.6 ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH

Consider a point mass m at a height h above the surface of the earth as shown in Fig. 8.8(a). The radius of the earth is denoted by R_E. Since this point is outside the earth,



(a)

Fig. 8.8 (a) *g at a height h above the surface of the earth.*

its distance from the centre of the earth is $(R_E + h)$. If $F(h)$ denoted the magnitude of the force on the point mass m , we get from Eq. (8.5) :

$$F(h) = \frac{GM_E m}{(R_E + h)^2} \tag{8.13}$$

The acceleration experienced by the point mass is $F(h)/m \equiv g(h)$ and we get

$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2} \,. \qquad (8.14)$$

This is clearly less than the value of g on the surface of earth : $g = \frac{GM_E}{R_E^2}$. For $h << R_E$, we can expand the RHS of Eq. (8.14) :

$$g(h) = \frac{GM_E}{R_E^2(1 + h/R_E)^2} = g\,(1 + h/R_E)^{-2}$$

For $\frac{h}{R_E} << 1$, using binomial expression,

$$g(h) \cong g\left(1 - \frac{2h}{R_E}\right). \qquad (8.15)$$

Equation (8.15) thus tells us that for small heights h above the value of g decreases by a factor $(1 - 2h/R_E)$.

Now, consider a point mass m at a depth d below the surface of the earth (Fig. 8.8(b)), so that its distance from the centre of the earth is $(R_E - d)$ as shown in the figure. The earth can be thought of as being composed of a smaller sphere of radius $(R_E - d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero because the result quoted in the previous section. As far as the smaller sphere of radius $(R_E - d)$ is concerned, the point mass is outside it and hence according to the result quoted earlier, the force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_s is the mass of the smaller sphere, then,

$$M_s / M_E = (R_E - d)^3 / R_E^3 \qquad (8.16)$$

Since mass of a sphere is proportional to be cube of its radius.

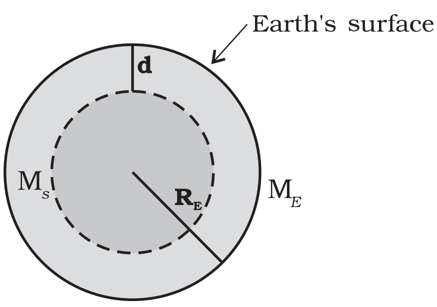


Fig. 8.8 (b) *g at a depth d. In this case only the smaller sphere of radius $(R_E–d)$ contributes to g.*

Thus the force on the point mass is

$$F(d) = G\, M_s\, m / (R_E - d)^2 \qquad (8.17)$$

Substituting for M_s from above , we get

$$F(d) = G\, M_E\, m\,(R_E - d) / R_E^3 \qquad (8.18)$$

and hence the acceleration due to gravity at a depth d,

$$g(d) = \frac{F(d)}{m} \text{ is}$$

$$g(d) = \frac{F(d)}{m} = \frac{GM_E}{R_E^3}(R_E - d)$$

$$= g\frac{R_E - d}{R_E} = g(1 - d/R_E) \qquad (8.19)$$

Thus, as we go down below earth's surface, the acceleration due gravity decreases by a factor $(1 - d/R_E)$. The remarkable thing about acceleration due to earth's gravity is that it is maximum on its surface decreasing whether you go up or down.

8.7 GRAVITATIONAL POTENTIAL ENERGY

We had discussed earlier the notion of potential energy as being the energy stored in the body at its given position. If the position of the particle changes on account of forces acting on it, then the change in its potential energy is just the amount of work done on the body by the force. As we had discussed earlier, forces for which the work done is independent of the path are the conservative forces.

The force of gravity is a conservative force and we can calculate the potential energy of a body arising out of this force, called the gravitational potential energy. Consider points close to the surface of earth, at distances from the surface much smaller than the radius of the earth. In such cases, the force of gravity is practically a constant equal to mg, directed towards the centre of the earth. If we consider a point at a height h_1 from the surface of the earth and another point vertically above it at a height h_2 from the surface, the work done in lifting the particle of mass m from the first to the second position is denoted by W_{12}

$$W_{12} = \text{Force} \times \text{displacement}$$
$$= mg\,(h_2 - h_1) \qquad (8.20)$$

If we associate a potential energy $W(h)$ at a point at a height h above the surface such that

$$W(h) = mgh + W_o \tag{8.21}$$

(where W_o = constant) ;
then it is clear that

$$W_{12} = W(h_2) - W(h_1) \tag{8.22}$$

The work done in moving the particle is just the difference of potential energy between its final and initial positions. Observe that the constant W_o cancels out in Eq. (8.22). Setting $h = 0$ in the last equation, we get $W(h = 0) = W_o$. $h = 0$ means points on the surface of the earth. Thus, W_o is the potential energy on the surface of the earth.

If we consider points at arbitrary distance from the surface of the earth, the result just derived is not valid since the assumption that the gravitational force mg is a constant is no longer valid. However, from our discussion we know that a point outside the earth, the force of gravitation on a particle directed towards the centre of the earth is

$$F = \frac{G M_E m}{r^2} \tag{8.23}$$

where M_E = mass of earth, m = mass of the particle and r its distance from the centre of the earth. If we now calculate the work done in lifting a particle from $r = r_1$ to $r = r_2$ ($r_2 > r_1$) along a vertical path, we get instead of Eq. (8.20)

$$W_{12} = \int_{r_1}^{r_2} \frac{G M m}{r^2} \mathrm{d}r$$

$$= -G M_E m \left(\frac{1}{r_2} - \frac{1}{r_1}\right) \tag{8.24}$$

In place of Eq. (8.21), we can thus associate a potential energy $W(r)$ at a distance r, such that

$$W(r) = -\frac{G M_E m}{r} + W_1 , \tag{8.25}$$

valid for $r > R$,

so that once again $W_{12} = W(r_2) - W(r_1)$. Setting r = infinity in the last equation, we get W (r = infinity) = W_1 . Thus, W_1 is the potential energy at infinity. One should note that only the difference of potential energy between two points has a definite meaning from Eqs. (8.22) and (8.24). One conventionally sets W_1 equal to zero, so that the potential energy at a point is just the amount of work done in displacing the particle from infinity to that point.

We have calculated the potential energy at a point of a particle due to gravitational forces on it due to the earth and it is proportional to the mass of the particle. The gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point. From the earlier discussion, we learn that the gravitational potential energy associated with two particles of masses m_1 and m_2 separated by distance by a distance r is given by

$$V = -\frac{G m_1 m_2}{r} \quad \text{(if we choose } V = 0 \text{ as } r \to \infty)$$

It should be noted that an isolated system of particles will have the total potential energy that equals the sum of energies (given by the above equation) for all possible pairs of its constituent particles. This is an example of the application of the superposition principle.

Example 8.3 Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Answer Consider four masses each of mass m at the corners of a square of side l; See Fig. 8.9. We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\, l$

Hence,

$$W(r) = -4 \frac{G m^2}{l} - 2 \frac{G m^2}{\sqrt{2}\, l}$$

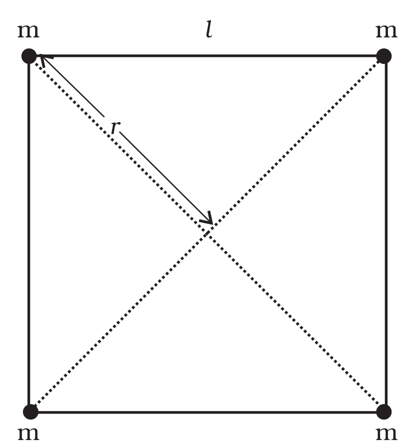


Fig. 8.9

$$= -\frac{2Gm^2}{l}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{l}$$

The gravitational potential at the centre of the square $\left(r = \sqrt{2}\, l/2\right)$ is

$$U(r) = -4\sqrt{2}\,\frac{Gm}{l}.$$

8.8 ESCAPE SPEED

If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: 'can we throw an object with such high initial speeds that it does not fall back to the earth?'

The principle of conservation of energy helps us to answer this question. Suppose the object did reach infinity and that its speed there was V_f. The energy of an object is the sum of potential and kinetic energy. As before W_1 denotes that gravitational potential energy of the object at infinity. The total energy of the projectile at infinity then is

$$E(\infty) = W_1 + \frac{mV_f^2}{2} \tag{8.26}$$

If the object was thrown initially with a speed V_i from a point at a distance $(h+R_E)$ from the centre of the earth (R_E = radius of the earth), its energy initially was

$$E(h+R_E) = \frac{1}{2}mV_i^2 - \frac{GmM_E}{(h+R_E)} + W_1 \tag{8.27}$$

By the principle of energy conservation Eqs. (8.26) and (8.27) must be equal. Hence

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} = \frac{mV_f^2}{2} \tag{8.28}$$

The R.H.S. is a positive quantity with a minimum value zero hence so must be the L.H.S. Thus, an object can reach infinity as long as V_i is such that

$$\frac{mV_i^2}{2} - \frac{GmM_E}{(h+R_E)} \geq 0 \tag{8.29}$$

The minimum value of V_i corresponds to the case when the L.H.S. of Eq. (8.29) equals zero.

Thus, the minimum speed required for an object to reach infinity (i.e. escape from the earth) corresponds to

$$\frac{1}{2}m\left(V_i^2\right)_{\min} = \frac{GmM_E}{h+R_E} \tag{8.30}$$

If the object is thrown from the surface of the earth, $h = 0$, and we get

$$(V_i)_{\min} = \sqrt{\frac{2GM_E}{R_E}} \tag{8.31}$$

Using the relation $g = GM_E / R_E^2$, we get

$$(V_i)_{\min} = \sqrt{2gR_E} \tag{8.32}$$

Using the value of g and R_E, numerically $(V_i)_{\min} \approx 11.2$ km/s. This is called the escape speed, sometimes loosely called the escape velocity.

Equation (8.32) applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r_E replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon.

Example 8.4 Two uniform solid spheres of equal radii R, but mass M and 4 M have a centre to centre separation 6 R, as shown in Fig. 8.10. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.

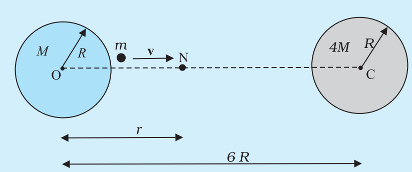


Fig. 8.10

Answer The projectile is acted upon by two mutually opposing gravitational forces of the two spheres. The neutral point N (see Fig. 8.10) is defined as the position where the two forces cancel each other exactly. If ON = r, we have

$$\frac{GMm}{r^2} = \frac{4GMm}{(6R-r)^2}$$

$$(6R - r)^2 = 4r^2$$

$$6R - r = \pm 2r$$

$$r = 2R \quad \text{or} - 6R.$$

The neutral point $r = -6R$ does not concern us in this example. Thus ON = r = $2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of $4M$ would suffice. The mechanical energy at the surface of M is

$$E_i = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R}.$$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{GMm}{2R} - \frac{4GMm}{4R}.$$

From the principle of conservation of mechanical energy

$$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

or

$$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{1/2}$$

◀

A point to note is that the speed of the projectile is zero at N, but is nonzero when it strikes the heavier sphere $4M$. The calculation of this speed is left as an exercise to the students.

8.9 EARTH SATELLITES

Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis. Since, 1957, advances in technology have enabled many countries including India to launch artificial earth satellites for practical use in fields like telecommunication, geophysics and meteorology.

We will consider a satellite in a circular orbit of a distance $(R_E + h)$ from the centre of the earth, where R_E = radius of the earth. If m is the mass of the satellite and V its speed, the centripetal force required for this orbit is

$$\text{F(centripetal)} = \frac{mV^2}{(R_E + h)} \tag{8.33}$$

directed towards the centre. This centripetal force is provided by the gravitational force, which is

$$\text{F(gravitation)} = \frac{G m M_E}{(R_E + h)^2} \tag{8.34}$$

where M_E is the mass of the earth.

Equating R.H.S of Eqs. (8.33) and (8.34) and cancelling out m, we get

$$V^2 = \frac{GM_E}{(R_E + h)} \tag{8.35}$$

Thus V decreases as h increases. From equation (8.35), the speed V for $h = 0$ is

$$V^2 \quad (h = 0) = GM/R_E = gR_E \tag{8.36}$$

where we have used the relation $g = GM/R_E^{\,2}$. In every orbit, the satellite traverses a distance $2\pi(R_E + h)$ with speed V. Its time period T therefore is

$$T = \frac{2\pi(R_E + h)}{V} = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{GM_E}} \tag{8.37}$$

on substitution of value of V from Eq. (8.35). Squaring both sides of Eq. (8.37), we get

$$T^2 = k\ (R_E + h)^3 \quad (\text{where } k = 4\pi^2/GM_E) \tag{8.38}$$

which is Kepler's law of periods, as applied to motion of satellites around the earth. For a satellite very close to the surface of earth h can be neglected in comparison to R_E in Eq. (8.38). Hence, for such satellites, T is T_o, where

$$T_0 = 2\pi\sqrt{R_E/g} \tag{8.39}$$

If we substitute the numerical values $g \simeq 9.8$ m s^{-2} and R_E = 6400 km., we get

$$T_0 = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}}\ \text{s}$$

Which is approximately 85 minutes.

Example 8.5 The planet Mars has two moons, phobos and delmos. (i) phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of mars. (ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days ?

Answer (i) We employ Eq. (8.38) with the sun's mass replaced by the martian mass M_m

$$T^2 = \frac{4\pi^2}{GM_m} R^3$$

$$M_m = \frac{4\pi^2}{G} \frac{R^3}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$= 6.48 \times 10^{23}$ kg.

(ii) Once again Kepler's third law comes to our aid,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.

$\therefore T_M = (1.52)^{3/2} \times 365$

$= 684$ days

We note that the orbits of all planets except Mercury, Mars and Pluto* are very close to being circular. For example, the ratio of the semi-minor to semi-major axis for our Earth is, $b/a = 0.99986$.

Example 8.6 **Weighing the Earth** : You are given the following data: $g = 9.81$ ms^{-2}, $R_E = 6.37 \times 10^6$ m, the distance to the moon $R = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Answer From Eq. (8.12) we have

$$M_E = \frac{g R_E^2}{G}$$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$= 5.97 \times 10^{24}$ kg.

The moon is a satellite of the Earth. From the derivation of Kepler's third law [see Eq. (8.38)]

$$T^2 = \frac{4\pi^2 R^3}{G M_E}$$

$$M_E = \frac{4\pi^2 R^3}{G T^2}$$

$$= \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$= 6.02 \times 10^{24}$ kg

Both methods yield almost the same answer, the difference between them being less than 1%.

Example 8.7 Express the constant k of Eq. (8.38) in days and kilometres. Given $k = 10^{-13}$ s^2 m^{-3}. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Answer Given

$k = 10^{-13}$ s^2 m^{-3}

$$= 10^{-13} \left[\frac{1}{(24 \times 60 \times 60)^2} \mathrm{d}^2 \right] \left[\frac{1}{(1/1000)^3 \mathrm{km}^3} \right]$$

$= 1.33 \times 10^{-14}$ d^2 km^{-3}

Using Eq. (8.38) and the given value of k, the time period of the moon is

$T^2 = (1.33 \times 10^{-14})(3.84 \times 10^5)^3$

$T = 27.3$ d

Note that Eq. (8.38) also holds for elliptical orbits if we replace (R_E+h) by the semi-major axis of the ellipse. The earth will then be at one of the foci of this ellipse.

8.10 ENERGY OF AN ORBITING SATELLITE

Using Eq. (8.35), the kinetic energy of the satellite in a circular orbit with speed v is

$$K{\bullet}E = \frac{1}{2} m v^2$$

$$= \frac{Gm M_E}{2(R_E + h)}, \qquad (8.40)$$

* *Refer to information given in the Box on Page 182*

Considering gravitational potential energy at infinity to be zero, the potential energy at distance (R+h) from the centre of the earth is

$$P.E = -\frac{G m M_E}{(R_E + h)} \tag{8.41}$$

The K.E is positive whereas the P.E is negative. However, in magnitude the K.E. is half the P.E, so that the total E is

$$E = K.E + P.E = -\frac{G m M_E}{2(R_E + h)} \tag{8.42}$$

The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy.

When the orbit of a satellite becomes elliptic, both the *K.E.* and *P.E.* vary from point to point. The total energy which remains constant is negative as in the circular orbit case. This is what we expect, since as we have discussed before if the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

Example 8.8 A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies ?

Answer Initially,

$$E_i = -\frac{G M_E m}{4 R_E}$$

While finally

$$E_f = -\frac{G M_E m}{8 R_E}$$

The change in the total energy is

$$\Delta E = E_f - E_i$$

$$= \frac{G M_E m}{8 R_E} = \left(\frac{G M_E}{R_E^2}\right)\frac{m R_E}{8}$$

$$\Delta E = \frac{g m R_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8} = 3.13 \times 10^9 \text{ J}$$

The kinetic energy is reduced and it mimics ΔE, namely, $\Delta K = K_f - K_i = -3.13 \times 10^9$ J.

The change in potential energy is twice the change in the total energy, namely

$\Delta V = V_f - V_i = -6.25 \times 10^9$ J

8.11 GEOSTATIONARY AND POLAR SATELLITES

An interesting phenomenon arises if in we arrange the value of $(R_E + h)$ such that T in Eq. (8.37) becomes equal to 24 hours. If the circular orbit is in the equatorial plane of the earth, such a satellite, having the same period as the period of rotation of the earth about its own axis would appear stationery viewed from a point on earth. The $(R_E + h)$ for this purpose works out to be large as compared to R_E:

$$R_E + h = \left(\frac{T^2 G M_E}{4\pi^2}\right)^{1/3} \tag{8.43}$$

and for T = 24 hours, h works out to be 35800 km. which is much larger than R_E. Satellites in a circular orbits around the earth in the

India's Leap into Space

India entered the space age with the launching of the low orbit satellite Aryabhatta in 1975. In the first few years of its programme the launch vehicles were provided by the erstwhile Soviet Union. Indigenous launch vehicles were employed in the early 1980's to send the Rohini series of satellites into space. The programme to send polar satellites into space began in late 1980's. A series of satellites labelled IRS (Indian Remote Sensing Satellites) have been launched and this programme is expected to continue in future. The satellites have been employed for surveying, weather prediction and for carrying out experiments in space. The INSAT (Indian National Satellite) series of satellites were designed and made operational for communications and weather prediction purposes beginning in 1982. European launch vehicles have been employed in the INSAT series. India tested its geostationary launch capability in 2001 when it sent an experimental communications satellite (GSAT-1) into space. In 1984 Rakesh Sharma became the first Indian astronaut. The Indian Space Research Organisation (ISRO) is the umbrella organisation that runs a number of centre. Its main lauch centre at Sriharikota (SHAR) is 100 km north of Chennai. The National Remote Sensing Agency (NRSA) is near Hyderabad. Its national centre for research in space and allied sciences is the Physical Research Laboratory (PRL) at Ahmedabad.

equatorial plane with T = 24 hours are called Geostationery Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications.

It is known that electromagnetic waves above a certain frequency are not reflected from ionosphere. Radio waves used for radio broadcast which are in the frequency range 2 MHz to 10 MHz, are below the critical frequency. They are therefore reflected by the ionosphere. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fail to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationery satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India are one such group of Geostationary satellites widely used for telecommunications in India.

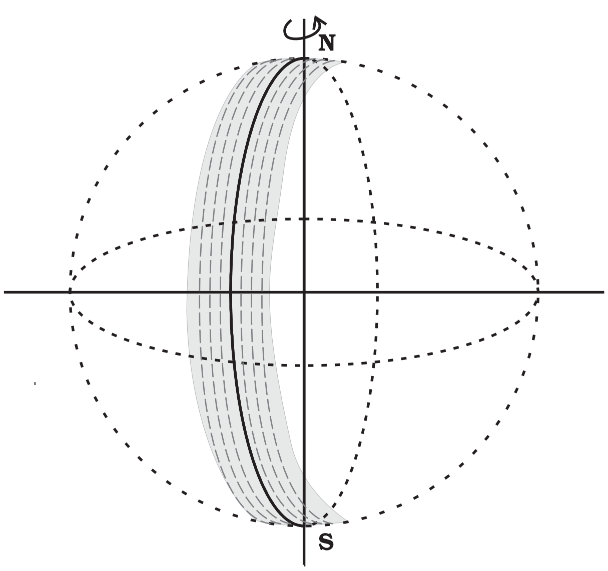


Fig. 8.11 *A Polar satellite. A strip on earth's surface (shown shaded) is visible from the satellite during one cycle. For the next revolution of the satellite, the earth has rotated a little on its axis so that an adjacent strip becomes visible.*

Another class of satellites are called the Polar satellites (Fig. 8.11). These are low altitude ($h \approx$ 500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

8.12 WEIGHTLESSNESS

Weight of an object is the force with which the earth attracts it. We are conscious of our own weight when we stand on a surface, since the surface exerts a force opposite to our weight to keep us at rest. The same principle holds good when we measure the weight of an object by a spring balance hung from a fixed point e.g. the ceiling. The object would fall down unless it is subject to a force opposite to gravity. This is exactly what the spring exerts on the object. This is because the spring is pulled down a little by the gravitational pull of the object and in turn the spring exerts a force on the object vertically upwards.

Now, imagine that the top end of the balance is no longer held fixed to the top ceiling of the room. Both ends of the spring as well as the object move with identical acceleration g. The spring is not stretched and does not exert any upward force on the object which is moving down with acceleration g due to gravity. The reading recorded in the spring balance is zero since the spring is not stretched at all. If the object were a human being, he or she will not feel his weight since there is no upward force on him. Thus, when an object is in free fall, it is weightless and this phenomenon is usually called the phenomenon of weightlessness.

In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the centre of the earth which is exactly the value of earth's acceleration due to gravity at that position. Thus in the satellite everything inside it is in a state of free fall. This is just as if we were falling towards the earth from a height. Thus, in a manned satellite, people inside experience no gravity. Gravity for us defines the vertical direction and thus for them there are no horizontal or vertical directions, all directions are the same. Pictures of astronauts floating in a satellite show this fact.

SUMMARY

1. Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant, which has the value 6.672×10^{-11} N m^2 kg^{-2}.

2. If we have to find the resultant gravitational force acting on the particle m due to a number of masses M_1, M_2, ...M_n etc. we use the principle of superposition. Let F_1, F_2, ...F_n be the individual forces due to M_1, M_2, ...M_n, each given by the law of gravitation. From the principle of superposition each force acts independently and uninfluenced by the other bodies. The resultant force F_R is then found by vector addition

$$F_R = F_1 + F_2 + \ldots\ldots + F_n = \sum_{i=1}^{n} F_i$$

where the symbol 'Σ' stands for summation.

3. Kepler's laws of planetary motion state that
   (a) All planets move in elliptical orbits with the Sun at one of the focal points
   (b) The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.
   (c) The square of the orbital period of a planet is proportional to the cube of the semi-major axis of the elliptical orbit of the planet

   The period T and radius R of the circular orbit of a planet about the Sun are related by

$$T^2 = \left(\frac{4\pi^2}{G\,M_s}\right) R^3$$

where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.

4. The acceleration due to gravity.
   (a) at a height h above the Earth's surface

$$g(h) = \frac{G\,M_E}{(R_E + h)^2}$$

$$\approx \frac{G\,M_E}{R_E^2}\left(1 - \frac{2h}{R_E}\right) \quad \text{for } h << R_E$$

$$g(h) = g(0)\left(1 - \frac{2h}{R_E}\right) \quad \text{where } g(0) = \frac{G\,M_E}{R_E^2}$$

(b) at depth d below the Earth's surface is

$$g(d) = \frac{G\,M_E}{R_E^2}\left(1 - \frac{d}{R_E}\right) = g(0)\left(1 - \frac{d}{R_E}\right)$$

5. The gravitational force is a conservative force, and therefore a potential energy function can be defined. The *gravitational potential energy* associated with two particles separated by a distance r is given by

$$V = -\frac{G\,m_1\,m_2}{r}$$

where V is taken to be zero at $r \to \infty$. The total potential energy for a system of particles is the sum of energies for all pairs of particles, with each pair represented by a term of the form given by above equation. This prescription follows from the principle of superposition.

6. If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = \frac{1}{2}m\,v^2 - \frac{G\,M\,m}{r}$$

That is, the total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion.

7. If m moves in a circular orbit of radius a about M, where $M >> m$, the total energy of the system is

$$E = -\frac{G\,M\,m}{2a}$$

with the choice of the arbitrary constant in the potential energy given in the point 5., above. The total energy is negative for any bound system, that is, one in which the orbit is closed, such as an elliptical orbit. The kinetic and potential energies are

$$K = \frac{G\,M\,m}{2a}$$

$$V = -\frac{G\,M\,m}{a}$$

8. The escape speed from the surface of the Earth is

$$v_e = \sqrt{\frac{2\,G\,M_E}{R_E}} = \sqrt{2gR_E}$$

and has a value of 11.2 km s^{-1}.

9. If a particle is outside a uniform spherical shell or solid sphere with a spherically symmetric internal mass distribution, the sphere attracts the particle as though the mass of the sphere or shell were concentrated at the centre of the sphere.

10. If a particle is inside a uniform spherical shell, the gravitational force on the particle is zero. If a particle is inside a homogeneous solid sphere, the force on the particle acts toward the centre of the sphere. This force is exerted by the spherical mass interior to the particle.

11. A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at a approximate distance of 4.22×10^4 km from the Earth's centre.

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Gravitational Constant	G	$[M^{-1} L^3 T^{-2}]$	N m^2 kg^{-2}	6.67×10^{-11}
Gravitational Potential Energy	$V(r)$	$[M L^2 T^{-2}]$	J	$-\frac{GMm}{r}$ (scalar)
Gravitational Potential	$U(r)$	$[L^2 T^{-2}]$	J kg^{-1}	$-\frac{GM}{r}$ (scalar)
Gravitational Intensity	**E** or **g**	$[LT^{-2}]$	m s^{-2}	$\frac{GM}{r^2}\hat{r}$ (vector)

POINTS TO PONDER

1. In considering motion of an object under the gravitational influence of another object the following quantities are conserved:
   (a) Angular momentum
   (b) Total mechanical energy
   Linear momentum is **not** conserved
2. Angular momentum conservation leads to Kepler's second law. However, it is not special to the inverse square law of gravitation. It holds for any central force.
3. In Kepler's third law (see Eq. (8.1) and $T^2 = K_S R^3$. The constant K_S is the same for all planets in circular orbits. This applies to satellites orbiting the Earth [(Eq. (8.38)].
4. An astronaut experiences weightlessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and the satellite are in "free fall" towards the Earth.
5. The *gravitational potential energy* associated with two particles separated by a distance r is given by

$$V = -\frac{G m_1 m_2}{r} + \text{constant}$$

   The constant can be given any value. The simplest choice is to take it to be zero. With this choice

$$V = -\frac{G m_1 m_2}{r}$$

   This choice implies that $V \to 0$ as $r \to \infty$. Choosing location of zero of the gravitational energy is the same as choosing the arbitrary constant in the potential energy. Note that the gravitational force is not altered by the choice of this constant.
6. The total mechanical energy of an object is the sum of its kinetic energy (which is always positive) and the potential energy. Relative to infinity (i.e. if we presume that the potential energy of the object at infinity is zero), the gravitational potential energy of an object is negative. The total energy of a satellite is negative.
7. The commonly encountered expression mgh for the potential energy is actually an approximation to the difference in the gravitational potential energy discussed in the point 6, above.
8. Although the gravitational force between two particles is central, the force between two finite rigid bodies is not necessarily along the line joining their centre of mass. For a spherically symmetric body however the force on a particle external to the body is as if the mass is concentrated at the centre and this force is therefore central.
9. The gravitational force on a particle inside a spherical shell is zero. However, (unlike a metallic shell which shields electrical forces) the shell does not shield other bodies outside it from exerting gravitational forces on a particle inside. *Gravitational shielding is not possible.*

EXERCISES

8.1 Answer the following :

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means ?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity ?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why ?

8.2 Choose the correct alternative :

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

8.3 Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

8.4 Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

8.5 Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the Milky Way to be 10^5 ly.

8.6 Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

8.7 Does the escape speed of a body from the earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, (d) the height of the location from where the body is launched?

8.8 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

8.9 Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem.

8.10 In the following two exercises, choose the correct answer from among the given ones: The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) 0.

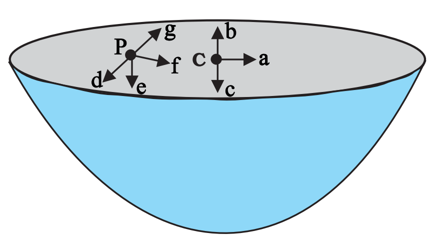


Fig. 8.12

8.11 For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

8.12 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero ? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

8.13 How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

8.14 A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50×10^8 km away from the sun ?

8.15 A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?

8.16 Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface ?

8.17 A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6 m; $G = 6.67 \times 10^{-11}$ N $m^2 kg^{-2}$.

8.18 The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

8.19 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; $G = 6.67 \times 10^{-11}$ N $m^2 kg^{-2}$.

8.20 Two stars each of one solar mass (= 2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

8.21 Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres ? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable ?

Additional Exercises

8.22 As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite ? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

8.23 A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity ? (mass of the sun = 2×10^{30} kg).

8.24 A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system ? Mass of the space ship = 1000 kg; mass of the sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^8 km; $G = 6.67\times10^{-11}$ N m^2 kg^{-2}.

8.25 A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it ? Mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; $G = 6.67\times10^{-11}$ N $m^2 kg^{-2}$.

APPENDICES

APPENDIX A 1

THE GREEK ALPHABET

Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Y	υ
Epsilon	E	ε	Nu	N	ν	Phi	Φ	ϕ, φ
Zeta	Z	ς	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

APPENDIX A 2

COMMON SI PREFIXES AND SYMBOLS FOR MULTIPLES AND SUB-MULTIPLES

Multiple			Sub-Multiple		
Factor	**Prefix**	**Symbol**	**Factor**	**Prefix**	**symbol**
10^{18}	Exa	E	10^{-18}	atto	a
10^{15}	Peta	P	10^{-15}	femto	f
10^{12}	Tera	T	10^{-12}	pico	p
10^{9}	Giga	G	10^{-9}	nano	n
10^{6}	Mega	M	10^{-6}	micro	μ
10^{3}	kilo	k	10^{-3}	milli	m
10^{2}	Hecto	h	10^{-2}	centi	c
10^{1}	Deca	da	10^{-1}	deci	d

APPENDIX A 3

SOME IMPORTANT CONSTANTS

Name	Symbol	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Charge of electron	e	1.602×10^{-19} C
Gravitational constant	G	6.673×10^{-11} N m^2 kg^{-2}
Planck constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.381×10^{-23} J K^{-1}
Avogadro number	N_A	6.022×10^{23} mol^{-1}
Universal gas constant	R	8.314 J mol^{-1} K^{-1}
Mass of electron	m_e	9.110×10^{-31} kg
Mass of neutron	m_n	1.675×10^{-27} kg
Mass of proton	m_p	1.673×10^{-27} kg
Electron-charge to mass ratio	e/m_e	1.759×10^{11} C/kg
Faraday constant	F	9.648×10^{4} C/mol
Rydberg constant	R	1.097×10^{7} m^{-1}
Bohr radius	a_0	5.292×10^{-11} m
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Wien's Constant	b	2.898×10^{-3} m K
Permittivity of free space	ε_0 $1/4\pi\,\varepsilon_0$	8.854×10^{-12} C^2 $N^{-1}m^{-2}$ 8.987×10^{9} N m^2 C^{-2}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m A^{-1} $\cong 1.257 \times 10^{-6}$ Wb A^{-1} m^{-1}

Other useful constants

Name	Symbol	Value
Mechanical equivalent of heat	J	4.186 J cal^{-1}
Standard atmospheric pressure	1 atm	1.013×10^{5} Pa
Absolute zero	0 K	−273.15 °C
Electron volt	1 eV	1.602×10^{-19} J
Unified Atomic mass unit	1 u	1.661×10^{-27} kg
Electron rest energy	mc^2	0.511 MeV
Energy equivalent of 1 u	1 uc^2	931.5 MeV
Volume of ideal gas(0 °C and 1atm)	V	22.4 L mol^{-1}
Acceleration due to gravity (sea level, at equator)	g	9.78049 m s^{-2}

APPENDIX A 4
CONVERSION FACTORS

Conversion factors are written as equations for simplicity.

Length

1 km = 0.6215 mi

1mi = 1.609 km

1m = 1.0936 yd = 3.281 ft = 39.37 in

1 in = 2.54 cm

1 ft = 12 in = 30.48 cm

1 yd = 3ft = 91.44 cm

1 lightyear = 1 ly = 9.461 x 10^{15}m

1 Å= 0.1nm

Area

1 m^2 = 10^4 cm^2

1km^2 = 0.3861 mi^2 = 247.1 acres

1 in^2= 6.4516 cm^2

1ft^2= 9.29 x $10^{-2}$$m^2$

1 m^2= 10.76 ft^2

1 acre = 43,560 ft^2

1 mi^2= 460 acres = 2.590 km^2

Volume

1m^3= $10^6$$cm^3$

1 L = 1000 cm^3 = 10^{-3} m^3

1 gal = 3.786 L

1 gal = 4 qt = 8 pt = 128 oz = 231 in^3

1 in^3 = 16.39 cm^3

1ft^3 = 1728 in^3 = 28.32 L = 2.832 × 10^4 cm^3

Speed

1 km h^{-1} = 0.2778 m s^{-1} = 0.6215 mi h^{-1}

1mi h^{-1} = 0.4470 m s^{-1} = 1.609 km h^{-1}

1mi h^{-1} = 1.467 ft s^{-1}

Magnetic Field

1 G = 10^{-4} T

1 T = 1 Wb m^{-2} = 10^4 G

Angle and Angular Speed

π rad = 180°

1 rad = 57.30°

1° = 1.745 × 10^{-2} rad

1 rev min^{-1} = 0.1047 rad s^{-1}

1 rad s^{-1} = 9.549 rev min^{-1}

Mass

1 kg = 1000 g

1 tonne = 1000 kg = 1 Mg

1 u = 1.6606 × 10^{-27} kg

1 kg = 6.022 × 10^{26} u

1 slug = 14.59 kg

1 kg = 6.852 × 10^{-2} slug

1 u = 931.50 MeV/c^2

Density

1 g cm^{-3} = 1000 kg m^{-3} = 1 kg L^{-1}

Force

1 N = 0.2248 lbf = 10^5 dyn

1 lbf = 4.4482 N

1 kgf = 2.2046 lbf

Time

1 h = 60 min = 3.6 ks

1 d = 24 h = 1440 min = 86.4 ks

1y = 365.24 d = 31.56 Ms

Pressure

1 Pa = 1 N m^{-2}

1 bar = 100 kPa

1 atm = 101.325 kPa = 1.01325 bar

1atm = 14.7 lbf/in^2 = 760 mm Hg

= 29.9 in Hg = 33.8 ft H_2O

1 lbf in^{-2} = 6.895 kPa

1 torr = 1mm Hg = 133.32 Pa

Energy

1 kW h = 3.6 MJ

1 cal = 4.186 J

1ft lbf = 1.356 J = 1.286 × 10^{-3} Btu

1 L atm = 101.325 J

1 L atm = 24.217 cal

1 Btu = 778 ft lb = 252 cal = 1054.35 J

1 eV = 1.602 × 10^{-19}J

1 u c^2 = 931.50 MeV

1 erg = 10^{-7}J

Power

1 horsepower (hp) = 550 ft lbf/s

= 745.7 W

1 Btu min^{-1} = 17.58 W

1 W = 1.341 × 10^{-3} hp

= 0.7376 ft lbf/s

Thermal Conductivity

1 W m^{-1} K^{-1} = 6.938 Btu in/hft^2 °F

1 Btu in/hft^2 °F = 0.1441 W/m K

APPENDIX A 5
MATHEMATICAL FORMULAE

Geometry

Circle of radius r: circumference = $2\pi r$;
area = πr^2

Sphere of radius r: area = $4\pi r^2$;

volume = $\frac{4}{3}\pi r^3$

Right circular cylinder of radius r and height h: area = $2\pi r^2 + 2\pi r h$;

volume = $\pi r^2 h$;

Triangle of base a and altitude h.

area = $\frac{1}{2} a h$

Quadratic Formula

If $ax^2 + bx + c = 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric Functions of Angle θ

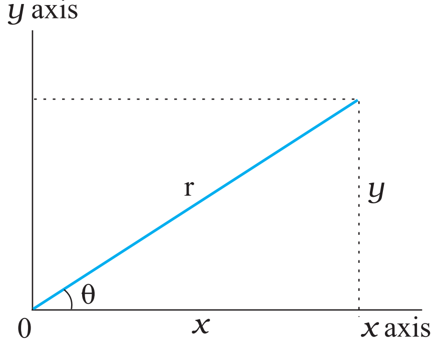


Fig. A 5.1

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$\sec\theta = \frac{r}{x} \qquad \csc\theta = \frac{r}{y}$$

Pythagorean Theorem

In this right triangle, $a^2 + b^2 = c^2$

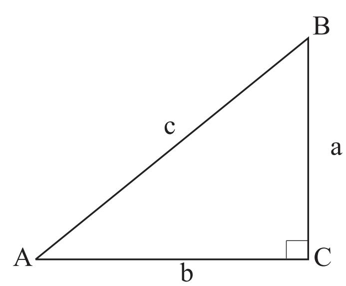


Fig. A 5.2

Triangles

Angles are A, B, C

Opposite sides are a, b, c

Angles $A + B + C = 180^0$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$c^2 = a^2 + b^2 - 2ab \cos C$

Exterior angle $D = A + C$

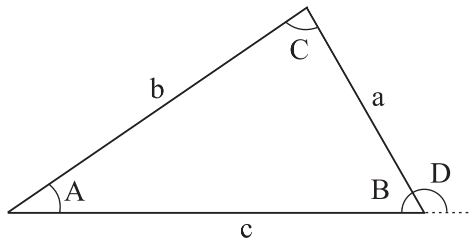


Fig. A 5.3

Mathematical Signs and Symbols

= equals
≅ equals approximately
~ is the order of magnitude of
≠ is not equal to
≡ is identical to, is defined as
> is greater than (>> is much greater than)
< is less than (<< is much less than)
≥ is greater than or equal to (or, is no less than)
≤ is less than or equal to (or, is no more than)
± plus or minus
∝ is proportional to
Σ the sum of
$\overline{x}$ or $<x>$ or x_{av} the average value of x

Trigonometric Identities

$\sin(90^0 - \theta) = \cos\theta$

$\cos(90^0 - \theta) = \sin\theta$

$\sin\theta / \cos\theta = \tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

$\cos\alpha + \cos\beta$

$= 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$

$\cos\alpha - \cos\beta$

$= -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$

Binomial Theorem

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \ldots\ldots(x^2 < 1)$$

$$(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} + \ldots\ldots(x^2 < 1)$$

Exponential Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

Logarithmic Expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ldots\ldots \quad (|x| < 1)$$

Trigonometric Expansion
(θ in radians)

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots\ldots$$
$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots\ldots$$
$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} - \ldots\ldots$$

Products of Vectors

Let $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ be unit vectors in the x, y and z directions. Then

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1,\ \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0,\ \hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}, \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}, \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Any vector **a** with components a_x, a_y, and a_z along the x,y, and z axes can be written,

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

Let **a**, **b** and **c** be arbitary vectors with magnitudes a, b and c. Then

$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$$

$$(s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b}) = s(\mathbf{a} \times \mathbf{b}) \quad \text{(s is a scalar)}$$

Let θ be the smaller of the two angles between **a** and **b**. Then

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$$

$$|\mathbf{a} \times \mathbf{b}| = ab\sin\theta$$

$$\mathbf{a} \times \mathbf{b} = -\,\mathbf{b} \times \mathbf{a} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\,\hat{\mathbf{i}} + (a_z b_x - b_z a_x)\,\hat{\mathbf{j}} + (a_x b_y - b_x a_y)\,\hat{\mathbf{k}}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\,\mathbf{c}$$

APPENDIX A 6

SI DERIVED UNITS

A 6.1 Some SI Derived Units expressed in SI Base Units

Physical quantity	SI Unit	
	Name	Symbol
Area	square metre	m^2
Volume	cubic metre	m^3
Speed, velocity	metre per second	m/s or $m\ s^{-1}$
Angular velocity	radian per second	rad/s or rad s^{-1}
Acceleration	metre per second square	m/s^2 or $m\ s^{-2}$
Angular acceleration	radian per second square	rad/s^2 or rad s^{-2}
Wave number	per metre	m^{-1}
Density, mass density	kilogram per cubic metre	kg/m^3 or kg m^{-3}
Current density	ampere per square metre	A/m^2 or A m^{-2}
Magnetic field strength, magnetic intensity, magnetic moment density	ampere per metre	A/m or A m^{-1}
Concentration (of amount of substance)	mole per cubic metre	mol/m^3 or mol m^{-3}
Specific volume	cubic metre per kilogram	m^3/kg or m^3 kg^{-1}
Luminance, intensity of illumination	candela per square metre	cd/m^2 or cd m^{-2}
Kinematic viscosity	square metre per second	m^2/s or $m^2\ s^{-1}$
Momentum	kilogram metre per second	kg m s^{-1}
Moment of inertia	kilogram square metre	kg m^2
Radius of gyration	metre	m
Linear/superficial/volume expansivities	per kelvin	K^{-1}
Flow rate	cubic metre per second	$m^3\ s^{-1}$

A 6.2 SI Derived Units with special names

Physical quantity	SI Unit			
	Name	Symbol	Expression in terms of other units	Expression in terms of SI base Units
Frequency	hertz	Hz	-	s^{-1}
Force	newton	N	-	kg m s^{-2} or kg m/s^2
Pressure, stress	pascal	Pa	N/m^2 or N m^{-2}	kg m^{-1} s^{-2} or kg /s^2 m
Energy, work, quantity of heat	joule	J	N m	kg m^2 s^{-2} or kg m^2/s^2
Power, radiant flux	watt	W	J/s or J s^{-1}	kg m^2 s^{-3}or kg m^2/s^3
Quantity of electricity, electric charge	coulomb	C	-	A s
Electric potential, potential difference, electromotive force	volt	V	W/A or W A^{-1}	kg $m^2$$s^{-3}$ A^{-1} or kg m^2/s^3 A
Capacitance	farad	F	C/V	A^2 s^4 kg^{-1} m^{-2}
Electric resistance	ohm	Ω	V/A	kg m^2 s^{-3} A^{-2}
Conductance	siemens	S	A/V	m^{-2} kg^{-1} s^3 A^2
Magnetic flux	weber	Wb	V s or J/A	kg m^2 s^{-2} A^{-1}
Magnetic field, magnetic flux density, magnetic induction	tesla	T	Wb/m^2	kg s^{-2} A^{-1}
Inductance	henry	H	Wb/A	kg m^2 s^{-2} A^{-2}
Luminous flux, luminous power	lumen	lm	-	cd /sr
Illuminance	lux	lx	lm/m^2	m^{-2} cd sr^{-1}
Activity (of a radio nuclide/radioactive source)	becquerel	Bq	-	s^{-1}
Absorbed dose, absorbed dose index	gray	Gy	J/kg	m^2/s^2 or m^2 s^{-2}

A 6.3 Some SI Derived Units expressed by means of SI Units with special names

Physical quantity	SI Unit		
	Name	Symbol	Expression in terms of SI base units
Magnetic moment	joule per tesla	J T^{-1}	m^2 A
Dipole moment	coulomb metre	C m	s A m
Dynamic viscosity	poiseiulles or pascal second or newton second per square metre	Pl or Pa s or N s m^{-2}	m^{-1} kg s^{-1}
Torque, couple, moment of force	newton metre	N m	m^2 kg s^{-2}
Surface tension	newton per metre	N/m	kg s^{-2}
Power density, irradiance, heat flux density	watt per square metre	W/m^2	kg s^{-3}

Heat capacity, entropy	joule per kelvin	J/K	m^2 kg s^{-2} K^{-1}
Specific heat capacity, specific entropy	joule per kilogram kelvin	J/kg K	m^2 s^{-2} K^{-1}
Specific energy, latent heat	joule per kilogram	J/kg	m^2 s^{-2}
Radiant intensity	watt per steradian	W sr^{-1}	kg m^2 s^{-3} sr^{-1}
Thermal conductivity	watt per metre kelvin	W m^{-1} K^{-1}	m kg s^{-3} K^{-1}
Energy density	joule per cubic metre	J/m^3	kg m^{-1} s^{-2}
Electric field strength	volt per metre	V/m	m kg s^{3} A^{-1}
Electric charge density	coulomb per cubic metre	C/m^3	m^{-3} A s
Electric flux density	coulomb per square metre	C/m^2	m^{-2} A s
Permittivity	farad per metre	F/m	m^{-3} kg^{-1} s^4 A^2
Permeability	henry per metre	H/m	m kg s^{-2} A^{-2}
Molar energy	joule per mole	J/mol	m^2 kg $s^{-2}$$mol^{-1}$
Angular momentum, Planck's constant	joule second	J s	kg m^2 s^{-1}
Molar entropy, molar heat capacity	joule per mole kelvin	J/mol K	m^2 kg s^{-2} K^{-1} mol^{-1}
Exposure (x-rays and γ-rays)	coulomb per kilogram	C/kg	kg^{-1} s A
Absorbed dose rate	gray per second	Gy/s	m^2 s^{-3}
Compressibility	per pascal	Pa^{-1}	m kg^{-1} s^2
Elastic moduli	newton per square metre	N/m^2 or N m^{-2}	kg m^{-1} s^{-2}
Pressure gradient	pascal per metre	Pa/m or N m^{-3}	kg m^{-2} s^{-2}
Surface potential	joule per kilogram	J/kg or N m/kg	m^2 s^{-2}
Pressure energy	pascal cubic metre	Pa m^3 or N m	kg m^2 s^{-2}
Impulse	newton second	N s	kg m s^{-1}
Angular impulse	newton metre second	N m s	kg m^2 s^{-1}
Specific resistance	ohm metre	Ωm	kg m^3 s^{-3} A^{-2}
Surface energy	joule per square metre	J/m^2 or N/m	kg s^{-2}

APPENDIX A 7
GENERAL GUIDELINES FOR USING SYMBOLS FOR PHYSICAL QUANTITIES, CHEMICAL ELEMENTS AND NUCLIDES

- Symbols for physical quantities are normally single letters and printed in italic (or sloping) type. However, in case of the two letter symbols, appearing as a factor in a product, some spacing is necessary to separate this symbol from other symbols.
- Abbreviations, i.e., shortened forms of names or expressions, such as p.e. for potential energy, are not used in physical equations. These abbreviations in the text are written in ordinary normal/roman (upright) type.
- Vectors are printed in bold and normal/roman (upright) type. However, in class room situations, vectors may be indicated by an arrow on the top of the symbol.
- Multiplication or product of two physical quantities is written with some spacing between them. Division of one physical quantity by another may be indicated with a horizontal bar or with

solidus, a slash or a short oblique stroke mark (/) or by writing it as a product of the numerator and the inverse first power of the denominator, using brackets at appropriate places to clearly distinguish between the numerator and the denominator.

- Symbols for chemical elements are written in normal/roman (upright) type. The symbol is not followed by a full stop.
  For example, Ca, C, H, He, U, etc.
- The attached numerals specifying a nuclide are placed as a left subscript (atomic number) and superscript (mass number).
  For example, a U-235 nuclide is expressed as ${}^{235}_{92}U$ (with 235 expressing the mass number and 92 as the atomic number of uranium with chemical symbol U).
- The right superscript position is used, if required, for indicating a state of ionisation (in case of ions).
  For example, Ca^{2+}, PO_4^{3-}

APPENDIX A 8
GENERAL GUIDELINES FOR USING SYMBOLS FOR SI UNITS, SOME OTHER UNITS, AND SI PREFIXES

- Symbols for units of physical quantities are printed/written in Normal/Roman (upright) type.
- Standard and recommended symbols for units are written in lower case roman (upright) type, starting with small letters. The shorter designations for units such as kg, m, s, cd, etc., are symbols and not the abbreviations. The unit names are never capitalised. However, the unit symbols are capitalised only if the symbol for a unit is derived from a proper name of scientist, beginning with a capital, normal/roman letter.
  For example, m for the unit 'metre', d for the unit 'day', atm for the unit 'atmospheric pressure', Hz for the unit 'hertz', Wb for the unit 'weber', J for the unit 'joule', A for the unit 'ampere', V for the unit 'volt', etc. The single exception is L, which is the symbol for the unit 'litre'. This exception is made to avoid confusion of the lower case letter l with the Arabic numeral l.
- Symbols for units do not contain any final full stop at the end of recommended letter and remain unaltered in the plural, using only singular form of the unit.
  For example, for a length of 25 centimetres the unit symbol is written as 25 cm and not 25 cms or 25 cm. or 25 cms., etc.
- Use of solidus (/) is recommended only for indicating a division of one letter unit symbol by another unit symbol. Not more than one solidus is used.
  For example :
  m/s^2 or m s^{-2} (with a spacing between m and s^{-2}) but not m/s/s;
  1 Pl =1 N s m^{-2} = 1 N s/m^2 = 1 kg/s m=1 kg m^{-1} s^{-1}, but not 1 kg/m/s;
  J/K mol or J K^{-1} mol^{-1}, but not J/K/mol; etc.
- Prefix symbols are printed in normal/roman (upright) type without spacing between the prefix symbol and the unit symbol. Thus certain approved prefixes written very close to the unit symbol are used to indicate decimal fractions or multiples of a SI unit, when it is inconveniently small or large.
  For example :

megawatt (1MW = 10^6 W);	nanosecond (1 ns = 10^{-9} s);
centimetre (1 cm = 10^{-2} m);	picofarad (1 pF = 10^{-12} F);.
kilometre (1 km = 10^3 m);	microsecond (1µs = 10^{-6} s);
millivolt (1 mV= 10^{-3} V);	gigahertz (1GHz = 10^9 Hz);

kilowatt-hour (1 kW h = 10^3 W h = 3.6 MJ = 3.6×10^6 J);
microampere (1μ A = 10^{-6} A); micron (1μm = 10^{-6} m);
angstrom (1 Å =0.1 nm = 10^{-10} m); etc.

The unit 'micron' which equals 10^{-6} m, i.e. a micrometre, is simply the name given to convenient sub-multiple of the metre. In the same spirit, the unit 'fermi', equal to a femtometre or 10^{-15} m has been used as the convenient length unit in nuclear studies. Similarly, the unit 'barn', equal to 10^{-28} m^2, is a convenient measure of cross-sectional areas in sub-atomic particle collisions. However, the unit 'micron' is preferred over the unit 'micrometre' to avoid confusion of the 'micrometre' with the length measuring instrument called 'micrometer'. These newly formed multiples or sub-multiples (cm, km, μm, μs, ns) of SI units, metre and second, constitute a new composite inseparable symbol for units.

- When a prefix is placed before the symbol of a unit, the combination of prefix and symbol is considered as a new symbol, for the unit, which can be raised to a positive or negative power without using brackets. These can be combined with other unit symbols to form compound unit. Rules for binding-in indices are not those of ordinary algebra.
  For example :
  cm^3 means always $(cm)^3 = (0.01\ m)^3 = (10^{-2}\ m)^3 = 10^{-6}\ m^3$, but never $0.01\ m^3$ or $10^{-2}\ m^3$ or 1cm^3 (prefix c with a spacing with m^3 is meaningless as prefix c is to be attached to a unit symbol and it has no physical significance or independent existence without attachment with a unit symbol).
  Similarly, mA^2 means always $(mA)^2 = (0.001A)^2 = (10^{-3}\ A)^2 = 10^{-6}\ A^2$, but never $0.001\ A^2$ or $10^{-3}\ A^2$ or m A^2;
  1 $cm^{-1} = (10^{-2}m)^{-1} = 10^2\ m^{-1}$, but not 1c m^{-1} or $10^{-2}\ m^{-1}$;
  1μs^{-1} means always $(10^{-6}s)^{-1} = 10^6\ s^{-1}$, but not $1 \times 10^{-6}\ s^{-1}$;
  1 km^2 means always $(km)^2 = (10^3\ m)^2 = 10^6\ m^2$, but not $10^3\ m^2$;
  1mm^2 means always $(mm)^2 = (10^{-3}\ m)^2 = 10^{-6}\ m^2$, but not $10^{-3}\ m^2$.
- A prefix is never used alone. It is always attached to a unit symbol and written or fixed before (pre-fix) the unit symbol.
  For example :
  $10^3/m^3$ means $1000/m^3$ or 1000 m^{-3}, but not k/m^3 or k m^{-3}.
  $10^6/m^3$ means $10{,}00{,}000/m^3$ or 10,00,000 m^{-3}, but not M/m^3 or M m^{-3}
- Prefix symbol is written very close to the unit symbol without spacing between them, while unit symbols are written separately with spacing when units are multiplied together.
  For example :
  m s^{-1} (symbols m and s^{-1}, in lower case, small letter m and s, are separate and independent unit symbols for metre and second respectively, with spacing between them) means 'metre per second', but not 'milli per second'.
  Similarly, ms^{-1} [symbol m and s are written very close to each other, with prefix symbol m (for prefix milli) and unit symbol s, in lower case, small letter (for unit 'second') without any spacing between them and making ms as a new composite unit] means 'per millisecond', but never 'metre per second'.
  mS^{-1}[symbol m and S are written very close to each other, with prefix symbol m (for prefix milli) and unit symbol S, in capital roman letter S (for unit 'siemens') without any spacing between them, and making mS as a new composite unit] means 'per millisiemens', but never 'per millisecond'.
  C m [symbol C and m are written separately, representing unit symbols C (for unit 'coulomb') and m (for unit 'metre'), with spacing between them] means 'coulomb metre', but never 'centimetre', etc.
- The use of double prefixes is avoided when single prefixes are available.
  For example :

10^{-9} m = 1nm (nanometre), but not 1mμm (millimicrometre),
10^{-6} m= 1μm (micron), but not 1mmm(millimillimetre),
10^{-12} F= 1 pF (picofarad), but not 1μμF (micromicrofarad),
10^{9} W=1 GW (giga watt), but not 1 kMW (kilomegawatt), etc.

- The use of a combination of unit and the symbols for units is avoided when the physical quantity is expressed by combining two or more units.
  For example :
  joule per mole kelvin is written as J/mol K or J mol^{-1} K^{-1}, but not joule/mole K or J/ mol kelvin or J/mole K, etc.
  joule per tesla is written as J/T or J T^{-1}, but not joule /T or J per tesla or J/tesla, etc.
  newton metre second is written as N m s, but not Newton m second or N m second or N metre s or newton metre s, etc.
  joule per kilogram kelvin is written as J/kg K or J kg^{-1} K^{-1}, but not J/kilog K or joule/kg K or J/kg kelvin or J/kilogram K, etc.
- To simplify calculations, the prefix symbol is attached to the unit symbol in the numerator and not to the denominator.
  For example :
  10^6 N/m^2 is written more conveniently as MN/m^2, in preference to N/mm^2.
  A preference has been expressed for multiples or sub-multiples involving the factor 1000, $10^{\pm 3n}$ where n is the integer.
- Proper care is needed when same symbols are used for physical quantities and units of physical quantities.
  For example :
  The physical quantity weight (*W*) expressed as a product of mass (*m*) and acceleration due to gravity (*g*) may be written in terms of symbols *W*, *m* and *g* printed in italic (or sloping) type as $W = m\ g$, preferably with a spacing between *m* and *g*. It should not be confused with the unit symbols for the units watt (W), metre (m) and gram (g). However, in the equation $W=m\ g$, the symbol *W* expresses the weight with a unit symbol J, *m* as the mass with a unit symbol kg and *g* as the acceleration due to gravity with a unit symbol m/s^2. Similarly, in equation $F = m\ a$, the symbol *F* expresses the force with a unit symbol N, *m* as the mass with a unit symbol kg, and *a* as the acceleration with a unit symbol m/s^2. These symbols for physical quantities should not be confused with the unit symbols for the units 'farad' (F), 'metre'(m) and 'are' (a).
  Proper distinction must be made while using the symbols h (prefix hecto, and unit hour), c (prefix centi, and unit carat), d (prefix deci and unit day), T (prefix tera, and unit tesla), a (prefix atto, and unit are), da (prefix deca, and unit deciare), etc.
- SI base unit 'kilogram' for mass is formed by attaching SI prefix (a multiple equal to 10^3) 'kilo' to a cgs (centimetre, gram, second) unit 'gram' and this may seem to result in an anomaly. Thus, while a thousandth part of unit of length (metre) is called a millimetre (mm), a thousandth part of the unit of mass (kg) is not called a millikilogram, but just a gram. This appears to give the impression that the unit of mass is a gram (g) which is not true. Such a situation has arisen because we are unable to replace the name 'kilogram' by any other suitable unit. Therefore, as an exception, name of the multiples and sub-multiples of the unit of mass are formed by attaching prefixes to the word 'gram' and not to the word 'kilogram'.
  For example :
  10^3 kg =1 megagram (1Mg), but not 1 kilo kilogram (1 kkg);
  10^{-6} kg = 1 milligram (1 mg), but not 1 microkilogram (1μkg);
  10^{-3} kg = 1 gram (1g), but not 1 millikilogram (1 mkg), etc.

It may be emphasised again that you should use the internationally approved and recommended symbols only. Continual practice of following general rules and guidelines in unit symbol writing would make you learn mastering the correct use of SI units, prefixes and related symbols for physical quantities in a proper perspective.

APPENDIX A 9
DIMENSIONAL FORMULAE OF PHYSICAL QUANTITIES

S.No	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
1.	Area	Length × breadth	$[L^2]$	$[M^0 L^2 T^0]$
2.	Volume	Length × breadth × height	$[L^3]$	$[M^0 L^3 T^0]$
3.	Mass density	Mass/volume	$[M]/[L^3]$ or $[M L^{-3}]$	$[M L^{-3} T^0]$
4.	Frequency	1/time period	$1/[T]$	$[M^0 L^0 T^{-1}]$
5.	Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0 L T^{-1}]$
6.	Acceleration	Velocity /time	$[LT^{-1}]/[T]$	$[M^0 L T^{-2}]$
7.	Force	Mass × acceleration	$[M][LT^{-2}]$	$[M LT^{-2}]$
8.	Impulse	Force × time	$[M LT^{-2}][T]$	$[M LT^{-1}]$
9.	Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[M L^2 T^{-2}]$
10.	Power	Work/time	$[ML^2 T^{-2}]/[T]$	$[M L^2 T^{-3}]$
11.	Momentum	Mass × velocity	$[M][LT^{-1}]$	$[M LT^{-1}]$
12.	Pressure, stress	Force/area	$[M LT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
13.	Strain	$\frac{\text{Change in dimension}}{\text{Oringinal dimension}}$	$[L]/[L]$ or $[L^3]/[L^3]$	$[M^o L^o T^o]$
14.	Modulus of elasticity	Stress/strain	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[M L^{-1} T^{-2}]$
15	Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[ML^0 T^{-2}]$
16.	Surface energy	Energy/area	$[ML^2 T^{-2}]/[L^2]$	$[ML^0T^{-2}]$
17.	Velocity gradient	Velocity/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
18.	Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[ML^{-2}T^{-2}]$
19.	Pressure energy	Pressure × volume	$[ML^{-1}T^{-2}][L^3]$	$[ML^2 T^{-2}]$
20.	Coefficient of viscosity	Force/area × velocity gradient	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1} T^{-1}]$
21.	Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0L^0T^0]$
22.	Trigonometric ratio (sinθ, cosθ, tanθ, etc.)	Length/length	$[L]/[L]$	$[M^0L^0T^0]$
23.	Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$

24.	Angular acceleration	Angular velocity/time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
25.	Radius of gyration	Distance	$[L]$	$[M^0LT^0]$
26.	Moment of inertia	Mass × (radius of gyration)^2	$[M]\,[L^2]$	$[ML^2\,T^0]$
27.	Angular momentum	Moment of inertia × angular velocity	$[ML^2]\,[T^{-1}]$	$[ML^2\,T^{-1}]$
28.	Moment of force, moment of couple	Force x distance	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
29.	Torque	Angular momentum/time, Or Force × distance	$[ML^2\,T^{-1}]\,/\,[T]$ or $[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
30.	Angular frequency	2π × Frequency	$[T^{-1}]$	$[M^0L^0T^{-1}]$
31.	Wavelength	Distance	$[L]$	$[M^0LT^0]$
32.	Hubble constant	Recession speed/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
33.	Intensity of wave	(Energy/time)/area	$[ML^2\,T^{-2}/T]/[L^2]$	$[ML^0T^{-3}]$
34.	Radiation pressure	$\frac{\text{Intensity of wave}}{\text{Speed of light}}$	$[MT^{-3}]/[LT^{-1}]$	$[ML^{-1}\,T^{-2}]$
35.	Energy density	Energy/volume	$[ML^2T^{-2}]/\,[L^3]$	$[ML^{-1}\,T^{-2}]$
36.	Critical velocity	$\frac{\text{Reynold's number} \times \text{coefficient of viscocity}}{\text{Mass density} \times \text{radius}}$	$\frac{[M^0L^0T^0][ML^{-1}\,T^{-1}]}{[ML^{-3}][L]}$	$[M^0LT^{-1}]$
37.	Escape velocity	(2 × acceleration due to gravity × earth's radius)$^{1/2}$	$[LT^{-2}]^{1/2}$ x $[L]^{1/2}$	$[M^0LT^{-1}]$
38.	Heat energy, internal energy	Work (= Force × distance)	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
39.	Kinetic energy	(1/2) mass × (velocity)^2	$[M]\,[LT^{-1}]^2$	$[ML^2T^{-2}]$
40	Potential energy	Mass × acceleration due to gravity × height	$[M]\,[LT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
41.	Rotational kinetic energy	½ × moment of inertia × (angular velocity)^2	$[M^0L^0T^0]\,[ML^2]$x$[T^{-1}]^2$	$[M\,L^2\,T^{-2}]$
42.	Efficiency	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$	$\frac{[ML^2\,T^{-2}]}{[ML^2\,T^{-2}]}$	$[M^0L^0T^0]$
43.	Angular impulse	Torque × time	$[ML^2\,T^{-2}]\,[T]$	$[M\,L^2\,T^{-1}]$
44.	Gravitational constant	$\frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{[M]\,[M]}$	$[M^{-1}L^3T^{-2}]$
45.	Planck constant	Energy/frequency	$[ML^2\,T^{-2}]\,/[T^{-1}]$	$[ML^2\,T^{-1}]$

46.	Heat capacity, entropy	Heat energy / temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
47.	Specific heat capacity	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{temperature}}$	$[ML^2T^{-2}]/[M][K]$	$[M^0L^2T^{-2}K^{-1}]$
48.	Latent heat	Heat energy/mass	$[ML^2T^{-2}]/[M]$	$[M^0L^2T^{-2}]$
49.	Thermal expansion coefficient or Thermal expansivity	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	$[L]/[L][K]$	$[M^0L^0K^{-1}]$
50.	Thermal conductivity	$\frac{\text{Heat energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$\frac{[ML^2T^{-2}][L]}{[L^2][K][T]}$	$[MLT^{-3}K^{-1}]$
51.	Bulk modulus or (compressibility)$^{-1}$	$\frac{\text{Volume} \times \text{(change in pressure)}}{\text{(change in volume)}}$	$\frac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	$[ML^{-1}T^{-2}]$
52.	Centripetal acceleration	$(\text{Velocity})^2/\text{radius}$	$[LT^{-1}]^2/[L]$	$[M^0LT^{-2}]$
53.	Stefan constant	$\frac{(\text{Energy / area} \times \text{time})}{(\text{Temperature})^4}$	$\frac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	$[ML^0T^{-3}K^{-4}]$
54.	Wien constant	Wavelength × temperature	$[L][K]$	$[M^0LT^0K]$
55.	Boltzmann constant	Energy/temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
56.	Universal gas constant	$\frac{\text{Pressure} \times \text{volume}}{\text{mole} \times \text{temperature}}$	$\frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$
57.	Charge	Current × time	$[A][T]$	$[M^0L^0TA]$
58.	Current density	Current /area	$[A]/[L^2]$	$[M^0L^{-2}T^0A]$
59.	Voltage, electric potential, electromotive force	Work/charge	$[ML^2T^{-2}]/[AT]$	$[ML^2T^{-3}A^{-1}]$
60.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$
61.	Capacitance	Charge/potential difference	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^4A^2]$
62.	Electrical resistivity or (electrical conductivity)$^{-1}$	$\frac{\text{Resistance} \times \text{area}}{\text{length}}$	$[ML^2T^{-3}A^{-2}]$ $[L^2]/[L]$	$[ML^3T^{-3}A^{-2}]$
63.	Electric field	Electrical force/charge	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
64.	Electric flux	Electric field × area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$

65.	Electric dipole moment	Torque/electric field	$\frac{[ML^2\ T^{-2}]}{[MLT^{-3}\ A^{-1}]}$	$[M^0\ LTA]$
66.	Electric field strength or electric intensity	$\frac{\text{Potential difference}}{\text{distance}}$	$\frac{[ML^2\ T^{-3}\ A^{-1}]}{[L]}$	$[MLT^{-3}\ A^{-1}]$
67.	Magnetic field, magnetic flux density, magnetic induction	$\frac{\text{Force}}{\text{Current} \times \text{length}}$	$[MLT^{-2}]/[A]\ [L]$	$[ML^0\ T^{-2}\ A^{-1}]$
68.	Magnetic flux	Magnetic field × area	$[MT^{-2}\ A^{-2}]\ [L^2]$	$[ML^2\ T^{-2}\ A^{-1}]$
69.	Inductance	$\frac{\text{Magnetic flux}}{\text{Current}}$	$\frac{[ML^2\ T^{-2}\ A^{-1}]}{[A]}$	$[ML^2\ T^{-2}\ A^{-2}]$
70.	Magnetic dipole moment	Torque/magnetic field or current × area	$[ML^2\ T^{-2}]\ /\ [MT^{-2}\ A^{-1}]$ or $[A]\ [L^2]$	$[M^0L^2T^0A]$
71.	Magnetic field strength, magnetic intensity or magnetic moment density	$\frac{\text{Magnetic moment}}{\text{Volume}}$	$\frac{[L^2A]}{[L^3]}$	$[M^0L^{-1}T^0A]$
72	Permittivity constant (of free space)	$\frac{\text{Charge} \times \text{charge}}{4\pi \times \text{electric force} \times (\text{distance})^2}$	$\frac{[AT][AT]}{[MLT^{-2}][L]^2}$	$[M^{-1}L^{-3}T^4\ A^2]$
73.	Permeability constant (of free space)	$\frac{2\pi \times \text{force} \times \text{distance}}{\text{current} \times \text{current} \times \text{length}}$	$\frac{[M^0\ L^0\ T^0][M\,L\,T^{-2}][L]}{[A][A][L]}$	$[MLT^{-2}\ A^{-2}]$
74.	Refractive index	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$	$[LT^{-1}]/LT^{-1}]$	$[M^0L^0T^0]$
75.	Faraday constant	Avogadro constant × elementary charge	$[AT]/[mol]$	$[M^0L^0TA\ mol^{-1}]$
76.	Wave number	2π/wavelength	$[M^0L^0T^0]\ /\ [L]$	$[M^0L^{-1}\ T^0]$
77.	Radiant flux, Radiant power	Energy emitted/time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
78.	Luminosity of radiant flux or radiant intensity	$\frac{\text{Radiant power or radiant flus of source}}{\text{Solid angle}}$	$[ML^2T^{-3}]\ /\ [M^0L^0T^0]$	$[ML^2\ T^{-3}]$
79.	Luminous power or luminous flux of source	$\frac{\text{Luminous energy emitted}}{\text{time}}$	$[ML^2\ T^{-2}]/[T]$	$[ML^2\ T^{-3}]$

80.	Luminous intensity or illuminating power of source	$\frac{\text{Luminous flux}}{\text{Soild angle}}$	$\frac{[ML^2\,T^{-3}]}{[M^0L^0T^0]}$	$[ML^2\,T^{-3}]$
81.	Intensity of illumination or luminance	$\frac{\text{Luminous intensity}}{(\text{distance})^2}$	$[ML^2\,T^{-3}]/[L^2]$	$[ML^0T^{-3}]$
82.	Relative luminosity	$\frac{\text{Luminous flux of a source of given wavelength}}{\text{luminous flux of peak sensitivity wavelength (555 nm) source of same power}}$	$\frac{[ML^2T^{-1}]}{[ML^2T^{-3}]}$	$[M^0L^0T^0]$
83.	Luminous efficiency	$\frac{\text{Total luminous flux}}{\text{Total radiant flux}}$	$[ML^2\,T^{-3}]\,/\,[\,ML^2T^{-3}]$	$[M^0L^0T^0]$
84.	Illuminance or illumination	$\frac{\text{Luminous flux incident}}{\text{area}}$	$[ML^2T^{-3}]/[L^2]$	$[ML^0T^{-3}]$
85.	Mass defect	(sum of masses of nucleons)-(mass of the nucleus)	$[M]$	$[ML^0T^0]$
86.	Binding energy of nucleus	Mass defect $\times$ (speed of light in vacuum)^2	$[M]\,[L\,T^{-1}]^2$	$[ML^2\,T^{-2}]$
87.	Decay constant	0.693/half life	$[T^{-1}]$	$[M^0L^0T^{-1}]$
88.	Resonant frequency	$(\text{Inductance} \times \text{capacitance})^{-\frac{1}{2}}$	$[ML^2T^{-2}A^{-2}]^{-\frac{1}{2}}$ x $[M^{-1}L^{-2}T^4A^2]^{-\frac{1}{2}}$	$[M^0L^0\,A^0\,T^{-1}]$
89.	Quality factor or Q-factor of coil	$\frac{\text{Resonant frequency} \times \text{inductance}}{\text{Resistance}}$	$\frac{[T^{-1}][ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]}$	$[M^0L^0T^0]$
90.	Power of lens	(Focal length)$^{-1}$	$[L^{-1}]$	$[M^0L^{-1}\,T^0]$
91.	Magnification	$\frac{\text{Image distance}}{\text{Object distance}}$	$[L]\,/[L]$	$[M^0L^0T^0]$
92.	Fluid flow rate	$\frac{(\pi/8)\,(\text{pressure})\times(\text{radius})^4}{(\text{viscosity coefficient})\times(\text{length})}$	$\frac{[ML^{-1}T^{-2}]\;[L^4]}{[ML^{-1}T^{-1}]\;[L]}$	$[M^0L^3T^{-1}]$
93	Capacitive reactance	(Angular frequency $\times$ capacitance)$^{-1}$	$[T^{-1}]^{-1}\,[M^{-1}\,L^{-2}T^4A^2]^{-1}$	$[ML^2\,T^{-3}\,A^{-2}]$
94.	Inductive reactance	(Angular frequency $\times$ inductance)	$[T^{-1}][ML^2\,T^{-2}\,A^{-2}]$	$[ML^2\,T^{-3}\,A^{-2}]$

ANSWERS

Chapter 2

2.1 (a) 10^{-6} ; (b) 1.5×10^4 ; (c) 5 ; (d) 11.3, 1.13×10^4.

2.2 (a) 10^7 ; (b) 10^{-16} ; (c) 3.9×10^4 ; (d) 6.67×10^{-8}.

2.5 500

2.6 (c)

2.7 0.035 mm

2.9 94.1

2.10 (a) 1 ; (b) 3 ; (c) 4 ; (d) 4 ; (e) 4 ; (f) 4.

2.11 8.72 m^2; 0.0855 m^3

2.12 (a) 2.3 kg ; (b) 0.02 g

2.13 13%; 3.8

2.14 (b) and (c) are wrong on dimensional grounds. Hint: The argument of a trigonometric function must always be dimensionless.

2.15 The correct formula is $m = m_0 (1 - v^2/c^2)^{-1/2}$

2.16 $\cong 3 \times 10^{-7}$ m^3

2.17 $\cong$ 10^4; intermolecular separation in a gas is much larger than the size of a molecule.

2.18 Near objects make greater angle than distant (far off) objects at the eye of the observer. When you are moving, the angular change is less for distant objects than nearer objects. So, these distant objects seem to move along with you, but the nearer objects in opposite direction.

2.19 $\cong 3 \times 10^{16}$ m; as a unit of length 1 parsec is defined to be equal to 3.084×10^{16} m.

2.20 1.32 parsec; 2.64″ (second of arc)

2.23 1.4×10^3 kg m^{-3}; the mass density of the Sun is in the range of densities of liquids / solids and *not* gases. This high density arises due to inward gravitational attraction on outer layers due to inner layers of the Sun.

2.24 1.429×10^5 km

2.25 Hint: tan θ must be dimensionless. The correct formula is tan $\theta = v/v'$ where v' is the speed of rainfall.

2.26 Accuracy of 1 part in 10^{11} to 10^{12}

2.27 $\cong$ 0.7 × 10^3 kg m^{-3}. In the solid phase atoms are tightly packed, so the atomic mass density is close to the mass density of the solid.

2.28 $\cong$0.3 × 10^{18} kg m^{-3} – Nuclear density is typically 10^{15} times atomic density of matter.

2.29 3.84 × 10^8 m

2.30 55.8 km

2.31 2.8 × 10^{22} km

2.32 3,581 km

2.33 Hint: the quantity $e^4/(16\,\pi^2\,\epsilon_0^2\,m_p\,m_e^2\,c^3\,G)$ has the dimension of time.

Chapter 3

3.1 (a), (b)

3.2 (a) A....B, (b) A....B, (c) B....A, (d) Same, (e) B....A....once.

3.4 37 s

3.5 1000 km/h

3.6 3.06 m s^{-2}; 11.4 s

3.7 1250 m (Hint: view the motion of B relative to A)

3.8 1 m s^{-2} (Hint: view the motion of B and C relative to A)

3.9 T = 9 min, speed = 40 km/h. Hint: $v\,T\,/\,(\,v - 20\,) = 18$; $v\,T\,/\,(\,v + 20\,) = 6$

3.10 (a) Vertically downwards; (b) zero velocity, acceleration of 9.8 m s^{-2} downwards; (c) $x > 0$ (upward and downward motion); $v < 0$ (upward), $v > 0$ (downward), $a > 0$ throughout; (d) 44.1 m, 6 s.

3.11 (a) True;, (b) False; (c) True (if the particle rebounds instantly with the same speed, it implies infinite acceleration which is unphysical); (d) False (true only when the chosen positive direction is along the direction of motion)

3.14 (a) 5 km h^{-1}, 5 km h^{-1}; (b) 0, 6 km h^{-1}; (c) $\frac{15}{8}$ km h^{-1}, $\frac{45}{8}$ km h^{-1}

3.15 Because, for an arbitrarily small interval of time, the magnitude of displacement is equal to the length of the path.

3.16 All the four graphs are impossible. (a) a particle cannot have two different positions at the same time; (b) a particle cannot have velocity in opposite directions at the same time; (c) speed is always non-negative; (d) total path length of a particle can never decrease with time. (Note, the arrows on the graphs are meaningless).

3.17 No, wrong. x-t plot does not show the trajectory of a particle. Context: A body is dropped from a tower (x = 0) at t = 0.

3.18 105 m s^{-1}

3.19 (a) A ball at rest on a smooth floor is kicked, it rebounds from a wall with reduced speed and moves to the opposite wall which stops it; (b) A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit; (c) A uniformly moving cricket ball turned back by hitting it with a bat for a very short time-interval.

3.20 $x < 0, v < 0, a > 0$; $x > 0, v > 0, a < 0$; $x < 0, v > 0, a > 0$.

3.21 Greatest in 3, least in 2; $v > 0$ in 1 and 2, $v < 0$ in 3.

3.22 Acceleration magnitude greatest in 2; speed greatest in 3; $v > 0$ in 1, 2 and 3; $a > 0$ in 1 and 3, $a < 0$ in 2; $a = 0$ at A, B, C, D.

3.23 A straight line inclined with the time-axis for uniformly accelerated motion; parallel to the time- axis for uniform motion.

3.24 10 s, 10 s

3.25 (a) 13 km h^{-1} ; (b) 5 km h^{-1}; (c) 20 s in either direction, viewed by any one of the parents, the speed of the child is 9 km h^{-1} in either direction; answer to (c) is unaltered.

3.26 $x_2 - x_1 = 15\,t$ (linear part); $x_2 - x_1 = 200 + 30\,t - 5\,t^2$ (curved part).

3.27 (a) 60 m, 6 m s^{-1} ; (b) 36 m, 9 m s^{-1}

3.28 (c), (d), (f)

Chapter 4

4.1 Volume, mass, speed, density, number of moles, angular frequency are scalars; the rest are vectors.

4.2 Work, current

4.3 Impulse

4.4 Only (c) and (d) are permissible

4.5 (a) T, (b) F, (c) F, (d) T, (e) T

4.6 Hint: The sum (difference) of any two sides of a triangle is never less (greater) than the third side. Equality holds for collinear vectors.

4.7 All statements except (a) are correct

4.8 400 m for each; B

4.9 (a) O; (b) O; (c) 21.4 km h^{-1}

4.10 Displacement of magnitude 1 km and direction 60° with the initial direction; total path length = 1.5 km (third turn); null displacement vector; path length = 3 km (sixth turn); 866 m, 30°, 4 km (eighth turn)

4.11 (a) 49.3 km h^{-1} ; (b) 21.4 km h^{-1}. No, the average speed equals average velocity magnitude only for a straight path.

4.12 About 18° with the vertical, towards the south.

4.13 15 min, 750 m

4.14 East (approximately)

4.15 150.5 m

4.16 50 m

4.17 9.9 m s^{-2}, along the radius at every point towards the centre.

4.18 6.4 g

4.19 (a) False (true only for uniform circular motion)

(b) True, (c) True.

4.20 (a) $\mathbf{v}(t) = (3.0\,\hat{\mathbf{i}} - 4.0t\,\hat{\mathbf{j}})$ $\hat{\mathbf{a}}(t) = -4.0\hat{\mathbf{j}}$

(b) 8.54 m s^{-1}, 70° with x-axis.

4.21 (a) 2 s, 24 m, 21.26 m s^{-1}

4.22 $\sqrt{2}$, 45° with the x-axis; $\sqrt{2}$, – 45° with the x - axis, $\left(5/\sqrt{2},\ -1/\sqrt{2}\right)$.

4.23 (b) and (e)

4.24 Only (e) is true

4.25 182 m s^{-1}

4.27 No. Rotations in *general* cannot be associated with vectors

4.28 A vector can be associated with a plane area

4.29 No

4.30 At an angle of $\sin^{-1}(1/3) = 19.5°$ with the vertical; 16 km.

4.31 0.86 m s^{-2}, 54.5° with the direction of velocity

Chapter 5

5.1 (a) to (d) No net force according to the First Law
(e) No force, since it is far away from all material agencies producing electromagnetic and gravitational forces.

5.2 The only force in each case is the force of gravity, (neglecting effects of air) equal to 0.5 N vertically downward. The answers do not change, even if the motion of the pebble is not along the vertical. The pebble is not at rest at the highest point. It has a constant horizontal component of velocity throughout its motion.

5.3 (a) 1 N vertically downwards (b) same as in (a)
(c) same as in (a); force at an instant depends on the situation at that instant, not on history.
(d) 0.1 N in the direction of motion of the train.

5.4 (i) T

5.5 $a = -2.5$ m s^{-2}. Using $v = u + at$, $0 = 15 - 2.5\,t$ i.e., $t = 6.0$ s

5.6 $a = 1.5/25 = 0.06$ m s^{-2}

$F = 3 \times 0.06 = 0.18$ N in the direction of motion.

5.7 Resultant force = 10 N at an angle of $\tan^{-1}(3/4) = 37°$ with the direction of 8 N force. Acceleration = 2 m s^{-2} in the direction of the resultant force.

5.8 $a = -2.5$ m s^{-2}, Retarding force = $465 \times 2.5 = 1.2 \times 10^3$ N

5.9 $F - 20{,}000 \times 10 = 20000 \times 5.0$, i.e., $F = 3.0 \times 10^5$ N

5.10 $a = -20$ m s^{-2} $0 \le t \le 30$ s

$t = -5$ s : $x = u\,t = -10 \times 5 = -50$ m

$t = 25$ s : $x = u\,t + (½)\ a\,t^2 = (10 \times 25 - 10 \times 625)$m $= -6$ km

$t = 100$ s : First consider motion up to 30 s

$x_1 = 10 \times 30 - 10 \times 900 = -8700$ m

At $t = 30$ s, $v = 10 - 20 \times 30 = -590$ m s^{-1}

For motion from 30 s to 100 s : $x_2 = -590 \times 70 = -41300$ m

$x = x_1 + x_2 = -50$ km

5.11 (a) Velocity of car (at $t = 10$ s) $= 0 + 2 \times 10 = 20$ m s^{-1}

By the First Law, the horizontal component of velocity is 20 m s^{-1} throughout.
Vertical component of velocity (at $t = 11$s) $= 0 + 10 \times 1 = 10$ m s^{-1}

Velocity of stone (at $t = 11$s) $= \sqrt{20^2 + 10^2} = \sqrt{500} = 22.4$ m s^{-1} at an angle of $\tan^{-1}$ (½) with the horizontal.
(b) 10 m s^{-2} vertically downwards.

5.12 (a) At the extreme position, the speed of the bob is zero. If the string is cut, it will fall vertically downwards.
(b) At the mean position, the bob has a horizontal velocity. If the string is cut, it will fall along a parabolic path.

5.13 The reading on the scale is a measure of the force on the floor by the man. By the Third Law, this is equal and opposite to the normal force N on the man by the floor.

(a) $N = 70 \times 10 = 700$ N ; Reading is 70 kg
(b) $70 \times 10 - N = 70 \times 5$; Reading is 35 kg
(c) $N - 70 \times 10 = 70 \times 5$; Reading is 105 kg
(d) $70 \times 10 - N = 70 \times 10$; Reading would be zero; the scale would read zero.

5.14 (a) In all the three intervals, acceleration and, therefore, force are zero.
(b) 3 kg m s^{-1} at $t = 0$; (c) -3 kg m s^{-1} at $t = 4$ s.

5.15 If the 20 kg mass is pulled,

$600 - T = 20\,a$, $T = 10\,a$

$a = 20$ m s^{-2}, $T = 200$ N

If the 10 kg mass is pulled, a = 20 m s^{-2}, $T = 400$ N

5.16 $T - 8 \times 10 = 8\,a, 12 \times 10 - T = 12a$
i.e. $a = 2$ m s^{-2}, $T = 96$ N

5.17 By momentum conservation principle, total final momentum is zero. Two momentum vectors cannot sum to a null momentum unless they are equal and opposite.

5.18 Impulse on each ball = $0.05 \times 12 = 0.6$ kg m s^{-1} in magnitude. The two impulses are opposite in direction.

5.19 Use momentum conservation : $100\,v = 0.02 \times 80$
$v = 0.016$ m s^{-1} = 1.6 cm s^{-1}

5.20 Impulse is directed along the bisector of the initial and final directions. Its magnitude is $0.15 \times 2 \times 15 \times \cos 22.5° = 4.2$ kg m s^{-1}

5.21 $$v = 2\pi \times 1.5 \times \frac{40}{60} = 2\pi \text{ m s}^{-1}$$

$$T = \frac{mv^2}{R} = \frac{0.25 \times 4\pi^2}{1.5} = 6.6 \text{ N}$$

$200 = \frac{mv_{max}^2}{R}$, which gives $v_{max} = 35 \text{ m s}^{-1}$

5.22 Alternative (b) is correct, according to the First Law

5.23 (a) The horse-cart system has no external force in empty space. The mutual forces between the horse and the cart cancel (Third Law). On the ground, the contact force between the system and the ground (friction) causes their motion from rest.

(b) Due to inertia of the body not directly in contact with the seat.

(c) A lawn mower is pulled or pushed by applying force at an angle. When you push, the normal force (N) must be more than its weight, for equilibrium in the vertical direction. This results in greater friction f ($f \propto N$) and, therefore, a greater applied force to move. Just the opposite happens while pulling.

(d) To reduce the rate of change of momentum and hence to reduce the force necessary to stop the ball.

5.24 A body with a constant speed of 1 cm s^{-1} receives impulse of magnitude 0.04 kg × 0.02 m s^{-1} = 8×10^{-4} kg m s^{-1} after every 2 s from the walls at $x = 0$ and $x = 2$ cm.

5.25 Net force = 65 kg × 1 m s^{-2} = 65 N

$a_{max} = \mu_s g = 2 \text{ m s}^{-2}$

5.26 Alternative (a) is correct. Note $mg + T_2 = m\boldsymbol{v}_2^2/\text{R}$; $T_1 - mg = m\boldsymbol{v}_1^2/\text{R}$

The moral is : do not confuse the actual material forces on a body (tension, gravitational force, etc) with the effects they produce : centripetal acceleration $\boldsymbol{v}_2^2/\text{R}$ or $\boldsymbol{v}_1^2/\text{R}$ in this example.

5.27 (a) 'Free body' : crew and passengers

Force on the system by the floor = F upwards; weight of system = mg downwards;

$\therefore F - mg = ma$

$F - 300 \times 10 = 300 \times 15$

$F = 7.5 \times 10^3$ N upward

By the Third Law, force on the floor by the crew and passengers = 7.5×10^3 N downwards.

(b) 'Free body' : helicopter plus the crew and passengers

Force by air on the system = R upwards; weight of system = mg downwards

$\therefore R - mg = ma$

$R - 1300 \times 10 = 1300 \times 15$

$R = 3.25 \times 10^4$ N upwards

By the Third Law, force (action) on the air by the helicopter = 3.25×10^4 N downwards.

(c) 3.25×10^4 N upwards

5.28 Mass of water hitting the wall per second

$= 10^3 \text{ kg m}^{-3} \times 10^{-2} \text{ m}^2 \times 15 \text{ m s}^{-1} = 150 \text{ kg s}^{-1}$

Force by the wall = momentum loss of water per second = 150 kg s^{-1} × 15 m s^{-1} = 2.25 × 10^3 N

5.29 (a) 3 $m\,g$ (down) (b) 3 $m\,g$ (down) (c) 4 $m\,g$ (up)

5.30 If N is the normal force on the wings,

$$N\cos\theta = mg, \qquad N\sin\theta = \frac{mv^2}{R}$$

which give $R = \dfrac{v^2}{g \tan \theta} = \dfrac{200\times200}{10\times\tan15^\circ} = 15\text{km}$

5.31 The centripetal force is provided by the lateral thrust by the rail on the flanges of the wheels. By the Third Law, the train exerts an equal and opposite thrust on the rail causing its wear and tear.

Angle of banking = $\tan^{-1}\left(\dfrac{v^2}{R\,g}\right) = \tan^{-1}\left(\dfrac{15\times15}{30\times10}\right) \simeq 37^\circ$

5.32 Consider the forces on the man in equilibrium : his weight, force due to the rope and normal force due to the floor.

(a) 750 N (b) 250 N; mode (b) should be adopted.

5.33 (a) $T - 400 = 240, \quad T = 640$ N

(b) $400 - T = 160, \quad T = 240$ N

(c) $T = 400$ N

(*d*) $T = 0$

The rope will break in case (a).

5.34 We assume perfect contact between bodies A and B and the rigid partition. In that case, the self-adjusting normal force on B by the partition (reaction) equals 200 N. There is no impending motion and no friction. The action-reaction forces between A and B are also 200 N. When the partition is removed, kinetic friction comes into play.

Acceleration of A + B = [200 – (150 × 0.15)] / 15 =11.8 m s^{-2}

Friction on A = 0.15 × 50 = 7.5 N

200 – 7.5 – F_{AB} = 5 × 11.8

$F_{AB} = 1.3 \times 10^2$ N; opposite to motion .

$F_{BA} = 1.3 \times 10^2$ N; in the direction of motion.

5.35 (a) Maximum frictional force possible for opposing impending relative motion between the block and the trolley = 150 × 0.18 = 27 N, which is more than the frictional force of 15 × 0.5 = 7.5 N needed to accelerate the box with the trolley. When the trolley moves with uniform velocity, there is no force of friction acting on the block.

(b) For the accelerated (non-inertial) observer, frictional force is opposed by the pseudo-force of the same magnitude, keeping the box at rest relative to the observer. When the trolley moves with uniform velocity there is no pseudo-force for the moving (inertial) observer and no friction.

5.36 Acceleration of the box due to friction = μg = 0.15 × 10 = 1.5 m s^{-2}. But the acceleration of the truck is greater. The acceleration of the box relative to the truck is 0.5 m s^{-2} towards the rear end. The time taken for the box to fall off the truck = $\sqrt{\dfrac{2\times5}{0.5}} = \sqrt{20}$ s.

During this time, the truck covers a distance = ½ × 2 × 20 = 20 m.

5.37 For the coin to revolve with the disc, the force of friction should be enough to provide the necessary centripetal force, i.e $\frac{mv^2}{r} \le \mu m g$. Now $v = r\omega$, where $\omega = \frac{2\pi}{T}$ is the angular frequency of the disc. For a given μ and ω, the condition is $r \le \mu g / \omega^2$. The condition is satisfied by the nearer coin (4 cm from the centre).

5.38 At the uppermost point, $N + mg = \frac{mv^2}{R}$, where N is the normal force (downwards) on the motorcyclist by the ceiling of the chamber. The minimum possible speed at the uppermost point corresponds to $N = 0$.

i.e. $v_{\min} = \sqrt{Rg} = \sqrt{25 \times 10} = 16 \,\mathrm{m\,s^{-1}}$

5.39 The horizontal force N by the wall on the man provides the needed centripetal force : $N = m R \omega^2$. The frictional force f (vertically upwards) opposes the weight mg. The man remains stuck to the wall after the floor is removed if $mg = f < \mu N$ i.e. $mg < \mu m R \omega^2$. The minimum angular speed of rotation of the cylinder is $\omega_{min} = \sqrt{g / \mu R} = 5 \,\mathrm{s^{-1}}$

5.40 Consider the free-body diagram of the bead when the radius vector joining the centre of the wire makes an angle θ with the vertical downward direction. We have $mg = N \cos\theta$ and $m R \sin\theta \;\omega^2 = N \sin\theta$. These equations give $\cos\theta = g/R\omega^2$. Since $\cos\theta \le 1$, the bead remains at its lowermost point for $\omega \le \sqrt{\frac{g}{R}}$.

For $\omega = \sqrt{\frac{2g}{R}}$, $\cos\theta = \frac{1}{2}$ i.e. $\theta = 60^0$.

Chapter 6

6.1 (a) +ve (b) –ve (c) –ve (d) + ve (e) – ve

6.2 (a) 882 J ; (b) –247 J; (c) 635 J ; (d) 635 J;
Work done by the net force on a body equals change in its kinetic energy.

6.3 (a) $x > a$; 0 (c) $x < a$, $x > b$; $-V_1$
(b) $-\infty < x < \infty$; V_1 (d) $-b/2 < x < -a/2$, $a/2 < x < b/2$; $-V_1$

6.5 (a) rocket; (b) For a conservative force work done over a path is minus of change in potential energy. Over a complete orbit, there is no change in potential energy; (c) K.E. increases, but P.E. decreases, and the sum decreases due to dissipation against friction; (d) in the second case.

6.6 (a) decrease; (b) kinetic energy; (c) external force; (d) total linear momentum, and also total energy (if the system of two bodies is isolated).

6.7 (a) F ; (b) F ; (c) F ; (d) F (true usually but not always, why?)

6.8 (a) No
(b) Yes
(c) Linear momentum is conserved during an inelastic collision, kinetic energy is, of course, not conserved even after the collision is over.
(d) elastic.

6.9 (b) t

6.10 (c) $t^{3/2}$

6.11 12 J

6.12 The electron is faster, $\upsilon_e / \upsilon_p = 13.5$

6.13 0.082 J in each half ; – 0.163 J

6.14 Yes, momentum of the molecule + wall system is conserved. The wall has a recoil momentum such that the momentum of the wall + momentum of the outgoing molecule equals momentum of the incoming molecule, assuming the wall to be stationary initially. However, the recoil momentum produces negligible velocity because of the large mass of the wall. Since kinetic energy is also conserved, the collision is elastic.

6.15 43.6 kW

6.16 (b)

6.17 It transfers its entire momentum to the ball on the table, and does not rise at all.

6.18 5.3 m s^{-1}

6.19 27 km h^{-1} (no change in speed)

6.20 50 J

6.21 (a) $m = \rho A\upsilon t$ (b) $K = \rho A\upsilon^3 t/2$ (c) $P = 4.5$ kW

6.22 (a) 49,000 J (b) 6.45 10^{-3} kg

6.23 (a) 200 m^2(b) comparable to the roof of a large house of dimension 14m × 14m.

6.24 21.2 cm, 28.5 J

6.25 No, the stone on the steep plane reaches the bottom earlier; yes, they reach with the same speed υ, [since $mgh = (1/2)\, m\,\upsilon^2$]

$\upsilon_B = \upsilon_C = 14.1$ m s^{-1} , $t_B = 2\sqrt{2}$ s , $t_C = 2\sqrt{2}$ s

6.26 0.125

6.27 8.82 J for both cases.

6.28 The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 m s^{-1} with respect to the trolley's new velocity. Apply momentum conservation for an observer outside. 10.36 m s^{-1}, 25.9 m.

6.29 All except (V) are impossible.

Chapter 7

7.1 The geometrical centre of each. No, the CM may lie outside the body, as in case of a ring, a hollow sphere, a hollow cylinder, a hollow cube etc.

7.2 Located on the line joining H and C1 nuclei at a distance of 1.24 Å from the H end.

7.3 The speed of the CM of the (trolley + child) system remains unchanged (equal to υ) because no external force acts on the system. The forces involved in running on the trolley are internal to this system.

7.6 $l_z = xp_y - yp_x$, $l_x = yp_z - zp_y$, $l_y = zp_x - xp_z$

7.8 72 cm

7.9 3675 N on each front wheel, 5145 N on each back wheel.

7.10 (a) 7/5 MR^2 (b) 3/2 MR^2

7.11 Sphere

7.12 Kinetic Energy = 3125 J; Angular Momentum = 62.5 J s

7.13 (a) 100 rev/min (use angular momentum conservation).

(b) The new kinetic energy is 2.5 times the initial kinetic energy of rotation. The child uses his internal energy to increase his rotational kinetic energy.

7.14 25 s^{-2}; 10 m s^{-2}

7.15 36 kW

7.16 at R/6 from the center of original disc opposite to the center of cut portion.

7.17 66.0 g

7.18 (a) Yes; (b) Yes, (c) the plane with smaller inclination ($\because a \propto \sin\theta$)

7.19 4J

7.20 6.75×10^{12} rad s^{-1}

7.21 (a) 3.8 m (b) 3.0 s

7.22 Tension = 98 N, N_B = 245 N, N_C = 147 N.

7.23 (a) 59 rev/min, (b) No, the K.E. is increased and it comes from work done by man in the process.

7.24 0.625 rad s^{-1}

7.27 (a) By angular momentum conservation, the common angular speed

$\omega = (I_1\,\omega_1 + I_2\,\omega_2)\ /\ (I_1 + I_2)$

(b) The loss is due to energy dissipation in frictional contact which brings the two discs to a common angular speed ω. However, since frictional torques are internal to the system, angular momentum is unaltered.

7.28 Velocity of A = $\omega_o R$ in the same direction as the arrow; velocity of B = $\omega_o R$ in the opposite direction to the arrow; velocity of C = $\omega_o R/2$ in the same direction as the arrow. The disc will not roll on a frictionless plane.

7.29 (a) Frictional force at B opposes velocity of B. Therefore, frictional force is in the same direction as the arrow. The sense of frictional torque is such as to oppose angular motion. ω_o and τ are both normal to the paper, the first into the paper, and the second coming out of the paper.

(b) Frictional force decreases the velocity of the point of contact B. Perfect rolling ensues when this velocity is zero. Once this is so, the force of friction is zero.

7.30 Frictional force causes the CM to accelerate from its initial zero velocity. Frictional torque causes retardation in the initial angular speed ω_o. The equations of motion are : $\mu_k\, m g = m a$ and $\mu_k\, m g R = -I\alpha$, which yield $v = \mu_k g t$, $\omega = \omega_o - \mu_k m g R t / I$. Rolling begins when $v = R\omega$. For a ring, $I = mR^2$, and rolling begins at $t = \omega_o R/2\mu_k g$. For a disc, $I = ½\, m R^2$ and rolling starts at break line $t = R\omega_o/3\mu_k g$. Thus, the disc begins to roll earlier than the ring, for the same R and ω_o. The actual times can be obtained for R = 10 cm, ω_o = 10 π rad s^{-1}, μ_k = 0.2

7.31 (a) 16.4 N

(b) Zero

(c) 37° approx.

Chapter 8

8.1 (a) No.

(b) Yes, if the size of the space ship is large enough for him to detect the variation in g.

(c) Tidal effect depends inversely on the cube of the distance unlike force, which depends inversely on the square of the distance.

8.2 (a) decreases; (b) decreases; (c) mass of the body; (d) more.

8.3 Smaller by a factor of 0.63.

8.5 3.54×10^8 years.

8.6 (a) Kinetic energy, (b) less,

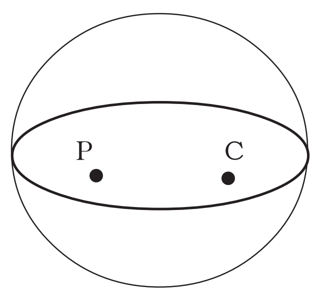
8.7 (a) No, (b) No, (c) No, (d) Yes

[The escape velocity is independent of mass of the body and the direction of projection. It depends upon the gravitational potential at the point from where the body is launched. Since this potential depends (slightly) on the latitude and height of the point, the escape velocity (speed) depends (slightly) on these factors.]

8.8 All quantities vary over an orbit except angular momentum and total energy.

8.9 (b), (c) and (d)

8.10 and **8.11** For these two problems, complete the hemisphere to sphere. At both P, and C, potential is constant and hence intensity = 0. Therefore, for the hemisphere, (c) and (e) are correct.



8.12 2.6×10^8 m

8.13 2.0×10^{30} kg

8.14 1.43×10^{12} m

8.15 28 N

8.16 125 N

8.17 8.0×10^6 m from the earth's centre

8.18 31.7 km/s

8.19 5.9×10^9 J

8.20 2.6×10^6 m/s

8.21 0, 2.7×10^{-8} J/kg; an object placed at the mid point is in an unstable equilibrium

8.22 -9.4×10^6 J/kg

8.23 $G\,M\,/\,R^2 = 2.3 \times 10^{12}$ m s^{-2} , $\omega^2 R = 1.1 \times 10^6$ m s^{-2}; here ω is the angular speed of rotation. Thus in the rotating frame of the star, the inward force is much greater than the outward centrifugal force at its equator. The object will remain stuck (and not fly off due to centrifugal force). Note, if angular speed of rotation increases say by a factor of 2000, the object will fly off.

8.24 3×10^{11} J

8.25 495 km