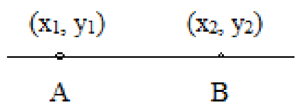
# Straight Lines

## What is a Straight Line?

A line is simply an object in geometry that is characterized under zero width object that extends on both sides. A straight line is just a line with no curves. So, a line that extends to both sides till in nity and has no curves is called a **straight line**.



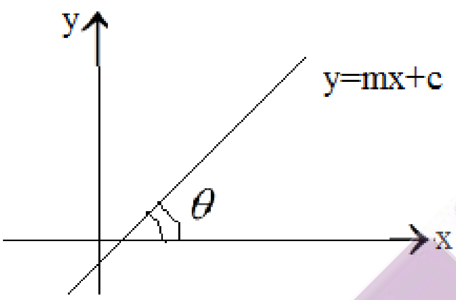

## Equation of Straight Line

The relation between variable x, y satisfy all points on the curve.

Straight line equation linear in x and constant terms.

ax + by + c = 0 { equation of straight lines.

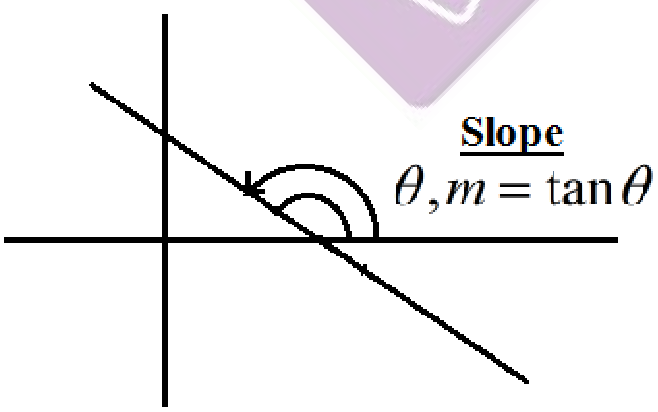
**Slope:-**




angle with + ve x-axis

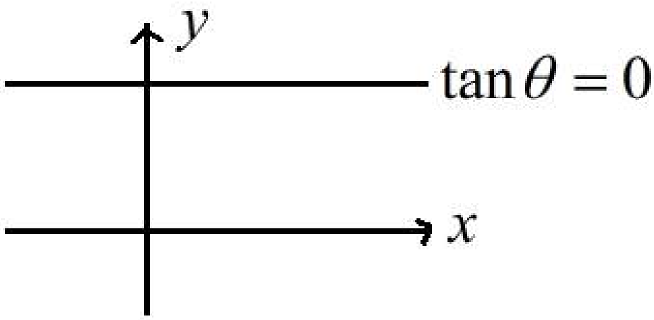


'tan θ' is called **slope of straight line**.

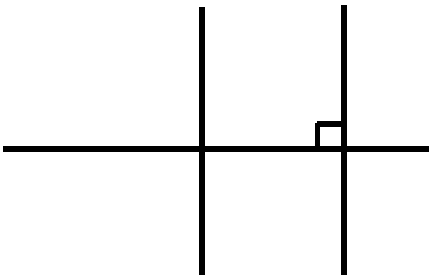



θ E[0,n)

**Note 1 –** If line is Horizontal, then slope = 0




**Note 2 –** If line is ⊥ to x-axis, i.e. vertical

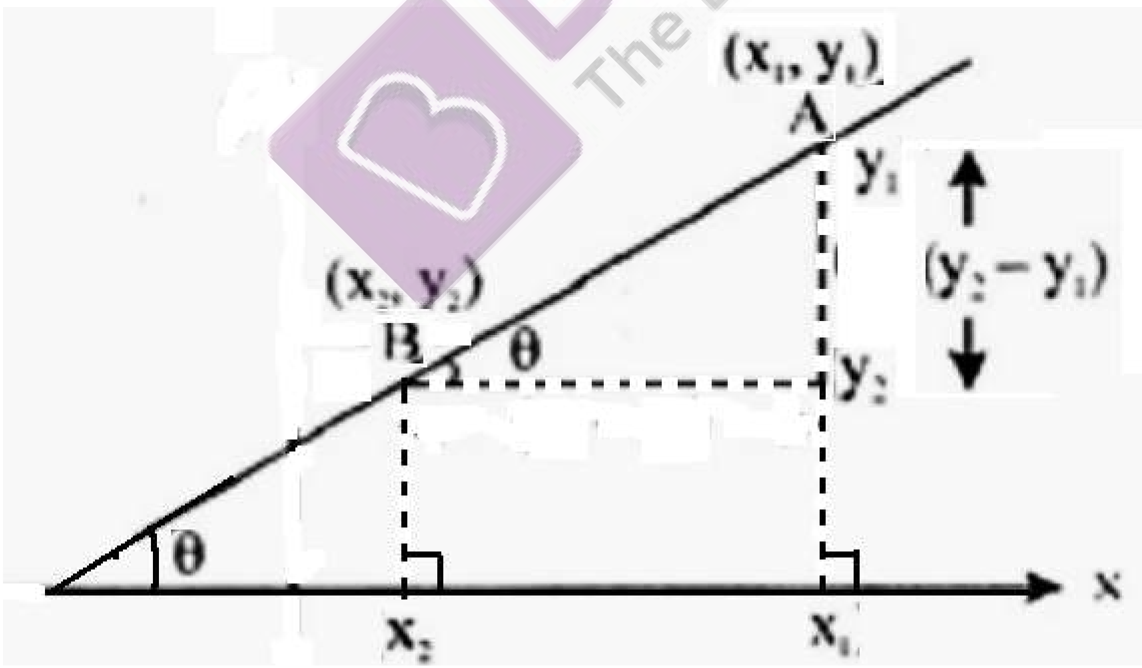


Slope = undefined

= $\frac{1}{0}$

= $\tan\frac{\pi}{2}$

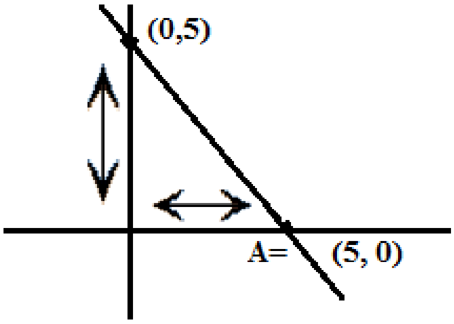
**Note 3 –**




$-(x_1 - x_2) \tan\theta = \frac{y_2 - y_2}{x_1 - x_2}$

## Intercept Form

x – co-ordinate of point of intersection of line with x-axis is called x-intercept




x – Intercept = 5

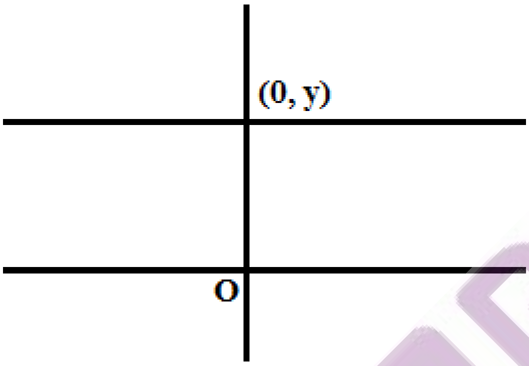
y – Intercept = 5

- Line passes through origin, intercept = 0

x – Intercept = 0

y – Intercept = 0

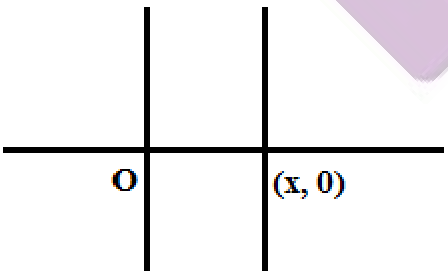
Similarly




y – intercept will be y-co-ordinate of point of intersection of line with y-axis.

x – Intercept =

y – Intercept = $y_1$




x – Intercept = $x_1$

y – Intercept =

Length of x – intercept = $|x_1|$

Length of y – intercept = $|y_1|$

## Point form

Equation of line passing through two points $(x_1, y_1)$ & $(x_2, y_2)$

$$y - y_1 \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) \rightarrow point$$

m = slope

$$point br\, y - y_2 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_2)$$ ––**Form – III**

**For example,**

**Example:** Find the equation of the lines that passes through the points (-2,4) and (1,2)

**Solution:**

Now we have a slope and two points. We can find the equation (by solving first for "b") if we have a point and the slope. So we need to choose one of the points and use it to solve for b. Using the point (−2, 4), we get:

y = mx + b

4 = (− 2/3)(−2) + b

4 = 4/3 + b

4 − 4/3 = b

12/3 − 4/3 = b

b = 8/3

so, y = ( − 2/3 ) x + 8/3.

On the other hand, if we use the point (1, 2), we get:

y = mx + b

2 = (− 2/3)(1) + b

2 = − 2/3 + b

2 + 2/3 = b

6/3 + 2/3 = b

b = 8/3

So it doesn't matter which point we choose. Either way, the answer is the same:

y = (− 2/3)x + 8/3



## Slope Point form (Equation of a Line with 2 Points)

Equation of line with slope (https://byjus.com/maths/slope-of-line/) 'm' and which passes through ($x_1$, $y_1$) can be given as

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = m(x - x_1) \text{ points}$$

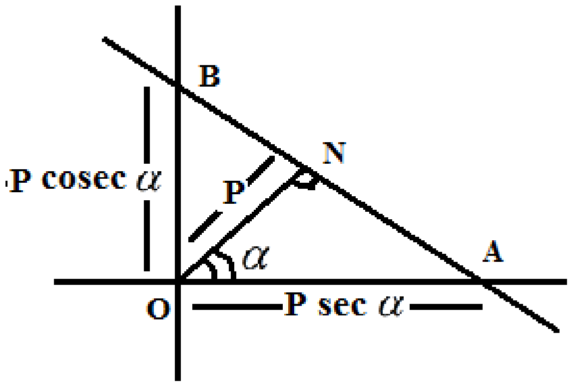
$$\searrow points \quad \searrow slope$$

**Form IV**

## Intercept form

Equation of line with x – intercept as ‘a’ and y – intercept as ‘b’ can be given as $\frac{x}{a} + \frac{y}{b} = 1$

**Form V**




ON = P

AON = α

Let length of ⊥r from origin to S.L is ‘P’ and let this ⊥r make an angle with + vex- axis ‘α’, then equation of line can be

$x \cos\alpha + y \sin\alpha = \frac{x}{p\sec\alpha} + \frac{y}{p\cos ec\alpha}$ $x\cos\alpha + y\sin\alpha = P$

Here ‘P’ should be the α E [0, 2π)

**Form – VI**

**Learn More:** Different Forms Of The Equation Of Line (https://byjus.com/maths/different-forms-of-the-equation-of-line/)

## Straight Lines Video Lesson

## General Form or Standard form of a Line

Equation of a straight line can be given as

$ax + by + c = 0$ a, b, c are Real numbers

## Slope form

$y = mx + c$ $\quad m = {}^{a}/_{b}$ $\quad y = \frac{-ax}{b} - \frac{c}{b},$ $\quad c = {}^{-c}/_{b}$

## Relation between two lines

- **Parallel line**

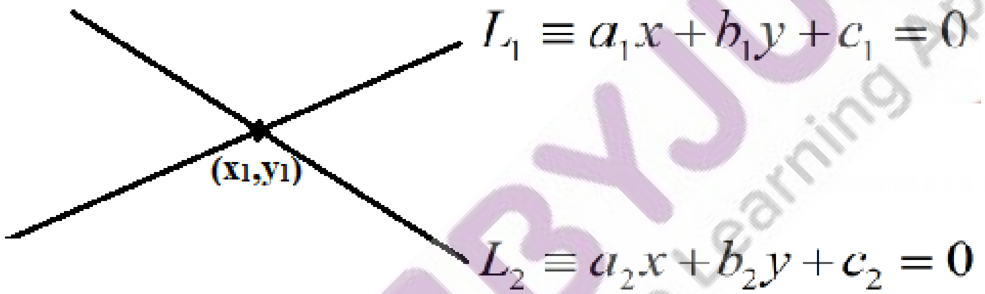
$L_1$ $a_1x + b_1y + c_1 = 0$

$L_2$ $a_2x + b_2y + c_2 = 0$

**Condition required:**

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

## Intersection of two lines

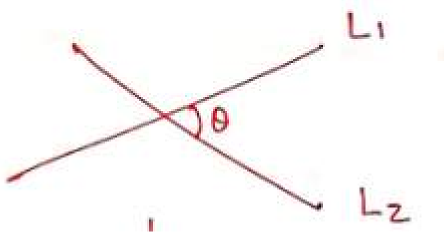



Solve $L_1$ & $L_2$

$a_1x + b_1y = -c_1$ $\quad a_2x + b_2y = -c_2$ $\quad x = x_1$, $y = y_1$ $\{from\ above\ equation)$

## Angle between Straight lines

$Let \quad L_1 \quad \equiv \quad y = m_1x + c_1$

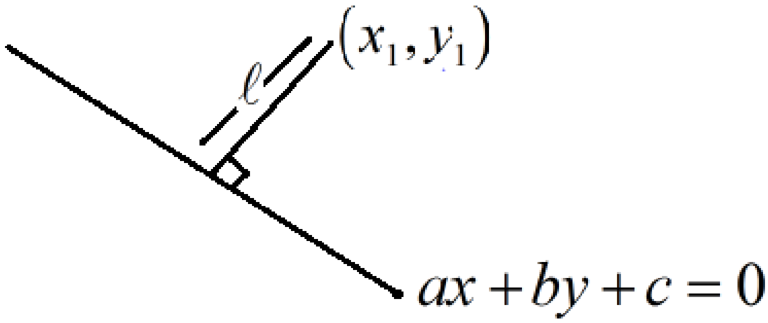



$L_2 \quad \equiv \quad y = m_2x + c_2$ $\quad \theta$ = acute angle $\quad \theta = \tan^{-1}\left|\left(\frac{m_2 - m_1}{1 + m_1 + m_2}\right)\right|$

$if \Rightarrow m_2 = m_1 \quad \rightarrow \quad lines\ are\ parallel$ $\quad m_1m_2 = -1, \quad lines\ L1\ \&L2\ are\ perpendicular\ to\ each\ other$

## Length of Perpendicular from a Point on a Line

The length of the perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is




$\ell = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

B (x, y) is foot of perpendicular is given by

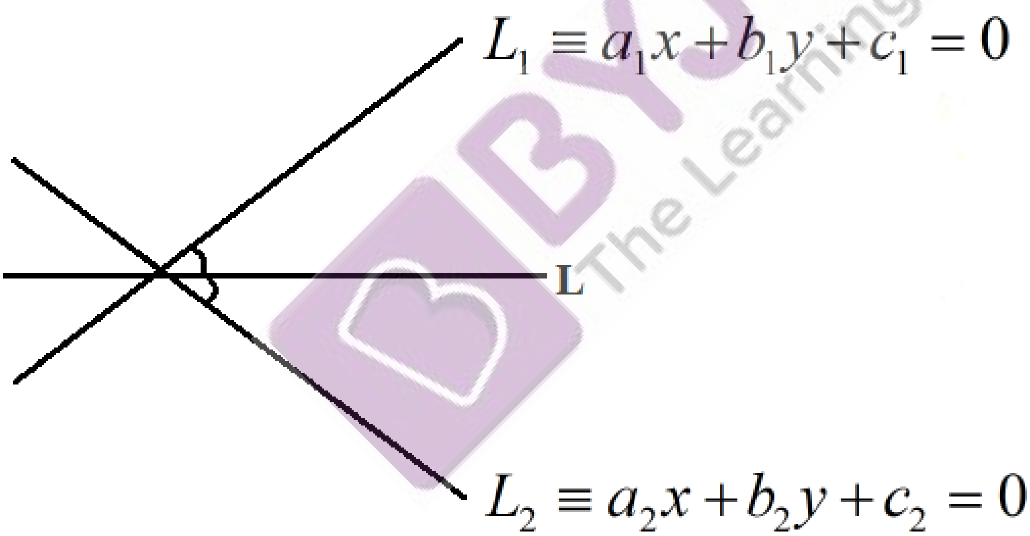
$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{(a^2+b^2)}$

A'(h, k) is mirror image, given by

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{(a^2+b^2)}$

## Angular Bisector of Straight lines

To find the equation of the bisectors of the angle between lines.




Equation of line L can be given

$\frac{a_1x+b_1y+c_1}{\sqrt{{a_1}^2+{b_1}^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{{a_2}^2+{b_2}^2}}$

## Family of Lines:

The general equation of the family of lines through the point of intersection of two given lines $L_1$ & $L_2$ is given by $L_1 + \lambda L_2 = 0$

Where λ is a parameter.

## Concurrency of Three Lines

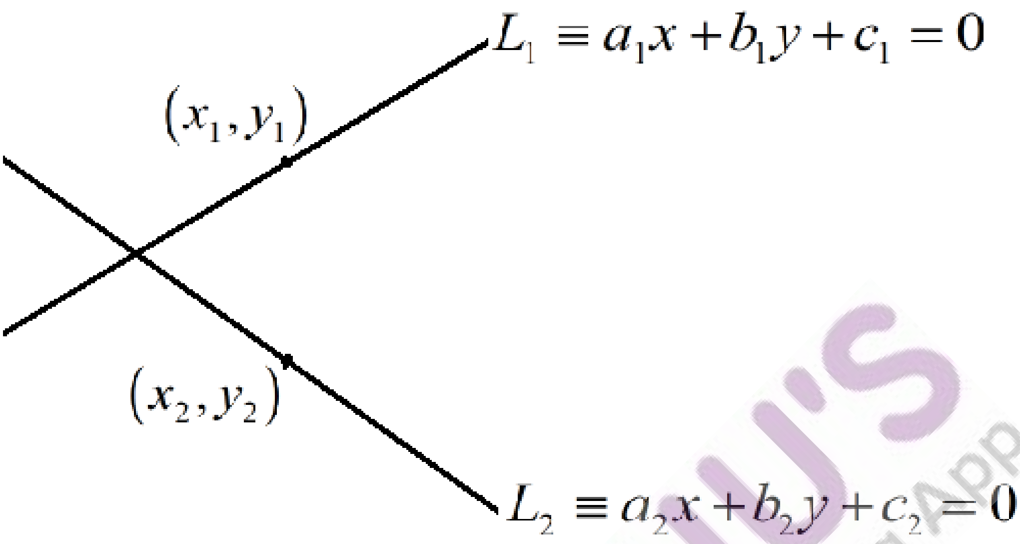
Let the lines be

$L_1 \equiv a_1x + b_1y + c_1 = 0 \quad L_2 \equiv a_2x + b_2y + c_2 = 0 \quad L_3 \equiv a_3x + b_3y + c_3 = 0$

So, condition for concurrency of linear is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

## Pair of Straight Lines

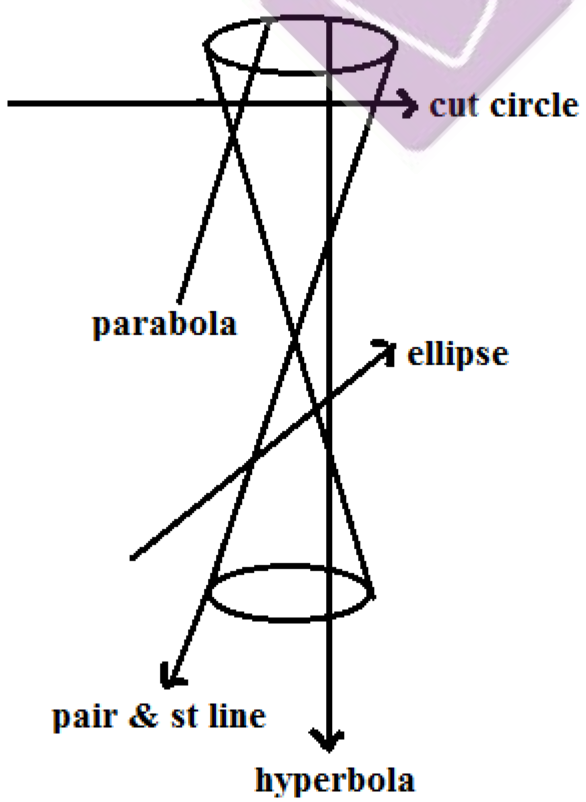



Join equation of line L1 & L2 represents P. S. L $(a_1x + b_1y+c_1)\ (a_2x+b_2y+c_2) = 0$

$f(x, y) . g(x, y) = 0 \quad represent\ P.O.S.L \quad \downarrow linear\ equation\ of\ line \quad linear\ eqution\ of\ line$

Let defines a standard form of equation:-

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represent conics curve equation



Condition for curve of being P.O.S.L $\Delta = abc + 2fgy - ag^2 - bf^2 - ch^2 = 0$

$$\Downarrow$$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

If $\Delta \neq 0$, (i) parabola $h^2 = ab$

(ii) hyperbola $h^2 < ab$

(iii) circle $h^2 = 0$, $a = b$

(iv) ellipse $h^2 > ab$

Now, lets see how did we get $\Delta = 0$

General equation $ax^2 + 2gx + hxy + by^2 + 2fy + c = 0$

$ax^2 + (2g+hy)x + (by^2 + 2fy + c) = 0$

we can consider equation 11 as quadratic equation in x keeping y as constant.

$$x = \frac{-(2g+hy) \pm \sqrt{(2g+hy)^2 - 4a(by^2+2fy+c)}}{2a} \qquad x = \frac{-(2g+hy) \pm \sqrt{Q(y)}}{2a}$$

Now, Q(y) has to be perfect square then only we can get two different line equation Q(y) in perfect square for that $\Delta$ value of Q(y) should be zero.

From there D = 0

$abc + 2fgh - bg^2 - af^2 - ch^2 = 0$

Or

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Hence proved.

⇒ point of intersection of two lines (P.O.S.L)

$$\frac{d}{du}(\text{equation (1)}) = \textit{Linear equation}\,(1)$$

$$\frac{d}{dy}(\textit{equation}(11)) = \textit{Linear equation}(11)$$

solving these two equation

We can get point of intersection.

Or

Solve the P.O.S.L, factorize it in $(L_1).(L_2) = 0$ or f(x, y) . g(u,y) = 0

Angle between the lines,

$$\tan\theta = \left(\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|\right)$$

$h^2 = ab \rightarrow$line is either parallel or confident

$h^2 < ab \rightarrow$imaginary line

$h^2 > ab \rightarrow$Two distinct lines

$a + b = 0 \Rightarrow$perpendicular line

P.O.S.L passing through origin

$\Rightarrow (y - m_1x) \times (y - m_2x) = 0$

$y^2 - m_2yx - m_1xy - m_1m_2x^2 = 0$

$y^2 - (m_1 + m_2)\, xy - m_1m_2x^2 = 0$

$\Rightarrow ax^2 + 2hxy + by^2 = 0$ − − − − − − − − (III)

$\Rightarrow y^2 + \frac{2h}{b}xy + \frac{ab}{b}x^2 = 0 - - - (IV)\ m1 + m2 = \frac{2h}{b}$

$m_1\, m_2 = \frac{a}{b} \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\sqrt{(m_1+m_2)^2 4m_1m_2}}{1+m_1m_2}\right| = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$

Proved.

## Straight Lines Formulas

| **All Formulas Related to Straight Lines** | |
|---|---|
| Equation of a Straight Line | ax + by + c = 0 |
| General form or Standard Form | y = mx + c |
| Equation of a Line with 2 Points (Slope Point Form) | (y − y1) = m(x − x1) |
| Angle Between Straight lines | $\theta = \tan^{-1}\left|\left(\frac{m_2 - m_1}{1 + m_1 + m_2}\right)\right|$ |

# Problems on Straight Lines

**Question 1:**

Find the equation to the straight line which passes through the point (-5, 4) and is such that the portion of it between the axes is divided by the given point in the ratio 1 : 2.

**Solution:**

Let the required straight line be $\frac{x}{a} + \frac{y}{b} = 1.$

Using the given conditions, $P\left(\frac{2a+1.0}{2+1}, \frac{2.0+1.b}{2+1}\right)$ is the point which divides (a, 0) and (0, b) internally in the ratio 1 : 2.

But P is (-5, 4)

Hence -5 = 2a/3, 4 = b/3 a = -15/2, b = 12.

Hence the required equation is $\frac{x}{(-15/2)} + \frac{y}{12} = 1$

**Question 2:**

Find the equation of the straight line which passes through the point (1, 2) and makes an angle θ with the positive direction of the x-axis where $\cos\theta = -\frac{1}{3}.$

**Solution:**

Here $\cos\theta = -\frac{1}{3}$ (a negative number) so that $\frac{\pi}{2} < \theta < \pi \Rightarrow \tan\theta = -\sqrt{8}$ = slope of line.

We know that the equation of the straight line passing through the point ($x_1$, $y_1$) having slope m is

$y - y_1 = m(x - x_1)$

Therefore the equation of the required line is $y - 2 = -\sqrt{8}\,(x - 1) \Rightarrow \sqrt{8}x + y - \sqrt{8} - 2 = 0.$

**Question 3:**

Find the equation of the line joining the points (-1, 3) and (4, -2).

**Solution:**

Equation of the line passing through the points ($x_1$, $y_1$) and ($x_2$, $y_2$) is $y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$

Hence equation of the required line will be $y - 3 = \frac{3+2}{-1-4}(x + 1) \Rightarrow x + y - 2 = 0$

**Question 4:**

Which line is having greatest inclination with positive direction of x-axis?

(i) line joining points (1, 3) and (4, 7)

(ii) line 3x − 4y + 3 = 0

**Solution:**

(i) Slope of line joining points A(1, 3) and B(4, 7) is $\frac{7-3}{4-1} = \frac{4}{3} = \tan\alpha$

(ii) Slope of line is $-\frac{3}{-4} = \frac{3}{4} = \tan\beta$

Now tan α > tan β

**Question 5:**

Angle of line positive direction of x-axis is θ. Line is rotated about some point on it in anticlockwise direction by angle 45° and its slope becomes 3. Find the angle θ.

**Solution:**

Originally slope of line is tan θ = m

Now slope of line after rotation is 3.

Angle between old position and new position of lines is 45°.

∴ we have $\tan 45^\circ = \frac{3-m}{1+3m}$

1 + 3m = 3 − m

4m = 2

m = 1/2 = tan θ

$\theta = \tan^{-1}(1/2)$

**Question 6:**

If line $3x - ay - 1 = 0$ is parallel to the line $(a + 2)x - y + 3 = 0$ then find the values of a.

**Solution:**

Slope of line $3x - ay - 1 = 0$ is $\frac{3}{a}$

Slope of line $(a+2)x - y + 3$s (a0+ 2)

Since lines are parallel then we have $a + 2 = \frac{3}{a}$

or $a^2 + 2a - 3 = 0$

or $(a-1)(a+3) = 0$

or a = 1 or a = 3.

**Question 7:**

Find the value of x for which the points (x, -1), (2, 1) and (4, 5) are collinear.

**Solution:**

If points A(x, -1) B(2, 1), and C(4, 5) are collinear, then

$\Rightarrow \frac{1-(-1)}{2-x} = \frac{5-1}{4-2} \Rightarrow \frac{2}{2-x} = 2 \Rightarrow x = 1$

**Question 8:**

The slope of a line is double of the slope of another line. It tangent of the angle between them is $\frac{1}{3}$. Find the slopes of the lines.

**Solution:**

Let $m_1$ and m be the slopes of the two given lines such that $m_1$ = 2m

We know that if θ is the angle between the lines $l_1$ and $l_2$ with slopes $m_1$ and $m_2$, then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

It is given that the tangent of the angle between the two lines is $\frac{1}{3}$

$\therefore \frac{1}{3} = \left|\frac{m-2m}{1+(2m).m}\right| \Rightarrow \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right| \Rightarrow 2|m|^2 - 3|m| + 1 = 0 \Rightarrow (|m| - 1)(2|m| - 1) = 0 \Rightarrow |m| = 1 \ or \ |m| = 1/2$
$\Rightarrow |m| \pm 1 \ or \ m = \pm 1/2$

**Question 9:**

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

**Solution:**

Line parallel to the line $3x - 4y + 2 = 0$ is $3x - 4y + t = 0$

It passes through the point (-2, 3), so 3(-2) − 4(3) + t = 0 or t = 18.

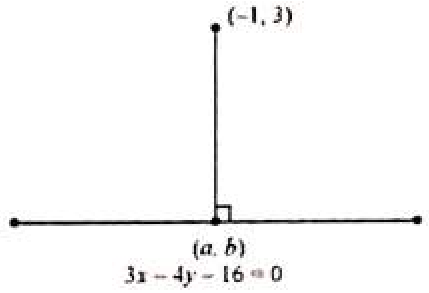
So equation of line is $3x - 4y + 18 = 0$

**Question 10:**

Find the coordinates of the foot of perpendicular from the point (-1, 3) to the line $3x - 4y - 16 = 0$.

**Solution:**

Let (a, b) be the coordinates of the foot of the perpendicular from the point (-1, 3) to the line $3x - 4y - 16 = 0$.




Slope of the line joining (-1, 3) and (a, b)

$m_1 = \frac{b-3}{a+1}$

Slope of the line $3x - 4y - 16 = 0$ is $\frac{3}{4}$

Since these two lines are perpendicular, $m_1m_2 = -1$

$\therefore \left(\frac{b-3}{a+1}\right) \times \left(\frac{3}{4}\right) = -1 \Rightarrow 4a + 3b = 5$ ...(1)

Point (a, b) lies on line $3x - 4y = 16.$

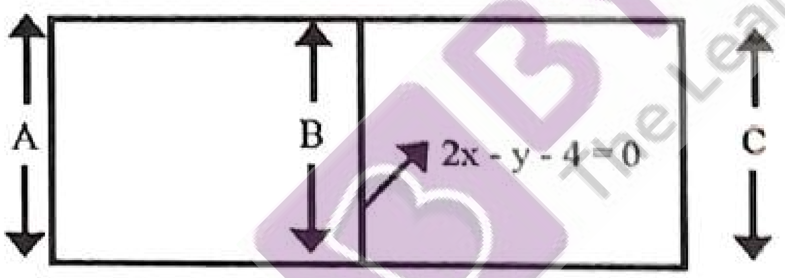
$\therefore\ 3a - 4b = 16$ ...(2)

On solving equations (1) and (2), we obtain

$a = \frac{68}{25}$ and $b = -\frac{49}{25}$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25}, \frac{49}{25}\right)$

**Question 11:**




Three lines

$x + 2y + 3 = 0, x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form 3 sides of two squares. Find the equations of remaining sides of these squares.

**Solution:**

Distance between the two parallel lines is $\frac{|7+3|}{\sqrt{5}} = 2\sqrt{5}$. The equations of the sides forming the square are of the form $2x - y + k = 0.$

Since the distance between sides A and B = distance between sides B and C,
$\frac{|k-(-4)|}{\sqrt{5}} = 2\sqrt{5} \Rightarrow \frac{k+4}{\sqrt{5}} = \pm 2\sqrt{5} \Rightarrow k = 6, -14.$

Hence the fourth side of the two squares is

(i) $2x - y + 6 = 0$, or (ii) $2x - y - 14 = 0.$

**Question 12:**

For the straight lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the

(i) bisector of the obtuse angle between them,

(ii) bisector of the acute angle between them,

(iii) bisector of the angle which contains (1, 2).

**Solution:**

Equations of bisectors of the angles between the given lines are

$\frac{4x+3y-6}{\sqrt{4^2+3^2}} = \pm \frac{5x+12y+9}{\sqrt{5^2+12^2}} \Rightarrow 9x-7y-41=0$ and $7x+9y-3=0.$

If θ is the angle between the line $4x+3y-6=0$ and the bisector $9x-7y-41=0$, then

$\tan\theta = \left|\frac{-\frac{4}{3}-\frac{9}{7}}{1+\left(\frac{-4}{3}\right)\frac{9}{7}}\right| = \frac{11}{3} > 1.$

Hence

(i) The bisector of the obtuse angle is $9x-7y-41=0.$

(ii) The bisector of the acute angle is $7x+9y-3=0.$

(iii) For the point (1, 2)

$4x+3y-6 = 4\times 1+3\times 2-6>0$ and $5x+12y+9 = 5\times 1+12\times 2+9>0.$

Hence equation of the bisector of the angle containing the point (1, 2) is

$\frac{4x+3y-6}{5} = \frac{5x+12y+9}{13} \Rightarrow 9x-7y-41=0.$

**Question 13:**

Find the value of λ if $2x^2+7xy+3y^2+8x+14y+\lambda=0$ will represent a pair of straight lines

**Solution:**

The given equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of lines

If abc + 2fgh − af$^2$ − bg$^2$ − bc$^2$ = 0 i.e., if

$6\lambda + 2(7)(4)\left(\frac{7}{2}\right) - 2(7)^2 - 3(4)^2 - \lambda\left(\frac{7}{2}\right)^2 = 0 \Rightarrow 6\lambda + 196 - 98 - 48 - \frac{49\lambda}{4} = 0 \Rightarrow \frac{49\lambda}{4} - 6\lambda = 196 - 146 = 50$

$\Rightarrow \frac{25\lambda}{4} = 50 \quad \lambda = \frac{200}{25} = 8$

**Question 14:**

If one of the lines of the pair $ax^2+2hxy+by^2=0$ bisects the angle between positive direction of the axes, then find relation for a, b and h.

**Solution:**

Bisector of the angle between the positive directions of the axes is y = x.

Since it is one of the lines of the given pair of lines $ax^2+2hxy+by^2=0,$

We have $x^2(a+2h+b)=0$ or $a+b=-2h.$

**Question 15:**

If the angle between the two lines represented by $2x^2+5xy+3y^2+6x+7y+4=0$ is $\tan^{-1}$(m), then find the value of m.

**Solution:**

The angle between the lines

$2x^2+5xy+3y^2+6x+7y+4=0$ is given by

$\tan\theta = \frac{\pm 2\sqrt{\frac{25}{4}-6}}{2+3} \Rightarrow \theta = \tan^{-1}\left(\pm\frac{1}{5}\right).$

**Question 16:**

The pair of lines $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ are rotated about the origin by

$$\frac{\pi}{6}$$

in the anticlockwise sense. Find the equation of the pair in the new position.

**Solution:**

The given equation of pair of straight lines can be rewritten as $\left(\sqrt{3}x - y\right)\left(x - \sqrt{3}y\right) = 0$. Their separate equations are $y = \sqrt{3}x$ and $y = 1/\sqrt{3}x$ or y = tan 60° x and y = tan 30° x

After rotation, the separate equations are

y = tan 90° x and y = tan 60° x

or x = 0 and $y = \sqrt{3}x$

the combined equation in the new position is $x\left(\sqrt{3}x - y\right) = 0$ or $\sqrt{3}x^2 - xy = 0$