12

PROBABILITY

It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge *-P.D. Laplace.*

- Introduction
- Classical Definition
- Addition Theorem

Pierre de Laplace

(1749-1827)
France

Laplace remembered as one of the greatest scientists of all time, sometimes referred to as a French Newton.

In 1812, Laplace established many fundamental results in statistics. He put forth a mathematical system of inductive reasoning based on probability. He only introduced the principles of probability, one among them is "probability is the ratio of the favoured events to the total possible events".

12.1 Introduction

In every day life, almost everything that we see or do is subject to chance. The occurrences of events like Earthquakes, Cyclones, Tsunami, Lightning, Epidemics, etc... are unpredictable. Most of the events occur quite unexpectedly and result in heavy loss to humanity. If we predict the occurrences of such events well in advance based on their past occurrences with a good amount of accuracy, one can think of preventive measures or damage control exercises much to the relief of human society. Such predictions well in advance of their actual happenings require the study of Probability theory.

A gambler's dispute-problem posed by Chevalier de Mere in 1654 led to exchange of letters between two famous French Mathematicians Blasie Pascal and Pierre de Fermat which created a mathematical theory of Probability. The family of major contributors to the development of Probability theory includes mathematicians like Christian Huggens (1629-1695), Bernoulli (1654-1705), De-Moivre (1667-1754), Pierre de Laplace (1749-1827), Gauss (1777-1855), Poisson (1781-1845), Chebyshev (1821-1894), Markov (1856-1922). In 1933, a Russian Mathematician A. Kolmogorov introduced an axiomatic approach which is considered as the basis for Modern Probability theory.

Probabilities always pertain to the occurrence or nonoccurrence of events. Let us define the terms random experiment, trial, sample space and different types of events used in the study of probability.

Mathematicians use the words "experiment" and "outcome" in a very wide sense. Any process of observation or measurement is called an experiment. Noting down whether a newborn baby is male or female, tossing a coin, picking up a ball from a bag containing balls of different colours and observing the number of accidents at a particular place in a day are some examples of experiments.

A random experiment is one in which the exact outcome cannot be predicted before conducting the experiment. However, one can list out all possible outcomes of the experiment.

The set of all possible outcomes of a random experiment is called its sample space and it is denoted by the letter S. Each repetition of the experiment is called a trial.

A subset of the sample space S is called an event.

Let A be a subset of S. If the experiment, when conducted, results in an outcome that belongs to A, then we say that the event A has occurred.

Let us illustrate random experiment, sample space, events with the help of some examples.

Random Experiment	Sample Space	Some Events
Tossing an unbiased coin once	$S = \{H, T\}$	The occurrence of head, $\{H\}$ is an event. The occurrence of tail, $\{T\}$ is another event.
Tossing an unbiased coin twice	$S = \{HT, HH, TT, TH\}$	$\{HT, HH\}$ and $\{TT\}$ are some of the events
Rolling an unbiased die once	$S = \{1, 2, 3, 4, 5, 6\}$	$\{1,3,5\}, \{2,4,6\}, \{3\}$ and $\{6\}$ are some of the events

Equally likely events

Two or more events are said to be equally likely if each one of them has an equal chance of occurrence.

In tossing a coin, the occurrence of Head and the occurrence of Tail are equally likely events.

Mutually exclusive events

Two or more events are said to be mutually exclusive if the occurrence of one event prevents the occurrence of other events. That is, mutually exclusive events can't occur simultaneously. Thus, if A and B are two mutually exclusive events, then $A \cap B = \phi$.

Fig. 12.1

In tossing a coin, the occurrence of head excludes the occurrence of tail. Similarly if an unbiased die is rolled, the six possible outcomes are mutually exclusive, since two or more faces cannot turn up simultaneously.

Complementary events

Let E be an event of a random experiment and S be its sample space. The set containing all the other outcomes which are not in E but in the sample space is called the complimentary event of E. It is denoted by $\overline{E}$. Thus, $\overline{E} = S - E$. Note that E and $\overline{E}$ are mutually exclusive events.

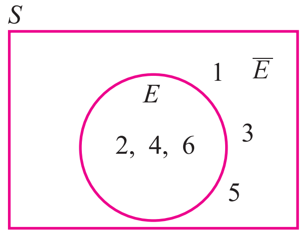

Fig. 12.2

In throwing a die, let $E = \{2, 4, 6\}$ be an event of getting a multiple of 2.

Then the complementary of the event E is given by $\overline{E} = \{1, 3, 5\}$.(see Figure 12.2)

Exhaustive events

Events $E_1, E_2, \cdots, E_n$ are exhaustive events if their union is the sample space S.

Sure event

The sample space of a random experiment is called sure or certain event as any one of its elements will surely occur in any trail of the experiment.

For example, getting one of 1, 2, 3, 4, 5, 6 in rolling a die is a sure event.

Impossible event

An event which will not occur on any account is called an impossible event.

It is denoted by ϕ.

For example, getting 7 in rolling a die once is an impossible event.

Favourable outcomes

The outcomes corresponding to the occurrence of the desired event are called favourable outcomes of the event.

For example, if E is an event of getting an odd number in rolling a die, then the outcomes 1, 3, 5 are favourable to the event E.

> *Note*
>
> In this chapter, we consider only random experiments all of whose outcomes are equally likely and sample spaces are finite. Thus, whenever we refer coins or dice, they are assumed to be unbiased.

12.2 Classical definition of probability

If a sample space contains n outcomes and if m of them are favourable to an event A, then, we write $n(S) = n$ and $n(A) = m$. The Probability of the event A, denoted by $P(A)$, is defined as the ratio of m to n.

That is $P(A) = \dfrac{\text{number of outcomes favourable to } A}{\text{total number of outcomes}}$.

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{m}{n}.$$

Note

(i) The above classical definition of probability is not applicable if the number of possible outcomes is infinite and the outcomes are not equally likely.

(ii) The probability of an event A lies between 0 and 1, both inclusive;

That is $0 \le P(A) \le 1$.

(iii) The probability of the sure event is 1. That is $P(S) = 1$.

(iv) The probability of an impossible event is 0. That is $P(\phi) = 0$.

(v) The probability that the event A will not occur is given by

$$P(\text{not } A) = P(\overline{A}) \text{ or } P(A') = \frac{n-m}{n} = \frac{n}{n} - \frac{m}{n}$$

$$\Longrightarrow \quad P(\overline{A}) = 1 - \frac{m}{n} = 1 - P(A).$$

(vi) $P(A) + P(\overline{A}) = 1$.

Example 12.1

A fair die is rolled. Find the probability of getting

(i) the number 4 (ii) an even number

(iii) a prime factor of 6 (iv) a number greater than 4.

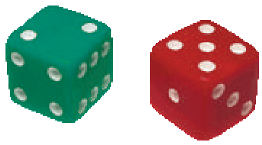

Fig. 12.3

Solution In rolling a die, the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

$\therefore n(S) = 6$.

(i) Let A be the event of getting 4.

$A = \{4\} \; \therefore n(A) = 1$.

$\therefore P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{6}$.

(ii) Let B be the event of getting an even number.

$B = \{2, 4, 6\} \quad \therefore \; n(B) = 3$.

Hence $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$.

(iii) Let C be the event of getting a prime factor of 6.

Then $C = \{2, 3\}$ $\therefore$ $n(C) = 2$.

Hence $P(C) = \frac{n(C)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

(iv) Let D be the event of getting a number greater than 4.

$D = \{5, 6\}$ $n(D) = 2$.

Hence, $P(D) = \frac{n(D)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

Example 12.2

In tossing a fair coin twice, find the probability of getting

(i) two heads (ii) atleast one head (iii) exactly one tail

Solution In tossing a coin twice, the sample space

$$S = \{HH, HT, TH, TT\}$$

$\therefore$ $n(S) = 4$.

(i) Let A be the event of getting two heads. Then $A = \{ HH \}$.

Thus, $n(A) = 1$.

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$.

(ii) Let B be the event of getting at least one head. Then $B = \{ HH, HT, TH \}$

Thus, $n(B) = 3$.

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$.

(iii) Let C be the event of getting exactly one tail. Then $C = \{ HT, TH \}$

Thus, $n(C) = 2$.

$\therefore$ $P(C) = \frac{n(C)}{n(S)} = \frac{2}{4} = \frac{1}{2}$.

Example 12.3

An integer is chosen from the first twenty natural numbers. What is the probability that it is a prime number?

Solution Here $S = \{ 1, 2, 3, \cdots, 20 \}$

$\therefore$ $n(S) = 20$.

Let A be the event of choosing a prime number.

Then, $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$.

$n(A) = 8$.

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$.

Example 12.4

There are 7 defective items in a sample of 35 items. Find the probability that an item chosen at random is non-defective.

Solution Total number of items $n(S) = 35$.

Number of defective items = 7.

Let A be the event of choosing a non-defective item.

Number of non-defective items, $n(A) = 35 - 7 = 28$.

$\therefore$ Probability that the chosen item is non-defective,

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{35} = \frac{4}{5}.$$

Example 12.5

Two unbiased dice are rolled once. Find the probability of getting

(i) a sum 8 (ii) a doublet (iii) a sum greater than 8.

Solution When two dice are thrown, the sample space is

$$\begin{aligned} S = \{ &(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),\\ &(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),\\ &(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),\\ &(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),\\ &(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),\\ &(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \} \end{aligned}$$

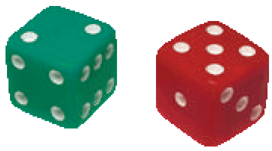

Fig. 12.4

$\therefore n(S) = 6 \times 6 = 36.$

(i) Let A be the event of getting a sum 8.

$\therefore \quad A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}.$

Then $n(A) = 5.$

Hence, $P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}.$

(ii) Let B be the event of getting a doublet

$\therefore \quad B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}.$

Thus, $n(B) = 6.$

$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$

(iii) Let C be the event of getting a sum greater than 8.

Then, $C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}.$

Thus, $n(C) = 10.$

$\therefore \quad P(C) = \frac{n(C)}{n(S)} = \frac{10}{36} = \frac{5}{18}.$

Example 12.6

From a well shuffled pack of 52 playing cards, one card is drawn at random. Find the probability of getting

(i) a king (ii) a black king

(iii) a spade card (iv) a diamond 10.

The 52 playing cards are classified as

Spade ♠	Hearts ♥	Clavor ♣	Diamond ♦
A	A	A	A
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
J	J	J	J
Q	Q	Q	Q
K	K	K	K
13	13	13	13

Solution Now, $n(S) = 52$.

(i) Let A be the event of drawing a king card

$$\therefore n(A) = 4.$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Let B be the event of drawing a black king card

Thus, $n(B) = 2$.

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

(iii) Let C be the event of drawing a spade card

Thus, $n(C) = 13$.

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{13}{52} = \frac{1}{4}.$$

(iv) Let D be the event of drawing a diamond 10 card.

Thus, $n(D) = 1$.

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{52}.$$

Example 12.7

There are 20 boys and 15 girls in a class of 35 students . A student is chosen at random. Find the probability that the chosen student is a (i) boy (ii) girl.

Solution Let S be the sample space of the experiment.

Let B and G be the events of selecting a boy and a girl respectively.

$$\therefore\ n(S) = 35,\ n(B) = 20 \text{ and } n(G) = 15.$$

(i) Probability of choosing a boy is $P(B) = \frac{n(B)}{n(S)} = \frac{20}{35}$

$$\Longrightarrow P(B) = \frac{4}{7}.$$

(ii) Probability of choosing a girl is $P(G) = \frac{n(G)}{n(S)} = \frac{15}{35}$

$$\Longrightarrow P(G) = \frac{3}{7}.$$

Example 12.8

The probability that it will rain on a particular day is 0.76. What is the probability that it will not rain on that day?

Solution Let A be the event that it will rain. Then $\overline{A}$ is the event that it will not rain.

Given that $P(A) = 0.76.$

Thus, $P(\overline{A}) = 1 - 0.76 \qquad \because P(A) + P(\overline{A}) = 1$

$= 0.24.$

$\therefore$ The probability that it will not rain is 0.24.

Example 12.9

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of drawing a red ball, then find the number of blue balls in the bag.

Solution Let the number of blue balls be x.

$\therefore$ Total number of balls, $n(S) = 5 + x$.

Let B be the event of drawing a blue ball and R be the event of drawing a red ball.

Given $P(B) = 3P(R)$

$$\Longrightarrow \quad \frac{n(B)}{n(S)} = 3\frac{n(R)}{n(S)}$$

$$\Longrightarrow \quad \frac{x}{5+x} = 3\left(\frac{5}{5+x}\right)$$

$$\Longrightarrow \quad x = 15$$

Thus, number of blue balls $= 15$.

Example 12.10

Find the probability that

(i) a leap year selected at random will have 53 Fridays

(ii) a leap year selected at random will have only 52 Fridays

(iii) a non-leap year selected at random will have 53 Fridays.

Solution (i) Number of days in a leap year = 366 days. i.e., 52 weeks and 2 days.

Now 52 weeks contain 52 Fridays and the remaining two days will be one of the following seven possibilities.

(Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thur), (Thur, Fri), (Fri, Sat) and (Sat, Sun).

The probability of getting 53 Fridays in a leap year is same as the probability of getting a Friday in the above seven possibilities.

Here $S = \{(\text{Sun},\text{Mon}),(\text{Mon},\text{Tue}),(\text{Tue},\text{Wed}),(\text{Wed},\text{Thur}),(\text{Thur},\text{Fri}),(\text{Fri},\text{Sat}),(\text{Sat},\text{Sun})\}$.

Then $n(S) = 7$.

Let A be the event of getting one Friday in the remaining two days.

$$A = \{(Thur, Fri),(Fri, Sat)\} \qquad \text{Then } n(A) = 2.$$

$$p(A) = \frac{n(A)}{n(S)} = \frac{2}{7}.$$

(ii) To get only 52 Fridays in a leap year, there must be no Friday in the remaining two days.

Let B be the event of not getting a Friday in the remaining two days. Then

$$B = \{(\text{Sun},\text{Mon}),(\text{Mon},\text{Tue}),(\text{Tue},\text{Wed}),(\text{Wed},\text{Thur}),(\text{Sat},\text{Sun})\}.$$

$$n(B) = 5.$$

Now, $P(B) = \frac{n(B)}{n(S)} = \frac{5}{7}.$

Note that A and B are complementary events.

(iii) Number of days in a non leap year = 365 days. i.e., 52 weeks and 1 day.

To get 53 Fridays in a non leap year, there must be a Friday in the seven possibilities: Sun, Mon, Tue, Wed, Thur, Fri and Sat.

Here $S = \{\text{Sun, Mon, Tue, Wed, Thur, Fri and Sat}\}$.

$\therefore \quad n(S) = 7.$

Let C be the event of getting a Friday in the remaining one day . Then

$$C = \{\text{Fri}\} \implies n(C) = 1.$$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{1}{7}.$$

Example 12.11

If A is an event of a random experiment such that $P(A):P(\overline{A}) = 7:12$, then find $P(A)$.

Solution Given that $P(A):P(\overline{A}) = 7:12$.

Let $P(A) = 7k$ and $P(\overline{A}) = 12k.,\ k > 0$

We know that $P(A) + P(\overline{A}) = 1$.

Then, $7k + 12k = 1 \implies 19k = 1.$

Thus, $k = \frac{1}{19}$

$\therefore \quad P(A) = 7k = \frac{7}{19}.$

Aliter

$$\frac{P(A)}{P(\overline{A})} = \frac{7}{12}$$

$$12P(A) = 7\times P(\overline{A})$$

$$= 7\,[1-P(A)]$$

$$19P(A) = 7$$

Thus, $P(A) = \frac{7}{19}$

Exercise 12. 1

1. A ticket is drawn from a bag containing 100 tickets. The tickets are numbered from one to hundred. What is the probability of getting a ticket with a number divisible by 10?

2. A die is thrown twice. Find the probability of getting a total of 9.

3. Two dice are thrown together. Find the probability that the two digit number formed with the two numbers turning up is divisible by 3.

4. Three rotten eggs are mixed with 12 good ones. One egg is chosen at random. What is the probability of choosing a rotten egg?

5. Two coins are tossed together. What is the probability of getting at most one head.

6. One card is drawn randomly from a well shuffled deck of 52 playing cards. Find the probability that the drawn card is

   (i) a Diamond (ii) not a Diamond (iii) not an Ace.

7. Three coins are tossed simultaneously. Find the probability of getting

   (i) at least one head (ii) exactly two tails (iii) at least two heads.

8. A bag contains 6 white balls numbered from 1 to 6 and 4 red balls numbered from 7 to 10. A ball is drawn at random. Find the probability of getting

   (i) an even-numbered ball (ii) a white ball.

9. A number is selected at random from integers 1 to 100. Find the probability that it is

   (i) a perfect square (ii) not a perfect cube.

10. For a sightseeing trip, a tourist selects a country randomly from Argentina, Bangladesh, China, Angola, Russia and Algeria. What is the probability that the name of the selected country will begin with A ?

11. A box contains 4 Green, 5 Blue and 3 Red balls. A ball is drawn at random. Find the probability that the selected ball is (i) Red in colour (ii) not Green in colour.

12. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the card is

    (i) a multiple of 4 (ii) not a multiple of 6.

13. A two digit number is formed with the digits 3, 5 and 7. Find the probability that the number so formed is greater than 57 (repetition of digits is not allowed).

14. Three dice are thrown simultaneously. Find the probability of getting the same number on all the three dice.

15. Two dice are rolled and the product of the outcomes (numbers) are found. What is the probability that the product so found is a prime number?

16. A jar contains 54 marbles each of which is in one of the colours blue, green and white. The probability of drawing a blue marble is $\frac{1}{3}$ and the probability of drawing a green marble is $\frac{4}{9}$. How many white marbles does the jar contain?

17. A bag consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. A trader A will accept only the shirt which are good, but the trader B will not accept the shirts which have major defects. One shirt is drawn at random. What is the probability that it is acceptable by (i) A (ii) B ?

18. A bag contains 12 balls out of which x balls are white. (i) If one ball is drawn at random, what is the probability that it will be a white ball. (ii) If 6 more white balls are put in the bag and if the probability of drawing a white ball will be twice that of in (i), then find x.

19. Piggy bank contains 100 fifty-paise coins, 50 one-rupee coins, 20 two-rupees coins and 10 five- rupees coins. One coin is drawn at random. Find the probability that the drawn coin (i) will be a fifty-paise coin (ii) will not be a five-rupees coin.

12.3 Addition theorem on probability

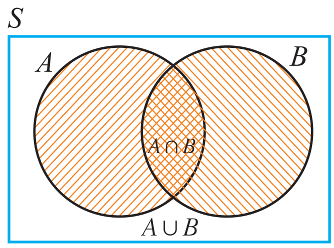

Fig. 12.5

Let A and B be subsets of a finite non-empty set S. Then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Divide both sides by $n(S)$,we get

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \quad (1)$$

If the subsets A and B correspond to two events A and B of a random experiment and if the set S corresponds to the sample space S of the experiment, then (1) becomes

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

This result is known as the addition theorem on probability.

Note

(i) The event $A \cup B$ is said to occur if the event A occurs or the event B occurs or both A and B occur simultaneously. The event $A \cap B$ is said to occur if both the events A and B occur simultaneously.

(ii) If A and B are mutually exclusive events, then $A \cap B = \emptyset$.

Thus, $P(A \cup B) = P(A) + P(B)$ $\quad \because P(A \cap B) = 0$.

(iii) $A \cap \overline{B}$ is same as $A \setminus B$ in the language of set theory.

Results (without proof)

(i) If A, B and C are any 3 events associated with a sample space S, then

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C).$$

(ii) If A_1, A_2 and A_3 are three mutually exclusive events, then

$$P(A_1\cup A_2\cup A_3) = P(A_1)+P(A_2)+P(A_3).$$

(iii) If $A_1, A_2, A_3, \cdots, A_n$ are mutually exclusive events, then

$$P(A_1\cup A_2\cup A_3\cup\cdots\cup A_n) = P(A_1)+P(A_2)+P(A_3)+\cdots+P(A_n).$$

(iv) $P(A\cap\overline{B}) = P(A)-P(A\cap B)$,

$P(\overline{A}\cap B) = P(B)-P(A\cap B)$

where $A\cap\overline{B}$ mean only A and not B;

Similarly $\overline{A}\cap B$ means only B and not A.

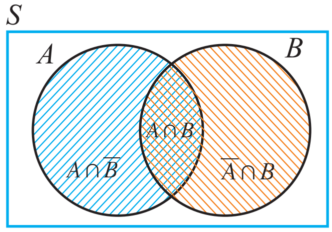

Fig. 12.6

Example 12.12

Three coins are tossed simultaneously. Using addition theorem on probability, find the probability that either exactly two tails or at least one head turn up.

Solution Now the sample space $S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$.

Hence, $n(S) = 8$.

Let A be the event of getting exactly two tails.

Thus, $A = \{HTT, TTH, THT\}$ and hence $n(A) = 3$.

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}.$$

Let B be the event of getting at least one head.

Thus, $B = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$ and hence $n(B) = 7$.

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}.$$

Now, the events A and B are not mutually exclusive.

Since $A\cap B = A$, $P(A\cap B) = P(A) = \frac{3}{8}$.

$\therefore\ P(A \text{ or } B) = P(A)+P(B)-P(A\cap B)$

Thus $P(A\cup B) = \frac{3}{8}+\frac{7}{8}-\frac{3}{8} = \frac{7}{8}$.

Note

In the above problem, we applied addition theorem on probability.

However, one can notice that $A\cup B = B$. Thus, $P(A\cup B) = P(B) = \frac{7}{8}$.

Example 12.13

A die is thrown twice. Find the probability that at least one of the two throws comes up with the number 5 (use addition theorem).

Solution In rolling a die twice, the size of the sample space , $n(S) = 36$.

Let A be the event of getting 5 in the first throw.

$\therefore \quad A = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}.$

Thus, $\quad n(A) = 6$, and $\quad P(A) = \frac{6}{36}.$

Let B be the event of getting 5 in the second throw.

$\therefore \quad B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}.$

Thus, $\quad n(B) = 6$ and $\quad P(B) = \frac{6}{36}.$

A and B are not mutually exclusive events , since $A \cap B = \{(5,5)\}.$

$\therefore \quad n(A \cap B) = 1$ and $P(A \cap B) = \frac{1}{36}.$

$\therefore$ By addition theorem,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}.$$

Example 12.14

The probability that a girl will be selected for admission in a medical college is 0.16. The probability that she will be selected for admission in an engineering college is 0.24 and the probability that she will be selected in both, is 0.11

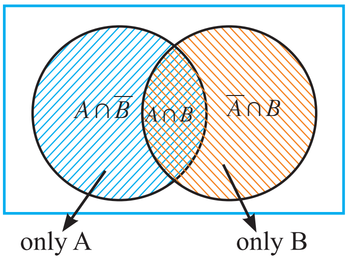

Fig. 12.7

(i) Find the probability that she will be selected in at least one of the two colleges.

(ii) Find the probability that she will be selected either in a medical college only or in an engineering college only.

Solution Let A be the event of getting selected in a medical college and B be the event of getting selected for admission in an engineering college.

(i) $P(A) = 0.16$, $P(B) = 0.24$ and $P(A \cap B) = 0.11$

P (she will be selected for admission in at least one of the two colleges) is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.16 + 0.24 - 0.11 = 0.29$$

(ii) P(she will be selected for admission in only one of the two colleges)

$$= P\ (\text{only } A \text{ or only } B)$$
$$= P(A\cap \overline{B}) + P(\overline{A}\cap B)$$
$$= [P(A) - P(A\cap B)] + [P(B) - P(A\cap B)]$$
$$= (0.16 - 0.11) + (0.24 - 0.11) = 0.18.$$

Example 12.15

A letter is chosen at random from the letters of the word "ENTERTAINMENT". Find the probability that the chosen letter is a vowel or T. (repetition of letters is allowed)

Solution There are 13 letters in the word ENTERTAINMENT.

$\therefore \quad n(S) = 13.$

Let A be the event of getting a vowel.

$\therefore \quad n(A) = 5.$

Hence, $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5}{13}.$

Let B be the event of getting the letter T.

$\therefore \quad n(B) = 3$

Hence, $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{13}$. Then

$P(A \text{ or } B) = P(A) + P(B) \qquad \because A$ and B are mutually exclusive events

$$= \frac{5}{13} + \frac{3}{13} = \frac{8}{13}.$$

Example 12.16

Let A, B, C be any three mutually exclusive and exhaustive events such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$. Find $P(A)$.

Solution

Let $\quad P(A) = p.$

Now, $P(B) = \frac{3}{2}P(A) = \frac{3}{2}p.$

Also, $P(C) = \frac{1}{2}P(B) = \frac{1}{2}\left(\frac{3}{2}p\right) = \frac{3}{4}p.$

Given that A, B and C are mutually exclusive and exhaustive events.

$\therefore\ P(A\cup B\cup C) = P(A) + P(B) + P(C)$ and $S = A\cup B\cup C$.

Now, $\quad P(S) = 1.$

That is, $P(A)+P(B)+P(C)=1$

$\Longrightarrow \quad p+\frac{3}{2}p+\frac{3}{4}p=1$

$\Longrightarrow \quad 4p+6p+3p=4$

Thus, $p=\frac{4}{13}$

Hence, $P(A)=\frac{4}{13}$.

Example 12.17

A card is drawn from a deck of 52 cards. Find the probability of getting a King or a Heart or a Red card.

Solution Let A, B and C be the events of getting a King, a Heart and a Red card respectively.

Now, $n(S) = 52$, $n(A) = 4$, $n(B) = 13$, $n(C) = 26$. Also,

$n(A\cap B) = 1$, $n(B\cap C) = 13$, $n(C\cap A) = 2$ and $n(A\cap B\cap C) = 1$.

$\therefore\ P(A) = \frac{4}{52},\ P(B) = \frac{13}{52},\ P(C) = \frac{26}{52}$.

$P(A\cap B) = \frac{1}{52},\ P(B\cap C) = \frac{13}{52},\ P(C\cap A) = \frac{2}{52}$ and $P(A\cap B\cap C) = \frac{1}{52}$.

Now $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$$= \frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52} = \frac{44-16}{52}$$

$$= \frac{7}{13}.$$

Example 12.18

A bag contains 10 white, 5 black, 3 green and 2 red balls. One ball is drawn at random. Find the probability that the ball drawn is white or black or green.

Solution Let S be the sample space.

$\therefore\quad n(S) = 20.$

Let W, B and G be the events of selecting a white, black and green ball respectively.

Probability of getting a white ball, $P(W) = \frac{n(W)}{n(S)} = \frac{10}{20}$.

Probability of getting a black ball, $P(B) = \frac{n(B)}{n(S)} = \frac{5}{20}$.

Probability of getting a green ball, $P(G) = \frac{n(G)}{n(S)} = \frac{3}{20}$.

$\therefore$ Probability of getting a white or black or green ball,

$P(W\cup B\cup G) = P(W)+P(B)+P(G)$ $\quad\because$ W, B and G are mutually exclusive.

$$= \frac{10}{20}+\frac{5}{20}+\frac{3}{20} = \frac{9}{10}.$$

(Note : $P(W\cup B\cup G) = P(R') = 1-P(R) = 1-\frac{2}{20} = \frac{9}{10}$)

Exercise 12.2

1. If A and B are mutually exclusive events such that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, then find $P(A \cup B)$.
2. If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$,then find $P(A \cap B)$.
3. If $P(A) = \frac{1}{2}, P(B) = \frac{7}{10}, P(A \cup B) = 1$. Find (i) $P(A \cap B)$ (ii) $P(A' \cup B')$.
4. If a die is rolled twice, find the probability of getting an even number in the first time or a total of 8 .
5. One number is chosen randomly from the integers 1 to 50. Find the probability that it is divisible by 4 or 6.
6. A bag contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If an item is chosen at random, find the probability that it is rusted or that it is a bolt.
7. Two dice are rolled simultaneously. Find the probability that the sum of the numbers on the faces is neither divisible by 3 nor by 4.
8. A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are rotten. If a person takes out one fruit at random, find the probability that the fruit is either an apple or a good fruit.
9. In a class, 40% of the students participated in Mathematics-quiz, 30% in Science-quiz and 10% in both the quiz programmes. If a student is selected at random from the class, find the probability that the student participated in Mathematics or Science or both quiz programmes.
10. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that it will be a spade or a king.
11. A box contains 10 white, 6 red and 10 black balls. A ball is drawn at random. Find the probability that the ball drawn is white or red.
12. A two digit number is formed with the digits 2, 5, 9 (repetition is allowed). Find the probability that the number is divisible by 2 or 5.
13. Each individual letter of the word "ACCOMMODATION" is written in a piece of paper, and all 13 pieces of papers are placed in a jar. If one piece of paper is selected at random from the jar, find the probability that
    (i) the letter '*A*' or '*O*' is selected.
    (ii) the letter '*M*' or '*C*' is selected.

14. The probability that a new car will get an award for its design is 0.25, the probability that it will get an award for efficient use of fuel is 0.35 and the probability that it will get both the awards is 0.15. Find the probability that

(i) it will get atleast one of the two awards

(ii) it will get only one of the awards.

15. The probability that A, B and C can solve a problem are $\frac{4}{5}, \frac{2}{3}$ and $\frac{3}{7}$ respectively. The probability of the problem being solved by A and B is $\frac{8}{15}$, B and C is $\frac{2}{7}$, A and C is $\frac{12}{35}$. The probability of the problem being solved by all the three is $\frac{8}{35}$. Find the probability that the problem can be solved by atleast one of them.

Exercise 12.3

Choose the correct answer

1. If ϕ is an impossible event, then $P(\phi) =$

(A) 1 (B) $\frac{1}{4}$ (C) 0 (D) $\frac{1}{2}$

2. If S is the sample space of a random experiment, then $P(S) =$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1

3. If p is the probability of an event A, then p satisfies

(A) $0 < p < 1$ (B) $0 \le p \le 1$ (C) $0 \le p < 1$ (D) $0 < p \le 1$

4. Let A and B be any two events and S be the corresponding sample space. Then $P(\overline{A} \cap B) =$

(A) $P(B) - P(A \cap B)$ (B) $P(A \cap B) - P(B)$

(C) $P(S)$ (D) $P[(A \cup B)']$

5. The probability that a student will score centum in mathematics is $\frac{4}{5}$. The probability that he will not score centum is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

6. If A and B are two events such that $P(A) = 0.25, P(B) = 0.05$ and $P(A \cap B) = 0.14$, then $P(A \cup B) =$

(A) 0.61 (B) 0.16 (C) 0.14 (D) 0.6

7. There are 6 defective items in a sample of 20 items. One item is drawn at random. The probability that it is a non-defective item is

(A) $\frac{7}{10}$ (B) 0 (C) $\frac{3}{10}$ (D) $\frac{2}{3}$

8. If A and B are mutually exclusive events and S is the sample space such that $P(A) = \frac{1}{3}P(B)$ and $S = A \cup B$, then $P(A) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{8}$

9. The probabilities of three mutually exclusive events A, B and C are given by $\frac{1}{3}, \frac{1}{4}$, and $\frac{5}{12}$. Then $P(A \cup B \cup C)$ is

(A) $\frac{19}{12}$ (B) $\frac{11}{12}$ (C) $\frac{7}{12}$ (D) 1

10. If $P(A) = 0.25, P(B) = 0.50, P(A \cap B) = 0.14$ then $P(\text{neither } A \text{ nor } B) =$

(A) 0.39 (B) 0.25 (C) 0.11 (D) 0.24

11. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected at random, the probability that it is not red is

(A) $\frac{5}{12}$ (B) $\frac{4}{12}$ (C) $\frac{3}{12}$ (D) $\frac{3}{4}$

12. Two dice are thrown simultaneously. The probability of getting a doublet is

(A) $\frac{1}{36}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$

13. A fair die is thrown once. The probability of getting a prime or composite number is

(A) 1 (B) 0 (C) $\frac{5}{6}$ (D) $\frac{1}{6}$

14. Probability of getting 3 heads or 3 tails in tossing a coin 3 times is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$

15. A card is drawn from a pack of 52 cards at random. The probability of getting neither an ace nor a king card is

(A) $\frac{2}{13}$ (B) $\frac{11}{13}$ (C) $\frac{4}{13}$ (D) $\frac{8}{13}$

16. The probability that a leap year will have 53 Fridays or 53 Saturdays is

(A) $\frac{2}{7}$ (B) $\frac{1}{7}$ (C) $\frac{4}{7}$ (D) $\frac{3}{7}$

17. The probability that a non-leap year will have 53 Sundays and 53 Mondays is

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) 0

18. The probability of selecting a queen of hearts when a card is drawn from a pack of 52 playing cards is

(A) $\frac{1}{52}$ (B) $\frac{16}{52}$ (C) $\frac{1}{13}$ (D) $\frac{1}{26}$

19. Probability of sure event is

(A) 1 (B) 0 (C) 100 (D) 0.1

20. The outcome of a random experiment results in either success or failure. If the probability of success is twice the probability of failure, then the probability of success is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) 0

Answers

1. SETS AND FUNCTIONS

Exercise 1.1

2. (i) A (ii) ϕ **3.** (i) {b, c} (ii) ϕ (iii) $\{a, e, f, s\}$

4. (i) {2, 4, 6, 7, 8, 9} (ii) {4, 6} (iii) {4, 6, 7, 8, 9}

10. {–5, –3, –2}, {–5, –3}, not associative

Exercise 1.2

2. Different answers are possible for (i) to (iv). One such answer is :

(i) $A' \cup (A \cap B)$ or $(A \backslash B)'$ (ii) $(A \cap B) \cup (A \cap C)$ (iii) $A \backslash (B \cup C)$ (iv) $(A \cap B) \backslash C$

5. (i) {12} (ii) {4, 8, 12, 20, 24, 28}

Exercise 1.3

1. 300 **2.** 430 **3.** 35 **5.** 100 **6.** 10%

7. (i) 10 (ii) 25 (iii) 15 **8.** (i) 450 (ii) 3550 (iii) 1850 **9.** 15

Exercise 1.4

1. (i) not a function (ii) function **2.** domain = {1, 2, 3, 4, 5}; range = {1, 3, 5, 7, 9}

3. (i) neither one to one nor onto (ii) constant function (iii) one-one and onto function

4. (i) not a function (ii) one-one function (iii) not a function (iv) bijective

5. $a = -2,\ b = -5,\ c = 8,\ d = -1$ **6.** range is $\left\{-\frac{1}{2}, -1, 1, \frac{1}{2}\right\}$; f is not a function from A to A

7. one-one and onto function **8.** (i) 12 and 14 (ii) 13 and 15 **9.** $a = 9,\ b = 15$

10. (i) $f = \{(5, -7), (6, -9), (7, -11), (8, -13)\}$

(ii) co-domain $= \{-11, 4, 7, -10, -7, -9, -13\}$

(iii) range $= \{-7, -9, -11, -13\}$ (iv) one-one function

11. (i) function (ii) function (iii) not a function (iv) not a function (v) function

12.

x	–1	–3	–5	–4
$f(x)$	2	1	6	3

13. $\{(6,1), (9,2), (15,4), (18,5), (21,6)\}$

x	6	9	15	18	21
$f(x)$	1	2	4	5	6

14. $\{(4,3),(6,4),(8,5),(10,6)\}$

x	4	6	8	10
$f(x)$	3	4	5	6

15. (i) 16 (ii) –32 (iii) 5 (iv) $\frac{2}{3}$ **16.** (i) 23 (ii) 34 (iii) 2

Exercise 1.5

1	2	3	4	5	6	7	8	9	10
A	C	C	A	A	B	A	B	B	B
11	12	13	14	15	16	17	18	19	20
A	B	C	D	A	D	D	B	A	C

2. SEQUENCES AND SERIES OF REAL NUMBERS

Exercise 2.1

1. (i) $-\frac{1}{3}, 0, 1$ (ii) $-27, 81, -243$ (iii) $-\frac{3}{4}, 2, -\frac{15}{4}$

2. (i) $\frac{9}{17}, \frac{11}{21}$ (ii) –1536, 18432 (iii) 36, 78 (iv) –21, 57

3. 378, $\frac{25}{313}$ **4.** 195, 256 **5.** 2, 5, 15, 35, 75 **6.** 1, 1, 1, 2, 3, 5

Exercise 2.2

1. A.P : 6, 11, 16, $\cdots$; the general term is $5n+1$ **2.** common difference is –5, $t_{15} = 55$

3. $t_{29} = 3$ **4.** $t_{12} = 23\sqrt{2}$ **5.** $t_{17} = 84$ **6.** (i) 27 terms (ii) 34 terms

8. $t_{27} = 109$ **9.** $n = 10$ **10.** 7 **11.** First year : 100, $t_{15} = 2200$

12. 2560 **13.** 10, 2, –6 or –6, 2, 10 **14.** 2, 6, 10 or 10, 6, 2 **16.** A.P., ₹95,000

Exercise 2.3

1. (i) G.P. with $r = 2$ (ii) G.P. with $r = 5$ (iii) G.P. with $r = \frac{2}{3}$
(iv) G.P. with $r = \frac{1}{12}$ (v) G.P. with $r = \frac{1}{2}$ (vi) not a G.P.

2. -2^7 **3.** 2, 6, 18 , $\cdots$ **4.** $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \cdots$ **5.** (i) $n = 8$ (ii) $n = 11$ **6.** $n = 5$

7. $r = 5$ **8.** $r = \frac{5}{2}$ or $\frac{2}{5}$; the terms : $\frac{2}{5}, 1, \frac{5}{2}$. (or) $\frac{5}{2}, 1, \frac{2}{5}$. **9.** 18, 6, 2 (or) 2, 6, 18

10. 4, 2, 1 (or) 1, 2, 4 **11.** $1, 3, 9, \cdots$ (or) $9, 3, 1, \cdots$ **12.** ₹$1000\left(\frac{105}{100}\right)^{12}$ **13.** ₹$50,000\left(\frac{85}{100}\right)^{15}$

Exercise 2.4

1. (i) 2850 (ii) 7875 **2.** 1020 **3.** (i) 260 (ii) – 75 **4.** (i) 1890 (ii) 50 5. – 820

6. $\frac{39}{11} + \frac{40}{11} + \frac{41}{11} + \cdots$ **7.** 8 terms or 23 terms **8.** 55350 **9.** 740 **10.** 7227 **11.** 36

12. 12000 **13.** 15 days **14.** A.P., ₹37,200 **16.** 156 times **20.** 1225 bricks

Exercise 2.5

1. $s_{20} = \frac{15}{4}\left[1-\left(\frac{1}{3}\right)^{20}\right]$ **2.** $s_{27} = \frac{1}{6}\left[1-\left(\frac{1}{3}\right)^{27}\right]$ **3.** (i) 765 (ii) $\frac{5}{2}(3^{12}-1)$

4. (i) $\frac{1-(0.1)^{10}}{0.9}$ (ii) $\frac{10}{81}(10^{20}-1)-\frac{20}{9}$ **5.** (i) $n = 6$ (ii) $n = 6$ **6.** $\frac{75}{4}\left[1-\left(\frac{4}{5}\right)^{23}\right]$

7. $3+6+12+\cdots$ **8.** (i) $\frac{70}{81}[10^{n}-1]-\frac{7n}{9}$ (ii) $n-\frac{2}{3}\left[1-\left(\frac{1}{10}\right)^{n}\right]$

9. $s_{15} = \frac{5(4^{15}-1)}{3}$ **10.** 2^{nd} option; number of mangoes 1023. **11.** $r = 2$

Exercise 2.6

1. (i) 1035 (ii) 4285 (iii) 2550 (iv) 17395 (v) 10650 (vi) 382500

2. (i) $k = 12$ (ii) $k = 9$ **3.** 29241 **4.** 91 **5.** 3818 cm^2 **6.** 201825 cm^3

Exercise 2.7

1	2	3	4	5	6	7	8	9	10
B	D	C	D	D	A	B	B	B	B
11	12	13	14	15	16	17	18	19	20
B	A	B	D	A	B	B	A	C	A

3. ALGEBRA

Exercise 3.1

1. $\left(4, \frac{3}{2}\right)$ **2.** (1 , 5) **3.** (3 , 2) **4.** $\left(\frac{1}{3}, \frac{1}{2}\right)$ **5.** (1 , 5)

6. $\left(\frac{11}{23}, \frac{22}{31}\right)$ **7.** (2 , 4) **8.** (2 , 1) **9.** $\left(5, \frac{1}{7}\right)$ **10.** (6 , – 4)

Exercise 3.2

1. (i) (4 , 3) (ii) (0.4, 0.3) (iii) (2 , 3) (iv) $\left(\frac{1}{2}, \frac{1}{3}\right)$

2. (i) 23 , 7 (ii) ₹18,000 , ₹14,000 (iii) 42 (iv) ₹800 (v) 253cm^2 (vi) 720 km

Exercise 3.3

1. (i) $4, -2$ (ii) $\frac{1}{2}, \frac{1}{2}$ (iii) $\frac{3}{2}, -\frac{1}{3}$ (iv) $0, -2$

(v) $\sqrt{15}, -\sqrt{15}$ (vi) $\frac{2}{3}, 1$ (vii) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (viii) $-13, 11$

2. (i) x^2-3x+1 (ii) x^2-2x+4 (iii) x^2+4 (iv) $x^2-\sqrt{2}x+\frac{1}{5}$

(v) $x^2-\frac{x}{3}+1$ (vi) $x^2-\frac{x}{2}-4$ (vii) $x^2-\frac{x}{3}-\frac{1}{3}$ (viii) $x^2-\sqrt{3}x+2$

Exercise 3.4

1. (i) $x^2+2x-1,\ 4$ (ii) $3x^2-11x+40,\ -125$ (iii) $x^2+2x-2,\ 2$

(iv) $x^2-\frac{5}{3}x+\frac{5}{9},\ -\frac{50}{9}$ (v) $2x^3-\frac{x^2}{2}-\frac{3}{8}x+\frac{51}{32}, -\frac{211}{32}$

(vi) $x^3-3x^2-8x+\frac{55}{2},\ -\frac{41}{2}$

2. $a = -6,\ b = 11$, Remainder is 5 **3.** $p = -2,\ q = 0$, Remainder is -10

Exercise 3.5

1. (i) $(x-1)(x+2)(x-3)$ (ii) $(x-1)(2x+3)(2x-1)$ (iii) $(x-1)(x-12)(x-10)$

(iv) $(x-1)(4x^2-x+6)$ (v) $(x-1)(x-2)(x+3)$ (vi) $(x+1)(x+2)(x+10)$

(vii) $(x-2)(x-3)(2x+1)$ (viii) $(x-1)(x^2+x-4)$ (ix) $(x-1)(x+1)(x-10)$

(x) $(x-1)(x+6)(2x+1)$ (xi) $(x-2)(x^2+3x+7)$ (xii) $(x+2)(x-3)(x-4)$

Exercise 3.6

1. (i) $7x^2yz^3$ (ii) x^2y (iii) $5c^3$ (iv) $7xyz^2$

2. (i) $c-d$ (ii) $x-3a$ (iii) $m+3$ (iv) $x+11$ (v) $x+2y$

(vi) $2x+1$ (vii) $x-2$ (viii) $(x-1)(x^2+1)$ (ix) $4x^2(2x+1)$ (x) $(a-1)^3(a+3)^2$

3. (i) x^2-4x+3 (ii) $x+1$ (iii) $2(x^2+1)$ (iv) x^2+4

Exercise 3.7

1. x^3y^2z **2.** $12x^3y^3z$ **3.** $a^2b^2c^2$ **4.** $264a^4b^4c^4$ **5.** a^{m+3}

6. $xy(x+y)$ **7.** $6(a-1)^2(a+1)$ **8.** $10xy(x+3y)(x-3y)(x^2-3xy+9y^2)$

9. $(x+4)^2(x-3)^3(x-1)$ **10.** $420x^3(3x+y)^2(x-2y)(3x+1)$

Exercise 3.8

1. (i) $(x-3)(x-2)(x+6)$ (ii) $(x^2+2x+3)(x^4+2x^2+x+2)$

(iii) $(2x^2+x-5)(x^3+8x^2+4x-21)$ (iv) $(x^3-5x-8)(2x^3-3x^2-9x+5)$

2. (i) $(x+1)(x+2)^2$ (ii) $(3x-7)^3(4x+5)$ (iii) $(x^2-y^2)(x^4+x^2y^2+y^4)$

(iv) $x(x+2)(5x+1)$ (v) $(x-2)(x-1)$ (vi) $2(x+1)(x+2)$

Exercise 3.9

1. (i) $\frac{2x+3}{x-4}$ (ii) $\frac{1}{x^2-1}$ (iii) $(x-1)$ (iv) $\frac{x^2+3x+9}{x+3}$

(v) x^2-x+1 (vi) $\frac{x+2}{x^2+2x+4}$ (vii) $\frac{x-1}{x+1}$ (viii) $(x+3)$

(ix) $\frac{(x-1)}{(x+1)}$ (x) 1 (xi) $\frac{(x+1)}{(2x-1)}$ (xii) $(x-2)$

Exercise 3.10

1. (i) $3x$ (ii) $\frac{x+9}{x-2}$ (iii) $\frac{1}{x+4}$ (iv) $\frac{1}{x-1}$ (v) $\frac{2x+1}{x+2}$ (vi) 1

2. (i) $\frac{x-1}{x}$ (ii) $\frac{x-6}{x-7}$ (iii) $\frac{x+1}{x-5}$ (iv) $\frac{x-5}{x-11}$ (v) 1 (vi) $\frac{3x+1}{4(3x+4)}$ (vii) $\frac{x-1}{x+1}$

Exercise 3.11

1. (i) $x^2 + 2x + 4$ (ii) $\frac{2}{x+1}$ (iii) $\frac{2(x+4)}{x+3}$ (iv) $\frac{2}{x-5}$

(v) $\frac{x+1}{x-2}$ (vi) $\frac{4}{x+4}$ (vii) $\frac{2}{x+1}$ (viii) 0

2. $\frac{2x^3+2x^2+5}{x^2+2}$ **3.** $\frac{5x^2-7x+6}{2x-1}$ **4.** 1

Exercise 3.12

1. (i) $14|a^3b^4c^5|$ (ii) $17|(a-b)^2(b-c)^3|$ (iii) $|x-11|$

(iv) $|x+y|$ (v) $\frac{11}{9}\left|\frac{x^2}{y}\right|$ (vi) $\frac{8}{5}\left|\frac{(a+b)^2(x-y)^4(b-c)^3}{(x+y)^2(a-b)^3(b+c)^5}\right|$

2. (i) $|4x-3|$ (ii) $|(x+5)(x-5)(x+3)|$ (iii) $|2x-3y-5z|$

(iv) $\left|x^2+\frac{1}{x^2}\right|$ (v) $|(2x+3)(3x-2)(2x+1)|$ (vi) $|(2x-1)(x-2)(3x+1)|$

Exercise 3.13

1. (i) $|x^2-2x+3|$ (ii) $|2x^2+2x+1|$ (iii) $|3x^2-x+1|$ (iv) $|4x^2-3x+2|$

2. (i) $a=-42, b=49$ (ii) $a=12, b=9$ (iii) $a=49, b=-70$ (iv) $a=9,\ b=-12$

Exercise 3.14

1. $\{-6,3\}$ **2.** $\left\{-\frac{4}{3},3\right\}$ **3.** $\left\{-\sqrt{5},\frac{3}{\sqrt{5}}\right\}$ **4.** $\left\{-\frac{3}{2},5\right\}$ **5.** $\left\{-\frac{4}{3},2\right\}$

6. $\left\{5,\frac{1}{5}\right\}$ **7.** $\left\{-\frac{5}{2},\frac{3}{2}\right\}$ **8.** $\left\{\frac{1}{b^2},\frac{1}{a^2}\right\}$ **9.** $\left\{-\frac{5}{2},3\right\}$ **10.** $\left\{7,\frac{8}{3}\right\}$

Exercise 3.15

1. (i) $\{-7,1\}$ (ii) $\left\{\frac{-3+\sqrt{5}}{2},\frac{-3-\sqrt{5}}{2}\right\}$ (iii) $\left\{-3,\frac{1}{2}\right\}$

(iv) $\left\{\frac{a-b}{2},-\left(\frac{a+b}{2}\right)\right\}$ (v) $\{\sqrt{3},1\}$ (vi) $\{-1,3\}$

2. (i) $\{4,3\}$ (ii) $\left\{\frac{2}{5},\frac{1}{3}\right\}$ (iii) $\left\{\frac{1}{2},2\right\}$ (iv) $\left\{-\frac{2b}{3a},\frac{b}{a}\right\}$

(v) $\left\{\frac{1}{a},a\right\}$ (vi) $\left\{\frac{a+b}{6},\frac{a-b}{6}\right\}$ (vii) $\left\{\frac{(9+\sqrt{769})}{8},\frac{(9-\sqrt{769})}{8}\right\}$ (viii) $\left\{-1,\frac{b^2}{a^2}\right\}$

Exercise 3.16

1. 8 or $\frac{1}{8}$ **2.** 9 and 6 **3.** 20 m, 5m or 10m, 10m **4.** $\frac{3}{2}m$

5. 45km/hr **6.** 5 km/hr **7.** 49 years, 7 years **8.** 24 cm **9.** 12 days

10. Speed of the first train = 20 km / hr and the speed of the second train = 15 km / hr

Exercise 3.17

1. (i) Real (ii) Non-real (iii) Real and equal (iv) Real and equal (v) Non-real (vi) Real
2. (i) $\frac{25}{2}$ (ii) ± 3 (iii) -5 or 1 (iv) 0 or 3

Exercise 3.18

1. (i) 6,5 (ii) $-\frac{r}{k}, p$ (iii) $\frac{5}{3}, 0$ (iv) $0, -\frac{25}{8}$
2. (i) $x^2 - 7x + 12 = 0$ (ii) $x^2 - 6x + 2 = 0$ (iii) $4x^2 - 16x + 9 = 0$
3. (i) $\frac{13}{6}$ (ii) $\pm\frac{1}{3}$ (iii) $\frac{35}{18}$ **4.** $\frac{4}{3}$
5. $4x^2 - 29x + 25 = 0$ **6.** $x^2 + 3x + 2 = 0$ **7.** $x^2 - 11x + 1 = 0$
8. (i) $x^2 - 6x + 3 = 0$ (ii) $27x^2 - 18x + 1 = 0$ (iii) $3x^2 - 18x + 25 = 0$
9. $x^2 + 3x - 4 = 0$ **10.** $k = -18$ **11.** $a = \pm 24$ **12.** $p = \pm 3\sqrt{5}$

Exercise 3.19

1	2	3	4	5	6	7	8	9	10
B	C	A	A	C	D	B	C	C	C
11	12	13	14	15	16	17	18	19	20
D	B	A	A	A	D	D	D	B	C
21	22	23	24	25					
D	A	C	C	A					

4. MATRICES

Exercise 4.1

1. $\begin{pmatrix} 400 & 500 \\ 200 & 250 \\ 300 & 400 \end{pmatrix}, \begin{pmatrix} 400 & 200 & 300 \\ 500 & 250 & 400 \end{pmatrix}$, 3×2, 2×3 **2.** $\begin{pmatrix} 6 \\ 8 \\ 13 \end{pmatrix}$, $\begin{pmatrix} 6 & 8 & 13 \end{pmatrix}$
3. (i) 2×3 (ii) 3×1 (iii) 3×3 (iv) 1×3 (v) 4×2
4. $1\times 8,\ 8\times 1,\ 2\times 4,\ 4\times 2$
5. $1\times 30, 30\times 1, 2\times 15, 15\times 2, 3\times 10, 10\times 3, 5\times 6, 6\times 5$.
6. (i) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ (iii) $\begin{pmatrix} 0 & -\frac{1}{3} \\ \frac{1}{3} & 0 \end{pmatrix}$ **7.** (i) $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \\ 3 & \frac{3}{2} \end{pmatrix}$ (ii) $\begin{pmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (iii) $\begin{pmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \\ \frac{3}{2} & 0 \end{pmatrix}$
8. (i) 3×4 (ii) 4, 0 (iii) 2^{nd} row and 3^{rd} column **9.** $A^T = \begin{pmatrix} 2 & 4 & 5 \\ 3 & 1 & 0 \end{pmatrix}$

Exercise 4.2

1. $x = 2,\ y = -4,\ z = -1$ **2.** $x = 4,\ y = -3$

3. $\begin{pmatrix} -1 & 2 \\ 16 & -6 \end{pmatrix}$ **4.** $\begin{pmatrix} 14 & 3 \\ 14 & 5 \end{pmatrix}$ **5.** $\begin{pmatrix} 0 & -18 \\ 33 & -45 \end{pmatrix}$ **6.** $a = 3,\ b = -4$

7. $X = \begin{pmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{pmatrix},\ Y = \begin{pmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{pmatrix}$ **8.** $x = -3, -3,\ y = -1, 4$

11. $\begin{matrix} & \text{TV} & \text{DVD} & \text{Video} & \text{CD} & \\ & 55 & 27 & 20 & 16 & \text{store I} \\ & 72 & 30 & 25 & 27 & \text{store II} \\ & 47 & 33 & 18 & 22 & \text{store III} \end{matrix}$

12. $\begin{matrix} \text{child} & \text{adult} & \\ 5 & 5 & \text{Before 2.00p.m.} \\ 10 & 10 & \text{After 2.00p.m.} \end{matrix}$

Exercise 4.3

1. (i) 4×2 (ii) not defined (iii) 3×5 (iv) 2×2

2. (i) (6) (ii) $\begin{pmatrix} 8 & -11 \\ 22 & 12 \end{pmatrix}$ (iii) $\begin{pmatrix} -40 & 64 \\ 22 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 12 & -42 \\ -6 & 21 \end{pmatrix}$

3. $\begin{pmatrix} 1750 \\ 1600 \\ 1650 \end{pmatrix} \begin{matrix} \text{I day} \\ \text{II day} \\ \text{III day} \end{matrix}$, (5000) **4.** $x = 3,\ y = 0$ **5.** $x = 2,\ y = -5$

7. $AB = \begin{pmatrix} 15 & 4 \\ 12 & 0 \end{pmatrix},\ BA = \begin{pmatrix} 9 & 6 \\ 17 & 6 \end{pmatrix},\ AB \neq BA$ **11.** $x = -3, 5$

Exercise 4.4

1	2	3	4	5	6	7	8	9	10
D	D	A	D	B	D	B	C	C	A
11	12	13	14	15	16	17	18	19	20
B	D	D	B	C	B	A	C	B	D

5. COORDINATE GEOMETRY

Exercise 5.1

1. (i) (–2, 1) (ii) (0,2) **2.** (i) (5,–2) (ii) (2, –1) **3.** (–12, 8)

4. (2, –2) **6.** (–24,–2) **7.** (–2,3) **8.** (–6,–3) **9.** (–1,0),(–4,2)

10. $\left(-3, \frac{3}{2}\right),\ (-2, 3),\ \left(-1, \frac{9}{2}\right)$ **11.** 4 : 7 internally

12. 5:2 internally, $\left(0, \frac{17}{7}\right)$ **13.** $\frac{\sqrt{130}}{2},\ \sqrt{13},\ \frac{\sqrt{130}}{2}$

Exercise 5.2

1. (i) 3 sq. units (ii) 32 sq. units (iii) 19 sq. units

2. (i) $a = -3$ (ii) $a = \frac{13}{2}$ (iii) $a = 1, 3$

3. (i) collinear (ii) not collinear (iii) collinear

4. (i) k = 1 (ii) k = 2 (iii) $k = \frac{7}{3}$

5. (i) 17 sq. units (ii) 43 sq. units (iii) 60.5 sq. units **7.** 1 sq. units, 1 : 4

Exercise 5.3

1. (i) 45° (ii) 60° (iii) 0° **2.** (i) $\frac{1}{\sqrt{3}}$ (ii) $\sqrt{3}$ (iii) undefined

3. (i) 1 (ii) –2 (iii) 1 **4.** (i) 45° (ii) 30° (iii) $\tan\theta = \frac{b}{a}$

5. $-\frac{1}{2}$ **6.** (i) 0 (ii) undefined (iii) 1 **7.** $\sqrt{3}$, 0 **10.** $a = -1$

11. $b = 6$ **12.** $-\frac{9}{10}$ **13.** $\frac{11}{7}, -13, -\frac{1}{4}$ **14.** $\frac{1}{12}, -\frac{4}{5}, \frac{9}{2}$

Exercise 5.4

1. $y = 5$, $y = -5$ **2.** $y = -2$, $x = -5$ **3.** (i) $3x + y - 4 = 0$ (ii) $\sqrt{3}x - y + 3 = 0$

4. $x - 2y + 6 = 0$ **5.** (i) slope 1, y-intercept 1 (ii) slope $\frac{5}{3}$, y-intercept 0

(iii) slope 2, y-intercept $\frac{1}{2}$ (iv) slope $-\frac{2}{3}$, y-intercept $-\frac{2}{5}$

6. (i) $4x + y - 6 = 0$ (ii) $2x - 3y - 22 = 0$ **7.** $2x - 2\sqrt{3}y + (3\sqrt{3} - 7) = 0$

8. (i) $x - 5y + 27 = 0$ (ii) $x + y + 6 = 0$ **9.** $6x + 5y - 2 = 0$

11. (i) $3x + 2y - 6 = 0$ (ii) $9x - 2y + 3 = 0$ (iii) $15x - 8y - 6 = 0$

12. (i) 3,5 (ii) –8, 16 (iii) $-\frac{4}{3}, -\frac{2}{5},$ **13.** $2x + 3y - 18 = 0$

14. $2x + y - 6 = 0$, $x + 2y - 6 = 0$ **15.** $x - y - 8 = 0$

16. $x + 3y - 6 = 0$ **17.** $2x + 3y - 12 = 0$ **18.** $x + 2y - 10 = 0$, $6x + 11y - 66 = 0$

19. $x + y - 5 = 0$ **20.** $3x - 2y + 4 = 0$

Exercise 5.5

1. (i) $-\frac{3}{4}$ (ii) 7 (iii) $\frac{4}{5}$ **4.** $a = 6$ **5.** $a = 5$ **6.** $p = 1,2$ **7.** $h = \frac{22}{9}$

8. $3x - y - 5 = 0$ **9.** $2x + y = 0$ **10.** $2x + y - 5 = 0$ **11.** $x + y - 2 = 0$

12. $5x + 3y + 8 = 0$ **13.** $x + 3y - 7 = 0$ **14.** $x - 3y + 6 = 0$

15. $x - 4y + 20 = 0$ **16.** (3, 2) **17.** 5 units **18.** $x + 2y - 5 = 0$

19. $2x + 3y - 9 = 0$

Exercise 5.6

1	2	3	4	5	6	7	8	9	10	11	12
C	B	A	D	A	B	D	A	D	C	C	B
13	14	15	16	17	18	19	20	21	22	23	
C	C	C	D	B	B	D	A	A	B	B	

6. GEOMETRY

Exercise 6.1

1. (i) 20cm (ii) 6cm (iii) 1 **2.** 7.5cm **3.** (i) No (ii) Yes **4.** 10.5cm

6. 12cm, 10cm **9.** (i) 7.5cm (ii) 5.8cm (iii) 4 cm **10.** (i) Yes (ii) No **11.** 18 cm

Exercise 6.2

1. (i) $x = 4$cm, $y = 9$cm (ii) $x = 3.6$cm, $y = 2.4$cm, $z = 10$cm (iii) $x = 8.4$cm, $y = 2.5$cm

2. 3.6m **3.** 1.2m **4.** 140m **6.** 6cm **7.** 64cm^2 **8.** 166.25 cm

9. (i) $\frac{9}{64}$ (ii) $\frac{55}{64}$ **10.** 6.3km^2 **11.** 72cm **12.** 9m

13. (i) $\triangle XWY$, $\triangle YWZ$, $\triangle XYZ$ (ii) 4.8m

Exercise 6.3

1. 65° **2.** (i) 4 cm (ii) 12 cm **3.** (i) 12 cm (ii) 5 cm **6.** 30 cm

Exercise 6.4

1	2	3	4	5	6	7	8	9	10
B	B	A	D	B	C	B	D	B	B
11	12	13	14	15	16	17	18	19	20
D	D	C	D	D	A	B	B	D	C

7. TRIGONOMETRY

Exercise 7.1

1. (i) No (ii) No

Exercise 7.2

1. 1.8m **2.** 30° **3.** No **4.** 174.7 m **5.** 40 cm **6.** Crow B

7. $5\sqrt{6}$ m **8.** 1912.40 m **9.** $30\sqrt{2}$ m **10.** 1.098 m **11.** $19\sqrt{3}$ m

12. Yes **13.** 87m **14.** 3 Minutes **15.** 3464 km **16.** 40 m

17. 60 m ; $40\sqrt{3}$ m **18.** 90m

Exercise 7.3

1	2	3	4	5	6	7	8	9	10
B	C	C	A	A	B	A	A	C	B
11	12	13	14	15	16	17	18	19	20
B	C	A	D	C	C	D	B	B	D

8. MENSURATION

Exercise 8.1

1. $704 cm^2$, $1936 cm^2$
2. $h = 8$ cm, $352 cm^2$
3. $h = 40$ cm, d = 35 cm
4. ₹2640
5. $r = 3.5$ cm, h = 7 cm
6. $h = 28$ cm
7. $C_1 : C_2 = 5 : 2$
8. $1300\pi\ cm^2$
9. $3168\ cm^2$
10. $550 cm^2$, $704\ cm^2$
11. $h = 15\sqrt{3}$ cm, $l = 30$ cm
12. $1416 cm^2$
13. $23.1 m^2$
14. 10.5 cm
15. $301\frac{5}{7}\ cm^2$
16. 2.8 cm
17. $4158 cm^2$
18. $C_1 : C_2 = 9 : 25$, $T_1 : T_2 = 9 : 25$
19. $44.1\pi\ cm^2$, $57.33\pi\ cm^2$
20. ₹246.40

Exercise 8.2

1. $18480\ cm^3$
2. 38.5 litres
3. $4620\ cm^3$
4. $r = 2.1$ cm
5. $V_1 : V_2 = 20 : 27$
6. 10 cm
7. $4158\ cm^3$
8. $7.04\ cm^3$
9. $8800 cm^3$
10. $616 cm^3$
11. 5cm
12. $1408.6\ cm^3$
13. $314\frac{2}{7}\ cm^3$
14. $2\sqrt{13}$ cm
15. 8 cm
16. 2.29 Kg
17. $3050\frac{2}{3}\ cm^3$
18. $288\pi cm^2$
19. $718\frac{2}{3}\ cm^3$
20. 1: 8

Exercise 8.3

1. $11.88\pi\ cm^2$
2. $7623 cm^3$
3. $220 mm^2$
4. 1034 sq.m
5. 12 cm
6. 12.8 km
7. 2 cm
8. 1 cm
9. 1386 litres
10. 3 hrs. 12 mins.
11. 16 cm
12. 16 cm
13. 750 lead shots
14. 10 cones
15. 70 cm
16. $r = 36$ cm, $l = 12\sqrt{13}$ cm
17. 11m

Exercise 8.4

1	2	3	4	5	6	7	8	9	10	11
B	C	A	A	B	C	A	B	D	C	C
12	13	14	15	16	17	18	19	20	21	22
D	D	B	D	B	C	B	D	A	D	C

10. GRAPH

Exercise 10.1

2. (i) $\{-2, 2\}$ (ii) $\{-2, 5\}$ (iii) $\{5, 1\}$ (iv) $\left\{-\frac{1}{2}, 3\right\}$
3. {–1, 5}
4. {–2, 3}
5. {–2.5, 2}
6. {–3, 5}
7. No real solutions

Exercise 10.2

1. 120 kms
2. (i) ₹105 (ii) 11 note books
3. (i) $y = 8$ (ii) $x = 6$
4. (i) $k = 15$ (ii) ₹ 45
5. $y = 4$; $x = 2$
6. 24 days

11. STATISTICS

Exercise 11.1

1. (i) 36, 0.44 (ii) 44, 0.64 **2.** 71 **3.** 3.38 kg **4.** $2\sqrt{5}$, 20

5. 3.74 **6.** (i) 5.97 (ii) 4.69 **7.** 6.32 **8.** 1.107 **9.** 15.08

10. 36.76, 6.06 **11.** 416, 20. 39 **12.** 54.19 **13.** 4800, 240400 **14.** 10.2, 1.99

15. 25 **16.** 20.41 **17.** 12 **18.** 5.24 **19.** 1159, 70

20. *A* is more consistent

Exercise 11.2

1	2	3	4	5	6	7	8	9	10
D	A	C	B	D	C	C	B	A	D
11	12	13	14	15					
D	B	C	D	B					

12. PROBABILITY

Exercise 12.1

1. $\frac{1}{10}$ **2.** $\frac{1}{9}$ **3.** $\frac{1}{3}$ **4.** $\frac{1}{5}$ **5.** $\frac{3}{4}$

6. (i) $\frac{1}{4}$ (ii) $\frac{3}{4}$ (iii) $\frac{12}{13}$ **7.** (i) $\frac{7}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{2}$

8. (i) $\frac{1}{2}$ (ii) $\frac{3}{5}$ **9.** (i) $\frac{1}{10}$ (ii) $\frac{24}{25}$ **10.** $\frac{1}{2}$ **11.** (i) $\frac{1}{4}$ (ii) $\frac{2}{3}$

12. (i) $\frac{1}{4}$ (ii) $\frac{17}{20}$ **13.** $\frac{1}{3}$ **14.** $\frac{1}{36}$ **15.** $\frac{1}{6}$ **16.** 12

17. (i) $\frac{22}{25}$ (ii) $\frac{24}{25}$ **18.** (i) $\frac{1}{4}$ (ii) 3 **19.** (i) $\frac{5}{9}$ (ii) $\frac{17}{18}$

Exercise 12. 2

1. $\frac{4}{5}$ **2.** $\frac{3}{20}$ **3.** (i) $\frac{1}{5}$ (ii) $\frac{4}{5}$ **4.** $\frac{5}{9}$ **5.** $\frac{8}{25}$

6. $\frac{5}{8}$ **7.** $\frac{4}{9}$ **8.** $\frac{9}{10}$ **9.** $\frac{3}{5}$ **10.** $\frac{4}{13}$

11. $\frac{8}{13}$ **12.** $\frac{2}{3}$ **13.** $\frac{5}{13}, \frac{4}{13}$ **14.** (i) 0.45 (ii) 0.3 **15.** $\frac{101}{105}$

Exercise 12. 3

1	2	3	4	5	6	7	8	9	10
C	D	B	A	A	B	A	A	D	A
11	12	13	14	15	16	17	18	19	20
D	C	C	B	B	D	D	A	A	B

Miscellaneous problems

(Not for examination)

1. If $f(x) = \dfrac{x-1}{x+1}, x \neq -1$, then prove that $f(2x) = \dfrac{3f(x)+1}{f(x)+3}$.

2. Solve the equation $(x-1)(x-2)(x-3)(x-4) = 15$ for real values of x.
   $\left(\text{Ans}: x = \dfrac{5 \pm \sqrt{21}}{2}\right)$

3. For what values of x do the three numbers $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ taken in that order constitute an A.P.? (Ans: $x = \log_5 2$)

4. In a G.P. with common ratio r, the sum of first four terms is equal to 15 and the sum of their squares is equal to 85. Prove that $14r^4 - 17r^3 - 17r^2 - 17r + 14 = 0$.

5. Prove that the sequence $\{b_n\}$ is a G.P. if and only if $b_n^2 = b_{n-1}\, b_{n+1}$, $n > 1$.

6. Certain numbers appear in both arithmetic progressions 17, 21, $\cdots$ and 16, 21, $\cdots$. Find the sum of the first ten numbers appearing in both progressions.

7. Prove that the sequence $\{a_n\}$ is an A.P. if and only if $a_n = \dfrac{a_{n-1} + a_{n+1}}{2}$, $n > 1$.

8. Prove that $\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha\cos^2\alpha = 1$

9. Prove that $\dfrac{\sin x + \cos x}{\cos^2 x} = \tan^3 x + \tan^2 x + \tan x + 1$.

10. If we divide a two-digit number by the sum of its digits, we get 4 as a quotient and 3 as a remainder. Now if we divide that two-digit number by the product of its digits, we get 3 as a quotient and 5 as a remainder. Find the two-digit number. (Ans : 23)

11. Find the sum of all two-digit numbers which, being divided by 4, leave a remainder of 1. (Ans : 1210)

12. Simplify the expression $\dfrac{\dfrac{1}{a} + \dfrac{1}{b+c}}{\dfrac{1}{a} - \dfrac{1}{b+c}} \times \left(1 + \dfrac{b^2 + c^2 - a^2}{2bc}\right)(a+b+c)^{-2}$
    (Ans : $\dfrac{1}{2bc}$)

13. The quadratic equation $ax^2 + bx + c = 0$ has no real roots and $a + b + c < 0$. Find the sign of the number c. (Hint. If $f(x) = 0$ has no real roots, then $f(x)$ has same sign for all x) (Ans: $c < 0$)

14. Find all real numbers x such that $f(x) = \dfrac{x-1}{x^2 - x + 6} > 0$. (Ans $x > 1$)

15. Solve the equation $1 + a + a^2 + \cdots + a^x = (1+a)(1+a^2)(1+a^4)(1+a^8)$
    (Ans: $x = 15$)

16. Compute $\dfrac{6x_1^2x_2 - 4x_1^3 + 6x_1x_2^2 - 4x_2^3}{3x_1^2 + 5x_1x_2 + 3x_2^2}$, where x_1 and x_2 are the roots of the equation $x^2 - 5x + 2 = 0$. (Ans : $-\dfrac{320}{73}$)

17. Prove the identity: $\operatorname{cosec}\alpha - \cot\alpha - \dfrac{\sin\alpha + \cos\alpha}{\cos\alpha} + \dfrac{\sec\alpha - 1}{\sin\alpha} = -1$

18. One-fourths of a herd of camels was seen in the forest. Twice the square root of the number of herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels. (Ans: Number of camels is 36)

19. After covering a distance of 30 km with a uniform speed there is some defect in a train engine and therefore, its speed is reduced to $\frac{4}{5}$ of its original speed. Consequently, the train reaches its destination late by 45 minutes. Had it happened after covering 18 kilo metres more, the train would have reached 9 minutes earlier. Find the speed of the train and the distance of journey. (Ans: Speed of the train is 30 km/hr and the distance of the journey is 120 km.)

20. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

21. If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, prove that $l^2m^2(l^2 + m^2 + 3) = 1$

22. At the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain. (Ans: 1.366 km)

23. If the opposite angular points of a square are (3, 4) and (1, –1), then find the coordinates of the remaining angular points. (Ans: $\left(\frac{9}{2},\frac{1}{2}\right)$ and $\left(-\frac{1}{2},\frac{5}{2}\right)$)

24. In an increasing G.P. the sum of first and the last term is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in the progression. (Ans : 6)

25. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a height b just above A is β. Prove that the height of the tower is $b\cot\beta\tan\alpha$.

26. A rectangular pool has the dimensions 40 ft × 20 ft. We have exactly 99 cu.ft of concrete to be used to create a border of uniform width and depth around the pool. If the border is to have a depth of 3 inches and if we use all of the concrete, how wide the border will be ? (Ans : 3 ft)

27. Simplify $(1+\frac{2}{2})(1+\frac{2}{3})(1+\frac{2}{4})\cdots(1+\frac{2}{n})$. (Ans : $\frac{(n+1)(n+2)}{6}$)

28. There are three circular disks such that two of them has radius r inches and the third has radius $2r$ inches. These three disks are placed in a plane such that each of its boundary has exactly one point in common with any other boundary. Find the area of the triangle formed by the centers of these disks. (Ans : $2\sqrt{2}\,r^2$ sq.inches)

29. Six circular discs each having radius 8 inches are placed on the floor in a circular fashion so that in the center area we could place a seventh disk touching all six of these disks exactly at one point each and each disk is touching two other disks one point each on both sides. Find the area formed by these six disks in the center. (Ans : $192\sqrt{3}$ sq. inches)

30. From a cylinderical piece of wood of radius 4 cm and height 5cm, a right circular cone with same base radius and height 3 cms is carved out.Prove that the total surface area of the remaining wood is 76π cm^2.

31. Show that $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$ where $n! = 1\times2\times3\times\cdots\times n$.

Bibliography

1. Peter J. Eccles, Introduction to Mathematical Reasoning, Cambridge University Press 2007
2. Ann Xavier Gantert, Algebra 2 and Trigonometry, Amsco School Publications Inc., 2009
3. Boris A Kordemsky, The Moscow Puzzles: 359 Mathematical Recreations, Dover Publications
4. Imre Lakatos, Proofs and Refutations: The Logic of Mathematical Discovery, January 1976
5. Krishnan Namboodiri, Richard G. Niemi, Matrix Algebra, An Introduction, Sage Publications 1984
6. Alfred S. Posamentier, Charles T. Salkind, Challenging Problems in Geometry, Dover Publications
7. Alfred S. Posamentier, Charles T. Salkind, Challenging Problems in Algebra, Dover Publications
8. James Stewart, Lothar Redlin, Saleem Watson, College Algebra, Thomson Brooks/Cole, Jan 2010
9. Michael Sullivan, College Algebra, Pearson Publishing, January 2007
10. http://www-history.mcs.st-and.ac.uk/BiogIndex.html
11. V.Govorov, P.Dybov, N.Miroshin, S.Smirnova, Problems in Mathematics, G.K. Publications 2010
12. H.S.Hall, S.R. Knight, Elementary Algebra for Schools, Surjeet Publications 2007
13. H.S.Hall, S.R. Knight, Higher Algebra, A.I.T.B.S Publishers 2009
14. D.Dorokhin, Z.Plaksenko, G.Bazhora, Collection of Problems and Exercises in Mathematics, Mir Publications 1990

QUESTION PAPER DESIGN

Subject : Mathematics **Time: 2.30 Hrs**

Class : X **Max marks: 100**

Weightage of marks to Learning Objectives

Objectives	Percentage
Knowledge	19
Understanding	31
Application	23
Skill	27
Total	100

Weightage to the types of Question

Type of Questions	Section-A Very Short Answer (Objective)	Section-B Short Answer	Section-C Long Answer	Section-D Very Long Answer	Total
Number of Questions	15	10	9	2	36
Marks	15	20	45	20	100
Time (in minutes)	20	35	65	30	2.30 Hrs

Difficulty Level

Level	Percentage of Marks
Difficult	12
Average	28
Easy	60

Sections and Options

<table>
<tr><th rowspan="2">Sections</th><th colspan="2">Question numbers</th><th rowspan="2">Number of Questions</th><th rowspan="2">Questions to be answered</th></tr>
<tr><th>From</th><th>To</th></tr>
<tr><td>A</td><td>1</td><td>15</td><td>15</td><td>15</td></tr>
<tr><td>B</td><td>16</td><td>30</td><td>16
30th Question is compulsory and is in ‘either’ ‘or’ type</td><td>10</td></tr>
<tr><td>C</td><td>31</td><td>45</td><td>16
45th Question is compulsory and is in ‘either’ ‘or’ type</td><td>9</td></tr>
<tr><td rowspan="2">D</td><td colspan="2">46</td><td>2
This Question is in ‘either’ ‘or’ type</td><td>1</td></tr>
<tr><td colspan="2">47</td><td>2
This Question is in ‘either’ ‘or’ type</td><td>1</td></tr>
</table>

Weightage to Content

Chapter No.	Chapter	Number of Questions				Total Marks
		1 mark	2 marks	5 marks	10 marks	
1	Sets and Functions	1	2	2		15
2	Sequences and series of Real Numbers	2	1	2		14
3	Algebra	2	2	3		21
4	Matrices	1	2	1		10
5	Coordinate Geometry	2	2	2		16
6	Geometry	2	1	1		9
7	Trigonometry	2	2	1		11
8	Mensuration	1	2	2		15
9	Practical Geometry				2	20
10	Graphs				2	20
11	Statistics	1	1	1		8
12	Probability	1	1	1		8
Total		15	16	16	4	167

Distribution of Marks and Questions towards Examples, Exercises and Framed questions

	Sec A (1 mark)	Sec B (2 marks)	Sec C (5 marks)	Sec D (10 marks)	Total Marks	Percentage
From the Examples given in the Text Book	---	6 (2)	6 (5)	1 (10)	52	31
From the Exercises given in the Text Book	10 (1)	8 (2)	8 (5)	3 (10)	96	58
Framed questions from specified chapters	5 (1)	2 (2)	2 (5)	---	19	11
Total	15 (1)	16 (2)	16 (5)	4 (10)	167	100

- Numbers in brackets indicate the marks for each question.

Section - A

1. All the 15 questions numbered 1 to 15 are multiple choice questions each with 4 distractors and all are compulsory. Each question carries one mark.
2. Out of 15 questions, 10 questions are from the multiple choice questions given in the Text Book. The remaining 5 questions should be framed from the five different chapters 2, 3, 5, 6 and 7 on the basis of the Text Book theorems, results, examples and exercises.

Section - B

1. 10 questions are to be answered from the questions numbered 16 to 30. Each question carries two marks.
2. Answer any 9 questions from the first 14 questions. Question No. 30 is compulsory and is in either or type.
3. The order of the first 14 questions should be in the order of the chapters in the Text Book.
4. Out of first 14 questions, 6 questions are from the examples and 8 questions are from the exercises.
5. The two questions under question no. 30 should be framed based on the examples and problems given in the exercises from any two different chapters of 2, 3, 5 and 8.

Section - C

1. 9 questions are to be answered from the questions numbered 31 to 45. Each question carries five marks.
2. Answer any 8 questions from the first 14 questions. Question no. 45 is compulsory and is in either or type.
3. The order of the first 14 questions should be in the order of the chapters in the Text Book.
4. Out of first 14 questions, 6 questions are from the examples and 8 questions are from the exercises.
5. The two questions under question no. 45 should be framed based on the examples and problems given in the exercises from any two different chapters of 2, 3, 5 and 8.
6. Questions numbered 30(a), 30(b), 45(a) and 45(b) should be framed based on the examples and problems given in the exercises from the chapters 2, 3, 5 and 8 subject to the condition that all of them should be from different chapters.

Section - D

1. This section contains two questions numbered 46 and 47, one from the chapter 9 and the other from the chapter 10, each with two alternatives (‘either’ ‘or’ type) from the same chapter. Each question carries ten marks.
2. Answer both the questions.
3. One of the questions 46(a), 47(a), 46(b) and 47(b) should be from the examples given in the text book. The remaining three questions should be from the exercises.

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Chapter / Objective	Knowledge				Understanding				Application				Skill				Total marks
	VSA	SA	LA	VLA	VSA	SA	LA	VLA	VSA	SA	LA	VLA	VSA	SA	LA	VLA	
Sets and Functions	1(1)	2(1)	5(1)			2(1)					5(1)						15
Sequences and Series of Real Numbers		2(1)	5(1)		1(1)				1(1)		5(1)						14
Algebra		2(1)	5(1)		1(1)				1(1)	2(1)	5(1)				5(1)		21
Matrices						4(2)	5(1)		1(1)								10
Coordinate Geometry		2(1)			1(1)	2(1)	5(1)		1(1)		5(1)						16
Geometry					1(1)	2(1)	5(1)		1(1)								9
Trigonometry					1(1)	2(1)	5(1)		1(1)	2(1)							11
Mensuration	1(1)					2(1)	5(1)			2(1)	5(1)						15
Practical Geometry																10(2)	20
Graphs																10(2)	20
Statistics			5(1)			2(1)			1(1)								8
Probability		2(1)					5(1)		1(1)								8
Total	2(2)	10(5)	20(4)		5(5)	16(8)	30(6)		8(8)	6(3)	25(5)				5(1)	40(4)	167

- Numbers in brackets indicate the number of questions.
- Other numbers indicate the marks.