6

- Introduction
- Basic Proportionality Theorem
- Angle Bisector Theorem
- Similar Triangles
- Tangent chord theorem
- Pythagoras theorem

EUCLID
(300 BC)
Greece

Euclid's 'Elements' is one of the most influential works in the history of mathematics, serving as the main text book for teaching mathematics especially geometry.

Euclid's algorithm is an efficient method for computing the greatest common divisor.

GEOMETRY

There is geometry in the humming of the strings, there is music in the spacing of spheres - Pythagoras

6.1 Introduction

Geometry is a branch of mathematics that deals with the properties of various geometrical figures. The geometry which treats the properties and characteristics of various geometrical shapes with axioms or theorems, without the help of accurate measurements is known as theoretical geometry. The study of geometry improves one's power to think logically.

Euclid, who lived around 300 BC is considered to be the father of geometry. Euclid initiated a new way of thinking in the study of geometrical results by deductive reasoning based on previously proved results and some self evident specific assumptions called axioms or postulates.

Geometry holds a great deal of importance in fields such as engineering and architecture. For example, many bridges that play an important role in our lives make use of congruent and similar triangles. These triangles help to construct the bridge more stable and enables the bridge to withstand great amounts of stress and strain. In the construction of buildings, geometry can play two roles; one in making the structure more stable and the other in enhancing the beauty. Elegant use of geometric shapes can turn buildings and other structures such as the Taj Mahal into great landmarks admired by all. Geometric proofs play a vital role in the expansion and understanding of many branches of mathematics.

The basic proportionality theorem is attributed to the famous Greek mathematician Thales. This theorem is also called Thales theorem.

To understand the basic proportionality theorem, let us perform the following activity.

Activity

Draw any angle XAY and mark points (say five points) P_1, P_2, D, P_3 and B on arm AX such that $AP_1 = P_1P_2 = P_2D = DP_3 = P_3B = 1$ unit (say).

Through B draw any line intersecting arm AY at C. Again through D draw a line parallel to BC to intersect AC at E.

Now $\quad AD = AP_1 + P_1P_2 + P_2D = 3$ units

and $\quad DB = DP_3 + P_3B = 2$ units

$\therefore \quad \frac{AD}{DB} = \frac{3}{2}$

Measure AE and EC.

We observe that $\frac{AE}{EC} = \frac{3}{2}$

Thus, in ΔABC if $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$

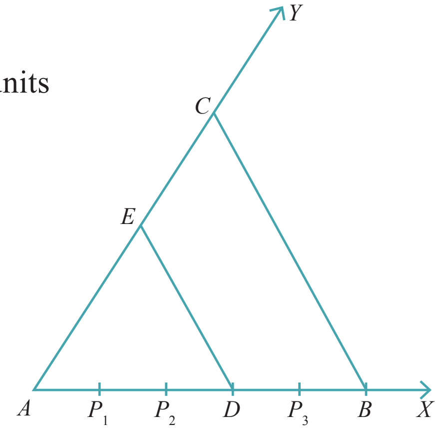


Fig. 6.1

We prove this result as a theorem known as Basic Proportionality Theorem or Thales Theorem as follows:

6.2 Basic proportionality and Angle Bisector theorems

Theorem 6.1 **Basic Proportionality theorem or Thales Theorem**

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

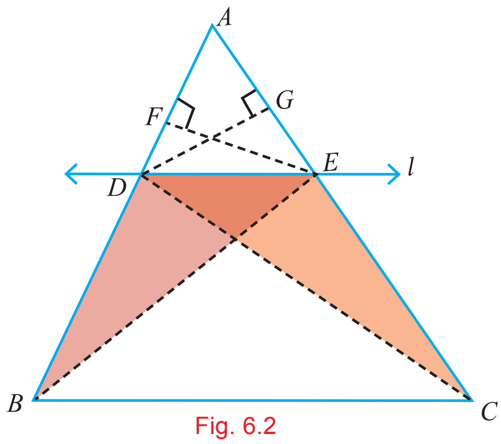


Fig. 6.2

Given: In a triangle ABC, a straight line l parallel to BC, intersects AB at D and AC at E.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, CD.

Draw $EF \perp AB$ and $DG \perp CA$.

Proof

Since, $EF \perp AB$, EF is the height of triangles ADE and DBE.

Area $(\Delta ADE) = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} AD \times EF$ and

Area $(\Delta DBE) = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} DB \times EF$

$$\therefore \quad \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DBE)} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} DB \times EF} = \frac{AD}{DB} \quad (1)$$

Similarly, we get

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DCE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \quad (2)$$

But, ΔDBE and ΔDCE are on the same base DE and between the same parallel straight lines BC and DE.

$$\therefore \quad \text{area } (\Delta DBE) = \text{area } (\Delta DCE) \quad (3)$$

Form (1), (2) and (3), we obtain $\frac{AD}{DB} = \frac{AE}{EC}$. Hence the theorem.

Corollary

If in a ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Proof

(i) From Thales theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Longrightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Longrightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Longrightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

Thus, $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) Similarly, we can prove

$$\frac{AB}{DB} = \frac{AC}{EC}$$

Do you know?

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$.

This is called componendo rule.

Here, $\frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

by componendo rule.

Is the converse of this theorem also true? To examine this let us perform the following activity.

Activity

Draw an angle $\angle XAY$ and on the ray AX, mark points P_1, P_2, P_3, P_4 and B such that

$AP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4B = 1$ unit (say).

Similarly, on ray AY, mark points Q_1, Q_2, Q_3, Q_4 and C such that

$AQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4C = 2$ units (say).

Now join P_1Q_1 and BC.

Then $\frac{AP_1}{P_1B} = \frac{1}{4}$ and $\frac{AQ_1}{Q_1C} = \frac{2}{8} = \frac{1}{4}$

Thus, $\frac{AP_1}{P_1B} = \frac{AQ_1}{Q_1C}$

We observe that the lines P_1Q_1 and BC are parallel to each other. i.e., $P_1Q_1 \parallel BC$ (1)

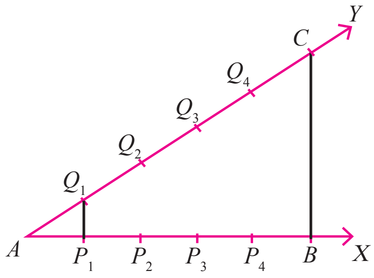


Fig. 6.3

Similarly, by joining P_2Q_2, P_3Q_3 and P_4Q_4 we see that

$$\frac{AP_2}{P_2B} = \frac{AQ_2}{Q_2C} = \frac{2}{3} \text{ and } P_2Q_2 \parallel BC \quad (2)$$

$$\frac{AP_3}{P_3B} = \frac{AQ_3}{Q_3C} = \frac{3}{2} \text{ and } P_3Q_3 \parallel BC \quad (3)$$

$$\frac{AP_4}{P_4B} = \frac{AQ_4}{Q_4C} = \frac{4}{1} \text{ and } P_4Q_4 \parallel BC \quad (4)$$

From (1), (2), (3) and (4) we observe that if a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

In this direction, let us state and prove a theorem which is the converse of Thales theorem.

Theorem 6.2

Converse of Basic Proportionality Theorem (Converse of Thales Theorem)

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

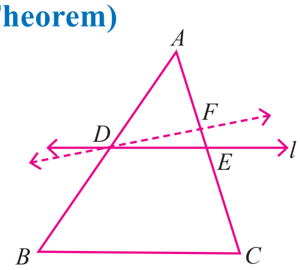


Fig. 6.4

Given: A line l intersects the sides AB and AC of ΔABC respectively at D and E such that $\frac{AD}{DB} = \frac{AE}{EC}$ (1)

To prove : $DE \parallel BC$

Construction : If DE is not parallel to BC, then draw a line $DF \parallel BC$.

Proof Since $DF \parallel BC$, by Thales theorem we get,

$$\frac{AD}{DB} = \frac{AF}{FC} \quad (2)$$

From (1) and (2), we get $\frac{AF}{FC} = \frac{AE}{EC} \implies \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$

$$\frac{AC}{FC} = \frac{AC}{EC} \quad \therefore\ FC = EC$$

This is possible only when F and E coincide. Thus, $DE \parallel BC$.

Theorem 6.3 Angle Bisector Theorem

The internal (external) bisector of an angle of a triangle divides the opposite side internally (externally) in the ratio of the corresponding sides containing the angle.

Case (i) (Internally)

Given : In $\triangle ABC$, AD is the internal bisector of $\angle BAC$ which meets BC at D.

To prove : $\dfrac{BD}{DC} = \dfrac{AB}{AC}$

Construction : Draw $CE \parallel DA$ to meet BA produced at E.

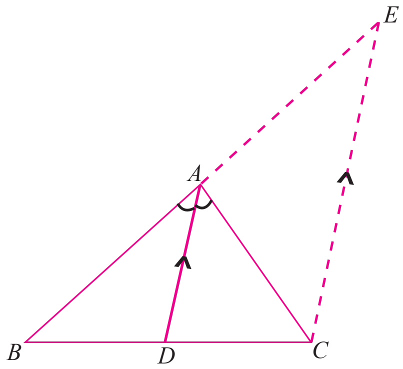


Fig. 6.5

Proof

Since $CE \parallel DA$ and AC is the transversal, we have

$\angle DAC = \angle ACE$ (alternate angles) (1)

and $\angle BAD = \angle AEC$ (corresponding angles) (2)

Since AD is the angle bisector of $\angle A$, $\angle BAD = \angle DAC$ (3)

From (1), (2) and (3), we have $\angle ACE = \angle AEC$

Thus in $\triangle ACE$, we have $AE = AC$ (sides opposite to equal angles are equal)

Now in $\triangle BCE$ we have, $CE \parallel DA$

$$\frac{BD}{DC} = \frac{BA}{AE} \qquad \text{(Thales theorem)}$$

$$\Longrightarrow \quad \frac{BD}{DC} = \frac{AB}{AC} \qquad (AE = AC)$$

Hence the theorem.

Case (ii) Externally (this part is not for examination)

Given: In $\triangle ABC$,

AD is the external bisector of $\angle BAC$

and intersects BC produced at D.

To prove: $\dfrac{BD}{DC} = \dfrac{AB}{AC}$

Construction: Draw $CE \parallel DA$ meeting AB at E.

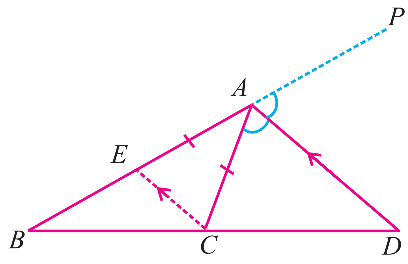


Fig. 6.6

Proof $CE \parallel DA$ and AC is a transversal,

$\angle ECA = \angle CAD$ (alternate angles) (1)

Also $CE \parallel DA$ and BP is a transversal

$$\angle CEA = \angle DAP \qquad \text{(corresponding angles)} \qquad (2)$$

But AD is the bisector of $\angle CAP$

$$\angle CAD = \angle DAP \qquad (3)$$

From (1), (2) and (3), we have

$$\angle CEA = \angle ECA$$

Thus, in $\triangle ECA$, we have $AC = AE$ (sides opposite to equal angles are equal)

In $\triangle BDA$, we have $EC \parallel AD$

$$\therefore \quad \frac{BD}{DC} = \frac{BA}{AE} \qquad \text{(Thales theorem)}$$

$$\Longrightarrow \quad \frac{BD}{DC} = \frac{BA}{AC} \qquad (AE = AC)$$

Hence the theorem.

Theorem 6.4 Converse of Angle Bisector Theorem

If a straight line through one vertex of a triangle divides the opposite side internally (externally) in the ratio of the other two sides, then the line bisects the angle internally (externally) at the vertex.

Case (i) : (Internally)

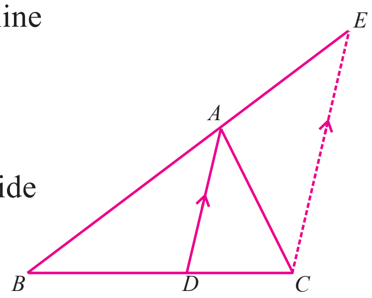


Fig. 6.7

Given : In $\triangle ABC$, the line AD divides the opposite side BC internally such that

$$\frac{BD}{DC} = \frac{AB}{AC} \qquad (1)$$

To prove : AD is the internal bisector of $\angle BAC$.

i.e., to prove $\angle BAD = \angle DAC$.

Construction :

Through C draw $CE \parallel AD$ meeting BA produced at E.

Proof Since $CE \parallel AD$, by Thales theorem, we have $\frac{BD}{DC} = \frac{BA}{AE}$ (2)

Thus, from (1) and (2) we have, $\frac{AB}{AE} = \frac{AB}{AC}$

$$\therefore \quad AE = AC$$

Now, in $\triangle ACE$, we have $\angle ACE = \angle AEC$ $(AE = AC)$ (3)

Since AC is a transversal of the parallel lines AD and CE,

we get, $\angle DAC = \angle ACE$ (alternate interior angles are equal) (4)

Also BE is a transversal of the parallel lines AD and CE.

we get $\angle BAD = \angle AEC$ (corresponding angles are equal) (5)

From (3), (4) and (5), we get

$$\angle BAD = \angle DAC$$

$\therefore$ AD is the angle bisector of $\angle BAC$.

Hence the theorem.

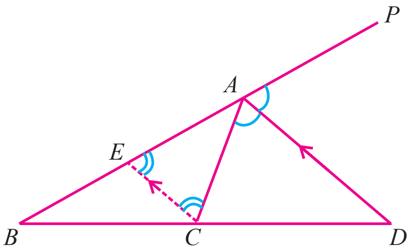


Fig. 6.8

Case (ii) Externally (this part is not for examination)

Given : In $\triangle ABC$, the line AD divides externally the opposite side BC produced at D.

such that $\dfrac{BD}{DC} = \dfrac{AB}{AC}$ (1)

To prove : AD is the bisector of $\angle PAC$,

i.e., to prove $\angle PAD = \angle DAC$

Construction : Through C draw $CE \parallel DA$ meeting BA at E.

Proof Since $CE \parallel DA$, by Thales theorem $\dfrac{BD}{DC} = \dfrac{BA}{EA}$ (2)

From (1) and (2), we have

$$\frac{AB}{AE} = \frac{AB}{AC} \quad \therefore\ AE = AC$$

In $\triangle ACE$, we have $\angle ACE = \angle AEC$ $(AE = AC)$ (3)

Since AC is a transversal of the parallel lines AD and CE, we have

$\angle ACE = \angle DAC$ (alternate interior angles) (4)

Also, BA is a transversal of the parallel lines AD and CE,

$\angle PAD = \angle AEC$ (corresponding angles) (5)

From (3) , (4) and (5), we get

$$\angle PAD = \angle DAC$$

$\therefore$ AD is the bisector of $\angle PAC$. Thus AD is the external bisector of $\angle BAC$

Hence the theorem.

Example 6.1

In $\triangle ABC$, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{2}{3}$. If $AE = 3.7$cm, find EC.

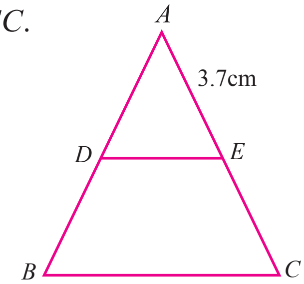


Fig. 6.9

Solution In $\triangle ABC$, $DE \parallel BC$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{(Thales theorem)}$$

$$\Longrightarrow \quad EC = \frac{AE \times DB}{AD}$$

Thus, $EC = \frac{3.7 \times 3}{2} = 5.55\,\text{cm}$

Example 6.2

In $\triangle PQR$, given that S is a point on PQ such that $ST \parallel QR$ and $\frac{PS}{SQ} = \frac{3}{5}$. If $PR = 5.6$cm, then find PT.

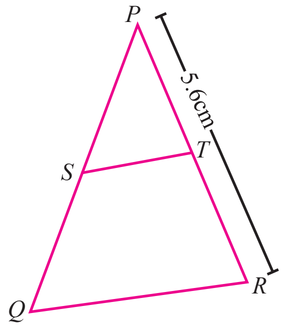


Fig. 6.10

Solution In $\triangle PQR$, we have $ST \parallel QR$ and by Thales theorem,

$$\frac{PS}{SQ} = \frac{PT}{TR} \quad (1)$$

Let $PT = x$. Thus, $TR = PR - PT = 5.6 - x$.

From (1), we get $PT = TR\left(\frac{PS}{SQ}\right)$

$$x = (5.6 - x)\left(\frac{3}{5}\right)$$

$$5x = 16.8 - 3x$$

Thus, $x = \frac{16.8}{8} = 2.1$ That is, PT = 2.1cm.

Example 6.3

In a $\triangle ABC$, D and E are points on AB and AC respectively such that $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle DEA$. Prove that $\triangle ABC$ is isosceles.

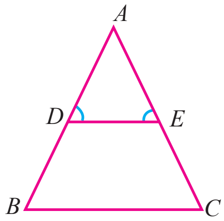


Fig. 6.11

Solution Since $\frac{AD}{DB} = \frac{AE}{EC}$, by converse of Thales theorem, $DE \parallel BC$

$$\therefore \quad \angle ADE = \angle ABC \text{ and} \quad (1)$$

$$\angle DEA = \angle BCA \quad (2)$$

But, given that $\angle ADE = \angle DEA$ (3)

From (1), (2) and (3), we get $\angle ABC = \angle BCA$

$\therefore$ $AC = AB$ (If opposite angles are equal, then opposite sides are equal).

Thus, $\triangle ABC$ is isosceles.

Example 6.4

The points D, E and F are taken on the sides AB, BC and CA *of* a $\triangle ABC$ respectively, such that $DE \parallel AC$ and $FE \parallel AB$.

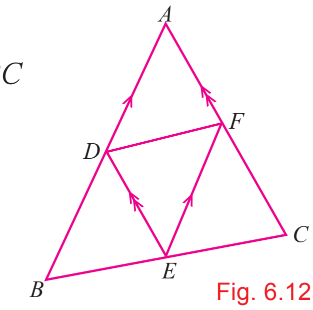


Fig. 6.12

Prove that $\dfrac{AB}{AD} = \dfrac{AC}{FC}$

Solution Given that in $\triangle ABC$, $DE \parallel AC$.

$$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC} \quad \text{(Thales theorem)} \qquad (1)$$

Also, given that $FE \parallel AB$.

$$\therefore \quad \frac{BE}{EC} = \frac{AF}{FC} \quad \text{(Thales theorem)} \qquad (2)$$

From (1) and (2), we get

$$\frac{BD}{AD} = \frac{AF}{FC}$$

$$\Longrightarrow \quad \frac{BD + AD}{AD} = \frac{AF + FC}{FC} \quad \text{(componendo rule)}$$

Thus, $$\frac{AB}{AD} = \frac{AC}{FC}.$$

Example 6.5

In $\triangle ABC$, the internal bisector AD of $\angle A$ meets the side BC at D. If $BD = 2.5$ cm, $AB = 5$ cm and $AC = 4.2$ cm, then find DC.

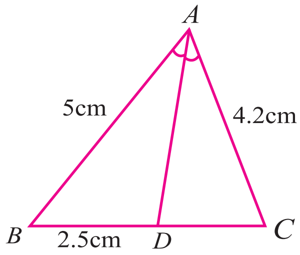


Fig. 6.13

Solution In $\triangle ABC$, AD is the internal bisector of $\angle A$.

$$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC} \quad \text{(angle bisector theorem)}$$

$$\Longrightarrow \quad DC = \frac{BD \times AC}{AB}$$

Thus, $$DC = \frac{2.5 \times 4.2}{5} = 2.1\text{cm}.$$

Example 6.6

In $\triangle ABC$, AE is the external bisector of $\angle A$, meeting BC produced at E. If $AB = 10$ cm, $AC = 6$ cm and $BC = 12$ cm, then find CE.

Solution In $\triangle ABC$, AE is the external bisector of $\angle A$ meeting BC produced at E.

Let $CE = x$ cm. Now, by the angle bisector theorem, we have

$$\frac{BE}{CE} = \frac{AB}{AC} \Longrightarrow \frac{12 + x}{x} = \frac{10}{6}$$

$$\Longrightarrow 3(12 + x) = 5x.$$

$$\Longrightarrow x = 18.$$

Hence, $CE = 18$ cm.

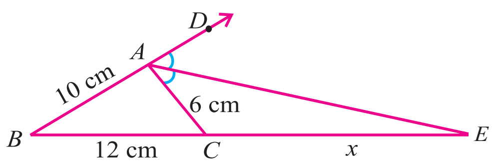


Fig. 6.14

Example 6.7

D is the midpoint of the side BC of $\triangle ABC$. If P and Q are points on AB and on AC such that DP bisects $\angle BDA$ and DQ bisects $\angle ADC$, then prove that $PQ \parallel BC$.

Solution In $\triangle ABD$, DP is the angle bisector of $\angle BDA$.

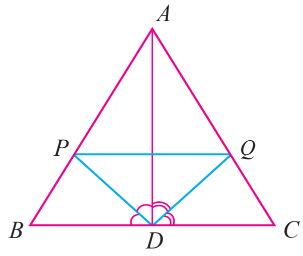


Fig. 6.15

$\therefore \quad \dfrac{AP}{PB} = \dfrac{AD}{BD}$ (angle bisector theorem) (1)

In $\triangle ADC$, DQ is the bisector of $\angle ADC$

$\therefore \quad \dfrac{AQ}{QC} = \dfrac{AD}{DC}$ (angle bisector theorem) (2)

But, $BD = DC$ (D is the midpoint of BC)

Now (2) $\Longrightarrow \dfrac{AQ}{QC} = \dfrac{AD}{BD}$ (3)

From (1) and (3) we get,

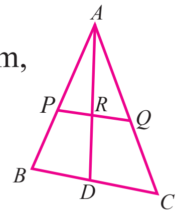
$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Thus, $PQ \parallel BC$. (converse of Thales theorem)

Exercise 6.1

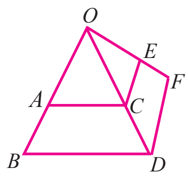
1. In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.
   (i) If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, then find AC.
   (ii) If $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm, then find CE.
   (iii) If $AD = 4x–3$, $BD = 3x–1$, $AE = 8x–7$ and $EC = 5x–3$, then find the value of x.

2. In the figure, $AP = 3$ cm, $AR = 4.5$ cm, $AQ = 6$ cm, $AB = 5$ cm, and $AC = 10$ cm. Find the length of AD.



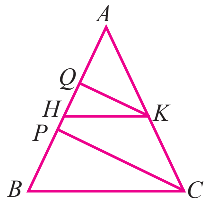
3. E and F are points on the sides PQ and PR respectively, of a $\triangle PQR$. For each of the following cases, verify $EF \parallel QR$.
   (i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm.
   (ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm.

4. In the figure, $AC \parallel BD$ and $CE \parallel DF$. If $OA = 12$cm, $AB = 9$ cm, $OC = 8$ cm and $EF = 4.5$ cm, then find FO.

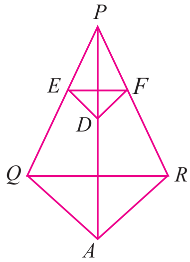


5. $ABCD$ is a quadrilateral with AB parallel to CD. A line drawn parallel to AB meets AD at P and BC at Q. Prove that $\frac{AP}{PD} = \frac{BQ}{QC}$.

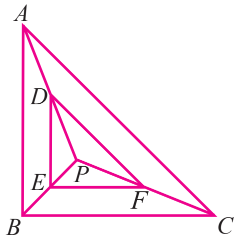
6. In the figure, $PC \parallel QK$ and $BC \parallel HK$. If $AQ = 6$ cm, $QH = 4$ cm, $HP = 5$ cm, $KC = 18$cm, then find AK and PB.



7. In the figure, $DE \parallel AQ$ and $DF \parallel AR$
Prove that $EF \parallel QR$.



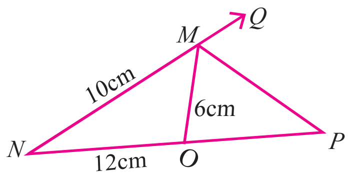
8. In the figure
$DE \parallel AB$ and $DF \parallel AC$.
Prove that $EF \parallel BC$.



9. In a $\triangle ABC$, AD is the internal bisector of $\angle A$, meeting BC at D.
   (i) If $BD = 2$ cm, $AB = 5$ cm, $DC = 3$ cm find AC.
   (ii) If $AB = 5.6$ cm, $AC = 6$ cm and $DC = 3$ cm find BC.
   (iii) If $AB = x$, $AC = x–2$, $BD = x+2$ and $DC = x–1$ find the value of x.

10. Check whether AD is the bisector of $\angle A$ of $\triangle ABC$ in each of the following.
   (i) $AB = 4$ cm, $AC = 6$ cm, $BD = 1.6$ cm, and $CD = 2.4$ cm.
   (ii) $AB = 6$ cm, $AC = 8$cm, $BD = 1.5$cm and $CD = 3$cm.

11. In a $\triangle MNO$, MP is the external bisector of $\angle M$ meeting NO produced at P. If $MN = 10$ cm, $MO = 6$ cm, $NO = 12$ cm, then find OP.



12. In a quadrilateral $ABCD$, the bisectors of $\angle B$ and $\angle D$ intersect on AC at E. Prove that $\frac{AB}{BC} = \frac{AD}{DC}$.

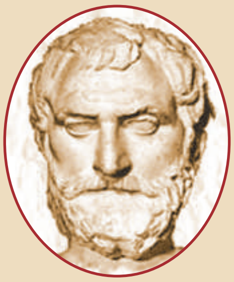
13. The internal bisector of $\angle A$ of ΔABC meets BC at D and the external bisector of $\angle A$ meets BC produced at E. Prove that $\frac{BD}{BE} = \frac{CD}{CE}$.

14. $ABCD$ is a quadrilateral with $AB = AD$. If AE and AF are internal bisectors of $\angle BAC$ and $\angle DAC$ respectively, then prove that $EF \parallel BD$.

6.3 Similar triangles

In class VIII, we have studied congruence of triangles in detail. We have learnt that two geometrical figures are congruent if they have the same size and shape. In this section, we shall study about those geometrical figures which have the same shape but not necessarily the same size. Such geometrical figures are called similar.

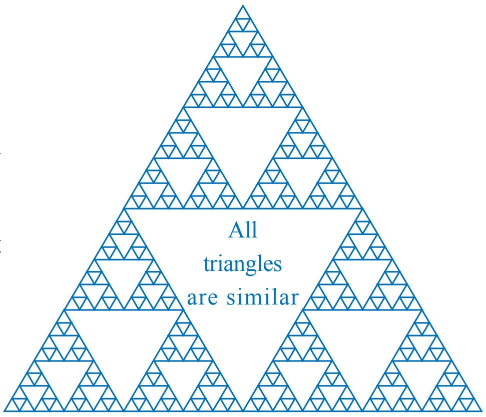
On looking around us, we see many objects which are of the same shape but of same or different sizes. For example, leaves of a tree have almost the same shapes but same or different sizes. Similarly photographs of different sizes developed from the same negative are of same shape but different sizes. All those objects which have the same shape but different sizes are called similar objects.



Thales of Miletus
(624-546 BC)
Greece

Thales was the first known philosopher, scientist and mathematician. He is credited with the first use of deductive reasoning applied to geometry. He discovered many prepositions in geometry. His method of attacking problems invited the attention of many mathematicians. He also predicted an eclipse of the Sun in 585 BC.

Thales said to have introduced Geometry in Greece, is believed to have found the heights of the Pyramids in Egypt, using shadows and the principle of similar triangles. Thus the use of similar triangles has made possible the measurements of heights and distances. He observed that the base angles of an isosceles triangle are equal. He used the idea of similar triangles and right triangles in practical geometry.



It is clear that the congruent figures are similar but the converse need not be true. In this section, we shall discuss only the similarity of triangles and apply this knowledge in solving problems. The following simple activity helps us to visualize similar triangles.

Activity

- Take a cardboard and make a triangular hole in it.
- Expose this cardboard to Sunlight at about one metre above the ground .
- Move it towards the ground to see the formation of a sequence of triangular shapes on the ground.
- Moving close to the ground, the image becomes smaller and smaller. Moving away from the ground, the image becomes larger and larger.
- You see that, the size of the angles forming the three vertices of the triangle would always be the same, even though their sizes are different.

Definition

Two triangles are similar if

(i) their corresponding angles are equal (or)

(ii) their corresponding sides have lengths in the same ratio (or proportional), which is equivalent to saying that one triangle is an enlargement of other.

Thus, two triangles $\triangle ABC$ and $\triangle DEF$ are similar if

(i) $\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$ (or)

(ii) $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$.

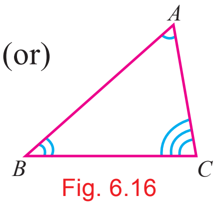


Fig. 6.16

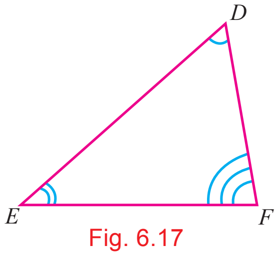


Fig. 6.17

Here, the vertices A, B and C correspond to the vertices D, E and F respectively. Symbolically, we write the similarity of these two triangles as $\triangle ABC \sim \triangle DEF$ and read it as $\triangle ABC$ is similar to $\triangle DEF$. The symbol '$\sim$' stands for 'is similar to'.

Remarks

Similarity of $\triangle ABC$ and $\triangle DEF$ can also be expressed symbolically using correct correspondence of their vertices as $\triangle BCA \sim \triangle EFD$ and $\Delta CAB \sim \Delta FDE$.

6.3.1 Criteria for similarity of triangles

The following three criteria are sufficient to prove that two triangles are similar.

(i) AA(Angle-Angle) similarity criterion

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

Remark If two angles of a triangle are respectively equal to two angles of another triangle then their third angles will also be equal. Therefore AA similarity criterion is also referred to AAA criteria.

(ii) SSS (Side-Side-Side) similarity criterion for Two Triangles

In two triangles, if the sides of one triangle are proportional (in the same ratio) to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

(iii) SAS (Side-Angle-Side) similarity criterion for Two Triangles

If one angle of a triangle is equal to one angle of the other triangle and if the corresponding sides including these angles are proportional, then the two triangles are similar.

Let us list out a few results without proofs on similarity of triangles.

(i) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

(ii) If a perpendicular is drawn from the vertex of a right angled triangle to its hypotenuse, then the triangles on each side of the perpendicular are similar to the whole triangle.

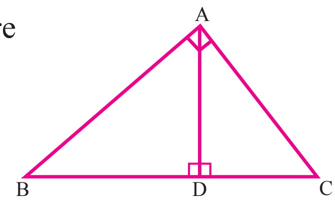


Fig. 6.18

Here, (a) $\Delta\, DBA \sim \Delta\, ABC$

(b) $\Delta\, DAC \sim \Delta\, ABC$

(c) $\Delta\, DBA \sim \Delta\, DAC$

(iii) If two triangles are similar, then the ratio of the corresponding sides is equal to the ratio of their corresponding altitudes.

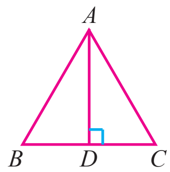


Fig. 6.19

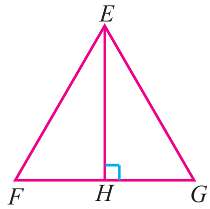


Fig. 6.20

i.e., if $\Delta ABC \sim \Delta EFG$, then $\dfrac{AB}{EF} = \dfrac{BC}{FG} = \dfrac{CA}{GE} = \dfrac{AD}{EH}$

(iv) If two triangles are similar, then the ratio of the corresponding sides is equal to the ratio of the corresponding perimeters.

If, $\Delta\, ABC \sim \Delta\, DEF$, then $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD} = \dfrac{AB + BC + CA}{DE + EF + FD}$.

Example 6.8

In ΔPQR, $AB \parallel QR$. If AB is 3 cm, PB is 2cm and PR is 6 cm, then find the length of QR.

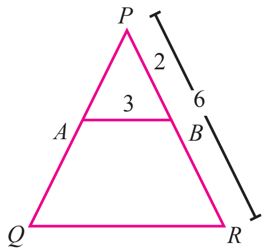


Fig. 6.21

Solution Given AB is 3 cm, PB is 2 cm PR is 6 cm and $AB \parallel QR$

In ΔPAB and ΔPQR

$\angle PAB = \angle PQR$ (corresponding angles)

and $\angle P$ is common.

$\therefore\ \Delta PAB \sim \Delta PQR$ (AA similarity criterion)

Since corresponding sides are proportional,

$$\frac{AB}{QR} = \frac{PB}{PR}$$

$$QR = \frac{AB \times PR}{PB}$$

$$= \frac{3 \times 6}{2}$$

Thus, $QR = 9$ cm.

Example 6.9

A man of height 1.8 m is standing near a Pyramid. If the shadow of the man is of length 2.7 m and the shadow of the Pyramid is 210 m long at that instant, find the height of the Pyramid.

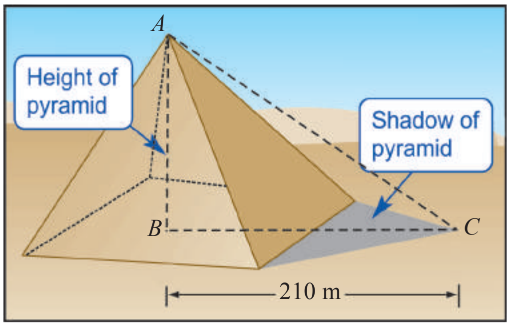


Fig. 6.22

Solution Let AB and DE be the heights of the Pyramid and the man respectively.

Let BC and EF be the lengths of the shadows of the Pyramid and the man respectively.

In ΔABC and ΔDEF, we have

$$\angle ABC = \angle DEF = 90^{\circ}$$

$$\angle BCA = \angle EFD$$

(angular elevation is same at the same instant)

$\therefore \Delta ABC \sim \Delta DEF$ (AA similarity criterion)

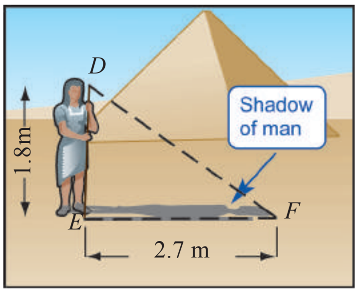


Fig. 6.23

Thus, $\dfrac{AB}{DE} = \dfrac{BC}{EF}$

$$\Longrightarrow \frac{AB}{1.8} = \frac{210}{2.7} \Longrightarrow AB = \frac{210}{2.7} \times 1.8 = 140.$$

Hence, the height of the Pyramid is 140 m.

Example 6.10

A man sees the top of a tower in a mirror which is at a distance of 87.6 m from the tower. The mirror is on the ground, facing upward. The man is 0.4 m away from the mirror, and the distance of his eye level from the ground is 1.5 m. How tall is the tower? (The foot of man, the mirror and the foot of the tower lie along a straight line).

Solution Let AB and ED be the heights of the man and the tower respectively. Let C be the point of incidence of the tower in the mirror.

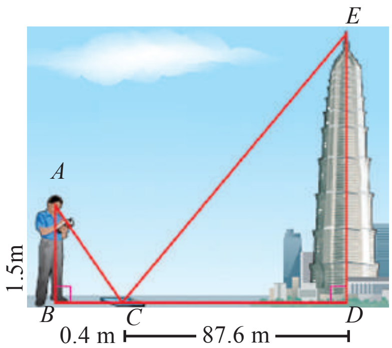


Fig. 6.24

In ΔABC and ΔEDC, we have

$$\angle ABC = \angle EDC = 90^{\circ}$$

$$\angle BCA = \angle DCE$$

(angular elevation is same at the same instant. i.e., the angle of incidence and the angle of reflection are same.)

$\therefore \quad \Delta ABC \sim \Delta EDC$ (AA similarity criterion)

Thus, $\dfrac{ED}{AB} = \dfrac{DC}{BC}$ (corresponding sides are proportional)

$$ED = \frac{DC}{BC} \times AB = \frac{87.6}{0.4} \times 1.5 = 328.5$$

Hence, the height of the tower is 328.5 m.

Example 6.11

The image of a tree on the film of a camera is of length 35 mm, the distance from the lens to the film is 42 mm and the distance from the lens to the tree is 6 m. How tall is the portion of the tree being photographed?

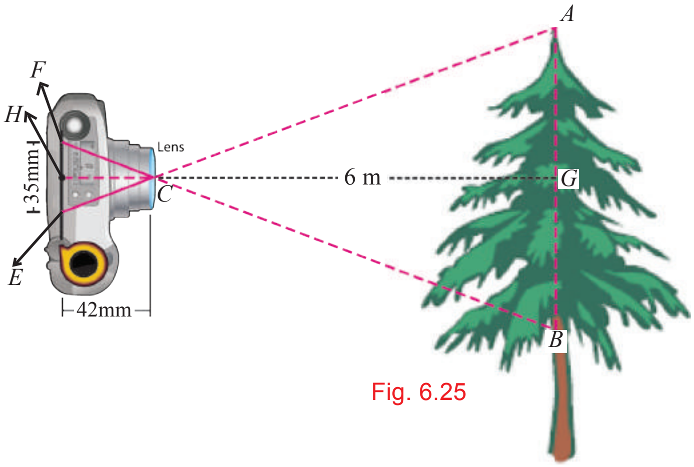


Fig. 6.25

Solution Let AB and EF be the heights of the portion of the tree and its image on the film respectively.

Let the point C denote the lens.

Let CG and CH be altitudes of ΔACB and ΔFEC.

Clearly, AB || FE.

In ΔACB and ΔFEC,

$\angle BAC = \angle FEC$

$\angle ECF = \angle ACB$ (vertically opposite angles)

$\therefore \quad \Delta ACB \sim \Delta ECF$ (AA criterion)

Thus, $\dfrac{AB}{EF} = \dfrac{CG}{CH}$

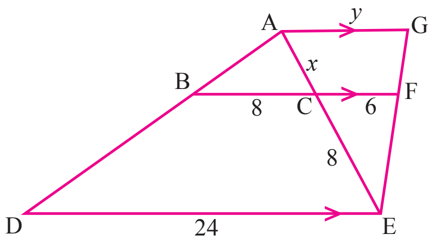
$\Longrightarrow \quad AB = \dfrac{CG}{CH} \times EF = \dfrac{6 \times 0.035}{0.042} = 5.$

Hence, the height of the tree photographed is 5m.

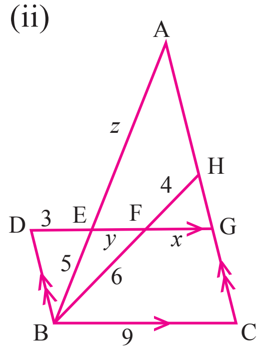
Exercise 6.2

1. Find the unknown values in each of the following figures. All lengths are given in centimetres. (All measures are not in scale)

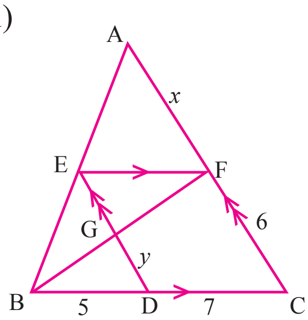
(i)



(ii)



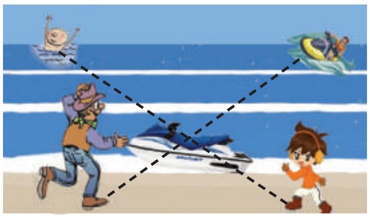
(iii)



2. The image of a man of height 1.8 m, is of length 1.5 cm on the film of a camera. If the film is 3 cm from the lens of the camera, how far is the man from the camera?

3. A girl of height 120 cm is walking away from the base of a lamp-post at a speed of 0.6 m/sec. If the lamp is 3.6 m above the ground level, then find the length of her shadow after 4 seconds.

4. A girl is in the beach with her father. She spots a swimmer drowning. She shouts to her father who is 50 m due west of her. Her father is 10 m nearer to a boat than the girl. If her father uses the boat to reach the swimmer, he has to travel a distance 126 m from that boat. At the same time, the girl spots a man riding a water craft who is 98 m away from the boat. The man on the water craft is due east of the swimmer. How far must the man travel to rescue the swimmer? (Hint : see figure). (Not for the examination)



5. P and Q are points on sides AB and AC respectively, of ΔABC. If $AP = 3$ cm, $PB = 6$ cm, $AQ = 5$ cm and $QC = 10$ cm, show that $BC = 3\,PQ$.

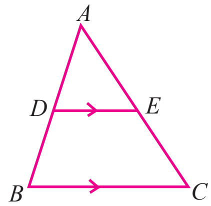
6. In ΔABC, $AB = AC$ and $BC = 6$ cm. D is a point on the side AC such that $AD = 5$ cm and $CD = 4$ cm. Show that $\Delta BCD \sim \Delta ACB$ and hence find BD.

7. The points D and E are on the sides AB and AC *of* ΔABC respectively, such that $DE \parallel BC$. If $AB = 3\,AD$ and the area of ΔABC is 72 cm^2, then find the area of the quadrilateral $DBCE$.

8. The lengths of three sides of a triangle ABC are 6 cm, 4 cm and 9 cm. $\triangle PQR \sim \triangle ABC$. One of the lengths of sides of $\triangle PQR$ is 35cm. What is the greatest perimeter possible for $\triangle PQR$?

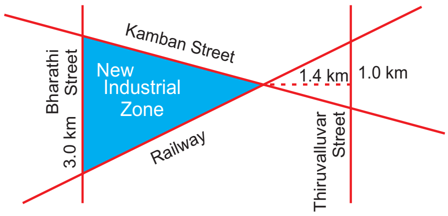
9. In the figure, $DE \parallel BC$ and $\dfrac{AD}{BD} = \dfrac{3}{5}$, calculate the value of

(i) $\dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC}$, (ii) $\dfrac{\text{area of trapezium } BCED}{\text{area of } \Delta ABC}$

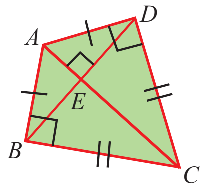


10. The government plans to develop a new industrial zone in an unused portion of land in a city.

The shaded portion of the map shown on the right, indicates the area of the new industrial zone. Find the area of the new industrial zone.

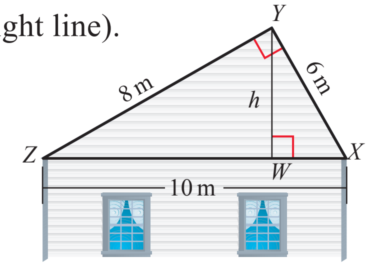


11. A boy is designing a diamond shaped kite, as shown in the figure where $AE = 16$ cm, $EC = 81$ cm. He wants to use a straight cross bar BD. How long should it be?



12. A student wants to determine the height of a flagpole. He placed a small mirror on the ground so that he can see the reflection of the top of the flagpole. The distance of the mirror from him is 0.5 m and the distance of the flagpole from the mirror is 3 m. If his eyes are 1.5 m above the ground level, then find the height of the flagpole.(The foot of student, mirror and the foot of flagpole lie along a straight line).

13. A roof has a cross section as shown in the diagram,
(i) Identify the similar triangles
(ii) Find the height h of the roof.



Theorem 6.5 Pythagoras theorem (Baudhayan theorem)

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : In a right angled $\triangle ABC$, $\angle A = 90^\circ$.

To prove : $BC^2 = AB^2 + AC^2$

Construction : Draw $AD \perp BC$

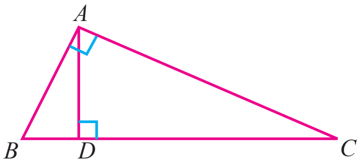


Fig. 6.26

Proof

In triangles ABC and DBA, $\angle B$ is the common angle.

Also, we have $\angle BAC = \angle ADB = 90^\circ$.

$\therefore \triangle ABC \sim \triangle DBA$ (AA similarity criterion)

Thus, their corresponding sides are proportional.

Hence, $$\frac{AB}{DB} = \frac{BC}{BA}$$

$$\therefore \quad AB^2 = DB \times BC \qquad (1)$$

Similarly, we have $\triangle ABC \sim \triangle DAC$.

Thus, $$\frac{BC}{AC} = \frac{AC}{DC}$$

$$\therefore \quad AC^2 = BC \times DC \qquad (2)$$

Adding (1) and (2) we get,

$$AB^2 + AC^2 = BD \times BC + BC \times DC$$
$$= BC(BD + DC)$$
$$= BC \times BC = BC^2$$

Thus, $BC^2 = AB^2 + AC^2$. Hence the Pythagoras theorem.

Remarks

The Pythagoras theorem has two fundamental aspects; one is about areas and the other is about lengths. Hence this landmark thorem connects Geometry and Algebra.The converse of Pythagoras theorem is also true. It was first mentioned and proved by Euclid.

The statement is given below. (Proof is left as an exercise.)

Theorem 6.6 **Converse of Pythagoras theorem**

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

6.4 Circles and Tangents

A straight line associated with circles is a tangent line which touches the circle at just one point. In geometry, tangent lines to circles play an important role in many geometrical constructions and proofs. . In this section, let us state some results based on circles and tangents and prove an important theorem known as Tangent-Chord thorem. If we consider a straight line and a circle in a plane, then there are three possibilities- they may not intersect at all, they may intersect at two points or they may touch each other at exactly one point. Now look at the following figures.

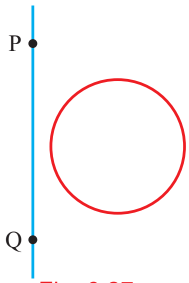


Fig. 6.27

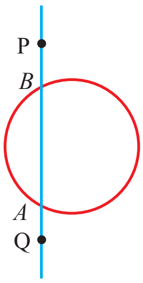


Fig. 6.28

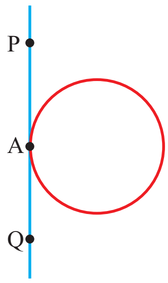


Fig. 6.29

In Fig. 6.27, the circle and the straight line PQ have no common point.

In Fig. 6.28, the straight line PQ cuts the circle at two distinct points A and B. In this case, PQ is called a secant to the circle.

In Fig. 6.29, the straight line PQ and the circle have exactly one common point. Equivalently the straight line touches the circle at only one point. The straight line PQ is called the tangent to the circle at A.

Definition

A straight line which touches a circle at only one point is called a tangent to the circle and the point at which it touches the circle is called its point of contact.

Theorems based on circles and tangents (without proofs)

1. A tangent at any point on a circle is perpendicular to the radius through the point of contact .
2. Only one tangent can be drawn at any point on a circle. However, from an exterior point of a circle two tangents can be drawn to the circle.
3. The lengths of the two tangents drawn from an exterior point to a circle are equal.
4. If two circles touch each other, then the point of contact of the circles lies on the line joining the centres.
5. If two circles touch externally, the distance between their centres is equal to the sum of their radii.
6. If two circles touch internally, the distance between their centres is equal to the difference of their radii.

Theorem 6.7 Tangent-Chord theorem

If from the point of contact of tangent (of a circle), a chord is drawn, then the angles which the chord makes with the tangent line are equal respectively to the angles formed by the chord in the corresponding alternate segments.

Given : O is the centre of the circle. ST is the tangent at A, and AB is a chord. P and Q are any two points on the circle in the opposite sides of the chord AB.

To prove : (i) $\angle BAT = \angle BPA$ (ii) $\angle BAS = \angle AQB$.

Construction: Draw the diameter AC of the circle. Join B and C.

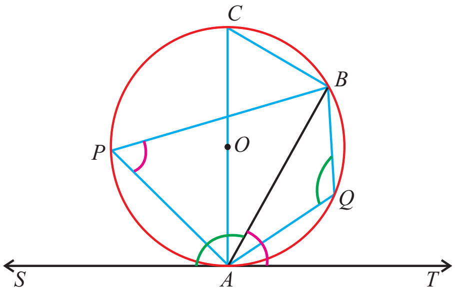


Fig. 6.30

Proof

Statement	Reason	
$\angle ABC = 90^\circ$	angle in a semi-circle is 90°	
$\angle CAB + \angle BCA = 90^\circ$	sum of two acute angles of a right $\triangle ABC$.	(1)
$\angle CAT = 90^\circ$	diameter is $\perp$ to the tangent at the point of contact.	
$\Longrightarrow \angle CAB + \angle BAT = 90^\circ$		(2)
$\angle CAB + \angle BCA = \angle CAB + \angle BAT$	from (1) and (2)	
$\Longrightarrow \quad \angle BCA = \angle BAT$		(3)

$\angle BCA = \angle BPA$	angles in the same segment	(4)
$\angle BAT = \angle BPA$. Hence (i).	from (3) and (4)	(5)
Now $\angle BPA + \angle AQB = 180^\circ$	opposite angles of a cyclic quadrilateral	
$\Longrightarrow \angle BAT + \angle AQB = 180^\circ$	from (5)	(6)
Also $\angle BAT + \angle BAS = 180^\circ$	linear pair	(7)
$\angle BAT + \angle AQB = \angle BAT + \angle BAS$	from (6) and (7)	

$\angle BAS = \angle AQB$. Hence (ii).

Thus, the Tangent -Chord theorem is proved.

Theorem 6.8 Converse of Tangent-Chord theorem

If in a circle, through one end of a chord, a straight line is drawn making an angle equal to the angle in the alternate segment, then the straight line is a tangent to the circle.

Definition

Let P be a point on a line segment AB. The product $PA \times PB$ represents the area of the rectangle whose sides are PA and PB.

This product is called the area of the rectangle contained by the parts PA and PB of the line segment AB.

Theorem 6.9

If two chords of a circle intersect either inside or outside of the circle, then the area of the rectangle contained by the segments of one chord is equal to the area of the rectangle contained by the segments of the other chord.

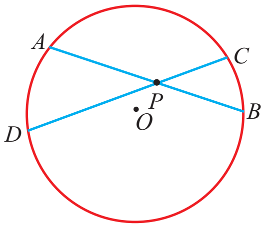


Fig. 6.31

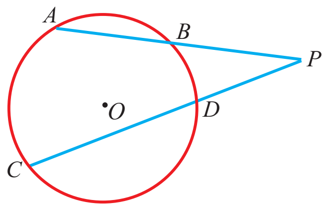


Fig. 6.32

In Fig.6.31, two chords AB and CD intersect at P inside the circle with centre at O. Then $PA \times PB = PC \times PD$. In Fig. 6.32, the chords AB and CD intersect at P outside the circle with centre O. Then $PA \times PB = PC \times PD$.

Example 6.12

Let PQ be a tangent to a circle at A and AB be a chord. Let C be a point on the circle such that $\angle BAC = 54^\circ$ and $\angle BAQ = 62^\circ$. Find $\angle ABC$..

Solution Since PQ is a tangent at A and AB is a chord, we have

$$\angle BAQ = \angle ACB = 62^\circ. \quad \text{(tangent-chord theorem)}$$

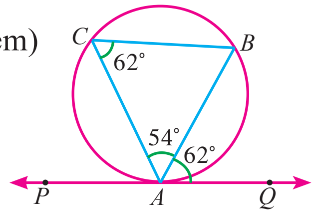


Fig. 6.33

Also, $\angle BAC + \angle ACB + \angle ABC = 180^\circ.$

(sum of all angles in a triangle is 180°)

Thus, $\angle ABC = 180^\circ - (\angle BAC + \angle ACB)$

Hence, $\angle ABC = 180^\circ - (54^\circ + 62^\circ) = 64^\circ.$

Example 6.13

Find the value of x in each of the following diagrams.

(i)

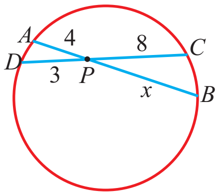


Fig. 6.34

(ii)

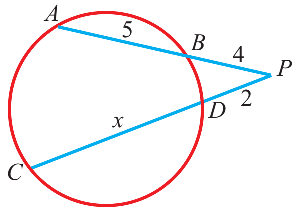


Fig. 6.35

Solution (i) We have $PA \,.\, PB = PC \,.\, PD$

$$PB = \frac{PC.PD}{PA}$$

Thus, $x = \dfrac{8 \times 3}{4} = 6.$

(ii) We have $PC \,.\, PD = PA.\, PB$

$$(2+x)\, 2 = 9 \times 4$$

$x + 2 = 18$. Thus, $x = 16$.

Example 6.14

In the figure, tangents PA and PB are drawn to a circle with centre O from an external point P. If CD is a tangent to the circle at E and $AP = 15$ cm, find the perimeter of ΔPCD

Solution We know that the lengths of the two tangents from an exterior point to a circle are equal.

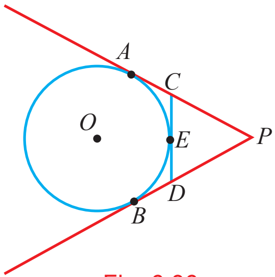


Fig. 6.36

$\therefore \; CA = CE, \; DB = DE$ and $PA = PB$.

Now, the perimeter of $\Delta PCD = PC + CD + DP$

$= PC + CE + ED + DP$

$= PC + CA + DB + DP$

$= PA + PB = 2\,PA \qquad (PB = PA)$

Thus, the perimeter of $\Delta PCD = 2 \times 15 = 30$ cm.

Example 6.15

$ABCD$ is a quadrilateral such that all of its sides touch a circle. If $AB = 6$ cm, $BC = 6.5$ cm and $CD = 7$ cm, then find the length of AD.

Solution Let P, Q, R and S be the points where the circle touches the quadrilateral. We know that the lengths of the two tangents drawn from an exterior point to a circle are equal.

Thus, we have, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$.

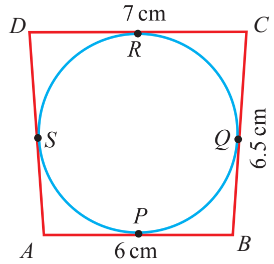


Fig. 6.37

Hence, $AP + BP + CR + DR = AS + BQ + CQ + DS$

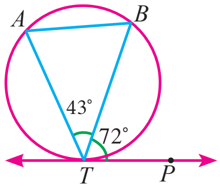
$\Longrightarrow \quad AB + CD = AD + BC.$

$\Longrightarrow \quad AD = AB + CD - BC$

$= 6 + 7 - 6.5 = 6.5$

Thus, $AD = 6.5$ cm.

Exercise 6.3



1. In the figure TP is a tangent to a circle. A and B are two points on the circle. If $\angle BTP = 72^\circ$ and $\angle ATB = 43^\circ$ find $\angle ABT$.

2. AB and CD are two chords of a circle which intersect each other internally at P. (i) If $CP = 4$ cm, $AP = 8$ cm, $PB = 2$ cm, then find PD.

   (ii) If $AP = 12$ cm, $AB = 15$ cm, $CP = PD$, then find CD

3. AB and CD are two chords of a circle which intersect each other externally at P

   (i) If $AB = 4$ cm $BP = 5$ cm and $PD = 3$ cm, then find CD.

   (ii) If $BP = 3$ cm, $CP = 6$ cm and $CD = 2$ cm, then find AB

4. A circle touches the side BC of ΔABC at P, AB and AC produced at Q and R respectively, prove that $AQ = AR = \frac{1}{2}$(perimeter of ΔABC)

5. If all sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

6. A lotus is 20 cm above the water surface in a pond and its stem is partly below the water surface. As the wind blew, the stem is pushed aside so that the lotus touched the water 40 cm away from the original position of the stem. How much of the stem was below the water surface originally?

7. A point O in the interior of a rectangle $ABCD$ is joined to each of the vertices A, B, C and D. Prove that $OA^2 + OC^2 = OB^2 + OD^2$

Exercise 6.4

Choose the correct answer

1. If a straight line intersects the sides AB and AC of a $\triangle ABC$ at D and E respectively and is parallel to BC, then $\frac{AE}{AC} =$

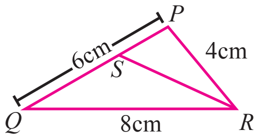
   (A) $\frac{AD}{DB}$ (B) $\frac{AD}{AB}$ (C) $\frac{DE}{BC}$ (D) $\frac{AD}{EC}$

2. In $\triangle ABC$, DE is $\parallel$ to BC, meeting AB and AC at D and E.

If $AD = 3$ cm, $DB = 2$ cm and $AE = 2.7$ cm, then AC is equal to

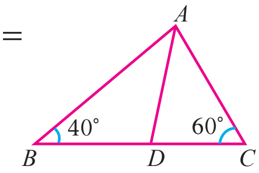
(A) 6.5 cm (B) 4.5 cm (C) 3.5 cm (D) 5.5 cm

3. In $\triangle PQR$, RS is the bisector of $\angle R$. If $PQ = 6$ cm, $QR = 8$ cm, $RP = 4$ cm then PS is equal to



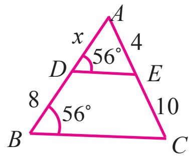
(A) 2 cm (B) 4 cm (C) 3 cm (D) 6 cm

4. In figure, if $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 40^\circ$, and $\angle C = 60^\circ$, then $\angle BAD =$



(A) 30° (B) 50° (C) 80° (D) 40°

5. In the figure, the value x is equal to



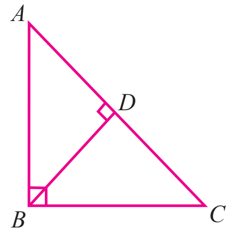
(A) $4 \cdot 2$ (B) $3 \cdot 2$

(C) $0 \cdot 8$ (D) $0 \cdot 4$

6. In triangles ABC and DEF, $\angle B = \angle E$, $\angle C = \angle F$, then

(A) $\frac{AB}{DE} = \frac{CA}{EF}$ (B) $\frac{BC}{EF} = \frac{AB}{FD}$ (C) $\frac{AB}{DE} = \frac{BC}{EF}$ (D) $\frac{CA}{FD} = \frac{AB}{EF}$

7. From the given figure, identify the wrong statement.



(A) $\Delta ADB \sim \Delta ABC$ (B) $\Delta ABD \sim \Delta ABC$

(C) $\Delta BDC \sim \Delta ABC$ (D) $\Delta ADB \sim \Delta BDC$

8. If a vertical stick 12 m long casts a shadow 8 m long on the ground and at the same time a tower casts a shadow 40 m long on the ground, then the height of the tower is

(A) 40 m (B) 50 m (C) 75 m (D) 60 m

9. The sides of two similar triangles are in the ratio 2:3, then their areas are in the ratio

(A) 9:4 (B) 4:9 (C) 2:3 (D) 3:2

10. Triangles ABC and DEF are similar. If their areas are 100 cm^2 and 49 cm^2 respectively and BC is 8.2 cm then EF =

(A) 5.47 cm (B) 5.74 cm (C) 6.47 cm (D) 6.74 cm

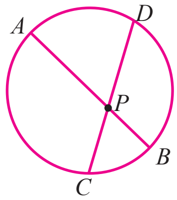
11. The perimeters of two similar triangles are 24 cm and 18 cm respectively. If one side of the first triangle is 8 cm, then the corresponding side of the other triangle is

(A) 4 cm (B) 3 cm (C) 9 cm (D) 6 cm

12. AB and CD are two chords of a circle which when produced to meet at a point P such that $AB = 5$ cm, $AP = 8$ cm, and $CD = 2$ cm then $PD =$

(A) 12 cm (B) 5 cm (C) 6 cm (D) 4 cm

13. In the adjoining figure, chords AB and CD intersect at P. If $AB = 16$ cm, $PD = 8$ cm, $PC = 6$ and AP >PB, then AP =

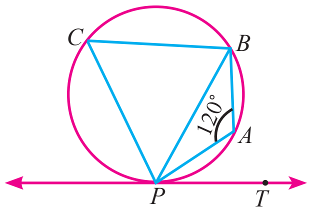


(A) 8 cm (B) 4 cm (C) 12 cm (D) 6 cm

14. A point P is 26 cm away from the centre O of a circle and PT is the tangent drawn from P to the circle is 10 cm, then OT is equal to

(A) 36 cm (B) 20 cm (C) 18 cm (D) 24 cm

15. In the figure, if $\angle PAB = 120^\circ$ then $\angle BPT =$

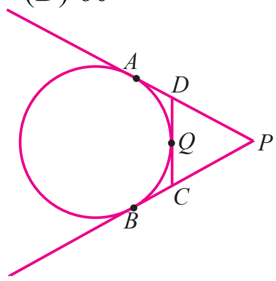


(A) 120° (B) 30° (C) 40° (D) 60°

16. If the tangents PA and PB from an external point P to circle with centre O are inclined to each other at an angle of 40°, then $\angle POA =$

(A) 70° (B) 80° (C) 50° (D) 60°

17. In the figure, PA and PB are tangents to the circle drawn from an external point P. Also CD is a tangent to the circle at Q. If $PA = 8$ cm and $CQ = 3$ cm, then PC is equal to



(A) 11 cm (B) 5 cm (C) 24 cm (D) 38 cm

18. ΔABC is a right angled triangle where $\angle B = 90^\circ$ and $BD \perp AC$. If BD = 8 cm, $AD = 4$ cm, then CD is

(A) 24 cm (B) 16 cm (C) 32 cm (D) 8 cm

19. The areas of two similar triangles are 16 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 3 cm, then the corresponding altitude of the other triangle is

(A) 6.5 cm (B) 6 cm (C) 4 cm (D) 4.5 cm

20. The perimeter of two similar triangles ΔABC and ΔDEF are 36 cm and 24 cm respectively. If $DE = 10$ cm, then AB is

(A) 12 cm (B) 20 cm (C) 15 cm (D) 18 cm