8

MENSURATION

Measure what is measurable, and make measurable what is not so

-Galileo Galilei

- Introduction
- Surface area and volume
  - Cylinder
  - Cone
  - Sphere
- Combined figures and invariant volumes

Archimedes

(287 BC - 212 BC)

Greece

Archimedes is remembered as the greatest mathematician of the ancient era.

He contributed significantly in geometry regarding the areas of plane figures and the areas as well as volumes of curved surfaces.

8.1 Introduction

The part of geometry which deals with measurement of lengths of lines, perimeters and areas of plane figures and surface areas and volumes of solid objects is called "Mensuration". The study of measurement of objects is essential because of its uses in many aspects of every day life. In elementary geometry one considers plane, multifaced surfaces as well as certain curved surfaces of solids (for example spheres).

"Surface Area to Volume" ratio has been widely acknowledged as one of the big ideas of Nanoscience as it lays the foundation for understanding size dependent properties that characterise Nanoscience scale and technology.

In this chapter, we shall learn how to find surface areas and volumes of solid objects such as cylinder, cone, sphere and combined objects

8.2 Surface Area

Archimedes of Syracuse, Sicily was a Greek Mathematician who proved that of the volume of a sphere is equal to two-thirds the volume of a circumscribed cylinder. He regarded this as his most vital achievement. He used the method of exhaustion to calculate the area under the arc of a parabola.

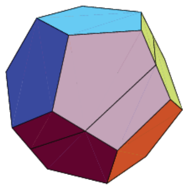


Fig. 8.1

Surface area is the measurement of exposed area of a solid object. Thus, the surface area is the area of all outside surfaces of a 3-dimensional object. The adjoining figures illustrate surface areas of some solids.

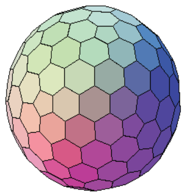


Fig. 8.2

8.2.1 Right Circular Cylinder

If we take a number of circular sheets of paper or cardboard of the same shape and size and stack them up in a vertical pile, then by this process, we shall obtain a solid object known as a Right Circular Cylinder. Note that it has been kept at right angles to the base, and the base is circular. (See Fig. 8.3)

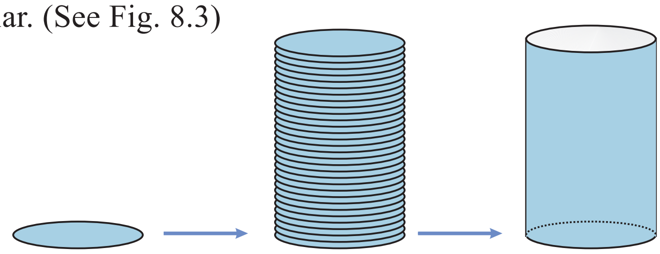


Fig. 8.3

Definition

If a rectangle revolves about its one side and completes a full rotation, the solid thus formed is called a right circular cylinder.

Activity

Let $ABCD$ be a rectangle. Assume that it revolves about its side AB and completes a full rotation. This revolution generates a right circular cylinder as shown in the figures. AB is called the axis of the cylinder. The length AB is the length or the height of the cylinder and AD or BC is called its radius.

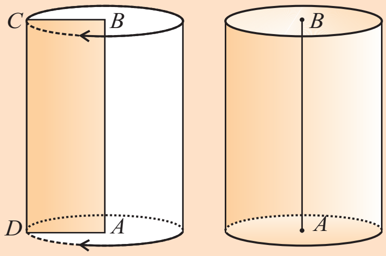


Fig. 8.4

Note

(i) If the base of a cylinder is not circular then it is called oblique cylinder.

(ii) If the base is circular but not perpendicular to the axis of the cylinder, then the cylinder is called circular cylinder.

(iii) If the axis is perpendicular to the circular base, then the cylinder is called right circular cylinder.

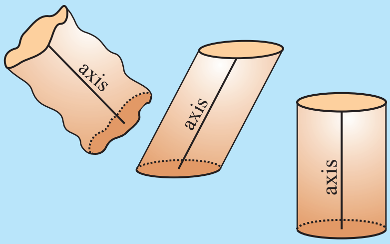


Fig. 8.5

(i) Curved Surface area of a solid right circular cylinder

In the adjoining figure, the bottom and top face of the right circular cylinder are concurrent circular regions, parallel to each other. The vertical surface of the cylinder is curved and hence its area is called the curved surface or lateral surface area of the cylinder.

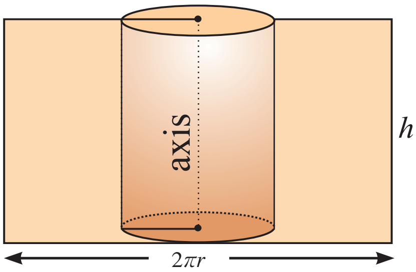


Fig. 8.6

Curved Surface Area of a cylinder, CSA = Circumference of the base $\times$ Height $= 2\pi r \times h$

$= 2\pi rh$ sq. units.

(ii) Total Surface Area of a solid right circular cylinder

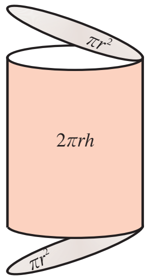


Fig. 8.7

Total Surface Area, TSA $=$ Area of the Curved Surface Area $+\ 2\times$ Base Area

$$= 2\pi rh + 2\times \pi r^2$$

Thus, TSA $= 2\pi r(h+r)$ sq.units.

(iii) Right circular hollow cylinder

Solids like iron pipe, rubber tube, etc., are in the shape of hollow cylinders. For a hollow cylinder of height h with external and internal radii R and r respectively,

we have, curved surface area, CSA $=$ External surface area $+$ Internal surface area

$$= 2\pi Rh + 2\pi rh$$

Thus, CSA $= 2\pi h(R+r)$ sq.units

Total surface area, TSA $=$ CSA $+\ 2\times$ Base area

$$= 2\pi h(R+r) + 2\times[\pi R^2 - \pi r^2]$$

$$= 2\pi h(R+r) + 2\pi(R+r)(R-r)$$

$\therefore$ TSA $= 2\pi(R+r)(R-r+h)$ sq.units.

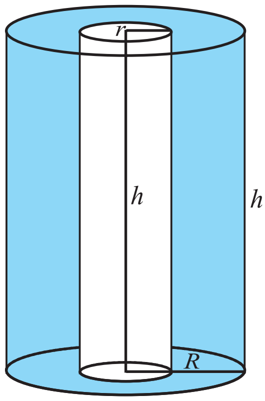


Fig. 8.8

Remark

Thickness of the hollow cylinder, w $= R - r$.

> *Note*
> In this chapter, for π we take an approximate value $\frac{22}{7}$ whenever it is required.

Example 8.1

A solid right circular cylinder has radius 7cm and height 20cm. Find its (i) curved surface area and (ii) total surface area. (Take $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the solid right circular cylinder respectively.

Given that $r = 7$cm and $h = 20$cm

Curved surface area, CSA $= 2\pi rh$

$$= 2\times\frac{22}{7}\times 7\times 20$$

Thus, the curved surface area $= 880$ sq.cm

Now, the total surface area $= 2\pi r(h+r)$

$$= 2\times\frac{22}{7}\times 7\times[20+7] = 44\times 27$$

Thus, the total surface area $= 1188$ sq.cm.

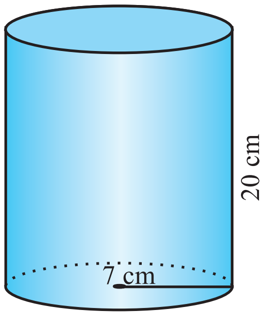


Fig. 8.9

Example 8.2

If the total surface area of a solid right circular cylinder is 880 sq.cm and its radius is 10 cm, find its curved surface area. (Take $\pi = \frac{22}{7}$)

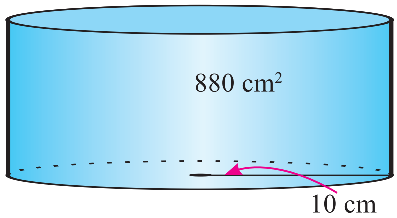


Fig. 8.10

Solution Let r and h be the radius and height of the solid right circular cylinder respectively.

Let S be the total surface area of the solid right circular cylinder.

Given that $r = 10$ cm and $S = 880$ cm^2

Now, $S = 880 \implies 2\pi r[h + r] = 880$

$$\implies \quad 2 \times \frac{22}{7} \times 10[h + 10] = 880$$

$$\implies \quad h + 10 = \frac{880 \times 7}{2 \times 22 \times 10}$$

$$\implies \quad h + 10 = 14$$

Thus, the height of the cylinder, $h = 4$ cm

Now, the curved surface area, CSA is

$$2\pi rh = 2 \times \frac{22}{7} \times 10 \times 4 = \frac{1760}{7}$$

Aliter :

$$\text{CSA} = \text{TSA} - 2\times \text{ Area of the base}$$
$$= 880 - 2 \times \pi r^2$$
$$= 880 - 2 \times \frac{22}{7} \times 10^2$$
$$= \frac{1760}{7} = 251\frac{3}{7} \text{ sq.cm.}$$

Thus, the curved surface area of the cylinder = $251\frac{3}{7}$ sq.cm.

Example 8.3

The ratio between the base radius and the height of a solid right circular cylinder is 2 : 5. If its curved surface area is $\frac{3960}{7}$ sq.cm, find the height and radius. (use $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the right circular cylinder respectively.

Given that $r : h = 2 : 5 \implies \frac{r}{h} = \frac{2}{5}$. Thus, $r = \frac{2}{5}h$

Now, the curved surface area, CSA = $2\pi rh$

$$\implies \quad 2 \times \frac{22}{7} \times \frac{2}{5} \times h \times h = \frac{3960}{7}$$

$$\implies \quad h^2 = \frac{3960 \times 7 \times 5}{2 \times 22 \times 2 \times 7} = 225$$

Thus, $h = 15 \implies r = \frac{2}{5}h = 6.$

Hence, the height of the cylinder is 15 cm and the radius is 6 cm.

Example 8.4

The diameter of a road roller of length 120 cm is 84 cm. If it takes 500 complete revolutions to level a playground, then find the cost of levelling it at the cost of 75 paise per square metre. (Take $\pi = \frac{22}{7}$)

Solution Given that $r = 42$ cm, $h = 120$ cm

$$\left.\begin{array}{r}\text{Area covered by the roller}\\ \text{in one revolution}\end{array}\right\} = \left\{\begin{array}{l}\text{Curved Surface Area}\\ \text{of the road roller.}\end{array}\right.$$

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 42 \times 120$$

$$= 31680 \text{ cm}^2.$$

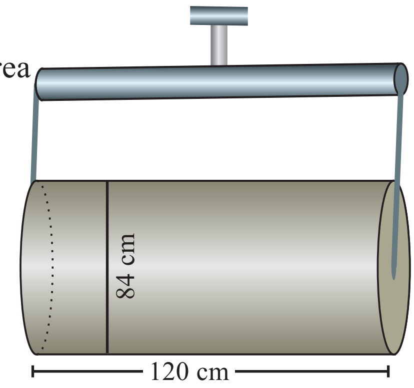


Fig. 8.11

$$\left.\begin{array}{r}\text{Area covered by the}\\ \text{roller in 500 revolutions}\end{array}\right\} = 31680 \times 500$$

$$= 15840000 \text{ cm}^2$$

$$= \frac{15840000}{10000} = 1584 \text{ m}^2 \qquad (10,000 \text{ cm}^2 = 1 \text{ sq.m})$$

Cost of levelling per 1sq.m. $= ₹ \frac{75}{100}$

Thus, cost of levelling the play ground $= \frac{1584 \times 75}{100} = ₹\ 1188.$

Example 8.5

The internal and external radii of a hollow cylinder are 12 cm and 18 cm respectively. If its height is 14cm, then find its curved surface area and total surface area. (Take $\pi = \frac{22}{7}$)

Solution Let r, R and h be the internal and external radii and the height of a hollow cylinder respectively.

Given that $r = 12$ cm, $R = 18$ cm, $h = 14$ cm

Now, curved surface area, CSA $= 2\pi h(R+r)$

Thus, CSA $= 2 \times \frac{22}{7} \times 14 \times (18 + 12)$

$= 2640$ sq.cm

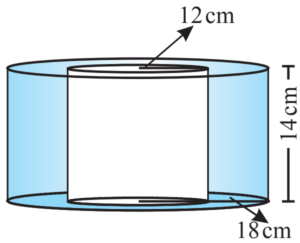


Fig. 8.12

Total surface area, TSA $= 2\pi(R + r)(R - r + h)$

$$= 2 \times \frac{22}{7} \times (18 + 12)(18 - 12 + 14)$$

$$= 2 \times \frac{22}{7} \times 30 \times 20 = \frac{26400}{7}.$$

Thus, the total surface area $= 3771\frac{3}{7}$ sq.cm.

8.2.2 Right Circular Cone

In our daily life we come across many solids or objects like ice cream container, the top of the temple car, the cap of a clown in a circus, the mehandi container. Mostly the objects mentioned above are in the shape of a right circular cone.

A cone is a solid object that tapers smoothly from a flat base to a point called vertex. In general, the base may not be of circular shape. Here, cones are assumed to be right circular, where right means that the axis that passes through the centre of the base is at right angles to its plane, and circular means that the base is a circle. In this section, let us define a right circular cone and find its surface area. One can visualise a cone through the following activity.

Activity

Take a thick paper and cut a right angled $\triangle ABC$, right angled at B. Paste a long thick string along one of the perpendicular sides say AB of the triangle. Hold the string with your hands on either side of the triangle and rotate the triangle about the string.

What happens? Can you recognize the shape formed on the rotation of the triangle around the string?. The shape so formed is a right circular cone.

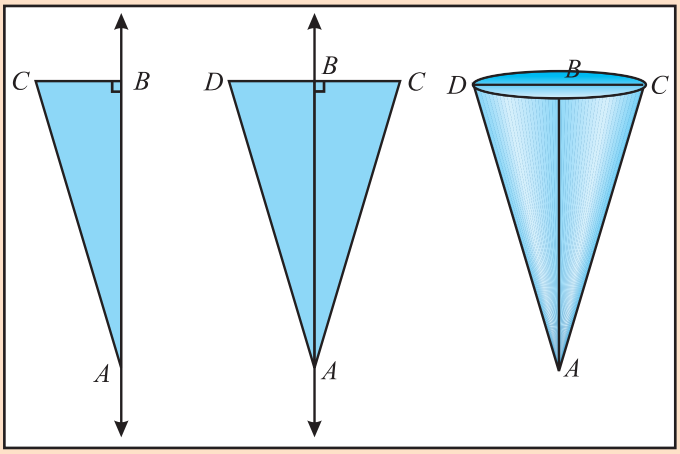


Fig. 8.13

If a right angled $\triangle ABC$ is revolved 360° about the side AB containing the right angle, the solid thus formed is called a right circular cone.

The length AB is called the height of the cone.

The length BC is called the radius of its base ($BC = r$). The length AC is called the slant height l of the cone ($AC = AD = l$).

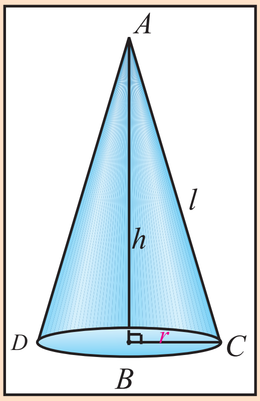


Fig. 8.14

In the right angled $\triangle ABC$

We have, $l = \sqrt{h^2 + r^2}$ (Pythagoras theorem)

$$h = \sqrt{l^2 - r^2}$$

$$r = \sqrt{l^2 - h^2}$$

Note

(i) If the base of a cone is not circular then, it is called **oblique cone.**

(ii) If the circular base is not perpendicular to the axis then, it is called **circular cone.**

(iii) If the vertex is directly above the centre of the circular base then, it is a **right circular cone.**

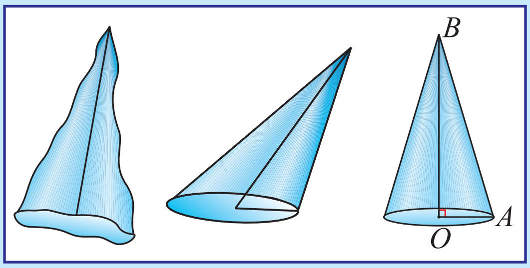


Fig. 8.15

(i) Curved surface area of a hollow cone

Let us consider a sector with radius l and central angle $\theta°$. Let L denote the length of the arc. Thus, $\frac{2\pi l}{L} = \frac{360°}{\theta°}$

$$\Longrightarrow \quad L = 2\pi l \times \frac{\theta°}{360°} \qquad (1)$$

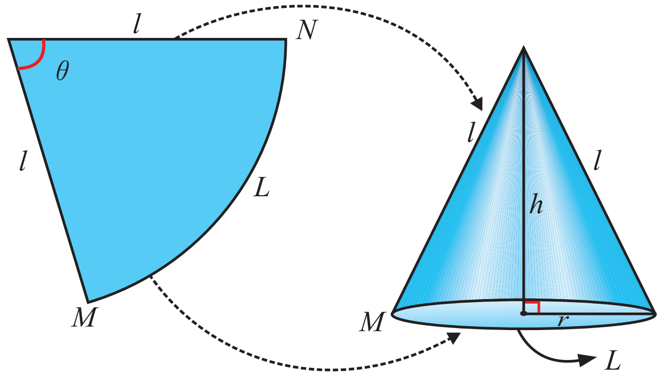


Fig. 8.16

Now, join the radii of the sector to obtain a right circular cone.

Let r be the radius of the cone.

Hence, $L = 2\pi r$

From (1) we obtain,

$$2\pi r = 2\pi l \times \frac{\theta°}{360°}$$

$$\Longrightarrow \quad r = l\left(\frac{\theta°}{360°}\right)$$

$$\Longrightarrow \quad \frac{r}{l} = \left(\frac{\theta°}{360°}\right)$$

Let A be the area of the sector. Then

$$\frac{\pi l^2}{A} = \frac{360°}{\theta°} \qquad (2)$$

Remarks

When a sector of a circle is folded into a cone, the following conversions are taking place:

Sector	Cone
Radius (l) →	Slant height (l)
Arc Length (L) →	Perimeter of the base $2\pi r$
Area →	Curved Surface Area πrl

Then the curved surface area of the cone $\}$ = Area of the sector

Thus, the area of the curved surface of the cone $\}$ $A = \pi l^2\left(\frac{\theta°}{360°}\right) = \pi l^2\left(\frac{r}{l}\right)$.

Hence, the curved surface area of the cone $= \pi rl$ sq.units.

(ii) Total surface area of the solid right circular cone

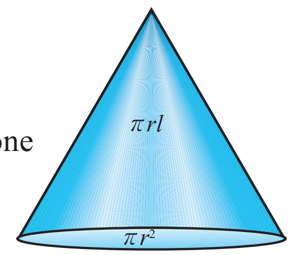


Fig. 8.17

Total surface area of the solid cone = $\{$ Curved surface area of the cone + Area of the base

$= \pi rl + \pi r^2$

Total surface area of the solid cone $= \pi r(l + r)$ sq.units.

Example 8.6

Radius and slant height of a solid right circular cone are 35cm and 37cm respectively. Find the curved surface area and total surface area of the cone. (Take $\pi = \frac{22}{7}$)

Solution Let r and l be the radius and the slant height of the solid right circular cone respectively.

$$r = 35 \text{ cm}, \ l = 37 \text{ cm}$$

Curved surface area, $\text{CSA} = \pi r l = \pi(35)(37)$

$\text{CSA} = 4070$ sq.cm

Total surface area, $\text{TSA} = \pi r[l + r]$

$$= \frac{22}{7} \times 35 \times [37 + 35]$$

Thus, $\text{TSA} = 7920$ sq.cm.

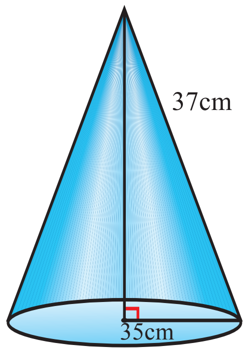


Fig. 8.18

Example 8.7

Let O and C be the centre of the base and the vertex of a right circular cone. Let B be any point on the circumference of the base. If the radius of the cone is 6 cm and if $\angle OBC = 60^o$, then find the height and curved surface area of the cone.

Solution Given that radius OB = 6 cm and $\angle OBC = 60^o$.

In the right angled $\triangle OBC$,

$$\cos 60^o = \frac{OB}{BC}$$

$$\Longrightarrow \quad BC = \frac{OB}{\cos 60^\circ}$$

$$\therefore \quad BC = \frac{6}{(\frac{1}{2})} = 12 \text{ cm}$$

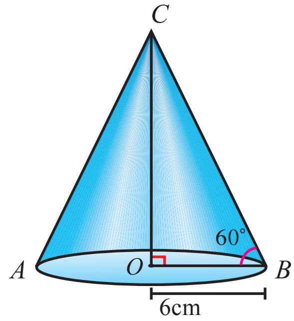


Fig. 8.19

Thus, the slant height of the cone, $l = 12$ cm

In the right angled $\triangle OBC$, we have

$$\tan 60^o = \frac{OC}{OB}$$

$$\Longrightarrow \quad OC = OB \tan 60^\circ = 6\sqrt{3}$$

Thus, the height of the cone, OC = $6\sqrt{3}$ cm

Now, the curved surface area is $\pi r l = \pi \times 6 \times 12 = 72\pi \text{ cm}^2$.

Example 8.8

A sector containing an angle of 120° is cut off from a circle of radius 21 cm and folded into a cone. Find the curved surface area of the cone. (Take $\pi = \frac{22}{7}$)

Solution Let r be the base radius of the cone.

Angle of the sector, $\theta = 120^\circ$

Radius of the sector, $R = 21$ cm

When the sector is folded into a right circular cone, we have

circumference of the base of the cone

$$= \text{Length of the arc}$$

$$\Longrightarrow \quad 2\pi r = \frac{\theta}{360^\circ} \times 2\pi R$$

$$\Longrightarrow \quad r = \frac{\theta}{360^\circ} \times R$$

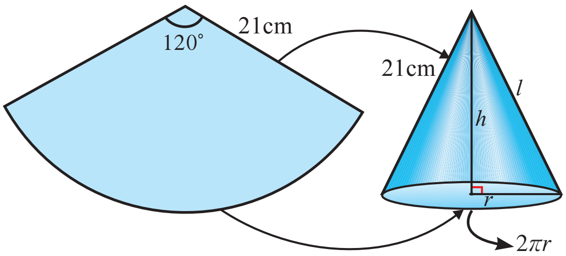


Fig. 8.20

Thus, the base radius of the cone, $r = \frac{120^\circ}{360^\circ} \times 21 = 7$ cm.

Also, the slant height of the cone ,

$$l = \text{Radius of the sector}$$

Thus, $\quad l = R \Longrightarrow l = 21$ cm.

Now , the curved surface area of the cone,

$$\text{CSA} = \pi r l$$

$$= \frac{22}{7} \times 7 \times 21 = 462.$$

Aliter :

CSA of the cone = Area of the sector

$$= \frac{\theta^\circ}{360^\circ} \times \pi \times R^2$$

$$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= 462 \text{ sq.cm.}$$

Thus, the curved surface area of the cone is 462 sq.cm.

8.2.3 Sphere

If a circular disc is rotated about one of its diameter, the solid thus generated is called sphere. Thus sphere is a 3- dimensional object which has surface area and volume.

(i) Curved surface area of a solid sphere

Activity

Take a circular disc, paste a string along a diameter of the disc and rotate it 360°. The object so created looks like a ball. The new solid is called sphere.

The following activity may help us to visualise the surface area of a sphere as four times the area of the circle with the same radius.

- Take a plastic ball.
- Fix a pin at the top of the ball.
- Wind a uniform thread over the ball so as to cover the whole curved surface area.
- Unwind the thread and measure the length of the thread used.
- Cut the thread into four equal parts.
- Place the strings as shown in the figures.

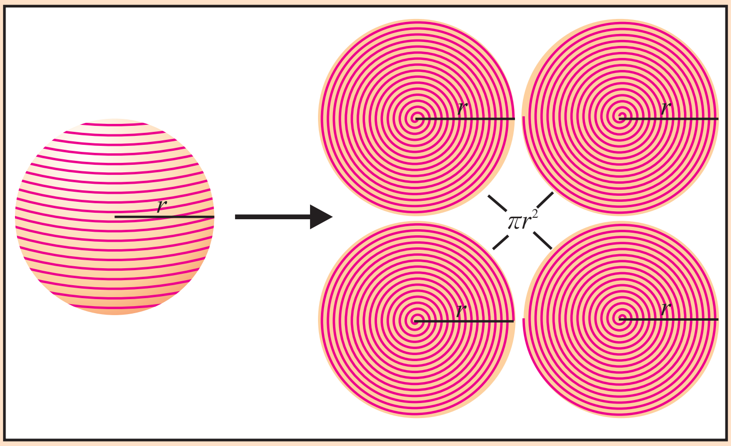


Fig. 8.21

- Measure the radius of the sphere and the circles formed.

Now, the radius of the sphere = radius of the four equal circles.

Thus, curved surface area of the sphere, CSA = $4 \times$ Area of the circle = $4 \times \pi r^2$

$\therefore$ The curved surface area of a sphere = $4\pi r^2$ sq. units.

(ii) Solid hemisphere

A plane passing through the centre of a solid sphere divides the sphere into two equal parts. Each part of the sphere is called a solid hemisphere.

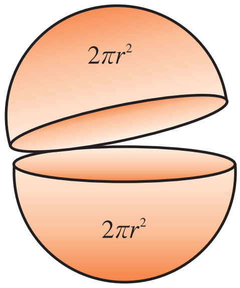


Fig. 8.22

Curved surface area of a hemisphere $= \dfrac{\text{CSA of the Sphere}}{2}$

$$= \frac{4\pi r^2}{2} = 2\pi r^2 \text{ sq.units.}$$

Total surface area of a hemisphere, TSA = Curved Surface Area + Area of the base Circle

$$= 2\pi r^2 + \pi r^2$$

$$= 3\pi r^2 \text{ sq.units.}$$

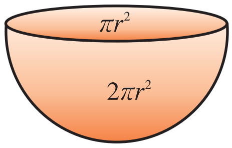


Fig. 8.23

(iii) Hollow hemisphere

Let R and r be the outer and inner radii of the hollow hemisphere.

Now, its curved surface area = Outer surface area + Inner surface area

$$= 2\pi R^2 + 2\pi r^2$$

$$= 2\pi(R^2 + r^2) \text{ sq.units.}$$

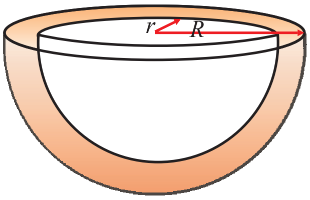


Fig. 8.24

The total surface area $= \begin{cases} \text{Outer surface area + Inner surface area} \\ + \text{ Area at the base} \end{cases}$

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= 2\pi(R^2 + r^2) + \pi(R + r)(R - r) \text{ sq.units.}$$

$$= \pi(3R^2 + r^2) \text{ sq. units}$$

Example 8.9

A hollow sphere in which a circus motorcyclist performs his stunts, has an inner diameter of 7 m. Find the area available to the motorcyclist for riding. (Take $\pi = \frac{22}{7}$)

Solution Inner diameter of the hollow sphere, $2r = 7$ m.

Available area to the motorcyclist for riding = Inner surface area of the sphere

$$= 4\pi r^2 = \pi(2r)^2$$

$$= \frac{22}{7} \times 7^2$$

Available area to the motorcyclist for riding = 154 sq.m.

Example 8.10

Total surface area of a solid hemisphere is 675π sq.cm. Find the curved surface area of the solid hemisphere.

Solution Given that the total surface area of the solid hemisphere,

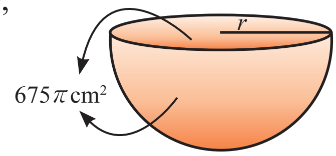


Fig. 8.25

$$3\pi r^2 = 675\pi \text{ sq. cm}$$

$$\Longrightarrow \quad r^2 = 225$$

Now, the curved surface area of the solid hemisphere,

$$\text{CSA} = 2\pi r^2 = 2\pi \times 225 = 450\pi \text{ sq.cm.}$$

Example 8.11

The thickness of a hemispherical bowl is 0.25 cm. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.(Take $\pi = \frac{22}{7}$)

Solution Let r, R and w be the inner and outer radii and thickness of the hemispherical bowl respectively.

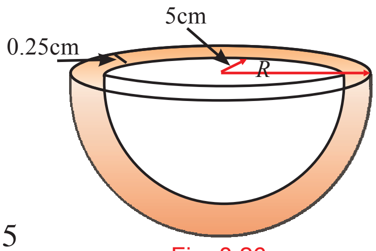


Fig. 8.26

Given that $r = 5\,\text{cm}$, $w = 0.25\,\text{cm}$

$\therefore \quad R = r + w = 5 + 0.25 = 5.25\,\text{cm}$

Now, outer surface area of the bowl $= 2\pi R^2$

$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$$

Thus, the outer surface area of the bowl = 173.25 sq.cm.

Exercise 8.1

1. A solid right circular cylinder has radius of 14 cm and height of 8 cm. Find its curved surface area and total surface area.

2. The total surface area of a solid right circular cylinder is 660 sq.cm. If its diameter of the base is 14 cm, find the height and curved surface area of the cylinder.

3. Curved surface area and circumference at the base of a solid right circular cylinder are 4400 sq.cm and 110 cm respectively. Find its height and diameter.

4. A mansion has 12 right cylindrical pillars each having radius 50 cm and height 3.5 m. Find the cost to paint the lateral surface of the pillars at ₹ 20 per square metre.

5. The total surface area of a solid right circular cylinder is 231 cm^2. Its curved surface area is two thirds of the total surface area. Find the radius and height of the cylinder.

6. The total surface area of a solid right circular cylinder is 1540 cm^2. If the height is four times the radius of the base, then find the height of the cylinder.`

7. The radii of two right circular cylinders are in the ratio of 3 : 2 and their heights are in the ratio 5 : 3. Find the ratio of their curved surface areas.

8. The external surface area of a hollow cylinder is 540π sq.cm. Its internal diameter is 16 cm and height is 15 cm. Find the total surface area.

9. The external diameter of a cylindrical shaped iron pipe is 25 cm and its length is 20 cm. If the thickness of the pipe is 1cm, find the total surface area of the pipe.

10. The radius and height of a right circular solid cone are 7 cm and 24 cm respectively. Find its curved surface area and total surface area.

11. If the vertical angle and the radius of a right circular cone are 60° and 15 cm respectively, then find its height and slant height.

12. If the circumference of the base of a solid right circular cone is 236 cm and its slant height is 12 cm, find its curved surface area.

13. A heap of paddy is in the form of a cone whose diameter is 4.2 m and height is 2.8 m. If the heap is to be covered exactly by a canvas to protect it from rain, then find the area of the canvas needed.

14. The central angle and radius of a sector of a circular disc are 180° and 21 cm respectively. If the edges of the sector are joined together to make a hollow cone, then find the radius of the cone.

15. Radius and slant height of a solid right circular cone are in the ratio 3 : 5. If the curved surface area is 60π sq.cm, then find its total surface area.

16. If the curved surface area of solid a sphere is 98.56 cm^2, then find the radius of the sphere..

17. If the curved surface area of a solid hemisphere is 2772 sq.cm, then find its total surface area.

18. Radii of two solid hemispheres are in the ratio 3 : 5. Find the ratio of their curved surface areas and the ratio of their total surface areas.

19. Find the curved surface area and total surface area of a hollow hemisphere whose outer and inner radii are 4.2 cm and 2.1 cm respectively.

20. The inner curved surface area of a hemispherical dome of a building needs to be painted. If the circumference of the base is 17.6 m, find the cost of painting it at the rate of ₹5 per sq.m.

8.3 Volume

So far we have seen the problems related to the surface area of some solids. Now we shall learn how to calculate volumes of some familiar solids. Volume is literally the 'amount of space filled'. The volume of a solid is a numerical characteristic of the solid.

For example, if a body can be decomposed into finite set of unit cubes (cubes of unit sides), then the volume is equal to the number of these cubes.

The cube in the figure, has a volume

$$= \text{length } \times \text{width} \times \text{ height}$$

$$= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 1\text{ cm}^3.$$

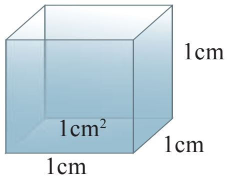


Fig. 8.27

If we say that the volume of an object is 100 cu.cm, then it implies that we need 100 cubes each of 1 cm^3 volume to fill this object completely.

Just like surface area, volume is a positive quantity and is invariant with respect to displacement. Volumes of some solids are illustrated below.

8.3.1 Volume of a right circular cylinder

(i) Volume of a solid right circular cylinder

The volume of a solid right circular cylinder is the product of the base area and height.

That is, the volume of the cylinder, V = Area of the base $\times$ height

$$= \pi r^2 \times h$$

Thus, the volume of a cylinder, $V = \pi r^2 h$ cu. units.

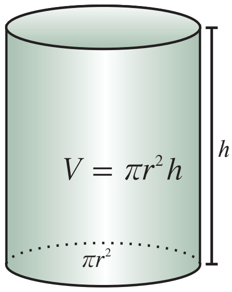


Fig. 8.28

(ii) Volume of a hollow cylinder (Volume of the material used)

Let R and r be the external and internal radii of a hollow right circular cylinder respectively. Let h be its height.

Then, the volume, $V = \left\{\begin{array}{c}\text{Volume of the}\\ \text{outer cylinder}\end{array}\right\} - \left\{\begin{array}{c}\text{Volume of the}\\ \text{inner cylinder}\end{array}\right\}$

$$= \pi R^2 h - \pi r^2 h$$

Hence, the volume of a hollow cylinder,

$$V = \pi h(R^2 - r^2) \text{ cu. units.}$$

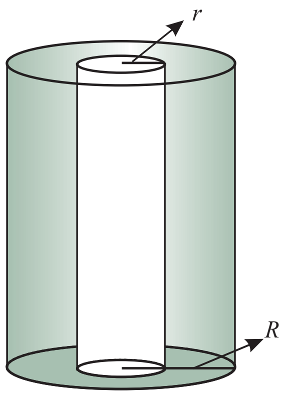


Fig. 8.29

Example 8.12

If the curved surface area of a right circular cylinder is 704 sq.cm, and height is 8 cm, find the volume of the cylinder in litres. (Take $\pi = \frac{22}{7}$)

Solution Let r and h be the radius and height of the right circular cylinder respectively.

Given that h = 8 cm and CSA = 704 sq.cm

Now, CSA = 704

$\Longrightarrow$ $2\pi rh = 704$

$2 \times \frac{22}{7} \times r \times 8 = 704$

$\therefore$ $r = \frac{704 \times 7}{2 \times 22 \times 8} = 14\text{ cm}$

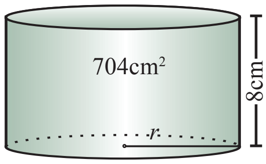


Fig. 8.30

Thus, the volume of the cylinder, $V = \pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 8$$

$$= 4928 \text{ cu.cm.}$$

Hence, the volume of the cylinder $= 4.928$ litres. (1000 cu.cm = l litre)

Example 8.13

A hollow cylindrical iron pipe is of length 28 cm. Its outer and inner diameters are 8 cm and 6 cm respectively. Find the volume of the pipe and weight of the pipe if 1 cu.cm of iron weighs 7 gm.(Take $\pi = \frac{22}{7}$)

Solution Let r, R and h be the inner, outer radii and height of the hollow cylindrical pipe respectively.

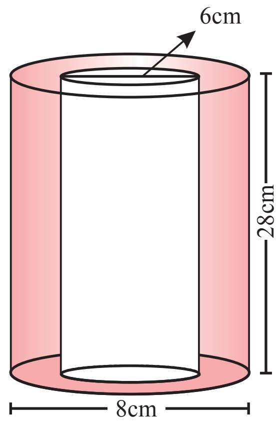


Fig. 8.31

Given that $2r = 6$ cm, $2R = 8$ cm, $h = 28$ cm

Now, the volume of the pipe, $V = \pi \times h \times (R + r)(R - r)$

$$= \frac{22}{7} \times 28 \times (4 + 3)(4 - 3)$$

$\therefore$ Volume, $V = 616$ cu. cm

Weight of 1 cu.cm of the metal $= 7$ gm

Weight of the 616 cu. cm of metal $= 7 \times 616$ gm

Thus, the weight of the pipe $= 4.312$ kg.

Example 8.14

Base area and volume of a solid right circular cylinder are 13.86 sq.cm, and 69.3 cu.cm respectively. Find its height and curved surface area.(Take $\pi = \frac{22}{7}$)

Solution Let A and V be the base area and volume of the solid right circular cylinder respectively.

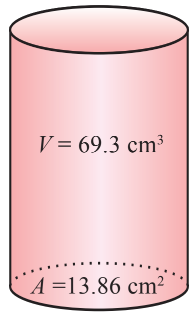


Fig. 8.32

Given that the base area, $A = \pi r^2 = 13.86$ sq.cm and

volume, $V = \pi r^2 h = 69.3$ cu.cm.

Thus, $\pi r^2 h = 69.3$

$\Longrightarrow$ $13.86 \times h = 69.3$

$\therefore$ $h = \frac{69.3}{13.86} = 5$ cm.

Now, the base area $= \pi r^2 = 13.86$

$$\frac{22}{7} \times r^2 = 13.86$$

$$r^2 = 13.86 \times \frac{7}{22} = 4.41 \quad \Longrightarrow \quad r = \sqrt{4.41} = 2.1 \text{ cm.}$$

Now, Curved surface area, CSA $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 5$$

Thus, CSA $= 66$ sq.cm.

8.3.2 Volume of a right circular cone

Let r and h be the base radius and the height of a right circular cone respectively.

The volume V of the cone is given by the formula: $V = \frac{1}{3} \times \pi r^2 h$ cu. units. To justify this formula, let us perform the following activity.

Activity

Make a hollow cone and a hollow cylinder like in the figure given below with the same height and same radius. Now, practically we can find out the volume of the cone by doing the process given below. Fill the cone with sand or liquid and then pour it into the cylinder. Continuing this experiment, we see that the cylinder will be filled completely by sand / liquid at the third time.

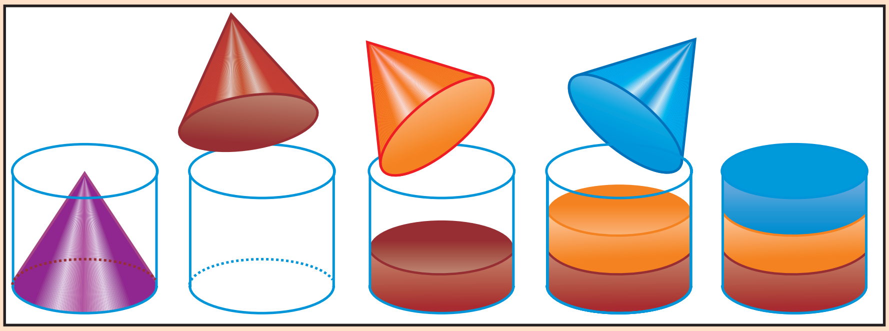


Fig. 8.33

From this simple activity, if r and h are the radius and height of the cylinder, then we find that $3 \times$(Volume of the cone) $=$ Volume of the cylinder $= \pi r^2 h$

Thus, the volume of the cone $= \frac{1}{3} \times \pi r^2 h$ cu. units.

Example 8.15

The volume of a solid right circular cone is 4928 cu. cm. If its height is 24 cm, then find the radius of the cone. (Take $\pi = \frac{22}{7}$)

Solution Let r, h and V be the radius, height and volume of a solid cone respectively.

Given that $V = 4928$ cu.cm and $h = 24$ cm

Thus, we have $\frac{1}{3}\pi r^2 h = 4928$

$$\implies \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 4928$$

$$\implies r^2 = \frac{4928 \times 3 \times 7}{22 \times 24} = 196.$$

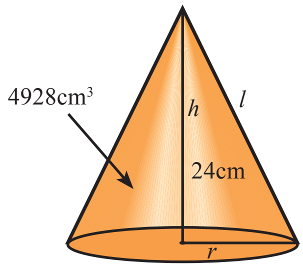


Fig. 8.34

Thus, the base radius of the cone, $r = \sqrt{196} = 14$ cm.

8.3.3 Volume of a Frustum of a Cone

Let us consider a right circular solid cone and cut it into two solids so as to obtain a smaller right circular cone. The other portion of the cone is called frustum of the cone. This is illustrated in the following activity.

Activity

Take some clay and form a right circular cone. Cut it with a knife parallel to its base. Remove the smaller cone. What are you left with? The left out portion of the solid cone is called frustum of the cone. The Latin word frustum means "piece cut off" and its plural is frusta.

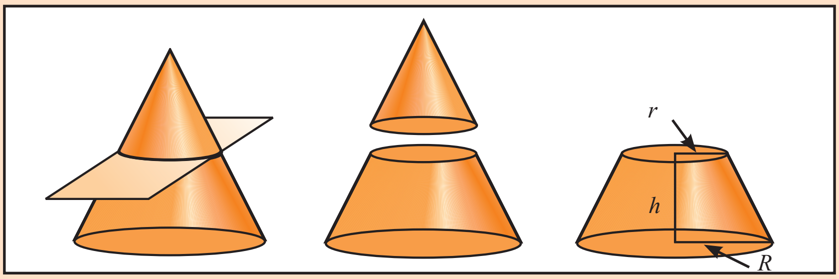


Fig. 8.35

Hence, if a solid right circular cone is sliced with a plane parallel to its base , the part of the cone containing the base is called a frustum of the cone. Thus a frustum has two circular discs, one at the bottom and the other at the top of it.

Let us find the volume of a frustum of a cone.

The volume of a frustum of a cone is nothing but the difference between volumes of two right circular cones. (See Fig. 8.35) Consider a frustum of a solid right circular cone.

Let R be the radius of the given cone. Let r and x be the radius and the height of the smaller cone obtained after removal of the frustum from the given cone.

Let h be the height of the frustum.

$$\text{Now, } \left.\begin{array}{l}\text{the volume of the}\\ \text{frustum of the cone}\end{array}\right\},\ V = \left.\begin{array}{c}\text{Volume of the}\\ \text{given cone}\end{array}\right\} - \left\{\begin{array}{l}\text{Volume of the}\\ \text{smaller cone}\end{array}\right.$$

$$= \frac{1}{3}\times\pi\times R^2\times(x+h) - \frac{1}{3}\times\pi\times r^2\times x$$

$$\text{Thus, } V = \frac{1}{3}\pi[x(R^2-r^2)+R^2h]. \qquad (1)$$

From the Fig. 8.36 we see that $\Delta BFE \sim \Delta DGE$

$$\therefore \quad \frac{BF}{DG} = \frac{FE}{GE}$$

$$\Longrightarrow \quad \frac{R}{r} = \frac{x+h}{x}$$

$$\Longrightarrow \quad Rx - rx = rh$$
$$\Longrightarrow \quad x(R - r) = rh$$

Thus, we get $\quad x = \dfrac{rh}{R - r} \qquad (2)$

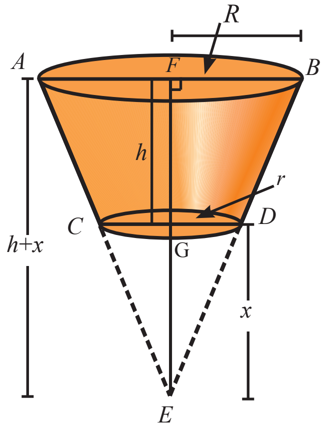


Fig. 8.36

Now, (1) $\Longrightarrow \quad V = \frac{1}{3}\pi[x(R^2 - r^2) + R^2 h]$

$$\Longrightarrow \quad = \frac{1}{3}\pi[x(R - r)(R + r) + R^2 h]$$

$$\Longrightarrow \quad = \frac{1}{3}\pi[rh(R + r) + R^2 h] \text{ using (2)}$$

Hence, the volume of the frustum of the cone,

$$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr) \text{ cu. units.}$$

Note

* Curved surface area of a frustum of a cone $= \pi(R + r)l$, where $l = \sqrt{h^2 + (R - r)^2}$

* Total surface area of a frustum of a the cone $= \pi l(R + r) + \pi R^2 + \pi r^2, l = \sqrt{h^2 + (R - r)^2}$

(* Not to be used for examination purpose)

Example 8.16

The radii of two circular ends of a frustum shaped bucket are 15 cm and 8 cm. If its depth is 63 cm, find the capacity of the bucket in litres. (Take $\pi = \frac{22}{7}$)

Solution Let R and r are the radii of the circular ends at the top and bottom and h be the depth of the bucket respectively.

Given that $R = 15$ cm , $r = 8$ cm and $h = 63$ cm.

Fig. 8.37

The volume of the bucket (frustum)

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 63 \times (15^2 + 8^2 + 15 \times 8)$$

$$= 26994 \text{ cu.cm.}$$

$$= \frac{26994}{1000} \text{ litres} \qquad (1000 \text{ cu.cm} = 1 \text{ litre})$$

Thus, the capacity of the bucket = 26.994 litres.

8.3.4 Volume of a Sphere

(i) Volume of a Solid Sphere

The following simple experiment justifies the formula for volume of a sphere,

$$V = \frac{4}{3}\pi r^3 \text{ cu.units.}$$

Activity

Take a cylindrical shaped container of radius R and height H. Fill the container with water. Immerse a solid sphere of radius r, where $R > r$, in the container and fill the displaced water into another cylindrical shaped container of radius r and height H. The height of the water level is equal to $\frac{4}{3}$ times of its radius ($h = \frac{4}{3}r$). Now, the volume of the solid sphere is same as that of the displaced water.

Volume of the displaced water, V = Base area x Height

$$= \pi r^2 \times \frac{4}{3}r \text{ (here, height of the water level } h = \frac{4}{3}r)$$

$$= \frac{4}{3}\pi r^3$$

Thus, the volume of the sphere, $V = \frac{4}{3}\pi r^3$ cu.units.

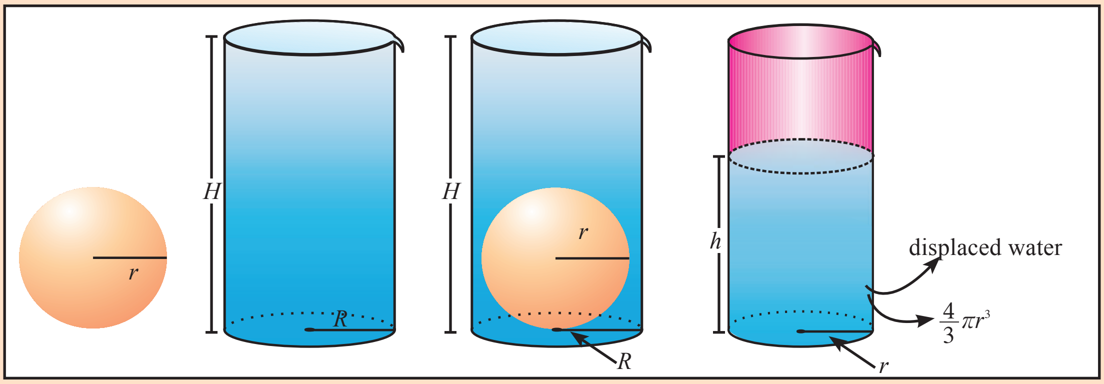


Fig. 8.38

(ii) Volume of a hollow sphere (Volume of the material used)

If the inner and outer radius of a hollow sphere are r and R respectively, then

$$\left.\begin{array}{l}\text{Volume of the}\\ \text{hollow sphere}\end{array}\right\} = \left.\begin{array}{l}\text{Volume of the}\\ \text{outer sphere}\end{array}\right\} - \left\{\begin{array}{l}\text{Volume of the}\\ \text{inner sphere}\end{array}\right.$$

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$\therefore$ Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$ cu. units.

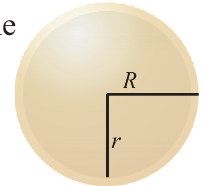


Fig. 8.39

(iii) Volume of a solid hemisphere

Volume of the solid hemisphere $= \frac{1}{2} \times$ volume of the sphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3 \text{ cu.units.}$$

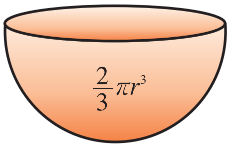


Fig. 8.40

(iv) Volume of a hollow hemisphere (Volume of the material used)

$$\left.\begin{array}{l}\text{Volume of a hollow}\\ \text{hemisphere}\end{array}\right\} = \left.\begin{array}{l}\text{Volume of outer}\\ \text{hemisphere}\end{array}\right\} - \left\{\begin{array}{l}\text{Volume of inner}\\ \text{hemisphere}\end{array}\right.$$

$$= \frac{2}{3} \times \pi \times R^3 - \frac{2}{3} \times \pi \times r^3$$

$$= \frac{2}{3}\pi(R^3 - r^3) \text{ cu.units.}$$

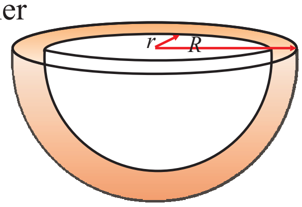


Fig. 8.41

Example 8.17

Find the volume of a sphere-shaped metallic shot-put having diameter of 8.4 cm.

(Take $\pi = \frac{22}{7}$)

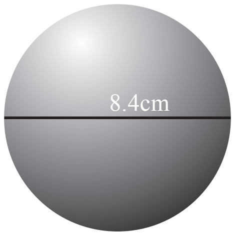


Fig. 8.42

Solution Let r be radius of the metallic shot-put.

Now, $2r = 8.4$ cm $\Longrightarrow$ $r = 4.2$ cm

Volume of the shot-put, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$$

Thus, the volume of the shot-put $= 310.464$ cu.cm.

Example 8.18

A cone, a hemisphere and cylinder have equal bases. If the heights of the cone and a cylinder are equal and are same as the common radius, then find the ratio of their respective volumes.

Solution Let r be the common radius of the cone, hemisphere and cylinder.

Let h be the common height of the cone and cylinder.

Given that $r = h$

Let V_1, V_2 and V_3 be the volumes of the cone, hemisphere and cylinder respectively.

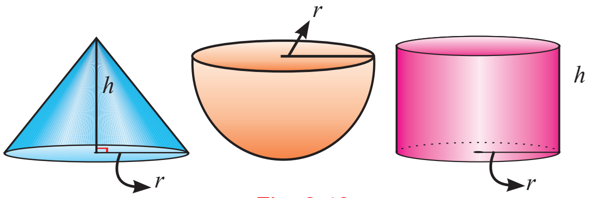


Fig. 8.43

Now, $V_1 : V_2 : V_3 = \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

$\Longrightarrow$ $= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3$ (here, $r = h$)

$\Longrightarrow V_1 : V_2 : V_3 = \frac{1}{3} : \frac{2}{3} : 1$

Hence, the required ratio is 1 : 2 : 3.

Example 8.19

If the volume of a solid sphere is $7241\frac{1}{7}$ cu.cm, then find its radius.

(Take $\pi = \frac{22}{7}$)

Solution Let r and V be the radius and volume of the solid sphere respectively.

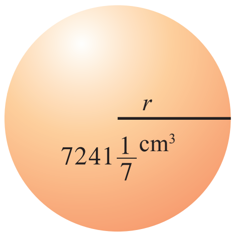


Fig. 8.44

Given that $V = 7241\frac{1}{7}$ cu.cm

$\Longrightarrow$ $\frac{4}{3}\pi r^3 = \frac{50688}{7}$

$\Longrightarrow$ $\frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{50688}{7}$

$$r^3 = \frac{50688}{7} \times \frac{3 \times 7}{4 \times 22}$$

$$= 1728 = 4^3 \times 3^3$$

Thus, the radius of the sphere, $r = 12$ cm.

Example 8.20

Volume of a hollow sphere is $\frac{11352}{7}$ cm^3. If the outer radius is 8 cm, find the inner radius of the sphere. (Take $\pi = \frac{22}{7}$)

Solution Let R and r be the outer and inner radii of the hollow sphere respectively.

Let V be the volume of the hollow sphere.

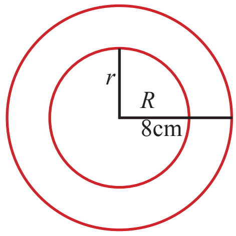


Fig. 8.45

Now, given that $V = \frac{11352}{7}$ cm^3

$$\Longrightarrow \quad \frac{4}{3}\pi(R^3 - r^3) = \frac{11352}{7}$$

$$\Longrightarrow \quad \frac{4}{3} \times \frac{22}{7}(8^3 - r^3) = \frac{11352}{7}$$

$$512 - r^3 = 387 \Longrightarrow r^3 = 125 = 5^3$$

Hence, the inner radius, $r = 5$ cm.

Exercise 8.2

1. Find the volume of a solid cylinder whose radius is 14 cm and height 30 cm.
2. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, then find the quantity of soup to be prepared daily in the hospital to serve 250 patients?
3. The sum of the base radius and the height of a solid right circular solid cylinder is 37 cm. If the total surface area of the cylinder is 1628 sq.cm, then find the volume of the cylinder.
4. Volume of a solid cylinder is 62.37 cu.cm. Find the radius if its height is 4.5 cm.
5. The radii of two right circular cylinders are in the ratio 2 : 3. Find the ratio of their volumes if their heights are in the ratio 5 : 3.
6. The radius and height of a cylinder are in the ratio 5 : 7. If its volume is 4400 cu.cm, find the radius of the cylinder.
7. A rectangular sheet of metal foil with dimension 66 cm $\times$ 12 cm is rolled to form a cylinder of height 12 cm. Find the volume of the cylinder.
8. A lead pencil is in the shape of right circular cylinder. The pencil is 28 cm long and its radius is 3 mm. If the lead is of radius 1 mm, then find the volume of the wood used in the pencil.

9. Radius and slant height of a cone are 20 cm and 29 cm respectively. Find its volume.

10. The circumference of the base of a 12 m high wooden solid cone is 44 m. Find the volume.

11. A vessel is in the form of a frustum of a cone. Its radius at one end and the height are 8 cm and 14 cm respectively. If its volume is $\frac{5676}{3}$ cm^3, then find the radius at the other end.

12. The perimeter of the ends of a frustum of a cone are 44 cm and 8.4π cm. If the depth is 14 cm., then find its volume.

13. A right angled $\triangle ABC$ with sides 5 cm, 12 cm and 13 cm is revolved about the fixed side of 12 cm. Find the volume of the solid generated.

14. The radius and height of a right circular cone are in the ratio 2 : 3. Find the slant height if its volume is 100.48 cu.cm. (Take $\pi = 3.14$)

15. The volume of a cone with circular base is 216π cu.cm. If the base radius is 9 cm, then find the height of the cone.

16. Find the mass of 200 steel spherical ball bearings, each of which has radius 0.7 cm, given that the density of steel is 7.95 g/cm^3. (Mass = Volume $\times$ Density)

17. The outer and the inner radii of a hollow sphere are 12 cm and 10 cm. Find its volume.

18. The volume of a solid hemisphere is 1152π cu.cm. Find its curved surface area.

19. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 14 cm.

20. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of volumes of the balloon in the two cases.

8.4 Combination of Solids

In our daily life we observe many objects like toys, vehicles, vessels, tools, etc., which are combination of two or more solids.

How can we find the surface areas and volumes of combination of solids?

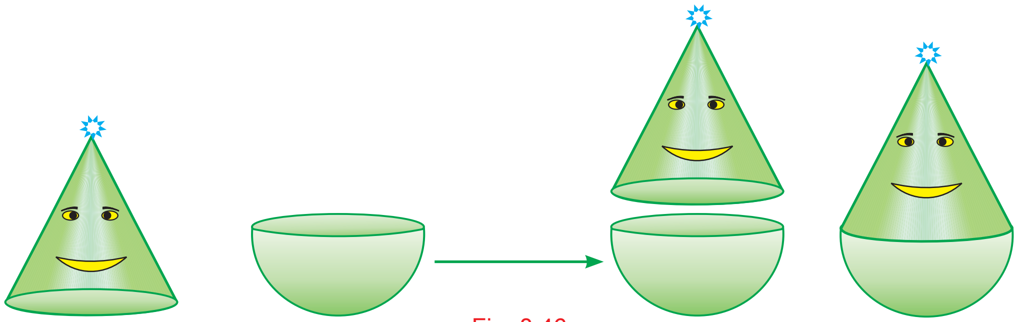


Fig. 8.46

The total surface area of the combination of solids need not be the sum of the surface areas of the solids which are combined together. However, in the above figure, the total surface area of the combined solid is equal to the sum of the curved surface area of the hemisphere and curved surface area of the cone. But the volume of the combined solid is equal to the sum of the volumes of the solids which are combined together. Thus, from the figure we have,

The total surface area of the solid $=$ $\left.\begin{matrix}\text{Curved surface area}\\ \text{of the hemisphere}\end{matrix}\right\} + \left\{\begin{matrix}\text{Curved surface area}\\ \text{of the cone}\end{matrix}\right.$

The total volume of the solid $=$ Volume of the hemisphere + Volume of the cone.

Example 8.21

A solid wooden toy is in the form of a cone surmounted on a hemisphere. If the radii of the hemisphere and the base of the cone are 3.5 cm each and the total height of the toy is 17.5 cm, then find the volume of wood used in the toy. (Take $\pi = \frac{22}{7}$)

Solution Hemispherical portion :

Radius, $r = 3.5$ cm

Conical portion :

Radius, $r = 3.5$ cm

Height, $h = 17.5 - 3.5 = 14$ cm

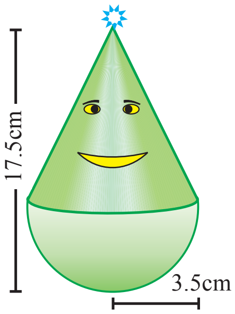


Fig. 8.47

Volume of the wood = Volume of the hemisphere + Volume of the cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi r^2}{3}(2r + h)$$

$$= \frac{22}{7} \times \frac{3.5 \times 3.5}{3} \times (2 \times 3.5 + 14) = 269.5$$

Hence, the volume of the wood used in the toy $= 269.5$ cu.cm.

Example 8.22

A cup is in the form of a hemisphere surmounted by a cylinder. The height of the cylindrical portion is 8 cm and the total height of the cup is 11.5 cm. Find the total surface area of the cup. (Take $\pi = \frac{22}{7}$)

Solution Hemispherical portion

Radius, r = Total height – 8

$\Longrightarrow r = 11.5 - 8 = 3.5$ cm

Cylindrical portion

Height, $h = 8$ cm.

Thus, radius $r = 3.5 \text{ cm} = \frac{7}{2}$ cm

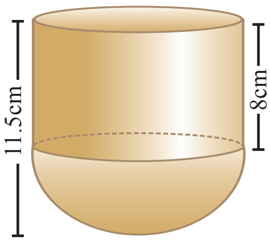


Fig. 8.48

Total surface area of the cup $= \left\{\begin{matrix}\text{CSA of the hemispherical portion}\\ +\ \text{CSA of the cylindrical portion}\end{matrix}\right.$

$$= 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(\frac{7}{2} + 8\right)$$

$\therefore$ Total surface area of the cup $= 253$ sq.cm.

Example 8.23

A circus tent is to be erected in the form of a cone surmounted on a cylinder. The total height of the tent is 49 m. Diameter of the base is 42 m and height of the cylinder is 21 m. Find the cost of canvas needed to make the tent, if the cost of canvas is ₹12.50/m^2. (Take $\pi = \frac{22}{7}$)

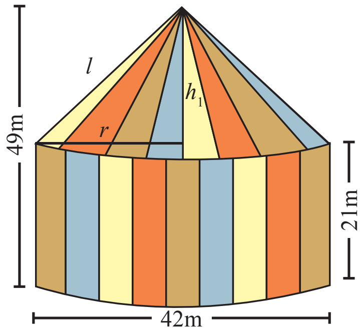


Fig. 8.49

Solution

Cylindrical Part	Conical Part
Diameter, $2r = 42$ m	Radius, $r = 21$ m
Radius, $r = 21$ m	Height, $h_1 = 49 - 21 = 28$ m
Height, $h = 21$ m	Slant height, $l = \sqrt{h_1^2 + r^2}$
	$= \sqrt{28^2 + 21^2}$
	$= 7\sqrt{4^2 + 3^2} = 35$ m

Total area of the canvas needed = CSA of the cylindrical part + CSA of the conical part

$$= 2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 21(2 \times 21 + 35) = 5082$$

Therefore, area of the canvas = 5082 m^2

Now, the cost of the canvas per sq.m = ₹12.50

Thus, the total cost of the canvas = 5082 × 12.5 = ₹63525.

Example 8.24

A hollow sphere of external and internal diameters of 8 cm and 4 cm respectively is melted and made into another solid in the shape of a right circular cone of base diameter of 8 cm. Find the height of the cone.

Solution Let R and r be the external and internal radii of the hollow sphere.

Let h and r_1 be the height and the radius of the cone to be made.

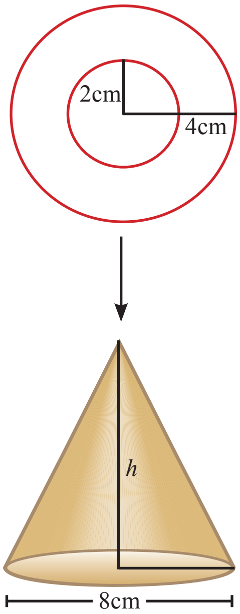


Fig. 8.50

Hollow Sphere: External	Hollow Sphere: Internal	Cone
$2R = 8$ cm	$2r = 4$ cm	$2r_1 = 8$
$\Longrightarrow R = 4$ cm	$\Longrightarrow r = 2$ cm	$\Longrightarrow r_1 = 4$

When the hollow sphere is melted and made into a solid cone, we have

Volume of the cone = Volume of the hollow sphere

$$\Longrightarrow \quad \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi[R^3 - r^3]$$

$$\Longrightarrow \quad \frac{1}{3}\times\pi\times 4^2\times h = \frac{4}{3}\times\pi\times(4^3-2^3)$$

$$\Longrightarrow \quad h = \frac{64-8}{4} = 14$$

Hence, the height of the cone $h = 14$ cm.

Example 8.25

Spherical shaped marbles of diameter 1.4 cm each, are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm.

Solution Let n be the number of marbles needed. Let r_1 and r_2 be the radii of the marbles and cylindrical beaker respectively.

Marbles	Cylindrical Beaker
Diameter, $2r_1 = 1.4$ cm	Diameter, $2r_2 = 7$ cm
Radius $r_1 = 0.7$ cm	Radius, $r_2 = \frac{7}{2}$ cm

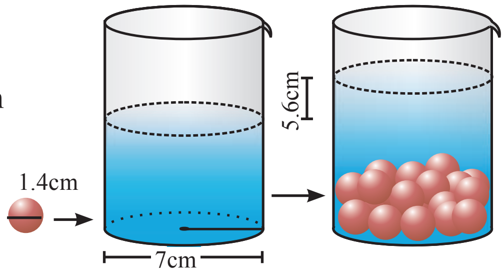


Fig. 8.51

Let h be the height of the water level raised.

Then, $h = 5.6$ cm

After the marbles are dropped into the beaker,

Volume of water raised = Volume of n marbles

$$\Longrightarrow \quad \pi r_2^2 h = n\times\frac{4}{3}\pi r_1^3$$

Thus, $$n = \frac{3r_2^2 h}{4r_1^3}$$

$$n = \frac{3\times\frac{7}{2}\times\frac{7}{2}\times 5.6}{4\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10}} = 150.$$

$\therefore$ The number of marbles needed is 150.

Example 8.26

Water is flowing at the rate of 15 km / hr through a cylindrical pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. In how many hours will the water level in the tank raise by 21 cm? (Take $\pi = \frac{22}{7}$)

Solution Speed of water = 15 km / hr

= 15000 m / hr

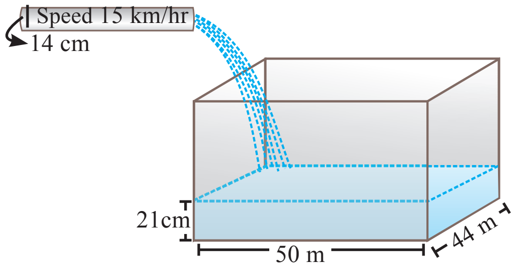


Fig. 8.52

Diameter of the pipe, $2r = 14$ cm

Thus, $r = \frac{7}{100}$ m.

Let h be the water level to be raised.

Thus, $h = 21 \text{ cm} = \frac{21}{100}$ m

Now, the volume of water discharged

$$= \text{Cross section area of the pipe} \times \text{Time} \times \text{Speed}$$

Volume of water discharged in one hour

$$= \pi r^2 \times 1 \times 15000$$
$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ cu.m}$$

Volume of required quantity of water in the tank is,

$$lbh = 50 \times 44 \times \frac{21}{100}$$

Assume that T hours are needed to get the required quantity of water.

$\therefore$ $\left.\begin{array}{r}\text{Volume of water discharged}\\ \text{in T hours}\end{array}\right\}$ = Required quantity of water in the tank

$$\Longrightarrow \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times T \times 15000 = 50 \times 44 \times \frac{21}{100}$$

Thus, $T = 2$ hours.

Hence, it will take 2 hours to raise the required water level.

Example 8.27

A cuboid shaped slab of iron whose dimensions are 55 cm$\times$40 cm$\times$15 cm is melted and recast into a pipe. The outer diameter and thickness of the pipe are 8 cm and 1 cm respectively. Find the length of the pipe. (Take $\pi = \frac{22}{7}$)

Solution Let h_1 be the length of the pipe.

Let R and r be the outer and inner radii of the pipe respectively.

Iron slab: Let $lbh = 55 \times 40 \times 15$.

Iron pipe:

Outer diameter, $2R = 8$ cm

$\therefore$ Outer radius, $R = 4$ cm

Thickness, $w = 1$ cm

$\therefore$ Inner radius, $r = R - w = 4 - 1 = 3$ cm

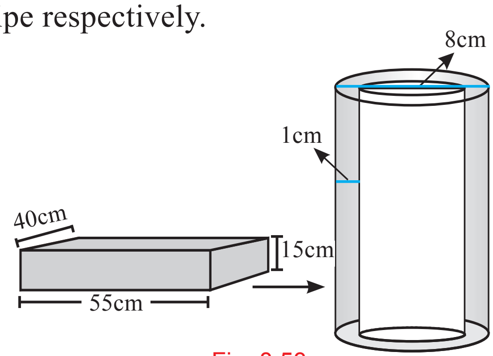


Fig. 8.53

Now, the volume of the iron pipe = Volume of iron slab

$$\Longrightarrow \pi h_1 (R + r)(R - r) = lbh$$

That is, $\frac{22}{7} \times h_1 (4 + 3)(4 - 3) = 55 \times 40 \times 15$

Thus, the length of the pipe, $h_1 = 1500$ cm $= 15$ m.

Exercise 8.3

1. A play-top is in the form of a hemisphere surmounted on a cone. The diameter of the hemisphere is 3.6 cm. The total height of the play-top is 4.2 cm. Find its total surface area.

2. A solid is in the shape of a cylinder surmounted on a hemisphere. If the diameter and the total height of the solid are 21 cm, 25.5 cm respectively, then find its volume.

3. A capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. If the length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm, find its surface area.

4. A tent is in the shape of a right circular cylinder surmounted by a cone. The total height and the diameter of the base are 13.5 m and 28 m. If the height of the cylindrical portion is 3 m, find the total surface area of the tent.

5. Using clay, a student made a right circular cone of height 48 cm and base radius 12 cm. Another student reshapes it in the form of a sphere. Find the radius of the sphere.

6. The radius of a solid sphere is 24 cm. It is melted and drawn into a long wire of uniform cross section. Find the length of the wire if its radius is 1.2 mm.

7. A right circular conical vessel whose internal radius is 5 cm and height is 24 cm is full of water. The water is emptied into an empty cylindrical vessel with internal radius 10 cm. Find the height of the water level in the cylindrical vessel.

8. A solid sphere of diameter 6 cm is dropped into a right circular cylindrical vessel with diameter 12 cm, which is partly filled with water. If the sphere is completely submerged in water, how much does the water level in the cylindrical vessel increase?.

9. Through a cylindrical pipe of internal radius 7 cm, water flows out at the rate of 5 cm/sec. Calculate the volume of water (in litres) discharged through the pipe in half an hour.

10. Water in a cylindrical tank of diameter 4 m and height 10 m is released through a cylindrical pipe of diameter 10 cm at the rate of 2.5 Km/hr. How much time will it take to empty the half of the tank? Assume that the tank is full of water to begin with.

11. A spherical solid material of radius 18 cm is melted and recast into three small solid spherical spheres of different sizes. If the radii of two spheres are 2cm and 12 cm, find the radius of the third sphere.

12. A hollow cylindrical pipe is of length 40 cm. Its internal and external radii are 4 cm and 12 cm respectively. It is melted and cast into a solid cylinder of length 20 cm. Find the radius of the new solid.

13. An iron right circular cone of diameter 8 cm and height 12 cm is melted and recast into spherical lead shots each of radius 4 mm. How many lead shots can be made?.

14. A right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 12 cm and diameter 6 cm, having a hemispherical shape on top. Find the number of such cones which can be filled with the ice cream available.

15. A container with a rectangular base of length 4.4 m and breadth 2 m is used to collect rain water. The height of the water level in the container is 4 cm and the water is transferred into a cylindrical vessel with radius 40 cm. What will be the height of the water level in the cylinder?

16. A cylindrical bucket of height 32 cm and radius 18 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

17. A cylindrical shaped well of depth 20 m and diameter 14 m is dug. The dug out soil is evenly spread to form a cuboid-platform with base dimension 20 m $\times$ 14 m. Find the height of the platform.

Exercise 8.4

Choose the correct answer

1. The curved surface area of a right circular cylinder of radius 1 cm and height 1 cm is equal to

   (A) π cm^2 (B) 2π cm^2 (C) 3π cm^3 (D) 2 cm^2

2. The total surface area of a solid right circular cylinder whose radius is half of its height h is equal to

   (A) $\frac{3}{2}\pi h$ sq. units (B) $\frac{2}{3}\pi h^2$ sq. units (C) $\frac{3}{2}\pi h^2$ sq.units (D) $\frac{2}{3}\pi h$ sq.units

3. Base area of a right circular cylinder is 80 cm^2. If its height is 5 cm, then the volume is equal to

   (A) 400 cm^3 (B) 16 cm^3 (C) 200 cm^3 (D) $\frac{400}{3}$ cm^3

4. If the total surface area a solid right circular cylinder is 200π cm^2 and its radius is 5 cm, then the sum of its height and radius is

   (A) 20 cm (B) 25 cm (C) 30 cm (D) 15 cm

5. The curved surface area of a right circular cylinder whose radius is a units and height is b units, is equal to

   (A) $\pi a^2 b$ sq.cm (B) $2\pi ab$ sq.cm (C) 2π sq.cm (D) 2 sq.cm

6. Radius and height of a right circular cone and that of a right circular cylinder are respectively, equal. If the volume of the cylinder is 120 cm^3, then the volume of the cone is equal to

   (A) 1200 cm^3 B) 360 cm^3 (C) 40 cm^3 (D) 90 cm^3

7. If the diameter and height of a right circular cone are 12 cm and 8 cm respectively, then the slant height is

(A) 10 cm (B) 20 cm (C) 30 cm (D) 96 cm

8. If the circumference at the base of a right circular cone and the slant height are 120π cm and 10 cm respectively, then the curved surface area of the cone is equal to

(A) 1200π cm^2 (B) 600π cm^2 (C) 300π cm^2 (D) 600 cm^2

9. If the volume and the base area of a right circular cone are 48π cm^3 and 12π cm^2 respectively, then the height of the cone is equal to

(A) 6 cm (B) 8 cm (C) 10 cm (D) 12 cm

10. If the height and the base area of a right circular cone are 5 cm and 48 sq.cm respectively, then the volume of the cone is equal to

(A) 240 cm^3 (B) 120 cm^3 (C) 80 cm^3 (D) 480 cm^3

11. The ratios of the respective heights and the respective radii of two cylinders are 1:2 and 2:1 respectively. Then their respective volumes are in the ratio

(A) 4 : 1 (B) 1 : 4 (C) 2 : 1 (D) 1 : 2

12. If the radius of a sphere is 2 cm , then the curved surface area of the sphere is equal to

(A) 8π cm^2 (B) 16 cm^2 (C) 12π cm^2 (D) 16π cm^2 .

13. The total surface area of a solid hemisphere of diameter 2 cm is equal to

(A) 12 cm^2 (B) 12π cm^2 (C) 4π cm^2 (D) 3π cm^2.

14. If the volume of a sphere is $\frac{9}{16}\pi$ cu.cm, then its radius is

(A) $\frac{4}{3}$ cm (B) $\frac{3}{4}$ cm (C) $\frac{3}{2}$ cm (D) $\frac{2}{3}$ cm.

15. The surface areas of two spheres are in the ratio of $9:25$. Then their volumes are in the ratio

(A) 81 : 625 (B) 729 : 15625 (C) 27 : 75 (D) 27 : 125.

16. The total surface area of a solid hemisphere whose radius is a units, is equal to

(A) $2\pi a^2$ sq.units (B) $3\pi a^2$ sq.units (C) $3\pi a$ sq.units (D) $3a^2$ sq.units.

17. If the surface area of a sphere is 100π cm^2, then its radius is equal to

(A) 25 cm (B) 100 cm (C) 5 cm (D) 10 cm .

18. If the surface area of a sphere is 36π cm^2, then the volume of the sphere is equal to

(A) 12π cm^3 (B) 36π cm^3 (C) 72π cm^3 (D) 108π cm^3.

19. If the total surface area of a solid hemisphere is 12π cm^2 then its curved surface area is equal to

(A) 6π cm^2 (B) 24π cm^2 (C) 36π cm^2 (D) 8π cm^2.

20. If the radius of a sphere is half of the radius of another sphere, then their respective volumes are in the ratio

(A) 1 : 8 (B) 2: 1 (C) 1 : 2 (D) 8 : 1

21. Curved surface area of solid sphere is 24 cm^2. If the sphere is divided into two hemispheres, then the total surface area of one of the hemispheres is

(A) 12 cm^2 (B) 8 cm^2 (C) 16 cm^2 (D) 18 cm^2

22. Two right circular cones have equal radii. If their slant heights are in the ratio 4 : 3, then their respective curved surface areas are in the ratio

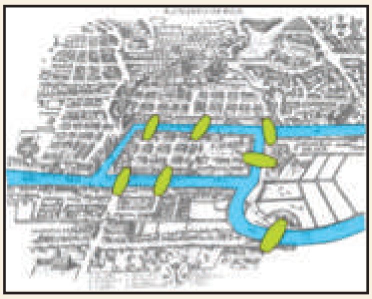
(A) 16 : 9 (B) 2 : 3 (C) 4 : 3 (D) 3 : 4

Do you know?

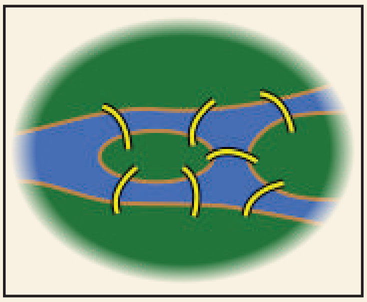
The Seven Bridges of Königsberg is a notable historical problem in mathematics. The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.(See Figure)

The problem was to find a route through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time (one could not walk half way onto the bridge and then turn around and later cross the other half from the other side).

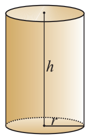
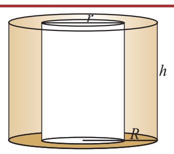
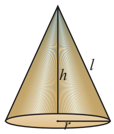
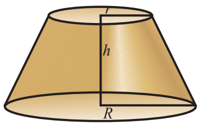
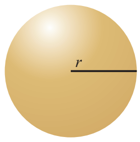
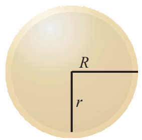
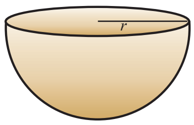
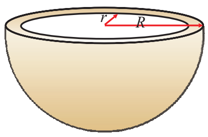
Leonhard Euler in 1735 proved that the problem has no solution. Its negative resolution by Euler laid the foundations of graph theory and presaged the idea of topology.



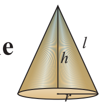
$\Longleftrightarrow$



Points to Remember

Sl. No	Name	Figure	Lateral or Curved Surface Area (sq.units)	Total Surface Area (sq.units)	Volume (cu.units)
1	**Solid right circular cylinder**		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2	**Right circular hollow cylinder**		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	Volume of the material used $\pi R^2 h - \pi r^2 h$ $= \pi h(R^2 - r^2)$ $= \pi h(R+r)(R-r)$
3	**Solid right circular cone**		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	**Frustum**		- - - - - - -	- - - - - - - - - - - - - - -	$\frac{1}{3}\pi h(R^2 + r^2 + Rr)$
5	**Sphere**		$4\pi r^2$	- - -	$\frac{4}{3}\pi r^3$
6	**Hollow sphere**		- - -	- - -	Volume of the material used $\frac{4}{3}\pi(R^3 - r^3)$
7	**Solid Hemisphere**		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
8	**Hollow Hemisphere**		$2\pi(R^2 + r^2)$	$2\pi(R^2 + r^2) + \pi(R^2 - r^2)$ $= \pi(3R^2 + r^2)$	Volume of the material used $\frac{2}{3}\pi(R^3 - r^3)$

9 **A sector of a circle converted into a Cone**



$l = \sqrt{h^2 + r^2}$

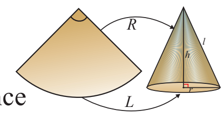
$h = \sqrt{l^2 - r^2}$

$r = \sqrt{l^2 - h^2}$

CSA of a cone = Area of the sector

$$\pi rl = \frac{\theta}{360} \times \pi r^2$$

Length of the sector = Base circumference of the cone



10. Volume of water flows out through a pipe = {Cross section area × Speed × Time }

11. No. of new solids obtained by recasting

$$= \frac{\text{Volume of the solid which is melted}}{\text{volume of one solid which is made}}$$

12 Conversions: 1 m³ = 1000 litres , 1 d.m³ = 1 litre , 1000 cm³ = 1 litre , 1000 litres = 1 *kl*