9 PRACTICAL GEOMETRY

Give me a place to stand, and I shall move the earth
-Archimedes

- Introduction
- Tangents
- Triangles
- Cyclic Quadrilaterals

Brahmagupta
(598-668 AD)
India
(Great Scientist of Ancient India)

Brahmagupta wrote the book "Brahmasphuta Siddhanta". His most famous result in geometry is a formula for cyclic quadrilateral :

Given the lengths p, q, r and s of the sides of any cyclic quadrilateral, he gave an approximate and an exact formula for the area.

Approximate area is

$$\left(\frac{p+r}{2}\right)\left(\frac{q+s}{2}\right).$$

Exact area is

$$\sqrt{(t-p)(t-q)(t-r)(t-s)},$$

where $2t = p+q+r+s$.

9.1 Introduction

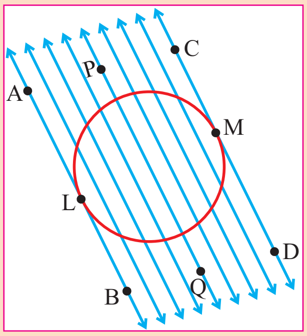
Geometry originated in Egypt as early as 3000 B.C., was used for the measurement of land. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes which were developed to meet some practical needs in surveying, construction, astronomy and various other crafts.

Recently there have been several new efforts to reform curricula to make geometry less worthy than its counterparts such as algebra, analysis, etc. But many mathematicians strongly disagree with this reform. In fact, geometry helps in understanding many mathematical ideas in other parts of mathematics. In this chapter, we shall learn how to draw tangents to circles, triangles and cyclic quadrilaterals with the help of given actual measurements.

In class IX, we have studied about various terms related to circle such as chord, segment, sector, etc. Let us recall some of the terms like secant, tangent to a circle through the following activities.

Activity

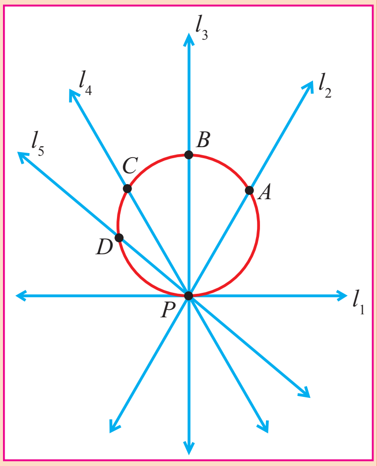
Take a paper and draw a circle of any radius. Draw a secant PQ to the circle. Now draw as many secants as possible parallel to PQ on both sides of PQ. Note that the points of contact of the secants are coming closer and closer on



either side. You can also note that at one stage, the two points will coincide on both sides. Among the secants parallel to *PQ*, the straight lines *AB* and *CD*, just touch the circle exactly at one point on the circle, say at *L* and *M* respectively. These lines *AB, CD* are called **tangents** to the circle at *L, M* respectively. We observe that AB is parallel to CD.

Activity

Let us draw a circle and take a point P on the circle. Draw many lines through the point P as shown in the figure. The straight lines which are passing through P, have two contact points on the circle. The straight lines l_2, l_3, l_4 and l_5 meet the circle at A, B, C and D respectively. So these lines l_2, l_3, l_4, l_5 are the secants to the circle. But the line l_1 touches the circle exactly at one point *P*. Now the line l_1 is called the **tangent** to the circle at *P*.



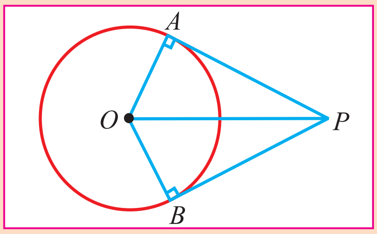
We know that in a circle, the radius drawn at the point of contact is perpendicular to the tangent at that point.

Let *AP* be a tangent at *A* drawn from an external point *P* to a circle

In a right angled ΔOPA , $OA \perp AP$

$$OP^2 = OA^2 + AP^2 \quad \text{[By Pythagoras theorem]}$$

$$AP = \sqrt{OP^2 - OA^2}.$$



9.2 Construction of tangents to a circle

Now let us learn how to draw a tangent to a circle

(i) using centre

(ii) using tangent-chord theorem .

9.2.1 Construction of a tangent to a circle (using the centre)

Result

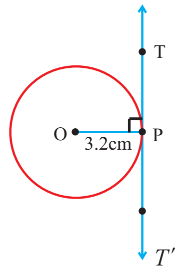
In a circle, the radius drawn at the point of contact is perpendicular to the tangent at that point.

Example 9.1

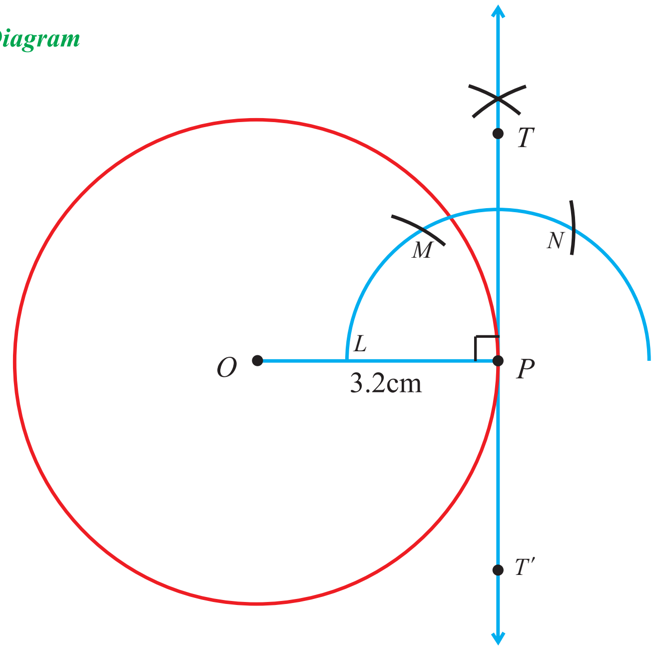
Draw a circle of radius 3.2cm. Take a point *P* on this circle and draw a tangent at *P*. (using the centre)

Given: Radius of the circle = 3.2 cm.

Rough Diagram



Fair Diagram



Construction

(i) With O as the centre draw a circle of radius 3.2 cm.

(ii) Take a point P on the circle and join OP.

(iii) Draw an arc of a circle with centre at P cutting OP at L.

(iv) Mark M and N on the arc such that $\overset{\frown}{LM} = \overset{\frown}{MN} = LP$.

(v) Draw the bisector PT of the angle $\angle MPN$.

(vi) Produce TP to T' to get the required tangent $T'PT$.

Remarks

One can draw the perpendicular line PT to the straight line OP through the point P on the circle. Now, PT is the tangent to the circle at the point P.

9.2.2 Construction of a tangent to a circle using the tangent-chord theorem

Result **The tangent-chord theorem**

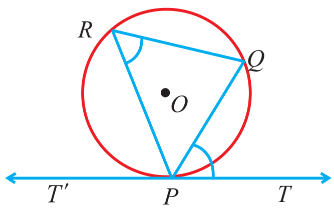
The angle between a chord of a circle and the tangent at one end of the chord is equal to the angle subtended by the chord on the alternate segment of the circle.

Example 9.2

Draw a circle of radius 3.2 cm At a point P on it, draw a tangent to the circle using the tangent-chord theorem.

Given : The radius of the circle = 3.2 cm.

Rough Diagram



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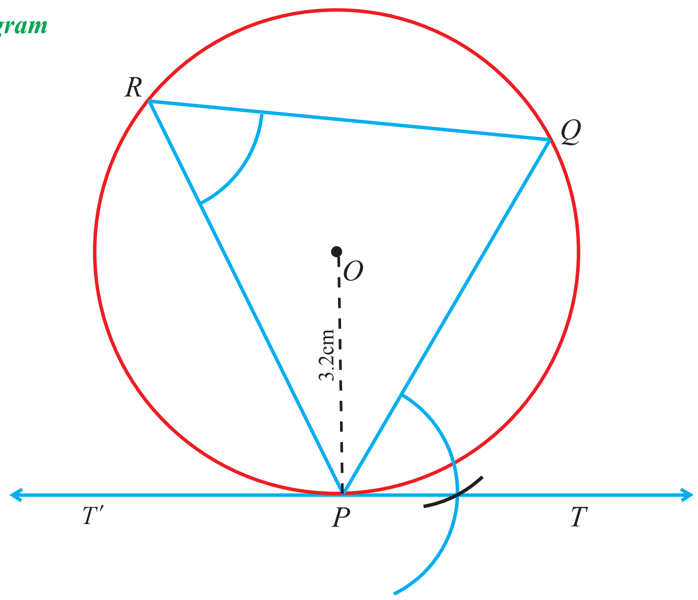


Construction

(i) With O as the centre, draw a circle of radius 3.2 cm.

(ii) Take a point P on the circle.

(iii) Through P, draw any chord PQ.

(iv) Mark a point R distinct from P and Q on the circle so that P, Q and R are in counter clockwise direction.

(v) Join PR and QR.

(vi) At P, construct $\angle QPT = \angle PRQ$.

(vii) Produce TP to T' to get the required tangent line $T'PT$.

9.2.3 Construction of pair of tangents to a circle from an external point

Results (i) Two tangents can be drawn to a circle from an external point.

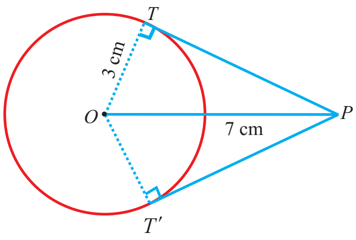
(ii) Diameters subtend 90° on the circumference of a circle.

Example 9.3

Draw a circle of radius 3 cm. From an external point 7 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Given: Radius of the circle = 3 cm. OP = 7 cm.

Rough Diagram



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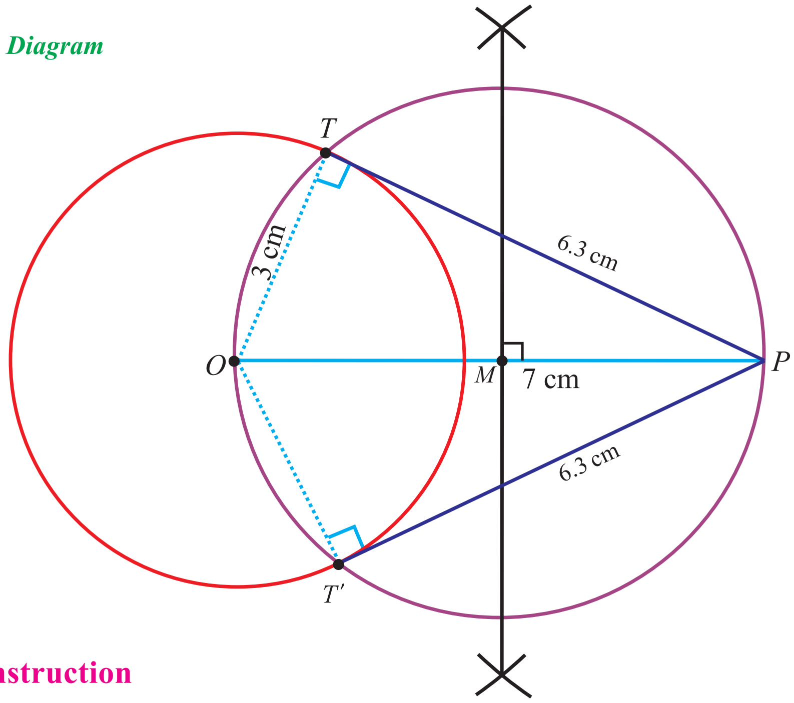


Construction

(i) With O as the centre draw a circle of radius 3 cm.

(ii) Mark a point P at a distance of 7 cm from O and join OP.

(iii) Draw the perpendicular bisector of OP. Let it meet OP at M.

(iv) With M as centre and MO as radius, draw another circle.

(v) Let the two circles intersect at T and T'.

(vi) Join PT and PT'. They are the required tangents.

Length of the tangent, PT = 6.3 cm

Verification

In the right angled ΔOPT,

$$PT = \sqrt{OP^2 - OT^2} = \sqrt{7^2 - 3^2}$$

$$= \sqrt{49 - 9} = \sqrt{40} \qquad \therefore \quad PT = 6.3 \text{ cm (approximately)}.$$

Exercise 9.1

1. Draw a circle of radius 4.2 cm, and take any point on the circle. Draw the tangent at that point using the centre.
2. Draw a circle of radius 4.8 cm. Take a point on the circle. Draw the tangent at that point using the tangent-chord theorem.
3. Draw a circle of diameter 10 cm. From a point *P*, 13 cm away from its centre, draw the two tangents *PA* and *PB* to the circle, and measure their lengths.
4. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 6 cm. Also, measure the lengths of the tangents.
5. Take a point which is 9 cm away from the centre of a circle of radius 3 cm, and draw the two tangents to the circle from that point.

9.3 Construction of triangles

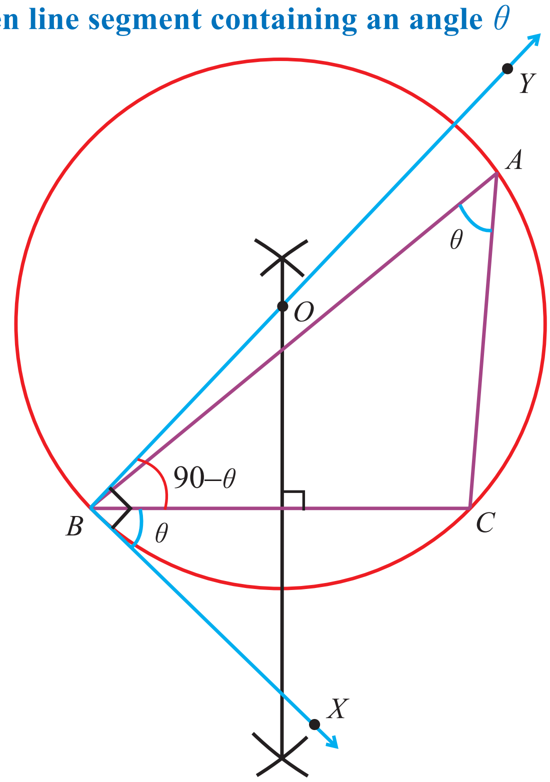
We have already learnt how to construct triangles when sides and angles are given. In this section, let us construct a triangle when

(i) the base, vertical angle and the altitude from the vertex to the base are given.

(ii) the base, vertical angle and the median from the vertex to the base are given.

First, let us describe the way of constructing a segment of a circle on a given line segment containing a given angle.

Construction of a segment of a circle on a given line segment containing an angle θ



Construction

(i) Draw a line segment $\overline{BC}$.

(ii) At *B*, make $\angle CBX = \theta$.

(iii) Draw $BY \perp BX$.

(iv) Draw the perpendicular bisector of *BC* which meets *BY* at O.

(v) With *O* as centre and *OB* as radius draw a circle.

(vi) Take any point *A* on the circle. By the tangent-chord theorem, the major arc *BAC* is the required segment of the circle containing the angle θ.

Construction of a triangle when its base and the vertical angle are given.

We shall describe the various steps involved in the construction of a triangle when its base and the vertical angle are given.

Construction

(i) Draw a line segment AB.

(ii) At A, make the given angle $\angle BAX = \theta$

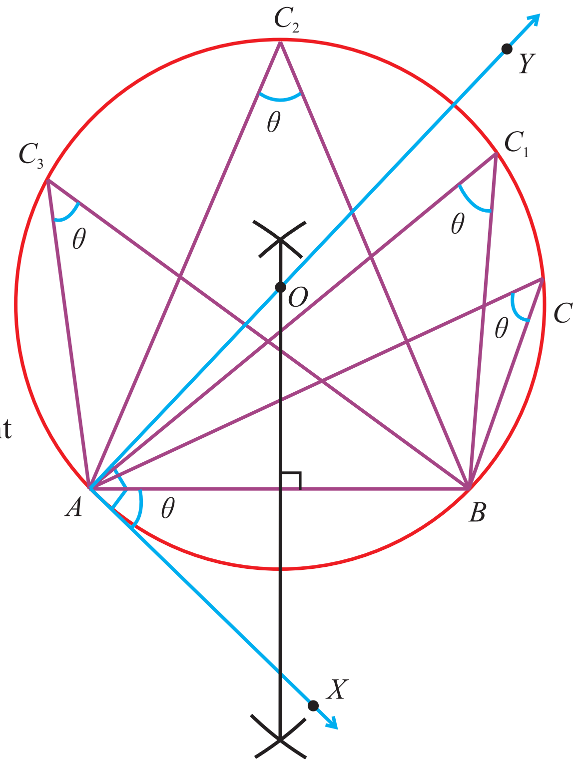
(iii) Draw $AY \perp AX$.

(iv) Draw the perpendicular bisector of AB which meets AY at O.

(v) With O as centre OA as radius, draw a circle.

(vi) Take any point C on the alternate segment of the circle and join AC and BC.

(vii) $\triangle ABC$ is the required triangle.



Now, one can justify that $\triangle ABC$ is one of the triangles, with the given base and the vertical angle.

Note that $AX \perp AY$. Thus, $\angle XAY = 90^\circ$.

Also, $OB = OA$. (the radii of the circle).

AX is the tangent to the circle at A and C is any point on the circle.

Hence, $\angle BAX = \angle ACB$. (tangent-chord theorem).

Remarks

If we take $C_1, C_2, C_3, \ldots$ are points on the circle, then all the triangle ΔABC_1, ΔABC_2, ΔABC_3, $\cdots$ are with same base and the same vertical angle.

9.3.1 Construction of a triangle when its base, the vertical angle and the altitude from the vertex to the base are given.

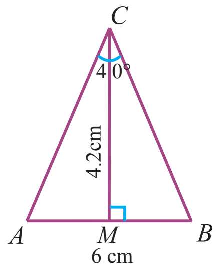
Example 9.4

Construct a ΔABC such that $AB = 6$ cm, $\angle C = 40^\circ$ and the altitude from C to AB is of length 4.2 cm.

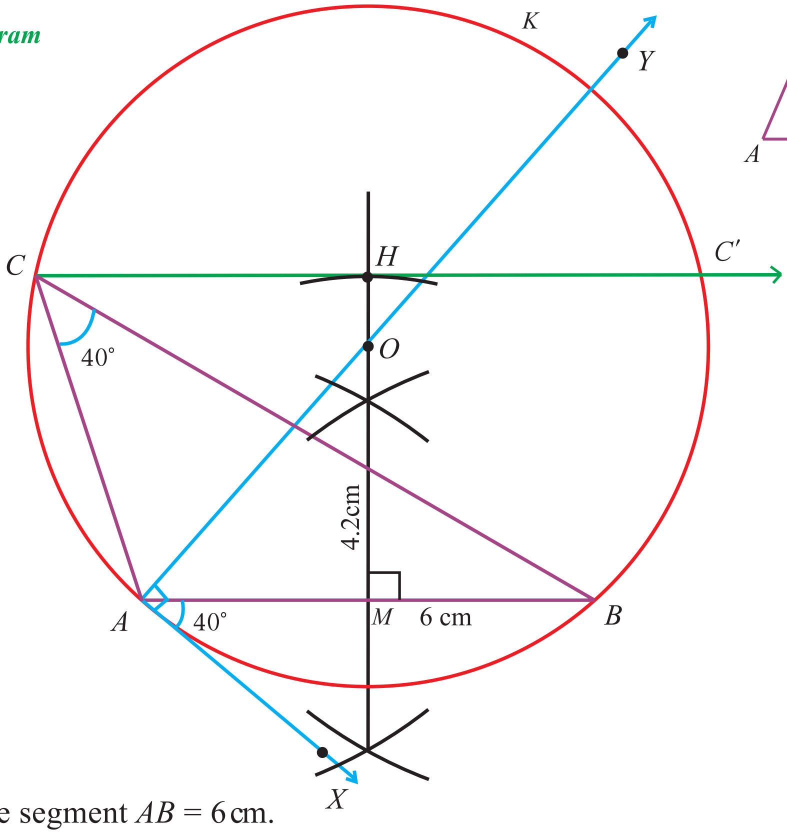
Given : In ΔABC, $AB = 6$ cm , $\angle C = 40^\circ$

The length of the altitude from C to AB is 4.2 cm.

Rough Diagram



Fair Diagram



Construction

(i) Draw a line segment $AB = 6$ cm.

(ii) Draw AX such that $\angle BAX = 40^\circ$.

(iii) Draw $AY \perp AX$.

(iv) Draw the perpendicular bisector of AB intersecting AY at O and AB at M.

(v) With O as centre and OA as radius, draw the circle .

(vi) The segment AKB contains the vertical angle 40°.

(vii) On the perpendicular bisector MO, mark a point H such that $MH = 4.2$ cm.

(viii) Draw CHC' parallel to AB meeting the circle at C and at C'.

(ix) Complete the ΔABC, which is one of the required triangles.

Remarks

$\triangle ABC'$ is also another required triangle.

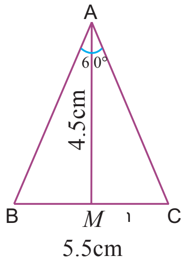
9.3.2 Construction of a triangle when its base, the vertical angle and the median from the vertex to the base are given.

Example 9.5

Construct a ΔABC in which $BC = 5.5$ cm., $\angle A = 60^\circ$ and the median AM from the vertex A is 4.5 cm.

Given : In ΔABC, $BC = 5.5$ cm , $\angle A = 60^\circ$, Median $AM = 4.5$ cm.

Rough Diagram



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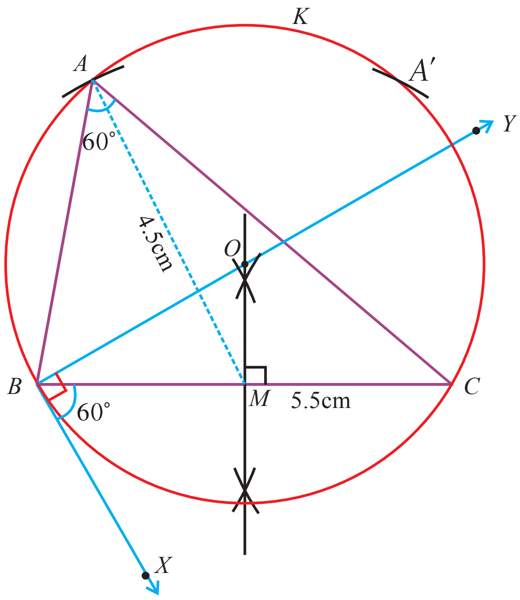


Construction

(i) Draw a line segment $BC = 5.5$ cm.

(ii) Through B draw BX such that $\angle CBX = 60^\circ$.

(iii) Draw $BY \perp BX$.

(iv) Draw the perpendicular bisector of BC intersecting BY at O and BC at M.

(v) With O as centre and OB as radius, draw the circle.

(vi) The major arc BKC of the circle, contains the vertical angle 60°.

(vii) With M as centre, draw an arc of radius 4.5 cm meeting the circle at A and A'.

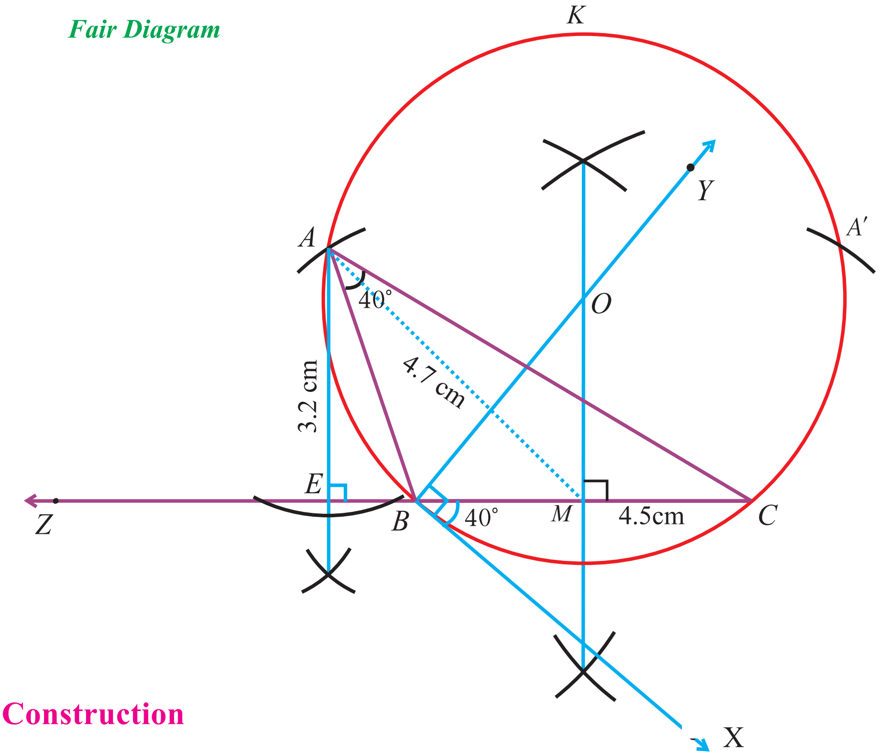
(viii) $\triangle ABC$ or $\Delta A'BC$ is the required triangle.

Example 9.6

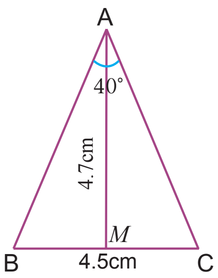
Construct a ΔABC, in which $BC = 4.5$ cm, $\angle A = 40^\circ$ and the median AM from A to BC is 4.7 cm. Find the length of the altitude from A to BC.

Given : In ΔABC, $BC = 4.5$ cm, $\angle A = 40^\circ$ and the median AM from A to BC is 4.7cm.

Fair Diagram



Rough Diagram



Construction

(i) Draw a line segment $BC = 4.5$ cm.

(ii) Draw BX such that $\angle CBX = 40^\circ$.

(iii) Draw $BY \perp BX$.

(iv) Draw the perpendicular bisector of BC intersecting BY at O and BC at M.

(v) With O as centre and OB as radius, draw the circle .

(vi) The major arc BKC of the circle, contains the vertical angle 40°.

(vii) With M as centre draw an arc of radius 4.7 cm meeting the circle at A and A'.

(viii) Complete ΔABC or $\triangle A'BC$, which is the required triangle.

(ix) Produce CB to CZ.

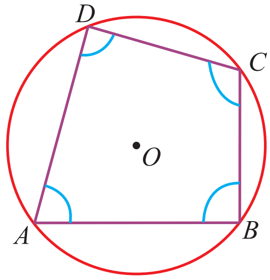
(x) Draw $AE \perp CZ$.

(xi) Length of the altitude AE is 3.2cm.

Exercise 9.2

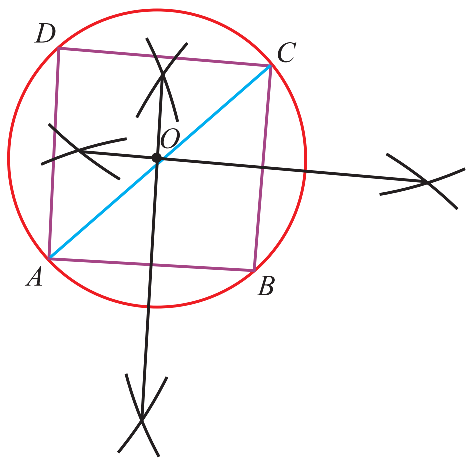
1. Construct a segment of a circle on a given line segment $AB = 5.2$ cm containing an angle 48°.
2. Construct a ΔPQR in which the base $PQ = 6$ cm, $\angle R = 60^\circ$ and the altitude from R to PQ is 4 cm.
3. Construct a ΔPQR such that $PQ = 4$ cm, $\angle R = 25^\circ$ and the altitude from R to PQ is 4.5 cm.
4. Construct a ΔABC such that $BC = 5$ cm. $\angle A = 45^\circ$ and the median from A to BC is 4cm.
5. Construct a ΔABC in which the base $BC = 5$ cm, $\angle BAC = 40^\circ$ and the median from A to BC is 6 cm. Also, measure the length of the altitude from A.

9.4 Construction of cyclic quadrilateral



If the vertices of a quadrilateral lie on a circle, then the quadrilateral is known as a cyclic quadrilateral. In a cyclic quadrilateral, the opposite angles are supplementary. That is, the sum of opposite angles is 180°. Thus, four suitable measurements (instead of five measurements) are sufficient for the construction of a cyclic quadrilateral.

Let us describe the various steps involved in the construction of a cyclic quadrilateral when the required measurements are given.



(i) Draw a rough figure and draw a $\triangle ABC$ or $\triangle ABD$ using the given measurements.

(ii) Draw the perpendicular bisectors of AB and BC intersecting each other at O. (one can take any two sides of $\triangle ABC$)

(iii) With O as the centre, and OA as radius, draw a circumcircle of $\triangle ABC$.

(iv) Using the given measurement, find the fourth vertex D and join AD and CD.

(v) Now, $ABCD$ is the required cyclic quadrilateral.

In this section, we shall construct a cyclic quadrilateral based on the different set of measurements of the cyclic quadrilateral as listed below.

(i) Three sides and one diagonal. (ii) Two sides and two diagonals. (iii) Three sides and one angle. (iv) Two sides and two angles. (v) One side and three angles. (vi) Two sides, one angle and one parallel line.

Type I (Three sides and one diagonal of a cyclic quadrilateral are given)

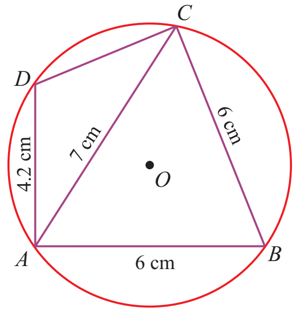
Example 9.7

Construct a cyclic quadrilateral *ABCD* in which *AB* = 6 cm, *AC* = 7 cm, *BC* = 6 cm, and *AD* = 4.2 cm.

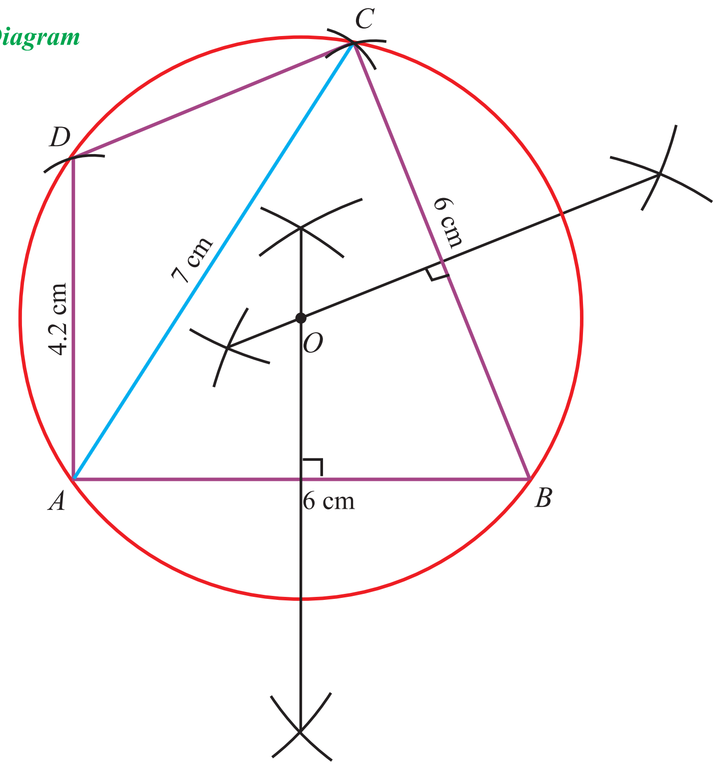
Given : In the cyclic quadrilateral *ABCD*, *AB* = 6 cm, *AC* = 7 cm.

BC = 6 cm, and *AD* = 4.2 cm.

Rough Diagram



Fair Diagram



Construction

(i) Draw a rough diagram and mark the measurements.

Draw a line segment *AB* = 6 cm.

(ii) With *A* and *B* as centres, draw arcs with radii 7 cm and 6 cm respectively, to intersect at *C*. Join *AC* and *BC*.

(iii) Draw the perpendicular bisectors of *AB* and *BC* to intersect at *O*.

(iv) With *O* as the centre and *OA* (= *OB* = *OC*) as radius draw the circumcircle of ΔABC

(v) With *A* as the centre and radius 4.2 cm. draw an arc intersecting the circumcircle at *D*.

(vi) Join *AD* and *CD*.

Now, *ABCD* is the required cyclic quadrilateral.

Type II (Two sides and two diagonals of a cyclic quadrilateral are given)

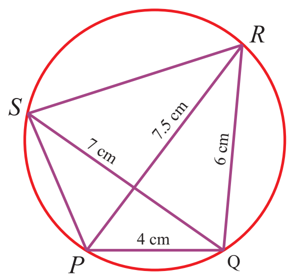
Example 9.8

Construct a cyclic quadrilateral $PQRS$ with $PQ = 4$ cm, $QR = 6$ cm, $PR = 7.5$ cm, $QS = 7$ cm

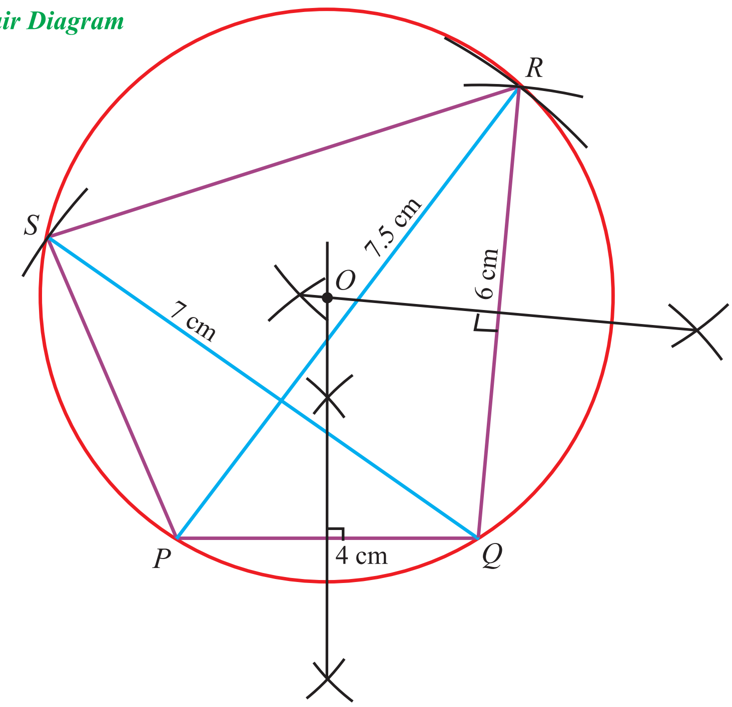
Given : In the cyclic quadrilateral $PQRS$, $PQ = 4$ cm, $QR = 6$ cm,

$PR = 7.5$ cm and $QS = 7$ cm

Rough diagram



Fair Diagram



Construction

(i) Draw a rough diagram and mark the measurements.
Draw a line segment $PQ = 4$ cm

(ii) With P as centre and radius 7.5 cm, draw an arc.

(iii) With Q as centre and radius 6 cm, draw another arc meeting the previous arc as in the figure at R.

(iv) Join PR and QR.

(v) Draw the perpendicular bisectors of PQ and QR intersecting each other at O.

(vi) With O as the centre $OP(=OQ=OR)$ as radius, draw the circumcircle of ΔPQR.

(vii) With Q as centre and 7cm radius , draw an arc intersecting the circle at S.

(viii) Join PS and RS.

(ix) Now, $PQRS$ is the required cyclic quadrilateral.

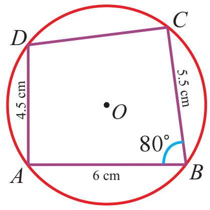
Type III (Three sides and one angle of a cyclic quadrilateral are given)

Example 9.9

Construct a cyclic quadrilateral $ABCD$ when $AB = 6$ cm, $BC = 5.5$ cm, $\angle ABC = 80^\circ$ and $AD = 4.5$ cm.

Given: In the Cyclic Quadrilateral $ABCD$, $AB = 6$ cm, $BC = 5.5$ cm, $\angle ABC = 80^\circ$ and $AD = 4.5$ cm.

Rough Diagram



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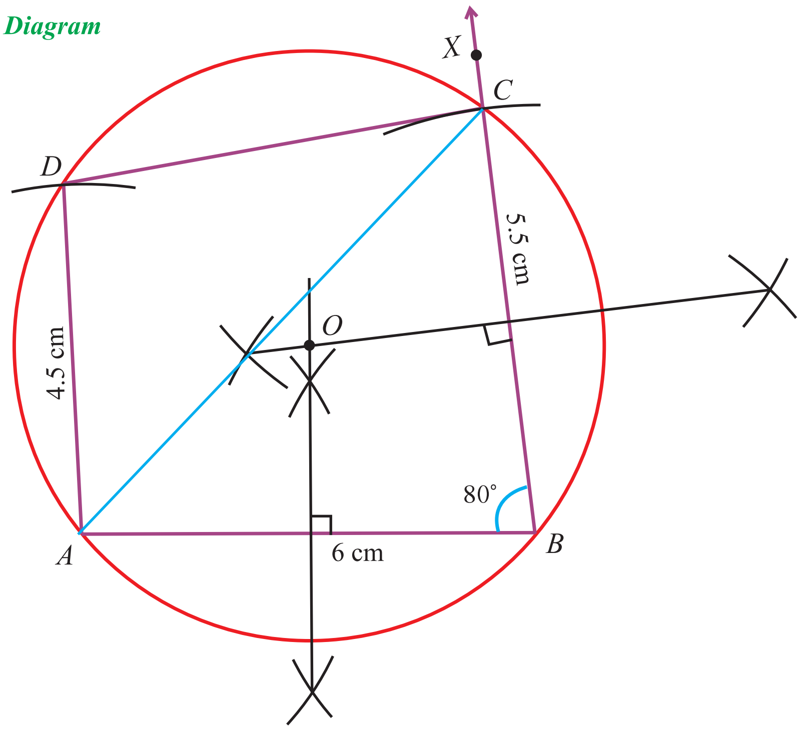


Construction

(i) Draw a rough diagram and mark the measurements.

Draw a line segment $AB = 6$ cm.

(ii) Through B draw BX such that $\angle ABX = 80^\circ$.

(iii) With B as centre and radius 5.5 cm, draw an arc intersecting BX at C and join AC.

(iv) Draw the perpendicular bisectors of AB and BC intersecting each other at O.

(v) With O as centre and OA $(= OB = OC)$ as radius, draw the circumcircle of ΔABC.

(vi) With A as centre and radius 4.5 cm, draw an arc intersecting the circle at D.

(vii) Join AD and CD.

(viii) Now, $ABCD$ is the required cyclic quadrilateral.

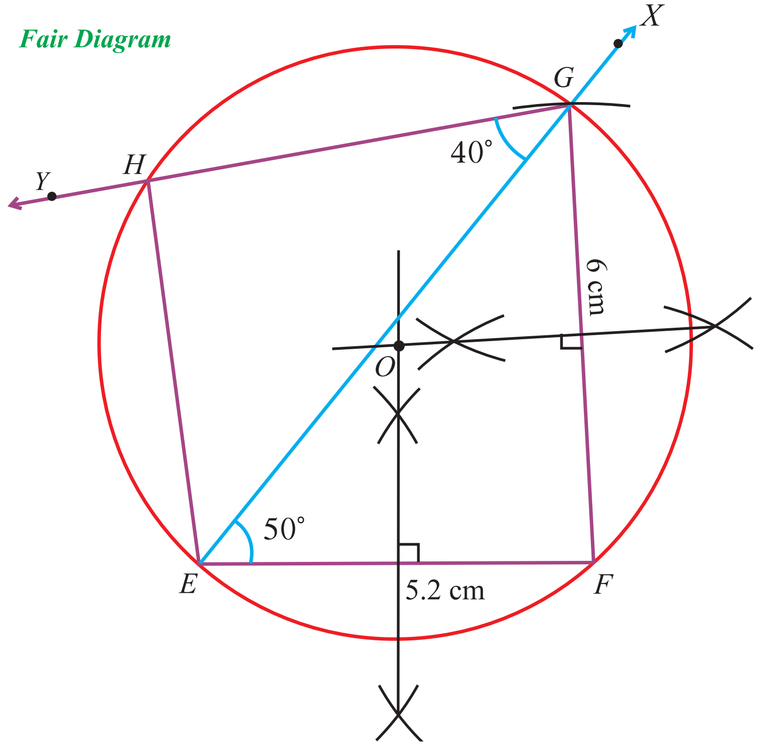
Type IV (Two sides and two angles of a cyclic quadrilateral are given)

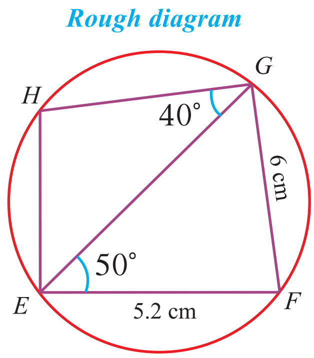
Example 9.10

Construct a cyclic quadrilateral *EFGH* with $EF = 5.2$ cm, $\angle GEF = 50^\circ$, $FG = 6$ cm and $\angle EGH = 40^\circ$.

Given: In the Cyclic Quadrilateral *EFGH*

$EF = 5.2$ cm, $\angle GEF = 50^\circ$, $FG = 6$ cm and $\angle EGH = 40^\circ$.





Construction

(i) Draw a rough diagram and mark the measurements.
Draw a line segment $EF = 5.2$ cm.

(ii) From *E,* draw *EX* such that $\angle FEX = 50^\circ$.

(iii) With *F* as centre and radius 6 cm, draw an arc intersecting *EX* at *G.*

(iv) Join *FG.*

(v) Draw the perpendicular bisectors of *EF* and *FG* intersecting each other at *O.*

(vi) With *O* as centre and *OE* $(= OF = OG)$ as radius, draw a circumcircle.

(vii) From *G,* draw *GY* such that $\angle EGY = 40^\circ$ which intersects the circle at *H.*

(viii) Join *EH.*

Now, *EFGH* is the required cyclic quadrilateral.

Type V **(One side and three angles of a cyclic quadrilateral are given)**

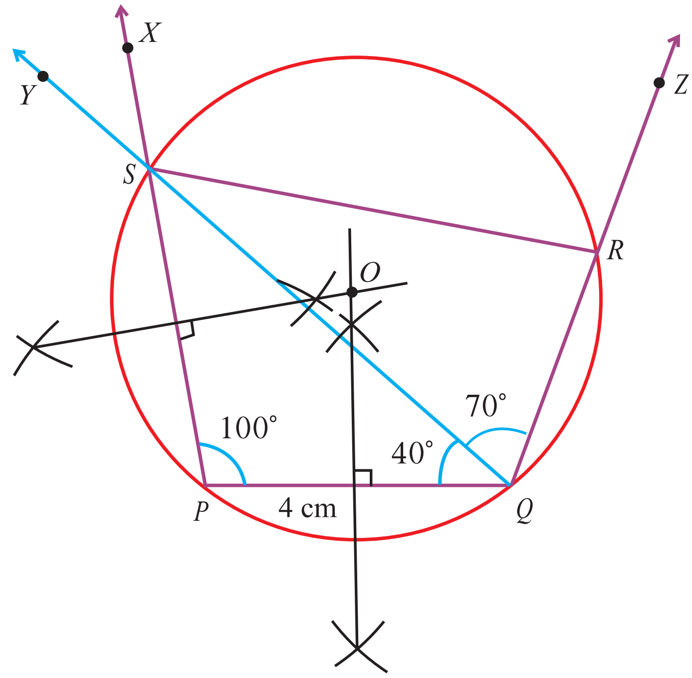
Example 9.11

Construct a cyclic quadrilateral $PQRS$ with $PQ = 4$ cm, $\angle P = 100^\circ$, $\angle PQS = 40^\circ$ and $\angle SQR = 70^\circ$.

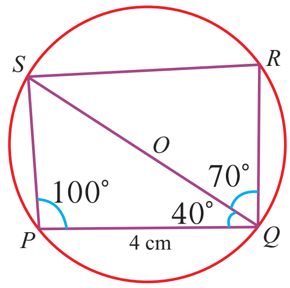
Given: In the cyclic quadrilateral $PQRS$,

$PQ = 4$ cm, $\angle P = 100^\circ$, $\angle PQS = 40^\circ$ and $\angle SQR = 70^\circ$.

Fair Diagram



Rough diagram



Construction

(i) Draw a rough diagram and mark the measurements.

Draw a line segment $PQ = 4$ cm.

(ii) From P draw PX such that $\angle QPX = 100^\circ$.

(iii) From Q draw QY such that $\angle PQY = 40^\circ$. Let QY meet PX at S.

(iv) Draw perpendicular bisectors of PQ and PS intersecting each other at O.

(v) With O as centre and $OP(= OQ = OS)$ as radius, draw a cicumcircle of $\triangle PQS$

(vi) From Q, draw QZ such that $\angle SQZ = 70^\circ$ which intersects the circle at R.

(vii) Join RS.

Now, $PQRS$ is the required cyclic quadrilateral.

Type VI

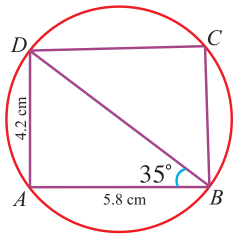
(Two sides , one angle and one parallel line are given)

Example 9.12

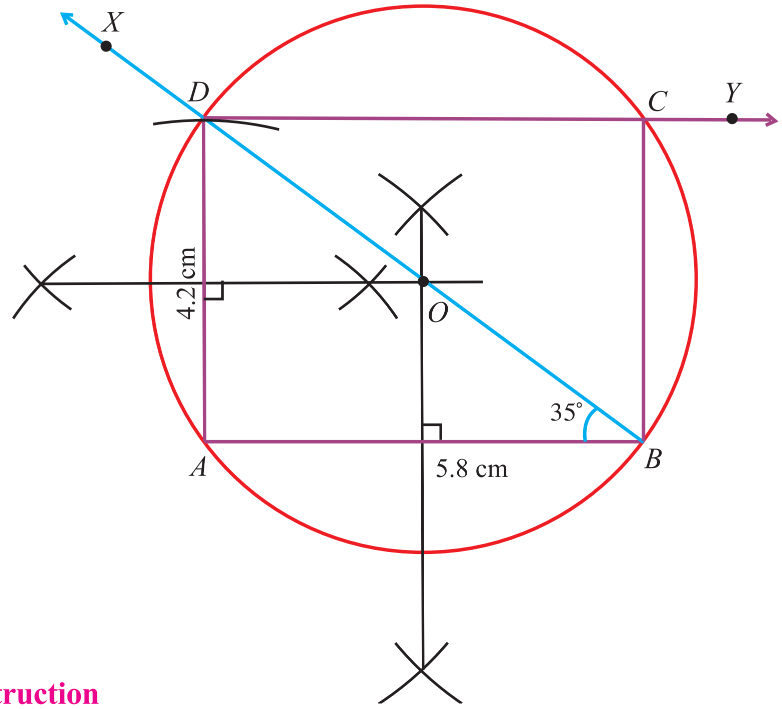
Construct a cyclic quadrilateral $ABCD$ when $AB = 5.8$ cm, $\angle ABD = 35^\circ$, $AD = 4.2$ cm and $AB \parallel CD$.

Given: In the cyclic quadrilateral $ABCD$, $AB = 5.8$ cm, $\angle ABD = 35^\circ$, $AD = 4.2$ cm and $AB \parallel CD$

Rough Diagram



Fair Diagram



Construction

(i) Draw a rough diagram and mark the measurements.

Draw a line segment $AB = 5.8$ cm.

(ii) From B, draw BX such that $\angle ABX = 35^\circ$.

(iii) With A as centre and radius 4.2 cm, draw an arc intersecting BX at D.

(iv) Draw perpendicular bisectors of AB and AD intersecting each other at O.

(v) With O as centre, and $OA\ (= OB = OD)$ as radius, draw a circumcircle of ΔABD.

(vi) Draw DY such that $DY \parallel AB$ intersecting the circle at C.

Join BC.

(vii) Now, $ABCD$ is the required cyclic quadrilateral.

Exercise 9.3

1. Construct a cyclic quadrilateral $PQRS$, with $PQ = 6.5$cm, $QR = 5.5$ cm, $PR = 7$cm and $PS = 4.5$cm.

2. Construct a cyclic quadrilateral $ABCD$ where $AB = 6$cm, AD = 4.8cm, $BD = 8$cm and $CD = 5.5$ cm.

3. Construct a cyclic quadrilateral $PQRS$ such that $PQ = 5.5$cm, $QR = 4.5$cm, $\angle QPR = 45^\circ$ and $PS = 3$cm.

4. Construct a cyclic quadrilateral $ABCD$ with $AB = 7$cm, $\angle A = 80^\circ$, $AD = 4.5$ cm and $BC = 5$ cm.

5. Construct a cyclic quadrilateral $KLMN$ such that $KL = 5.5$ cm, $KM = 5$ cm, $LM = 4.2$ cm and $LN = 5.3$ cm.

6. Construct a cyclic quadrilateral $EFGH$ where $EF = 7$ cm, $EH = 4.8$ cm, $FH = 6.5$ cm and $EG = 6.6$ cm.

7. Construct a cyclic quadrilateral $ABCD$, given $AB = 6$ cm, $\angle ABC = 70^\circ$, $BC = 5$cm and $\angle ACD = 30^\circ$

8. Construct a cyclic quadrilateral $PQRS$ given $PQ = 5$ cm, $QR = 4$ cm, $\angle QPR = 35^\circ$ and $\angle PRS = 70^\circ$

9. Construct a cyclic quadrilateral $ABCD$ such that $AB = 5.5$ cm $\angle ABC = 50^\circ$, $\angle BAC = 60^\circ$ and $\angle ACD = 30^\circ$

10. Construct a cyclic quadrilateral $ABCD$, where $AB = 6.5$ cm, $\angle ABC = 110^\circ$, $BC = 5.5$ cm and $AB \parallel CD$.

Do you know?

Every year since 1901, the prestigious Nobel Prize has been awarded to individuals for achievements in Physics, Chemistry, Physiology or medicine, Literature and for Peace. The Nobel Prize is an international award administered by the Nobel Foundation in Stockholm, Sweden. There is no Nobel Prize for Mathematics.

The Fields medal is a prize awarded to two , three or four Mathematicians not over 40 years of age at each International congress of the International Mathematical Union (IMU), a meeting that takes place every four years.

The Fields medal is often described as the Nobel Prize for Mathematics.