UNIT 10

OSCILLATIONS

Life is a constant oscillation between the sharp horns of a dilemma – H.L. Mencken

LEARNING OBJECTIVES

In this unit, the student is exposed to

- oscillatory motion – periodic motion and non-periodic motion
- simple harmonic motion
- angular harmonic motion
- linear harmonic oscillator – both horizontal and vertical
- combination of springs – series and parallel
- simple pendulum
- expression of energy – potential energy, kinetic energy and total energy
- graphical representation of simple harmonic motion
- types of oscillation – free, damped, maintained and forced oscillations
- concept of resonance

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10.1 INTRODUCTION

Figure 10.1. Thanjavur dancing doll

Have you seen the Thanjavur Dancing Doll (In Tamil, it is called 'Thanjavur thalayatti bommai')?. It is a world famous Indian cultural doll (Figure 10.1). What does this doll do when disturbed? It will dance such that the head and body move continuously in a to and fro motion, until the movement gradually stops. Similarly, when we walk on the road, our hands and legs will move front and back. Again similarly, when a mother swings a cradle to make her child sleep, the cradle is made to move in to and fro motion. All these motions are different from the motion that we have discussed so far. These motions are shown in Figure 10.2. Generally, they are known as oscillatory motion or vibratory motion. A similar motion occurs even at atomic levels. When the temperature is raised, the atoms in a solid vibrate about their rest position (mean position or equilibrium position). The study of vibrational motion is very important in engineering applications, such as, designing the structure of building, mechanical equipments, etc.

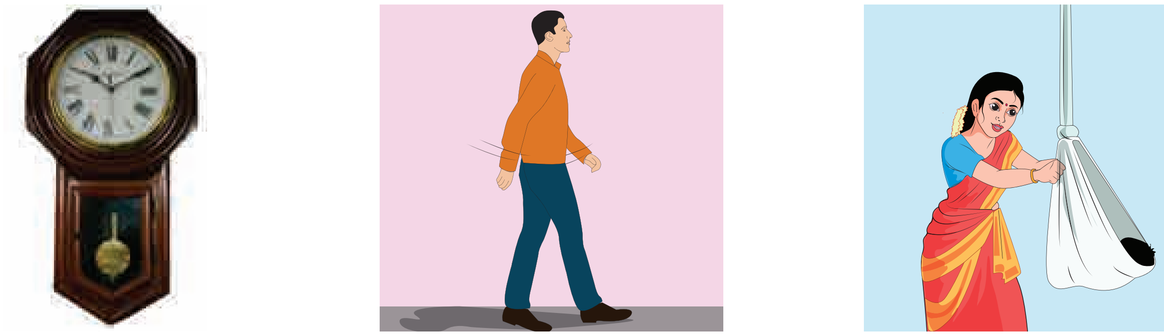

Figure 10.2. Motions

10.1.1 Periodic and non-periodic motion

Motion in physics can be classified as repetitive (periodic motion) and non-repetitive (non-periodic motion).

1. **Periodic motion**
   Any motion which repeats itself in a fixed time interval is known as periodic motion.
   Examples : Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.
2. **Non-Periodic motion**
   Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.
   Example : Occurance of Earth quake, eruption of volcano, etc.

EXAMPLE 10.1

Classify the following motions as periodic and non-periodic motions?.

a. Motion of Halley's comet.
b. Motion of clouds.
c. Moon revolving around the Earth.

Solution

a. Periodic motion
b. Non-periodic motion
c. Periodic motion

EXAMPLE 10.2

Which of the following functions of time represent periodic and non-periodic motion?.

a. $\sin \omega t + \cos \omega t$
b. $\ln \omega t$

Solution

a. Periodic
b. Non-periodic

Question to ponder

Discuss "what will happen if the motion of the Earth around the Sun is not a periodic motion".

10.1.2 Oscillatory motion

When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory). Examples; our heart beat, swinging motion of the wings of an insect, grandfather's clock (pendulum clock), etc. Note that all oscillatory motion are periodic whereas all periodic motions need not be oscillation in nature. see Figure 10.3

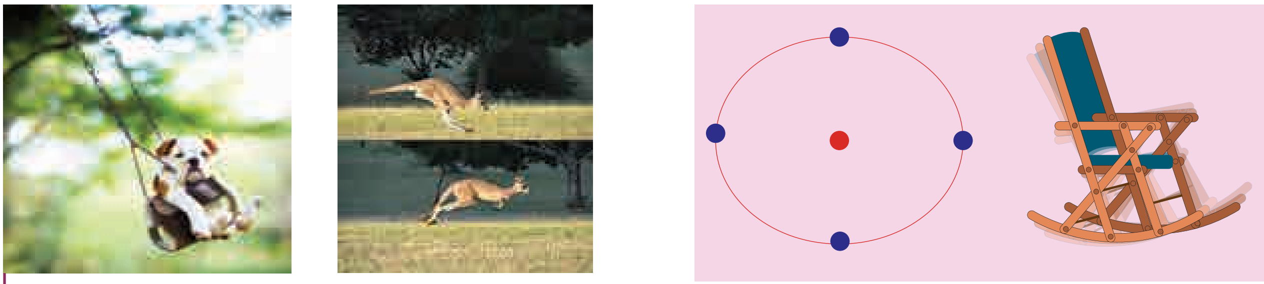

Figure 10.3 **Oscillatory or vibratory motions**

10.2 SIMPLE HARMONIC MOTION (SHM)

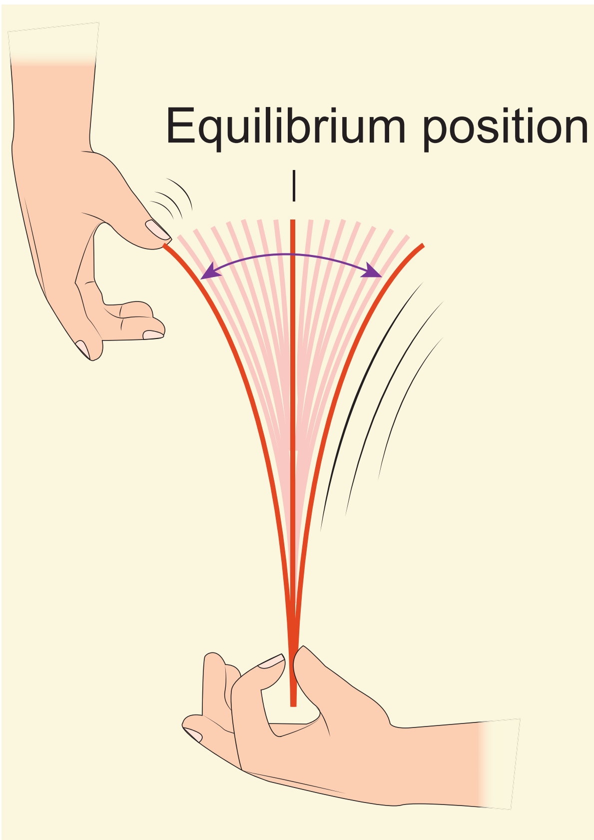

Figure 10.4 Simple Harmonic Motion

> **Note** A simple harmonic motion is a special type of oscillatory motion. But all oscillatory motions need not be simple harmonic .

Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point. In one dimensional case, let x be the displacement of the particle and a_x be the acceleration of the particle, then

$$a_x \propto x \tag{10.1}$$

$$a_x = -b\,x \tag{10.2}$$

where b is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}. By multiplying by mass of the particle on both sides of equation (10.2) and from Newton's second law, the force is

$$F_x = -k\,x \tag{10.3}$$

where k is a force constant which is defined as force per unit length. The negative sign indicates that displacement and force (or acceleration) are in opposite directions. This means that when the displacement of the particle is taken towards right of equilibrium position (x takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the displacement of the particle is taken towards left of equilibrium position (x takes negative value), the force (or acceleration) will point towards equilibrium (towards right). This type of force is known as ***restoring force*** *because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position.* This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

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In order to represent in two or three dimensions, we can write using vector notation

$$\vec{F} = -k\vec{r} \qquad (10.4)$$

where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship. This means that the exponent of force $\vec{F}$ and the exponent of displacement $\vec{r}$ are unity. The sketch between cause (magnitude of force $|\vec{F}|$) and effect (magnitude of displacement $|\vec{r}|$) is a straight line passing through second and fourth quadrant as shown in Figure 10.5. By measuring slope $\frac{1}{k}$, one can find the numerical value of force constant k.

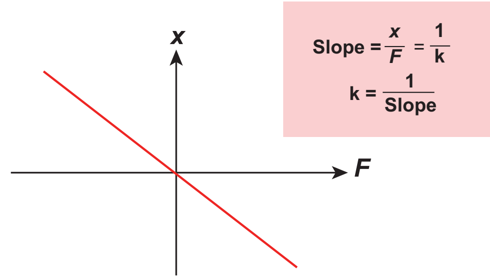

Figure 10.5 Force verses displacement graph

10.2.1 The projection of uniform circular motion on a diameter of SHM

Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction (as shown in Figure 10.6). Let us assume that the origin of the coordinate system coincides with the center O of the circle. If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$. By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate

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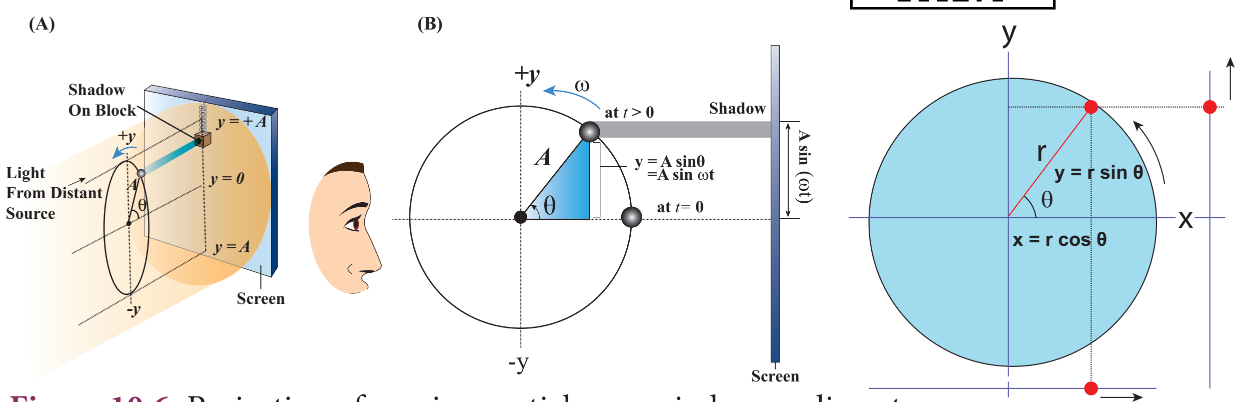

Figure 10.6 Projection of moving particle on a circle on a diameter

a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion. Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in Figure 10.7. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.

The following figures explain the position of particle at different time :

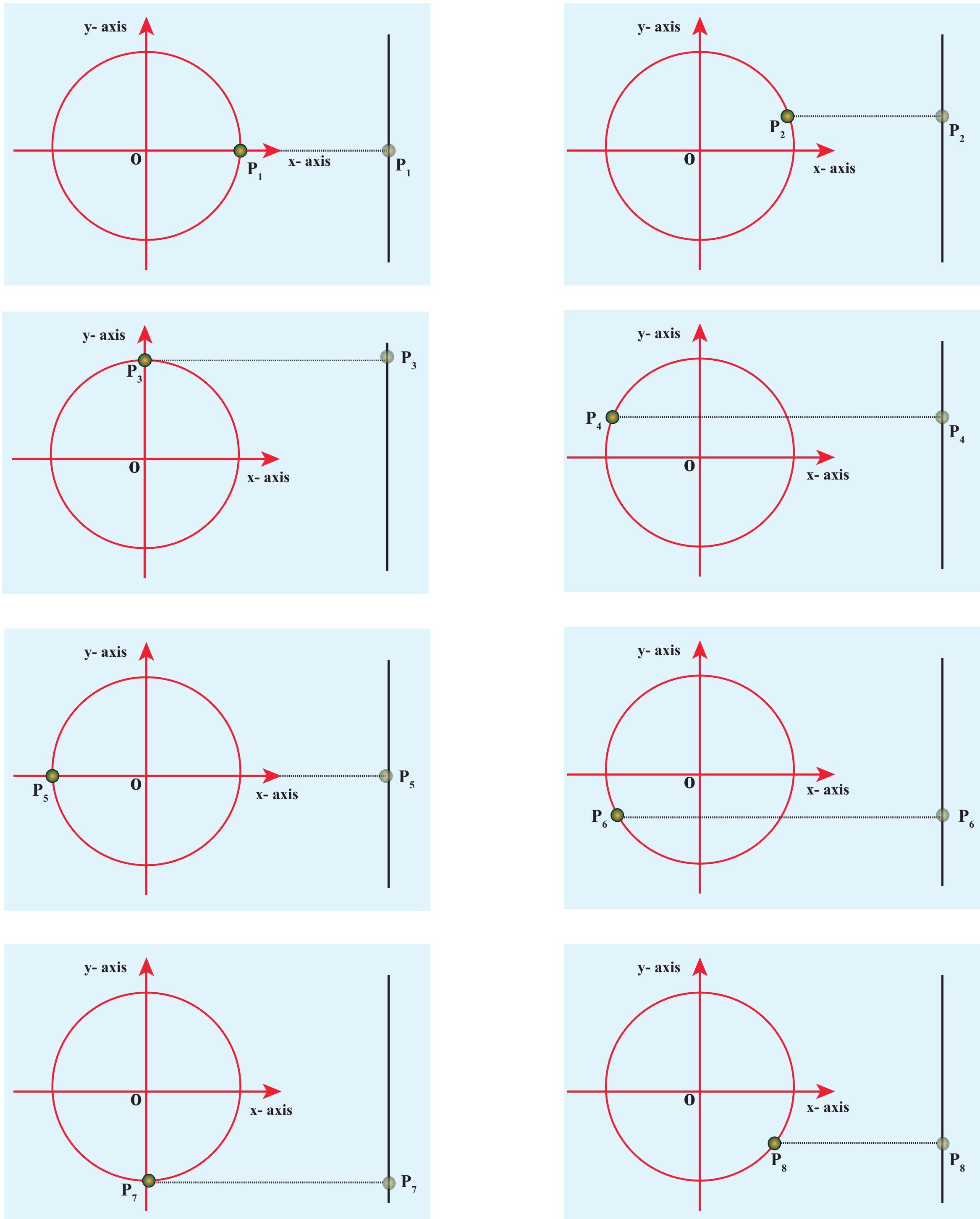

Figure 10.7 The location of a particle at each instant as projected on a vertical axis

As a specific example, consider a spring mass system (or oscillation of pendulum) as shown in Figure 10.8. When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion.

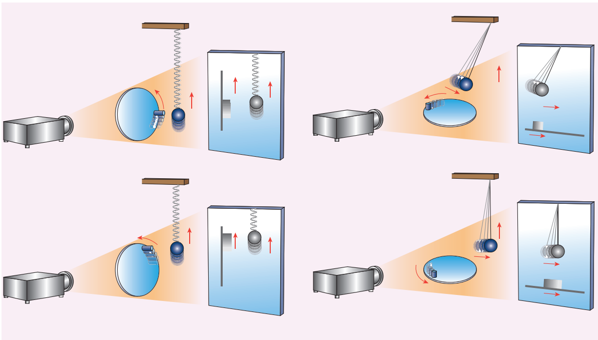

Figure 10.8 Motion of spring mass (or simple pendulum) related to uniform circular motion

Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straightline motion which is simple harmonic in nature. The circle is known as reference circle of the simple harmonic motion. *The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.*

ACTIVITY

a. Sketch the projection of ***spiral in*** motion as a wave form.
b. Sketch the projection of ***spiral out*** motion as a wave form.

10.2.2 Displacement, velocity, acceleration and its graphical representation - SHM

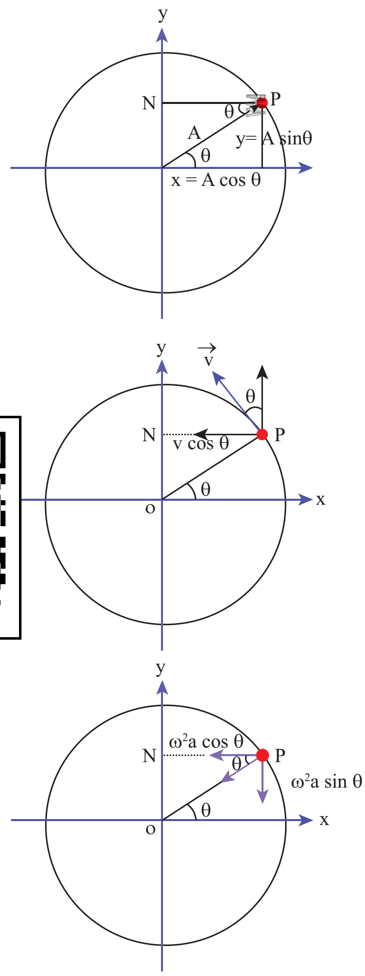

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Figure 10.9 Displacement, velocity and acceleration of a particle at some instant of time

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. Let *P* be the position of the particle on a circle of radius A at some instant of time *t* as shown in Figure 10.9. Then its displacement y at that instant of time *t* can be derived as follows

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$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta \qquad (10.5)$$

But $\theta = \omega t$, ON = y and OP = A

$$y = A \sin \omega t \quad (10.6)$$

The displacement y takes maximum value (which is equal to A) when $\sin \omega t = 1$. This *maximum displacement from the mean position is known as amplitude (A) of the vibrating particle.* For simple harmonic motion, the amplitude is constant. But, in general, for any motion other than simple harmonic, the amplitude need not be constant, it may vary with time.

Velocity

The rate of change of displacement is velocity. Taking derivative of equation (10.6) with respect to time, we get

$$v = \frac{dy}{dt} = \frac{d}{dt}(A \sin \omega t)$$

For circular motion (of constant radius), amplitude A is a constant and further, for uniform circular motion, angular velocity ω is a constant. Therefore,

$$v = \frac{dy}{dt} = A\,\omega \cos \omega t \quad (10.7)$$

Using trigonometry identity,

$\sin^2 \omega t + \cos^2 \omega t = 1 \Rightarrow \cos \omega t = \sqrt{1-\sin^2 \omega t}$

we get

$$v = A\omega\sqrt{1-\sin^2 \omega t}$$

From equation (10.6),

$$\sin \omega t = \frac{y}{A}$$

$$v = A\omega\sqrt{1-\left(\frac{y}{A}\right)^2}$$

$$v = \omega\sqrt{A^2 - y^2} \quad (10.8)$$

From equation (10.8), when the displacement $y = 0$, the velocity $v = \omega A$ (maximum) and for the maximum displacement $y = A$, the velocity $v = 0$ (minimum).

As displacement increases from zero to maximum, the velocity decreases from maximum to zero. This is repeated.

Since velocity is a vector quantity, equation (10.7) can also be deduced by resolving in to components.

Acceleration

The rate of change of velocity is acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega \cos \omega t\right)$$

$$a = -\omega^2 A \sin \omega t = -\omega^2 y \quad (10.9)$$

$$\therefore \quad a = \frac{d^2 y}{dt^2} = -\omega^2 y \quad (10.10)$$

From the Table 10.1 and figure 10.10, we observe that at the mean position

Table 10.1 Displacement, velocity and acceleration at different instant of time.

Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4} = T$
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement $y = A \sin \omega t$	0	A	0	$-A$	0
Velocity $v = A\,\omega \cos \omega t$	$A\,\omega$	0	$-A\,\omega$	0	$A\,\omega$
Acceleration $a = -A\,\omega^2 \sin \omega t$	0	$-A\,\omega^2$	0	$A\,\omega^2$	0

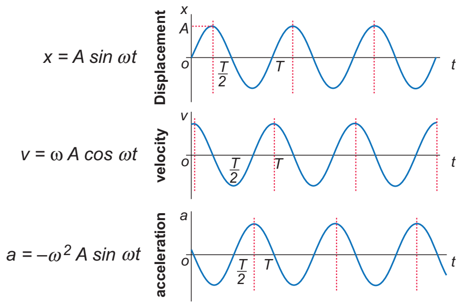


Figure 10.10 Variation of displacement, velocity and acceleration at different instant of time

(y = 0), velocity of the particle is maximum but the acceleration of the particle is zero. At the extreme position ($y = \pm A$), the velocity of the particle is zero but the acceleration is maximum $\pm A\omega^2$ acting in the opposite direction.

EXAMPLE 10.3

Which of the following represent simple harmonic motion?

(i) $x = A \sin \omega t + B \cos \omega t$

(ii) $x = A \sin \omega t + B \cos 2\omega t$

(iii) $x = A\, e^{i\omega t}$

(iv) $x = A \ln \omega t$

Solution

(i) $x = A \sin \omega t + B \cos \omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,\omega \sin \omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + B \cos \omega t)$$

$$\frac{d^2x}{dt^2} = -\,\omega^2 x$$

This differential equation is similar to the differential equation of SHM (equation 10.10).

Therefore, $x = A \sin \omega t + B \cos \omega t$ represents SHM.

(ii) $x = A \sin \omega t + B \cos 2\omega t$

$$\frac{dx}{dt} = A\,\omega \cos \omega t - B\,(2\omega) \sin 2\omega t$$

$$\frac{d^2x}{dt^2} = -\,\omega^2\,(A \sin \omega t + 4B \cos 2\omega t)$$

$$\frac{d^2x}{dt^2} \neq -\,\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \sin \omega t + B \cos 2\omega t$ does not represent SHM.

(iii) $x = A\, e^{i\omega t}$

$$\frac{dx}{dt} = A\,\omega e^{i\omega t}$$

$$\frac{d^2x}{dt^2} = -A\,\omega^2\, e^{i\omega t} = -\omega^2 x$$

This differential equation is like the differential equation of SHM (equation 10.10). Therefore, $x = A\, e^{i\omega t}$ represents SHM.

(iv) $x = A \ln \omega t$

$$\frac{dx}{dt} = \left(\frac{A}{\omega t}\right)\omega = \frac{A}{t}$$

$$\frac{d^2x}{dt^2} = -\frac{A}{t^2} \Rightarrow \frac{d^2x}{dt^2} \neq -\omega^2 x$$

This differential equation is not like the differential equation of a SHM (equation 10.10). Therefore, $x = A \ln \omega t$ does not represent SHM.

EXAMPLE 10.4

Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and velocity of the particle at position x_2 is v_2. Show that the ratio of time period and amplitude is

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

Solution

Using equation (10.8)

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v^2 = \omega^2 (A^2 - x^2)$

Therefore, at position x_1,

$$v_1^2 = \omega^2(A^2 - x_1^2) \quad (1)$$

Similarly, at position x_2,

$$v_2^2 = \omega^2(A^2 - x_2^2) \quad (2)$$

Subtrating (2) from (1), we get

$$v_1^2 - v_2^2 = \omega^2(A^2 - x_1^2) - \omega^2(A^2 - x_2^2)$$

$$= \omega^2(x_2^2 - x_1^2)$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad (3)$$

Dividing (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{\omega^2\left(A^2 - x_1^2\right)}{\omega^2\left(A^2 - x_2^2\right)} \Rightarrow A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \quad (4)$$

Dividing equation (3) and equation (4), we have

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

10.2.3 Time period, frequency, phase, phase difference and epoch in SHM.

(i) Time period

The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. For one complete revolution, the time taken is t = T, therefore

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} \quad (10.11)$$

Then, the displacement of a particle executing simple harmonic motion can be written either as sine function or cosine function.

$$y(t) = A \sin \frac{2\pi}{T}t \quad \text{or} \quad y(t) = A \cos \frac{2\pi}{T}t$$

where T represents the time period. Suppose the time t is replaced by $t + T$, then the function

$$y(t+T) = A \sin \frac{2\pi}{T}(t+T)$$

$$= A \sin(\frac{2\pi}{T}t + 2\pi)$$

$$= A \sin \frac{2\pi}{T}t = y(t)$$

$$y(t+T) = y(t)$$

Thus, the function repeats after one time period. This $y(t)$ is an example of periodic function.

(ii) Frequency and angular frequency

The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is s^{-1} or hertz (In symbol, Hz).

Mathematically, frequency is related to time period by

$$f = \frac{1}{T} \quad (10.12)$$

The number of cycles (or revolutions) per second is called angular frequency. It is usually denoted by the Greek small letter 'omega', ω. Comparing equation (10.11) and equation (10.12), angular frequency and frequency are related by

$$\omega = 2\pi f \quad (10.13)$$

SI unit for angular frequency is rad s^{-1}. (read it as radian per second)

(iii) Phase

The phase of a vibrating particle at any instant completely specifies the state of the particle. It expresses the position and direction of motion of the particle at that instant with respect to its mean position (Figure 10.11).

$$y = A \sin(\omega t + \varphi_0) \quad (10.14)$$

where $\omega t + \varphi_0 = \varphi$ *is called the phase of the vibrating particle. At time* t = 0 *s (initial time), the phase* $\varphi = \varphi_0$ *is called epoch (initial phase) where* φ_0 *is called the angle of epoch.*

Phase difference: Consider two particles executing simple harmonic motions. Their equations are $y_1 = A\sin(\omega t + \varphi_1)$ and $y_2 = A\sin(\omega t + \varphi_2)$, then the phase difference $\Delta\varphi = (\omega t + \varphi_2) - (\omega t + \varphi_1) = \varphi_2 - \varphi_1$.

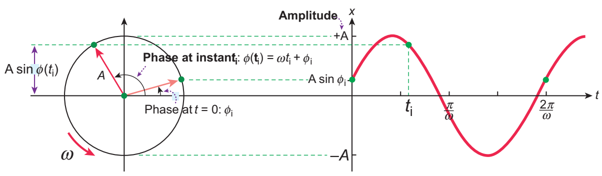


Figure 10.11 The phase of vibrating particle at two instant of time.

EXAMPLE 10.5

A nurse measured the average heart beats of a patient and reported to the doctor in terms of time period as 0.8 *s*. Express the heart beat of the patient in terms of number of beats measured per minute.

Solution

Let the number of heart beats measured be *f*. Since the time period is inversely proportional to the heart beat, then

$$f = \frac{1}{T} = \frac{1}{0.8} = 1.25\,s^{-1}$$

One minute is 60 second,

$$(1 \text{ second} = \frac{1}{60} \text{ minute} \Rightarrow 1\text{ s}^{-1} = 60 \text{ min}^{-1})$$

$f = 1.25\text{ s}^{-1} \Rightarrow f = 1.25 \times 60 \text{ min}^{-1} = 75$ beats per minute

EXAMPLE 10.6

Calculate the amplitude, angular frequency, frequency, time period and initial phase for the simple harmonic oscillation given below

a. $y = 0.3\sin(40\pi t + 1.1)$

b. $y = 2\cos(\pi t)$

c. $y = 3\sin(2\pi t - 1.5)$

Solution

Simple harmonic oscillation equation is $y = A\sin(\omega t + \varphi_0)$ or $y = A\cos(\omega t + \varphi_0)$

a. For the wave, $y = 0.3\sin(40\pi t + 1.1)$

Amplitude is $A = 0.3$ unit

Angular frequency $\omega = 40\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{20} = 0.05\,s$

Initial phase is $\varphi_0 = 1.1$ rad

b. For the wave, $y = 2\cos(\pi t)$

Amplitude is $A = 2$ unit

Angular frequency $\omega = \pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{0.5} = 2\,s$

Initial phase is $\varphi_0 = 0$ rad

c. For the wave, $y = 3\sin(2\pi t + 1.5)$

Amplitude is $A = 3$ unit

Angular frequency $\omega = 2\pi$ rad s^{-1}

Frequency $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

Time period $T = \frac{1}{f} = \frac{1}{1} = 1\,s$

Initial phase is $\varphi_0 = 1.5$ rad

EXAMPLE 10.7

Show that for a simple harmonic motion, the phase difference between

a. displacement and velocity is $\frac{\pi}{2}$ radian or 90°.

b. velocity and acceleration is $\frac{\pi}{2}$ radian or 90°.

c. displacement and acceleration is π radian or 180°.

Solution

a. The displacement of the particle executing simple harmonic motion $y = A \sin\omega t$

Velocity of the particle is

$$v = A\,\omega \cos\omega t = A\,\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between displacement and velocity is $\frac{\pi}{2}$.

b. The velocity of the particle is

$$v = A\,\omega \cos \omega t$$

Acceleration of the particle is

$$a = -A\omega^2 \sin\omega t = A\omega^2 \cos\left(\omega t + \frac{\pi}{2}\right)$$

The phase difference between velocity and acceleration is $\frac{\pi}{2}$.

c. The displacement of the particle is $y = A \sin\omega t$

Acceleration of the particle is

$$a = -A\,\omega^2 \sin \omega t = A\,\omega^2 \sin(\omega t + \pi)$$

The phase difference between displacement and acceleration is π.

10.3 ANGULAR SIMPLE HARMONIC MOTION

10.3.1 Time period and frequency of angular SHM

When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation. The point

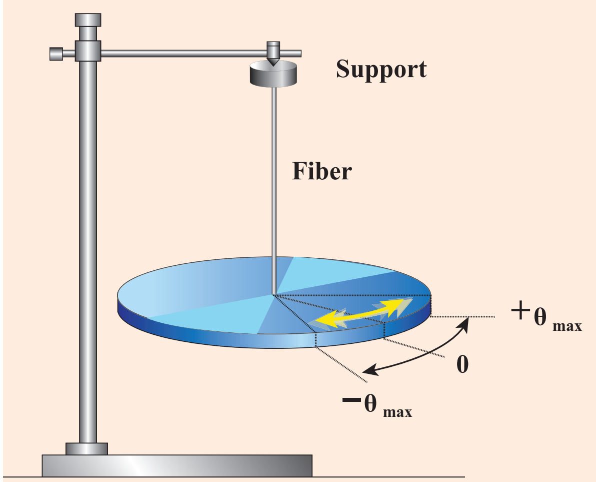


Figure 10.12 A body (disc) allowed to rotate freely about an axis

at which the resultant torque acting on the body is taken to be zero is called mean position. If the body is displaced from the mean position, then the resultant torque acts such that it is proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position. (Note: Torque is explained in unit 5)

Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is

$$\vec{\tau} \propto \vec{\theta} \quad (10.15)$$

$$\vec{\tau} = -\kappa\vec{\theta} \quad (10.16)$$

κ is the restoring torsion constant, which is torque per unit angular displacement. If I is the moment of inertia of the body and $\vec{\alpha}$ is the angular acceleration then

$$\vec{\tau} = I\vec{\alpha} = -\kappa\vec{\theta}$$

But $\vec{\alpha}=\frac{d^2\vec{\theta}}{dt^2}$ and therefore,

$$\frac{d^2\vec{\theta}}{dt^2}=-\frac{\kappa}{I}\vec{\theta} \qquad (10.17)$$

This differential equation resembles simple harmonic differential equation.

So, comparing equation (10.17) with simple harmonic motion given in equation (10.10), we have

$$\omega=\sqrt{\frac{\kappa}{I}}\,rad\,s^{-1} \qquad (10.18)$$

The frequency of the angular harmonic motion (from equation 10.13) is

$$f=\frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}\,Hz \qquad (10.19)$$

The time period (from equation 10.12) is

$$T=2\pi\sqrt{\frac{I}{\kappa}}\text{ second} \qquad (10.20)$$

10.3.2 Comparison of Simple Harmonic Motion and Angular Simple Harmonic Motion

In linear simple harmonic motion, the displacement of the particle is measured in terms of linear displacement $\vec{r}$ The restoring force is $\vec{F}=-k\vec{r}$, where k is a spring constant or force constant which is force per unit displacement. In this case, the inertia factor is mass of the body executing simple harmonic motion.

In angular simple harmonic motion, the displacement of the particle is measured in terms of angular displacement $\vec{\theta}$. Here, the spring factor stands for torque constant i.e., the moment of the couple to produce unit angular displacement or the restoring torque per unit angular displacement. In this case, the inertia factor stands for moment of inertia of the body executing angular simple harmonic oscillation.

Table 10.2 Comparision of simple harmonic motion and angular harmonic motion

S.No	Simple Harmonic Motion	Angular Harmonic Motion
1.	The displacement of the particle is measured in terms of linear displacement $\vec{r}$.	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$ (also known as angle of twist).
2.	Acceleration of the particle is $\vec{a}=-\omega^2\vec{r}$	Angular acceleration of the particle is $\vec{\alpha}=-\omega^2\vec{\theta}$.
3.	Force, $\vec{F}=m\vec{a}$, where m is called mass of the particle.	Torque, $\vec{\tau}=I\vec{\alpha}$, where I is called moment of inertia of a body.
4.	The restoring force $\vec{F}=-k\vec{r}$, where k is restoring force constant.	The restoring torque $\vec{\tau}=-\kappa\vec{\theta}$, where the symbol κ (Greek alphabet is pronounced as '*kappa*') is called restoring torsion constant. It depends on the property of a particular torsion fiber.
5.	Angular frequency, $\omega=\sqrt{\frac{k}{m}}$ rad s^{-1}	Angular frequency, $\omega=\sqrt{\frac{\kappa}{I}}$ rad s^{-1}

10.4 LINEAR SIMPLE HARMONIC OSCILLATOR (LHO)

10.4.1 Horizontal oscillations of a spring-mass system

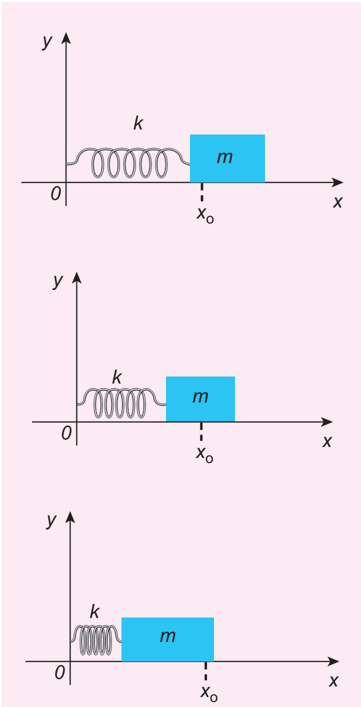


Figure 10.13 Horizontal oscillation of a spring-mass system

Consider a system containing a block of mass m attached to a massless spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in Figure 10.13. Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed. Suppose the mass is displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position x_0. Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block. For one dimensional motion, mathematically, we have

$$F \propto x$$
$$F = -k\,x$$

where negative sign implies that the restoring force will always act opposite to the direction of the displacement. This equation is called Hooke's law (refer to unit 7). Notice that, the restoring force is linear with the displacement (i.e., the exponent of force and displacement are unity). This is not always true; in case if we apply a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher powers of x) and therefore, the oscillation of the system is not linear and hence, it is called non-linear oscillation. We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).

From Newton's second law, we can write the equation for the particle executing simple harmonic motion

$$m\frac{d^2x}{dt^2} = -k\,x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (10.21)$$

Comparing the equation (10.21) with simple harmonic motion equation (10.10), we get

$$\omega^2 = \frac{k}{m}$$

which means the angular frequency or natural frequency of the oscillator is

$$\omega = \sqrt{\frac{k}{m}}\ \text{rad}\ s^{-1} \qquad (10.22)$$

The frequency of the oscillation is

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\ \text{Hertz} \qquad (10.23)$$

and the time period of the oscillation is

$$T=\frac{1}{f}=2\pi\sqrt{\frac{m}{k}}\ \text{seconds} \qquad (10.24)$$

Notice that ***in simple harmonic motion, the time period of oscillation is independent of amplitude***. This is valid only if the amplitude of oscillation is small.

The solution of the differential equation of a SHM may be written as

$$x(t) = A\sin(\omega t+\varphi) \qquad (10.25)$$

Or

$$x(t) = A\cos(\omega t+\varphi) \qquad (10.26)$$

where A, ω and ϕ are constants. General solution for differential equation 10.21 is $x(t) = A\sin(\omega t+\varphi) + B\cos(\omega t+\varphi)$ where A and B are contants.

Note

(a) Since, mass is inertial porperty and spring constant is an elastic property.

Time period is $T=2\pi\sqrt{\frac{m}{k}}$

$$T=2\pi\sqrt{\frac{\textit{Inertial property}}{\textit{Elasticproperty}}}=2\pi\sqrt{\left|\frac{\textit{displacement}}{\textit{acceleration}}\right|}$$

(b) $\frac{\textit{Displacement}}{\textit{acceleration}}=\frac{x}{\frac{d^2x}{dt^2}}=-\frac{m}{k}$, whose

modulus value or magnitude is $\frac{m}{k}$

hence, time period $T=2\pi\sqrt{\frac{m}{k}}$

10.4.2 Vertical oscillations of a spring

Figure 10.14 Springs

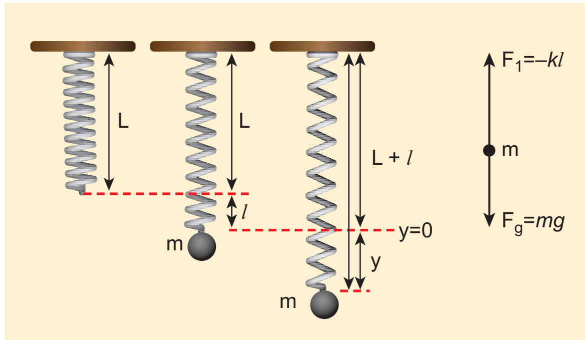


Figure 10.15 A massless spring with stiffness constant k

Let us consider a massless spring with stiffness constant or force constant k attached to a ceiling as shown in Figure 10.15. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l. Let F_1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in Figure 10.15. When the system is under equilibrium,

$$F_1 + mg = 0 \qquad (10.27)$$

But the spring elongates by small displacement l, therefore,

$$F_1 \propto l \Rightarrow F_1 = -kl \qquad (10.28)$$

Substituting equation (10.28) in equation (10.27), we get

$$-kl + mg = 0$$
$$mg = kl$$
or
$$\frac{m}{k} = \frac{l}{g} \qquad (10.29)$$

Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is $y + l$) is

$$F_2 \propto (y + l)$$
$$F_2 = -k(y + l) = -ky - kl \qquad (10.30)$$

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get

$$-ky - kl + mg = m\frac{d^2y}{dt^2} \qquad (10.31)$$

The net force acting on the mass due to this stretching is

$$F = F_2 + mg$$
$$F = -ky - kl + mg \qquad (10.32)$$

The gravitational force opposes the restoring force. Substituting equation (10.29) in equation (10.32), we get

$$F = -ky - kl + kl = -ky$$

Applying Newton's law, we get

$$m\frac{d^2y}{dt^2} = -ky$$
$$\frac{d^2y}{dt^2} = -\frac{k}{m}y \qquad (10.33)$$

The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ second} \qquad (10.34)$$

Note The time period obtained for horizontal oscillations of spring and for vertical oscillations of spring are found to be equal.

The time period can be rewritten using equation (10.29)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}} \text{ second} \qquad (10.35)$$

The acceleration due to gravity g can be computed from the formula

$$g = 4\pi^2\left(\frac{l}{T^2}\right) \text{ m s}^{-2} \qquad (10.36)$$

EXAMPLE 10.8

A spring balance has a scale which ranges from 0 to 25 kg and the length of the scale is 0.25m. It is taken to an unknown planet X where the acceleration due to gravity is 11.5 m s^{-1}. Suppose a body of mass M kg is suspended in this spring and made to oscillate with a period of 0.50 s. Compute the gravitational force acting on the body.

Solution

Let us first calculate the stiffness constant of the spring balance by using equation (10.29),

$$k = \frac{mg}{l} = \frac{25 \times 11.5}{0.25} = 1150\ N m^{-1}$$

The time period of oscillations is given by $T = 2\pi\sqrt{\frac{M}{k}}$, where M is the mass of the body.

Since, M is unknown, rearranging, we get

$$M = \frac{kT^2}{4\pi^2} = \frac{(1150)(0.5)^2}{4\pi^2} = 7.3\,kg$$

The gravitational force acting on the body is $W = Mg = 7.3 \times 11.5 = 83.95\ N \approx 84\ N$

10.4.3 Combinations of springs

Figure 10.16 Combination of spring as a shock-absorber in the motor cycle

Spring constant or force constant, also called as stiffness constant, is a measure of the stiffness of the spring. Larger the value of the spring constant, stiffer is the spring. This implies that we need to apply more force to compress or elongate the spring. Similarly, smaller the value of spring constant, the spring can be stretched (elongated) or compressed with lesser force. Springs can be connected in two ways. Either the springs can be connected end to end, also known as series connection, or alternatively, connected in parallel. In the following subsection, we compute the effective spring constant when

a. Springs are connected in series
b. Springs are connected in parallel

a. Springs connected in series

When two or more springs are connected in series, we can replace (by removing) all the springs in series with an equivalent spring (effective spring) whose net effect is the same as if all the springs are in series connection. Given the value of individual spring constants k_1, k_2, k_3,... (known quantity), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_s (unknown quantity). For simplicity, let us consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m as shown in Figure 10.17. The results thus obtained can be generalized for any number of springs in series.

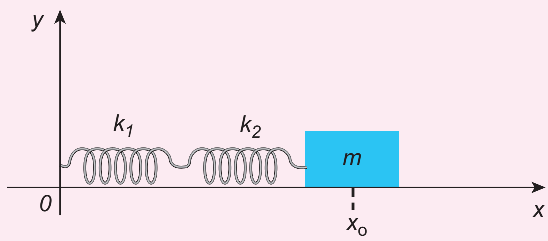


Figure 10.17 Springs are connected in series

Let F be the applied force towards right as shown in Figure 10.18. Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths. Let x_1 and x_2 be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force F. Then, the net displacement of the mass point is

$$x = x_1 + x_2 \qquad (10.37)$$

From Hooke's law, the net force

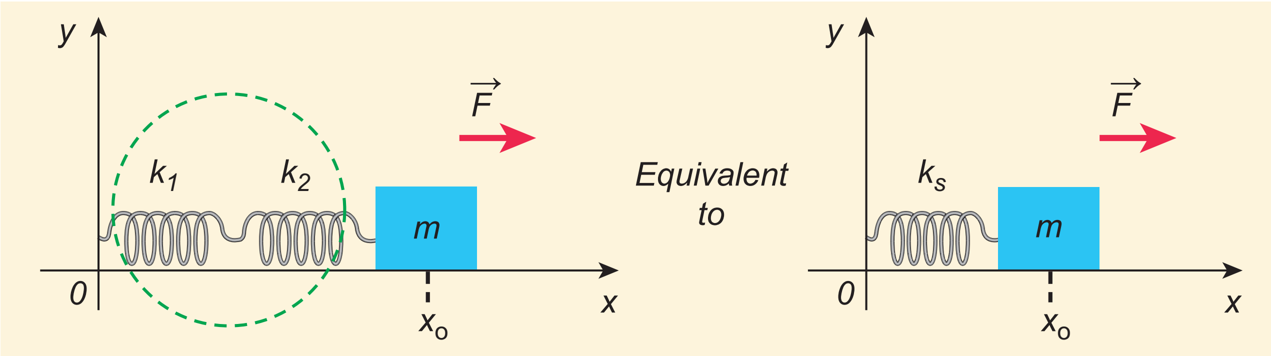


Figure 10.18 Effective spring constant in series connection

$$F = -k_s(x_1 + x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s} \quad (10.38)$$

For springs in series connection

$$-k_1x_1 = -k_2x_2 = F$$

$$\Rightarrow x_1 = -\frac{F}{k_1} \text{ and } x_2 = -\frac{F}{k_2} \quad (10.39)$$

Therefore, substituting equation (10.39) in equation (10.38), the *effective spring constant* can be calculated as

$$-\frac{F}{k_1} - \frac{F}{k_2} = -\frac{F}{k_s}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

Or

$$k_s = \frac{k_1k_2}{k_1 + k_2} \text{ Nm}^{-1} \quad (10.40)$$

Suppose we have n springs connected in series, the effective spring constant in series is

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=1}^{n} \frac{1}{k_i} \quad (10.41)$$

If all spring constants are identical i.e., $k_1 = k_2 = \ldots = k_n = k$ then

$$\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n} \quad (10.42)$$

This means that the effective spring constant reduces by the factor n. Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

From equation (10.39), we have,

$$k_1x_1 = k_2x_2$$

Then the ratio of compressed distance or elongated distance x_1 and x_2 is

$$\frac{x_2}{x_1} = \frac{k_1}{k_2} \quad (10.43)$$

The elastic potential energy stored in first and second springs are $V_1 = \frac{1}{2}k_1x_1^2$ and $V_2 = \frac{1}{2}k_2x_2^2$ respectively. Then, their ratio is

$$\frac{V_1}{V_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1}{k_2}\left(\frac{x_1}{x_2}\right)^2 = \frac{k_2}{k_1} \quad (10.44)$$

Note

The reciprocal of stiffness constant is called flexibility constant or compliance, denoted by C. It is measured in m N^{-1}

If n springs are connected in series :

$$\text{net compliance } C_s = \sum_{i=1}^{n} C_i$$

If n springs are connected in parallel :

$$\frac{1}{C_p} = \sum_{i=1}^{n} \frac{1}{C_i}$$

EXAMPLE 10.9

Consider two springs whose force constants are 1 N m^{-1} and 2 N m^{-1} which are connected in series. Calculate the effective spring constant (k_s) and comment on k_s.

Solution

$$k_1 = 1\text{ N m}^{-1}, k_2 = 2\text{ N m}^{-1}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}\text{ N m}^{-1}$$

$$k_s = \frac{1\times 2}{1+2} = \frac{2}{3}\text{ N m}^{-1}$$

$$k_s < k_1 \text{ and } k_s < k_2$$

Therefore, the effective spring constant is lesser than both k_1 and k_2.

b. Springs connected in parallel

When two or more springs are connected in parallel, we can replace (by removing) all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection. Given the values of individual spring constants to be k_1, k_2, k_3, ... (known quantities), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_p (unknown quantity). For simplicity, let us consider only two springs of spring constants k_1 and k_2 attached to a mass m as shown in Figure 10.19. The results can be generalized to any number of springs in parallel.

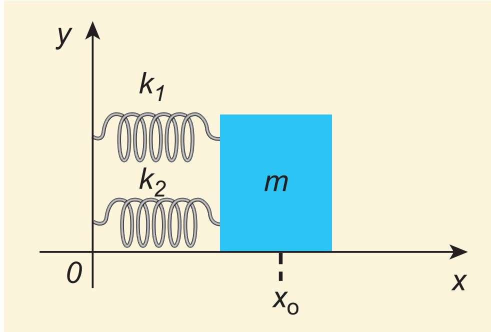


Figure 10.19 Springs connected in parallel

Let the force F be applied towards right as shown in Figure 10.20. In this case, both the springs elongate or compress by the same amount of displacement. Therefore, net force for the displacement of mass m is

$$F = -k_p x \qquad (10.45)$$

where k_p is called ***effective spring constant***. Let the first spring be elongated by a displacement x due to force F_1 and second spring be elongated by the same displacement x due to force F_2, then the net force

$$F = -k_1 x - k_2 x \qquad (10.46)$$

Equating equations (10.46) and (10.45), we get

$$k_p = k_1 + k_2 \qquad (10.47)$$

Generalizing, for n springs connected in parallel,

$$k_p = \sum_{i=1}^{n} k_i \qquad (10.48)$$

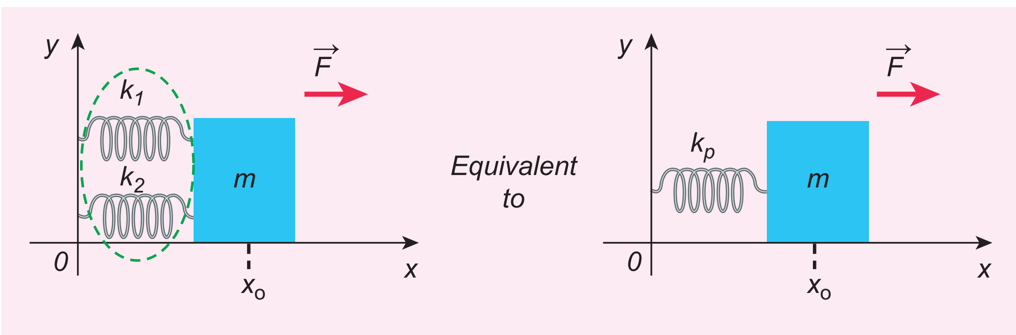


Figure 10.20 Effective spring constant in parallel connection

If all spring constants are identical i.e., $k_1 = k_2 = ... = k_n = k$ then

$$k_p = n\,k \qquad (10.49)$$

This implies that the effective spring constant increases by a factor n. Hence, for the springs in parallel connection, the effective spring constant is greater than individual spring constant.

Note The spring constant is inversely proportional to the length of the spring

$$k \propto \frac{1}{\textit{length of the spring}}$$

If the spring is cut into two pieces, one piece with length l_1 and other with length l_2, such that $l_1 = nl_2$, then spring constant of first length is $k_1 = \frac{k(n+1)}{n}$ and spring constant of second length is $k_2 = (n+1)\,k$, where k is the original spring constant before cutting into pieces.

EXAMPLE 10.10

Consider two springs with force constants 1 N m^{-1} and 2 N m^{-1} connected in parallel. Calculate the effective spring constant (k_p) and comment on k_p.

Solution

$$k_1 = 1\text{ N m}^{-1},\ k_2 = 2\text{ N m}^{-1}$$

$$k_p = k_1 + k_2 \text{ N m}^{-1}$$

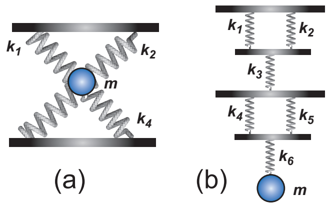
$$k_p = 1 + 2 = 3\text{ N m}^{-1}$$

$$k_p > k_1 \text{ and } k_p > k_2$$

Therefore, the effective spring constant is greater than both k_1 and k_2.

EXAMPLE 10.11

Calculate the equivalent spring constant for the following systems and also compute if all the spring constants are equal:



Solution

a. Since k_1 and k_2 are parallel, $k_u = k_1 + k_2$
Similarly, k_3 and k_4 are parallel, therefore, $k_d = k_3 + k_4$
But k_u and k_d are in series,

therefore, $k_{eq} = \frac{k_u k_d}{k_u + k_d}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k$
Which means, $k_u = 2k$ and $k_d = 2k$

Hence, $k_{eq} = \frac{4k^2}{4k} = k$

b. Since k_1 and k_2 are parallel, $k_A = k_1 + k_2$
Similarly, k_4 and k_5 are parallel, therefore, $k_B = k_4 + k_5$
But k_A, k_3, k_B, and k_6 are in series,

therefore, $\frac{1}{k_{eq}} = \frac{1}{k_A} + \frac{1}{k_3} + \frac{1}{k_B} + \frac{1}{k_6}$

If all the spring constants are equal then, $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = k$
which means, $k_A = 2k$ and $k_B = 2k$

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{2k} + \frac{1}{k} = \frac{3}{k}$$

$$k_{eq} = \frac{k}{3}$$

EXAMPLE 10.12

A mass m moves with a speed v on a horizontal smooth surface and collides with a nearly massless spring whose spring constant is k. If the mass stops after collision, compute the maximum compression of the spring.

Solution

When the mass collides with the spring, from the law of conservation of energy "the loss in kinetic energy of mass is gain in elastic potential energy by spring".

Let x be the distance of compression of spring, then the law of conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{k}}$$

10.4.4 Oscillations of a simple pendulum in SHM and laws of simple pendulum

Simple pendulum

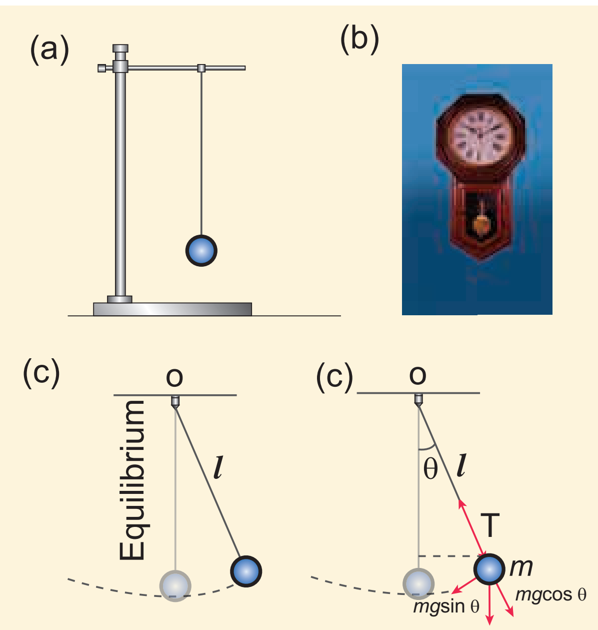


Figure 10.21 Simple pendulum

A pendulum is a mechanical system which exhibits periodic motion. It has a bob with mass m suspended by a long string (assumed to be massless and inextensible string) and the other end is fixed on a stand as shown in Figure 10.21 (a). At equilibrium, the pendulum does not oscillate and hangs vertically downward. Such a position is known as mean position or equilibrium position. When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion. Let l be the length of the pendulum which is taken as the distance between the point of suspension and the centre of gravity of the bob. Two forces act on the bob of the pendulum at any displaced position, as shown in the Figure 10.21 (d),

(i) The gravitational force acting on the body ($\vec{F} = m\vec{g}$) which acts vertically downwards.

(ii) The tension in the string $\vec{T}$ which acts along the string to the point of suspension.

Resolving the gravitational force into its components:

a. **Normal component:** The component along the string but in opposition to the direction of tension, $F_{as} = mg\cos\theta$.

b. **Tangential component:** The component perpendicular to the string i.e., along tangential direction of arc of swing, $F_{ps} = mg\sin\theta$.

Therefore, The normal component of the force is, along the string,

$$T - W_{as} = m\frac{v^2}{l}$$

Here v is speed of bob

$$T - mg\cos\theta = m\frac{v^2}{l} \qquad (10.50)$$

Note From Newton's 2nd law, $\vec{F} = m\vec{a}$ Here, the net force on the L.H.S is $T-W_{as}$
In R.H.S, $m\vec{a}$ is equivalent to the centripetal force $= \frac{mv^2}{l}$ which makes the bob oscillate.

From the Figure 10.21, we can observe that the tangential component W_{ps} of the gravitational force always points towards the equilibrium position i.e., the direction in which it always points opposite to the direction of displacement of the bob from the mean position. Hence, in this case, the tangential force is nothing but the restoring force. Applying Newton's second law along tangential direction, we have

$$m\frac{d^2s}{dt^2} + F_{ps} = 0 \Rightarrow m\frac{d^2s}{dt^2} = -F_{ps}$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta \qquad (10.51)$$

where, s is the position of bob which is measured along the arc. Expressing arc length in terms of angular displacement i.e.,

$$s = l\theta \qquad (10.52)$$

then its acceleration,

$$\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2} \qquad (10.53)$$

Substituting equation (10.53) in equation (10.51), we get

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad (10.54)$$

Because of the presence of $\sin\theta$ in the above differential equation, it is a non-linear differential equation (Here, homogeneous second order). Assume "the small oscillation approximation", $\sin\theta \approx \theta$, the above differential equation becomes linear differential equation.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \qquad (10.55)$$

This is the well known oscillatory differential equation. Therefore, the angular frequency of this oscillator (natural frequency of this system) is

$$\omega^2 = \frac{g}{l} \qquad (10.56)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{l}} \text{ in rad } s^{-1} \qquad (10.57)$$

The frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ in Hz} \qquad (10.58)$$

and time period of oscillations is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ in second} \qquad (10.59)$$

Laws of simple pendulum

The time period of a simple pendulum

a. Depends on the following laws

(i) Law of length

For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum.

$$T \propto \sqrt{l} \qquad (10.60)$$

(ii) Law of acceleration

For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}} \qquad (10.61)$$

b. Independent of the following factors

(i) Mass of the bob

The time period of oscillation is independent of mass of the simple pendulum. This is similar to free fall. Therefore, in a pendulum of fixed length, it does not matter whether an elephant swings or an ant swings. Both of them will swing with the same time period.

(ii) Amplitude of the oscillations

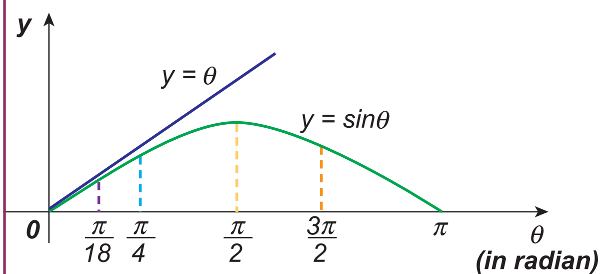
For a pendulum with small angle approximation (angular displacement is very small), the time period is independent of amplitude of the oscillation.

EXAMPLE 10.13

In simple pendulum experiment, we have used small angle approximation . Discuss the small angle approximation.

θ (in degrees)	θ(in radian)	sin θ
0	0	0
5	0.087	0.087
10	0.174	0.174
15	0.262	0.256
20	0.349	0.342
25	0.436	0.422
30	0.524	0.500
35	0.611	0.574
40	0.698	0.643
45	0.785	0.707

For θ in radian, $\sin\theta \approx \theta$ for very small angles



This means that "for θ as large as 10 degrees, sin θ is nearly the same as θ when θ is expressed in radians". As θ increases in value sinθ gradually becomes different from θ

Pendulum length due to effect of temperature

Suppose the suspended wire is affected due to change in temperature. The rise in temperature affects length by

$$l = l_o(1 + \alpha\,\Delta t)$$

where l_o is the original length of the wire and l is final length of the wire when the temperature is raised. Let Δt is the change in temperature and α is the co-efficient of linear expansion.

Then, $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l_0(1+\alpha\Delta t)}{g}}$

$$= 2\pi\sqrt{\frac{l_0}{g}}\sqrt{(1+\alpha\Delta t)}$$

$$T = T_0\,(1+\alpha\Delta t)^{\frac{1}{2}} \approx T_0\,(1+\frac{1}{2}\alpha\Delta t)$$

$$\Rightarrow \quad \frac{T}{T_0} - 1 = \frac{T-T_0}{T_0} = \frac{\Delta T}{T_0} = \frac{1}{2}\alpha\Delta t$$

where ΔT is the change in time period due to the effect of temperature and T_0 is the time period of the simple pendulum with original length l_0.

EXAMPLE 10.14

If the length of the simple pendulum is increased by 44% from its original length, calculate the percentage increase in time period of the pendulum.

Solution

Since

$$T \propto \sqrt{l}$$

Therefore,

$$T = \text{constant}\ \sqrt{l}$$

$$\frac{T_f}{T_i}=\sqrt{\frac{l+\frac{44}{100}l}{l}}=\sqrt{1.44}=1.2$$

Therefore, $T_f = 1.2\ T_i = T_i + 20\%\ T_i$

Oscillation of liquid in a U-tube:

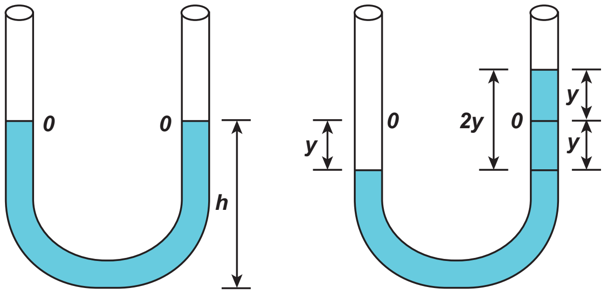


Figure 10.22 U-shaped glass tube

Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area A. Let us pour a non-viscous uniform incompressible liquid of density ρ in the U-shaped tube to a height *h* as shown in the Figure 10.22. If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position *O*. It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure. Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position *O*, which means, the pressure at blown arm is higher than the other arm. This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest.

Time period of the oscillation is

$$T = 2\pi\sqrt{\frac{l}{2g}}\ \text{second} \qquad (10.62)$$

10.5 ENERGY IN SIMPLE HARMONIC MOTION

a. Expression for Potential Energy

For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k\vec{r}$$

Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case

$$F = -\,k\,x \qquad (10.63)$$

As we have discussed in unit 4 of volume I, the work done by the conservative force field is independent of path. The potential energy U can be calculated from the following expression.

$$F = -\frac{dU}{dx} \qquad (10.64)$$

Comparing (10.63) and (10.64), we get

$$-\frac{dU}{dx} = -\,kx$$

$$dU = k\,x\,dx$$

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Dummy variable

The integrating variable x' (read x' as "x prime")is a dummy variable

$$\int_0^y tdt=\int_0^y xdx=\int_0^y pdp=\frac{y^2}{2}$$

Notice that the integrating variables like *t*, *x* and *p* are dummy variables because, in this integration, whether we put *t* or *x* or *p* as variable for integration, we get the same answer.

This work done by the force F during a small displacement dx stores as potential energy

$$U(x)=\int_0^x k\,x'\,dx'=\frac{1}{2}k\left(x'\right)^2\Big|_0^x=\frac{1}{2}kx^2 \quad (10.65)$$

From equation (10.22), we can substitute the value of force constant $k = m\,\omega^2$ in equation (10.65),

$$U\left(x\right)=\frac{1}{2}m\omega^2x^2 \quad (10.66)$$

where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation (10.6), we get

$$x = A \sin \omega t$$

$$U(t)=\frac{1}{2}m\omega^2A^2\sin^2\omega t \quad (10.67)$$

This variation of U is shown below.

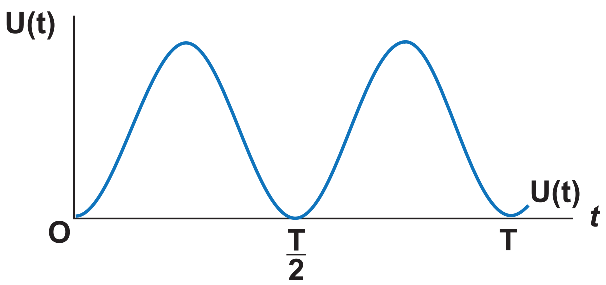


Figure 10.23 Variation of potential energy with time t

Question to think over

"If the potential energy is minimum then its second derivative is positive, why?"

b. Expression for Kinetic Energy

Kinetic energy

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \quad (10.68)$$

Since the particle is executing simple harmonic motion, from equation (10.6)

$$x = A \sin \omega t$$

Therefore, velocity is

$$v_x=\frac{dx}{dt}=A\omega\cos\omega t \quad (10.69)$$

$$=A\omega\sqrt{1-\left(\frac{x}{A}\right)^2}$$

$$v_x=\omega\sqrt{A^2-x^2} \quad (10.70)$$

Hence,

$$KE=\frac{1}{2}mv_x^2=\frac{1}{2}m\omega^2\left(A^2-x^2\right) \quad (10.71)$$

$$KE=\frac{1}{2}m\omega^2A^2\cos^2\omega t \quad (10.72)$$

This variation with time is shown below.

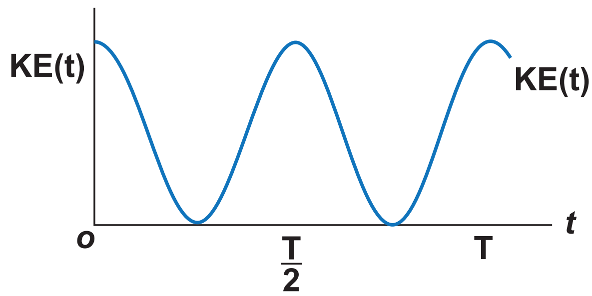


Figure 10.24 Variation of kinetic energy with time t.

c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy

$$E = KE + U \quad (10.73)$$

$$E=\frac{1}{2}m\omega^2(A^2-x^2)+\frac{1}{2}m\omega^2x^2$$

Hence, cancelling x^2 term,

$$E=\frac{1}{2}m\omega^2A^2=\text{constant} \quad (10.74)$$

Alternatively, from equation (10.67) and equation (10.72), we get the total energy as

$$E=\frac{1}{2}m\omega^2A^2\sin^2\omega t+\frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$=\frac{1}{2}m\omega^2A^2(\sin^2\omega t+\cos^2\omega t)$$

From trigonometry identity,

$(\sin^2 \omega t + \cos^2 \omega t) = 1$

$$E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$$

which gives the law of conservation of total energy. This is depicted in Figure 10.26

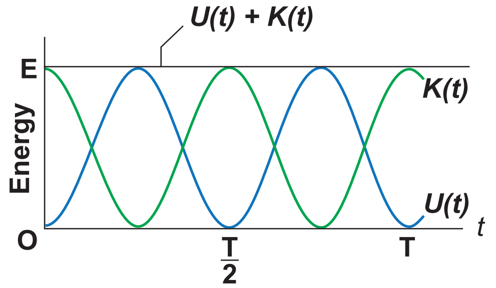


Figure 10.25 Both kinetic energy and potential energy vary but total energy is constant

Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}} \qquad (10.75)$$

EXAMPLE 10.15

Write down the kinetic energy and total energy expressions in terms of linear momentum, For one-dimensional case.

Solution

Kinetic energy is $KE = \frac{1}{2} m v_x^2$

Multiply numerator and denominator by m

$$KE = \frac{1}{2m} m^2 v_x^2 = \frac{1}{2m} (m v_x)^2 = \frac{1}{2m} p_x^2$$

where, p_x is the linear momentum of the particle executing simple harmonic motion.

Total energy can be written as sum of kinetic energy and potential energy, therefore, from equation (10.73) and also from equation (10.75), we get

$$E = KE + U(x) = \frac{1}{2m} p_x^2 + \frac{1}{2} m\omega^2 x^2 = \text{constant}$$

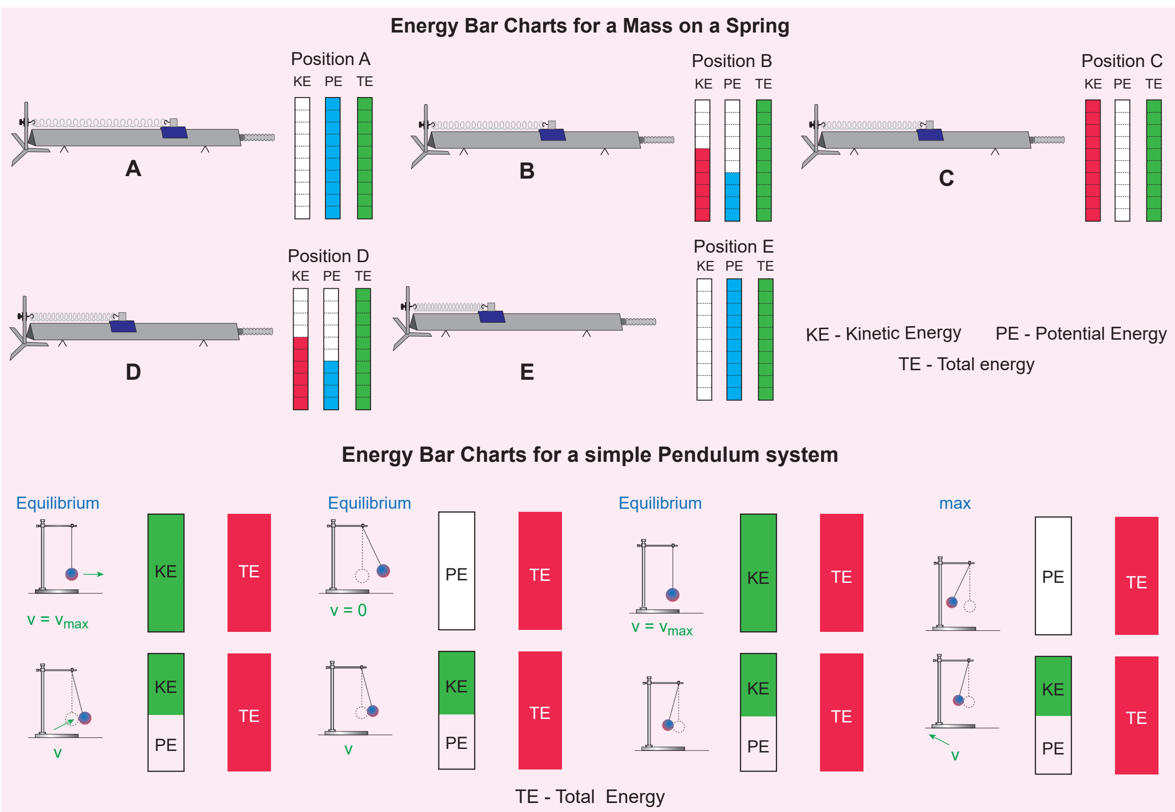
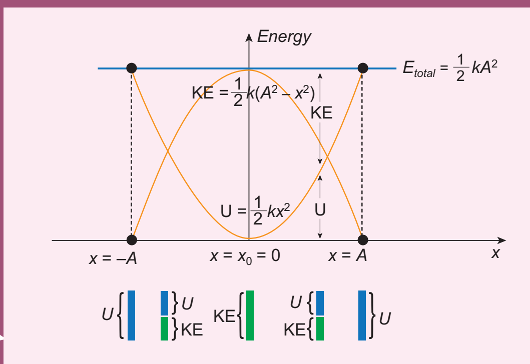


Figure 10.26 Conservation of energy – spring mass system and simple pendulum system

Note

Conservation of energy

Both the kinetic energy and potential energy are periodic functions, and repeat their values after a time period $\frac{T}{2}$. But total energy is constant for all the values of x or t. The kinetic energy and the potential energy for a simple harmonic motion are always positive. Note that kinetic energy cannot take negative value because it is proportional to the square of velocity. The measurement of any physical quantity must be a real number. Therefore, if kinetic energy is negative then the numerical value of velocity becomes an imaginary number, which is physically not acceptable. At equilibrium, it is purely kinetic energy and at extreme positions it is purely potential energy.



EXAMPLE 10.16

Compute the position of an oscillating particle when its kinetic energy and potential energy are equal.

Solution

Since the kinetic energy and potential energy of the oscillating particle are equal,

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

10.6 TYPES OF OSCILLATIONS:

10.6.1 Free oscillations

When the oscillator is allowed to oscillate by displacing its position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration. In this case, the amplitude, frequency and the energy of the vibrating object remains constant.

Examples:

(i) Vibration of a tuning fork.

(ii) Vibration in a stretched string.

(iii) Oscillation of a simple pendulum.

(iv) Oscillationsof a spring-mass system.

10.6.2 Damped oscillations

During the oscillation of a simple pendulum (in previous case), we have assumed that the amplitude of the oscillation is constant and also the total energy of the oscillator is constant. But in reality, in a medium, due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses. It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy. The energy lost is absorbed by the surrounding medium. This type of oscillatory motion is known as damped oscillation. In other words, if an oscillator moves in a resistive medium, its amplitude goes on decreasing and the energy of the oscillator is used to do work against the resistive medium. The motion of the oscillator is said to be damped and in this case, the resistive force (or damping force) is proportional to the velocity of the oscillator.

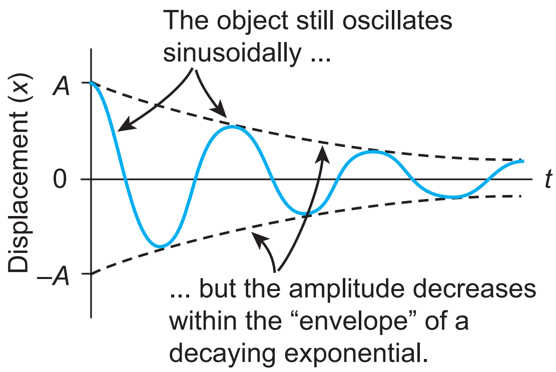


Figure 10.27 Damped harmonic oscillator – amplitude decreases as time increases.

Examples

(i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container.

(ii) Electromagnetic oscillations in a tank circuit.

(iii) Oscillations in a dead beat and ballistic galvanometers.

10.6.3 Maintained oscillations

While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example:

The vibration of a tuning fork getting energy from a battery or from external power supply.

10.6.4 Forced oscillations

Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator. -

In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

Example:

Sound boards of stringed instruments.

10.6.5 Resonance

It is a special case of forced vibrations where the frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

Example

The breaking of glass due to sound

Note

The concept of resonance is used in Tuning of station (or channel) in a radio (or Television) circuits.

DO YOU KNOW?

Soliders are not allowed to march on a bridge.

This is to avoid resonant vibration of the bridge.

While crossing a bridge, if the period of stepping on the ground by marching soldiers equals the natural frequency of the bridge, it may result in resonance vibrations. This may be so large that the bridge may collapse.

Extra:

Pendulum in a lift:

(i) Lift moving upwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g + a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g+a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will decrease when lift moves upward.

(ii) Lift moving downwards with acceleration a:

Effective acceleration due to gravity is $g_{eff} = g - a$

Then time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g-a)}}$

Since the time period is inversely related to acceleration due to gravity, time period will increase when lift moves downward.

(iii) Lift falls with acceleration $a > g$:

The effective acceleration is $g_{eff} = a - g$

Then time period is $T = \frac{1}{2\pi}\sqrt{\frac{l}{g_{eff}}} = \frac{1}{2\pi}\sqrt{\frac{l}{(a-g)}}$

in this case, the pendulum will turn upside down and will oscillate about higest point.

(iv) Lift falls with acceleration $a = g$:

The effective acceleration is $g_{eff} = g - g = 0$

Then time period is $T \rightarrow \infty$ which means pendulum does not oscillate and its motion is arrested.

(v) If the simple pendulum is kept in a car which moves horizontally with acceleration a:

The effective acceleration is $g_{eff} = \sqrt{g^2 + a^2}$

Time period is $T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$

SUMMARY

- When an object or a particle moves back and forth repeatedly about a reference point for some duration of time it is said to have Oscillatory (or vibratory) motion.
- For a SHM, the acceleration or force on the particle is directly proportional to its displacement from a fixed point and always directed towards that fixed point. The force is
$$F_x = -kx$$
where k is a constant whose dimension is force per unit length, called as force constant.
- In Simple harmonic motion, the displacement, $y = A \sin \omega t$.
- In Simple harmonic motion, the velocity, $v = A\,\omega \cos \omega t = \omega\sqrt{A^2 - y^2}$.
- In Simple harmonic motion, the acceleration, $a = \frac{d^2y}{dt^2} = -\omega^2 y$.
- The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. Time period $T = \frac{2\pi}{\omega}$.
- The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is S^{-1} or hertz (In symbol, Hz). Mathematically, frequency is related to time period by $f = \frac{1}{T}$.
- The frequency of the angular harmonic motion is $f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$ Hz
- For n springs connected in series, the effective spring constant in series is
$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_n} = \sum_{i=0}^{n} \frac{1}{k_i}$$
- For n springs connected in parallel, the effective spring constant is
$$k_p = \sum_{i=1}^{n} k_i$$
- The time period for U-tube oscillation is $T = 2\pi\sqrt{\frac{l}{2g}}$ second.
- For a conservative system in one dimension, the force field can be derived from a scalar potential energy: $F = -\frac{dU}{dx}$.
- In a simple harmonic motion, potential energy is $U(x) = \frac{1}{2}m\omega^2 x^2$.
- In a simple harmonic motion, kinetic energy is $KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$.
- Total energy for a simple harmonic motion is $E = \frac{1}{2}m\omega^2 A^2 = \text{constant}$.
- Types of oscillations – Free oscillations, Damped oscillations, Maintained oscillations and Forced oscillations.
- Resonance is a special case of forced oscillations.

CONCEPT MAP

Oscillation

Simple Harmonic Motion (SHM)

$F = -kx \quad x = +A\sin\omega t$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$a = -\omega^2 A \sin\omega t$

$a = -\omega^2 y$

$V = A\omega\cos\omega t$

$v = \omega\sqrt{A^2 - y^2}$

$F = -\frac{dU}{dx}$

$U = \frac{1}{2}kx^2$

$TE = \frac{1}{2}kA^2$

$KE = \frac{1}{2}mv^2$

$T = \frac{2\pi}{\omega}$

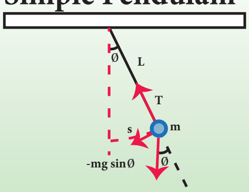
Angular SHM

$\omega = \sqrt{\frac{\kappa}{I}}$

Linear SHM

$\omega = \sqrt{\frac{k}{m}}$

Simple Pendulam



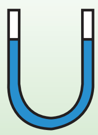
Differential equation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Time Period:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

U - Tube



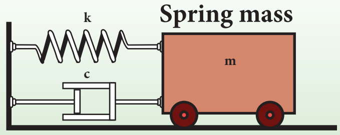
Differential equation:

$$\frac{d^2y}{dt^2} = -\frac{2g}{l}y$$

Time Period:

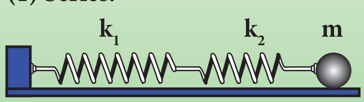
$$T = 2\pi\sqrt{\frac{l}{2g}}$$

Spring mass



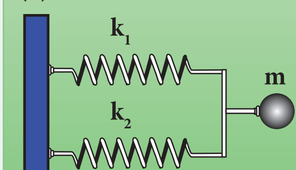
Combination of Springs

(1) Series:



$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

(2) Parallel:



$$k_p = k_1 + k_2$$

I. Multiple Choice Questions

1. In a simple harmonic oscillation, the acceleration against displacement for one complete oscillation will be

(model NSEP 2000-01)

a) an ellipse b) a circle

c) a parabola d) a straight line

2. A particle executing SHM crosses points A and B with the same velocity. Having taken 3 s in passing from A to B, it returns to B after another 3 s. The time period is

a) 15 s b) 6 s

c) 12 s d) 9 s

3. The length of a second's pendulum on the surface of the Earth is 0.9 m. The length of the same pendulum on surface of planet X such that the acceleration of the planet X is n times greater than the Earth is

a) $0.9n$ b) $\frac{0.9}{n}m$

c) $0.9n^2m$ d) $\frac{0.9}{n^2}$

4. A simple pendulum is suspended from the roof of a school bus which moves in a horizontal direction with an acceleration a, then the time period is

a) $T\alpha\frac{1}{g^2+a^2}$ b) $T\alpha\frac{1}{\sqrt{g^2+a^2}}$

c) $T\alpha\sqrt{g^2+a^2}$ d) $T\alpha(g^2+a^2)$

5. Two bodies A and B whose masses are in the ratio 1:2 are suspended from two separate massless springs of force constants k_A and k_B respectively. If the two bodies oscillate vertically such that their maximum velocities are in the ratio 1:2, the ratio of the amplitude A to that of B is

a) $\sqrt{\frac{k_B}{2k_A}}$ b) $\sqrt{\frac{k_B}{8k_A}}$

c) $\sqrt{\frac{2k_B}{k_A}}$ d) $\sqrt{\frac{8k_B}{k_A}}$

6. A spring is connected to a mass m suspended from it and its time period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is

a) $T'=\sqrt{2}T$ b) $T'=\frac{T}{\sqrt{2}}$

c) $T'=\sqrt{2T}$ d) $T'=\sqrt{\frac{T}{2}}$

7. The time period for small vertical oscillations of block of mass m when the masses of the pulleys are negligible and spring constant k_1 and k_2 is

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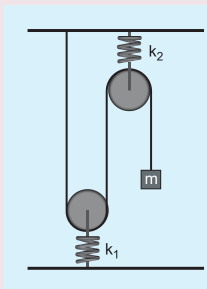


a) $T=4\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$

b) $T=2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$

c) $T=4\pi\sqrt{m(k_1+k_2)}$

d) $T=2\pi\sqrt{m(k_1+k_2)}$

8. A simple pendulum has a time period T_1. When its point of suspension is moved vertically upwards according as $y = k\,t^2$, where y is vertical distance covered and $k = 1\ ms^{-2}$, its time period becomes T_2. Then, $\frac{T_1^2}{T_2^2}$ is (g = 10 m s^{-2}) (IIT 2005)

a) $\frac{5}{6}$ b) $\frac{11}{10}$

c) $\frac{6}{5}$ d) $\frac{5}{4}$

9. An ideal spring of spring constant k, is suspended from the ceiling of a room and a block of mass M is fastened to its lower end. If the block is released when the spring is un-stretched, then the maximum extension in the spring is (IIT 2002)

a) $4\frac{Mg}{k}$ b) $\frac{Mg}{k}$

c) $2\frac{Mg}{k}$ d) $\frac{Mg}{2k}$

10. A pendulum is hung in a very high building oscillates to and fro motion freely like a simple harmonic oscillator. If the acceleration of the bob is 16 ms^{-2} at a distance of 4 m from the mean position, then the time period is (NEET 2018 model)

a) 2 s b) 1 s

c) 2πs (d) πs

11. A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will

a) first increase and then decrease

b) first decrease and then increase

c) increase continuously

d) decrease continuously

12. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are (AIPMT 2012)

a) kg m s^{-1} b) kg m s^{-2}

c) kg s^{-1} (d) kg s

13. When a damped harmonic oscillator completes 100 oscillations, its amplitude is reduced to $\frac{1}{3}$ of its initial value. What will be its amplitude when it completes 200 oscillations?.

a) $\frac{1}{5}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{9}$

14. Which of the following differential equations represents a damped harmonic oscillator ?

a) $\frac{d^2y}{dt^2}+y=0$ b) $\frac{d^2y}{dt^2}+\gamma\frac{dy}{dt}+y=0$

c) $\frac{d^2y}{dt^2}+k^2y=0$ d) $\frac{dy}{dt}+y=0$

15. If the inertial mass and gravitational mass of the simple pendulum of length l are not equal, then the time period of the simple pendulum is

a) $T=2\pi\sqrt{\frac{m_i l}{m_g g}}$

b) $T=2\pi\sqrt{\frac{m_g l}{m_i g}}$

c) $T=2\pi\frac{m_g}{m_i}\sqrt{\frac{l}{g}}$

d) $T=2\pi\frac{m_i}{m_g}\sqrt{\frac{l}{g}}$

Answers:

1) d	2) c	3) a	4) b
5) b	6) b	7) a	8) c
9) c	10) d	11) a	12) c
13) d	14) b	15) a	

II. Short Answers Questions

1. What is meant by periodic and non-periodic motion?. Give any two examples, for each motion.
2. What is meant by force constant of a spring?.
3. Define time period of simple harmonic motion.
4. Define frequency of simple harmonic motion.
5. What is an epoch?.
6. Write short notes on two springs connected in series.
7. Write short notes on two springs connected in parallel.
8. Write down the time period of simple pendulum.
9. State the laws of simple pendulum?.
10. Write down the equation of time period for linear harmonic oscillator.
11. What is meant by free oscillation?.
12. Explain damped oscillation. Give an example.
13. Define forced oscillation. Give an example.
14. What is meant by maintained oscillation?. Give an example.
15. Explain resonance. Give an example.

III. Long Answers Questions

1. What is meant by simple harmonic oscillation?. Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.
2. Describe Simple Harmonic Motion as a projection of uniform circular motion.
3. What is meant by angular harmonic oscillation?. Compute the time period of angular harmonic oscillation.
4. Write down the difference between simple harmonic motion and angular simple harmonic motion.
5. Discuss the simple pendulum in detail.
6. Explain the horizontal oscillations of a spring.
7. Describe the vertical oscillations of a spring.
8. Write short notes on the oscillations of liquid column in U-tube.
9. Discuss in detail the energy in simple harmonic motion.
10. Explain in detail the four different types of oscillations.

IV. Numerical Problems

1. Consider the Earth as a homogeneous sphere of radius R and a straight hole is bored in it through its centre. Show that a particle dropped into the hole will execute a simple harmonic motion such that its time period is

$$T=2\pi\sqrt{\frac{R}{g}}$$

2. Calculate the time period of the oscillation of a particle of mass m moving in the potential defined as

$$U(x)=\begin{cases}\frac{1}{2}kx^2, x<0\\ mgx, x>0\end{cases}$$

Answer: $\pi\sqrt{\frac{m}{k}}+2\sqrt{\frac{2E}{g^2m}}$, where E is the total energy of the particle.

3. Consider a simple pendulum of length $l = 0.9\ m$ which is properly placed on a trolley rolling down on a inclined plane which is at $\theta = 45°$ with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Answer: 0.86 s

4. A piece of wood of mass m is floating erect in a liquid whose density is ρ. If it is slightly pressed down and released, then executes simple harmonic motion. Show that its time period of oscillation is $T = 2\pi\sqrt{\frac{m}{Ag\rho}}$

5. Consider two simple harmonic motion along x and y-axis having same frequencies but different amplitudes as $x = A\sin(\omega t + \varphi)$ (along x axis) and $y = B\sin\omega t$ (along y axis). Then show that

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\varphi = \sin^2\varphi$$

and also discuss the special cases when

a. $\varphi = 0$ b. $\varphi = \pi$ c. $\varphi = \frac{\pi}{2}$

d. $\varphi = \frac{\pi}{2}$ and $A = B$ (e) $\varphi = \frac{\pi}{4}$

Note: *when a particle is subjected to two simple harmonic motion at right angle to each other the particle may move along different paths. Such paths are called* ***Lissajous figures****.*

Answer :

a. $y = \frac{B}{A}x$, equation is a straight line passing through origin with positive slope.

b. $y = -\frac{B}{A}x$ equation is a straight line passing through origin with negative slope.

c. $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, equation is an ellipse whose center is origin.

d. $x^2 + y^2 = A^2$, equation is a circle whose center is origin .

e. $\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\frac{1}{\sqrt{2}} = \frac{1}{2}$, equation is an ellipse (oblique ellipse which means tilted ellipse)

6. Show that for a particle executing simple harmonic motion

a. the average value of kinetic energy is equal to the average value of potential energy.

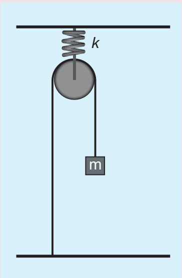
b. average potential energy = average kinetic energy = $\frac{1}{2}$(total energy)

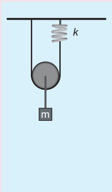
Hint : *average kinetic energy = <kinetic energy>* = $\frac{1}{T}\int_0^T (Kinetic\,energy)\,dt$

and

average Potential energy = <Potential energy> = $\frac{1}{T}\int_0^T (Potential\ energy)\,dt$

7. Compute the time period for the following system if the block of mass m is slightly displaced vertically down from its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.





Hint and answer:

Case(a)

Pulley is fixed rigidly here. When the mass displace by y and the spring will also stretch by y. Therefore, $F = T = ky$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case(b)

Mass displace by y, pulley also displaces by y. $T = 4ky$.

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

BOOKS FOR REFERENCE

1. Vibrations and Waves – A. P. French, CBS publisher and Distributors Pvt. Ltd.
2. Concepts of Physics – H. C. Verma, Volume 1 and Volume 2, Bharati Bhawan Publisher.
3. Fundamentals of Physics – Halliday, Resnick and Walker, Wiley Publishers, 10th edition.
4. Physics for Scientist and Engineers with Modern Physics – Serway and Jewett, Brook/ Coole Publishers, Eighth Edition.

ICT CORNER

Oscillations

Through this activity you will be able to learn about the resonance.

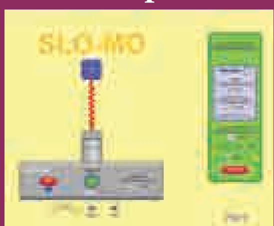
STEPS:

- Use the URL or scan the QR code to open ‘PhET’ simulation on ‘Resonance’. Click the play button.
- In the activity window a diagram of resonator is given. Click the play icon and move the slider on ‘sim speed’ given below to see the resonance.
- Move the slider to change ‘Number of Resonators’, ‘Mass’ and ‘Spring constant’ on the right side window and see the ‘frequency’.
- Select the ‘On’, ‘Off’ button on ‘Gravity’ to see the different resonance.

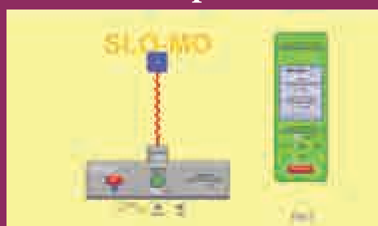
Step1

Step2

Step3



Step4



URL:

https://phet.colorado.edu/en/simulation/legacy/resonance

* Pictures are indicative only.

* If browser requires, allow **Flash Player** or **Java Script** to load the page.

B163_11_Phy_EM