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TERM - I

VOLUME 2

MATHEMATICS

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Department Of School Education

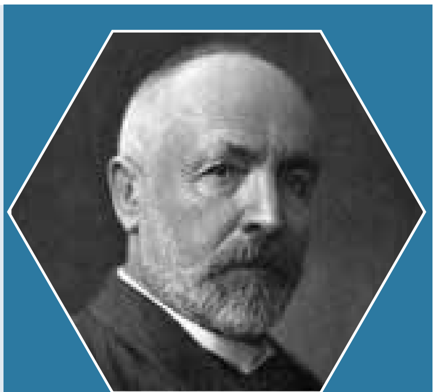
1 SET LANGUAGE

A set is a many that allows itself to thought of as a one

-Georg Cantor

E3RQRH

The theory of sets was developed by German mathematician Georg Cantor. Today it is used in almost every branch of Mathematics. In Mathematics, sets are convenient because all mathematical structures can be regarded as sets.



Georg Cantor
(1845 - 1918)

Learning Outcomes

- To describe a set.
- To represent sets in descriptive form, set builder form and roster form.
- To identify different types of sets.
- To understand and perform set operations.
- To use Venn diagrams to represent sets and set operations.
- To solve life oriented simple word problems.

1.1 Introduction

In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

Study the problem: 16 students play only Cricket, 18 students play only Volley ball and 3 students play both Cricket and Volley ball, while 2 students play neither Cricket nor Volley ball. Totally 39 students are there in a class.

We can describe this pictorially as follows:

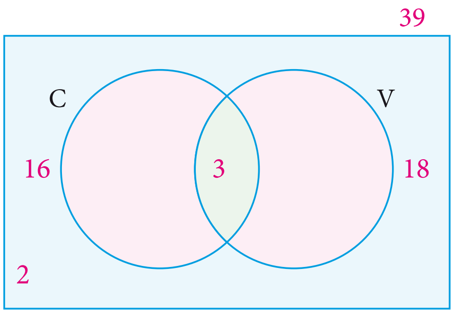


Fig. 1.1

What do the circles in the picture represent? They are **collections** of students who play games. We do not need to draw 39 students, or even 39 symbols, or use different colours to distinguish those who play Cricket from those who play Volleyball etc. Simply calling the collections C and V is enough; we can talk of those in both C and V, in neither and in one but not the other. This is **the language of sets**. A great deal of mathematics is written in this language and hence we are going to study it.

But **why** is this language so important, why should mathematicians want to use this language? One reason is that everyday language is imprecise and can cause confusion. For example, if I write 1, 2, 3,... what do the three dots at the end mean? You say,"Of course, they mean the list of natural numbers". What if I write 1, 2, 4,...? What comes next ? It could be 7, and then 11, and so on. (Can you see why ?) Or it could be continued as 8, 16, and so on. If we explicitly say, "The collection of numbers that are powers of 2", then we know that the latter is meant. So, in general, when we are talking of collections of numbers, we may refer to some collection in some short form, but writing out the collection may be difficult. It is here that the language of sets is of help. We can speak of the powers of 2 as a **set** of numbers.

Note, this is a list that goes on forever, so it is an **infinite** set. By now, we have come across several infinite sets: the set of natural numbers, the set of integers, the set of rational numbers, and many more. We also know the set of prime numbers, again an infinite set. But we know many **finite** sets too. The number of points of intersection of three lines on the plane is an example.

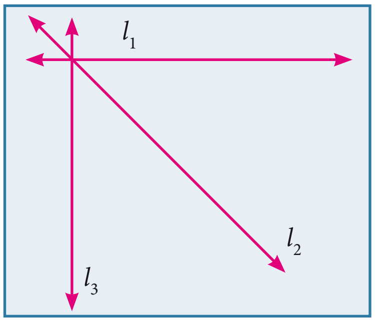


Fig. 1.2

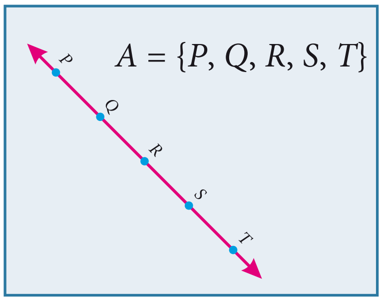


Fig. 1.3

We talk of a point being on a line and we know that a line contains inifinite number of points. For example, five points P,Q,R,S and T which lie on a line can be denoted by a set $A = \{P, Q, R, S, T\}$.

We can already see that many of these sets are important in whatever algebra and geometry we have learnt, and we expect that there will be more important sets coming along as we learn mathematics. That is why we are going to learn the language of sets. For now, we will work with small finite sets and learn its language.

Let us look at the following pictures. What do they represent?

Here, Fig.1.4 represents a collection of fruits and Fig. 1.5 represents a collection of house- hold items.

We observe in the above cases, our attention turns from one individual object to a collection of objects based on their characteristics. Any such collection is called a set.

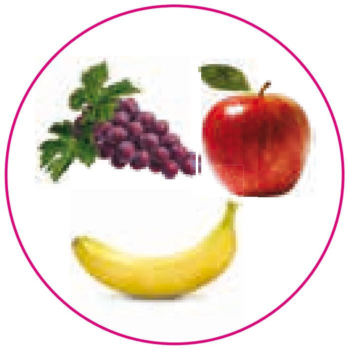


Fig. 1.4

Fig. 1.5

1.2 Set

A set is a well-defined collection of objects.

Here "well-defined collection of objects" means that given a specific object it must be possible for us to decide whether the object is an element of the given collection or not.

The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.
3. The collection of prime numbers.

We see that in the adjacent box, statements (1), (2), and (4) are well defined and therefore they are sets. Whereas (3) and (5) are not well defined because the words good and beautiful are difficult to agree on. I might consider a student to be good and you may not. I might consider Malligai is beautiful but you may not. So we will consider only those collections to be sets where there is no such ambiguity.

Which of the following are sets ?

1. Collection of Natural numbers.
2. Collection of English alphabets.
3. Collection of good students in a class.
4. Collection of States in our country.
5. Collection of beautiful flowers in a garden.

Therefore (3) and (5) are not sets.

Note

(i) Elements of a set are listed only once.

(ii) The order of listing the elements of the set does not change the set.

Both these conditions are natural. The collection 1,2,3,4,5,6,7,8, … as well as the collection 1, 3, 2, 4, 5, 7, 6, 8, … are the same though listed in different order. Since it is necessary to know whether an object is an element in the set or not, we do not want to list that element many times.

Activity-1

Discuss and give as many examples of collections from your daily life situations, which are sets and which are not sets.

Notation

A set is usually denoted by capital letters of the English Alphabets A, B, P, Q, X, Y, etc.

The elements of a set are denoted by small letters of the English alphabets a, b, p, q, x, y, etc.

The elements of a set is written within curly brackets "{ }"

If x is an element of a set A or x belongs to A, we write $x \in A$.

If x is not an element of a set A or x does not belongs to A, we write $x \notin A$.

For example,

Consider the set $A = \{2,3,5,7\}$ then

2 is an element of A; we write $2 \in A$

5 is an element of A; we write $5 \in A$

6 is not an element of A; we write $6 \notin A$

Example 1.1

Consider the set A = {Ashwin, Muralivijay, Vijay Shankar, Badrinath }.

Fill in the blanks with the appropriate symbol $\in$ or $\notin$.

(i) Muralivijay ____ A. (ii) Ashwin ______ A. (iii) Badrinath ______A.

(iv) Ganguly _____ A. (v) Tendulkar _____ A

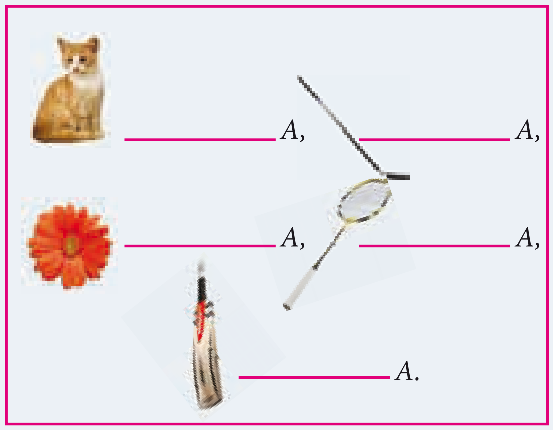
Solution

(i) Muralivijay $\in A$. (ii) Ashwin $\in A$ (iii) Badrinath $\in A$

(iv) Ganguly $\notin A$. (v) Tendulkar $\notin A$.

Activity-2

Insert the appropriate symbol $\in$ (belongs to) or $\notin$ (does not belong to) in the blanks.



1.3 Representation of a Set

The collection of odd numbers can be described in many ways:

(1) "The set of odd numbers" is a fine description, we understand it well.

(2) It can be written as $\{1, 3, 5, \ldots\}$ and you know what I mean.

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

All of them are equivalent and useful. For instance,the two descriptions "The collection of all solutions to the equation $x-5 = 3$" and $\{8\}$ refer to the same set.

A set can be represented in any one of the following three ways or forms:

(i) Descriptive Form.

(ii) Set-Builder Form or Rule Form.

(iii) Roster Form or Tabular Form.

1.3.1 Descriptive Form

In descriptive form, a set is described in words.

For example,

(i) The set of all vowels in English alphabets.

(ii) The set of whole numbers.

1.3.2 Set Builder Form or Rule Form

In set builder form, all the elements are described by a rule.

For example,

(i) $A = \{x : x \text{ is a vowel in English alphabets}\}$

(ii) $B = \{x | x \text{ is a whole number}\}$

Note

The symbol ':' or '|' stands for "such that".

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

(i) $A = \{a, e, i, o, u\}$

(ii) $B = \{0,1,2,3,\ldots\}$

Note

Three dots (…) in the example (ii) is called ellipsis. It indicates that the pattern of the listed elements continues in the same manner.

Can this form of representation be possible always?

Activity-3

Write the following sets in respective forms.

S.No.	Descriptive Form	Set Builder Form	Roster Form
1	The set of all natural numbers less than 10		
2		$\{x : x \text{ is a multiple of } 3, x \in \mathbb{N}\}$	
3			$\{2,4,6,8,10\}$
4	The set of all days in a week.		
5			$\{\ldots -3,-2,-1,0,1,2,3\ldots\}$

Example 1.2

Write the set of letters of the following words in Roster form

(i) ASSESSMENT (ii) PRINCIPAL

Solution

(i) ASSESSMENT

$A= \{A, S, E, M, N, T\}$

(ii) PRINCIPAL

$B=\{P, R, I, N, C, A, L\}$

Exercise 1.1

1. Which of the following are sets?

   (i) The Collection of prime numbers upto 100.

   (ii) The Collection of rich people in India.

   (iii) The Collection of all rivers in India.

   (iv) The Collection of good Hockey players.

2. List the set of letters of the following words in Roster form.

   (i) INDIA (ii) PARALLELOGRAM

   (iii) MISSISSIPPI (iv) CZECHOSLOVAKIA

3. Consider the following sets $A = \{0, 3, 5, 8\}$, $B = \{2, 4, 6, 10\}$ and $C = \{12, 14, 18, 20\}$.

   (a) State whether True or False:

   (i) $18 \in C$ (ii) $6 \notin A$ (iii) $14 \notin C$ (iv) $10 \in B$

   (v) $5 \in B$ (vi) $0 \in B$

   (b) Fill in the blanks:

   (i) $3 \in$ ____ (ii) $14 \in$ _____ (iii) 18 ____ B (iv) 4 _____ B

4. Represent the following sets in Roster form.

(i) A = The set of all even natural numbers less than 20.

(ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$

(iii) $C = \{x : x$ is perfect cube, $27 < x < 216\}$

(iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$

5. Represent the following sets in set builder form.

(i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.

(ii) $C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$

(iii) D = The set of all tamil months in a year.

(iv) E = The set of odd Whole numbers less than 9.

6. Represent the following sets in descriptive form.

(i) P ={ January, June, July}

(ii) $Q = \{7,11,13,17,19,23,29\}$

(iii) $R = \{x : x \in \mathbb{N}, x < 5\}$

(iv) $S = \{x : x$ is a consonant in English alphabets$\}$

1.4 Types of Sets

There is a very special set of great interest: the empty collection ! Why should one care about the empty collection? Consider the set of solutions to the equation $x^2+1 = 0$. It has no elements at all in the set of Real Numbers. Also consider all rectangles with one angle greater than 90 degrees. There is no such rectangle and hence this describes an empty set.

So, the empty set is important, interesting and deserves a special symbol too.

1.4.1 Empty Set or Null Set

A set consisting of no element is called the *empty set* or *null set* or *void set*.

It is denoted by $\varnothing$ or { }.

Thinking Corner

Are the sets $\{0\}$ and $\{\varnothing\}$ empty sets?

For example,

(i) $A=\{x : x$ is an odd integer and divisible by $2\}$

$\therefore A=\{\ \}$ or $\varnothing$

(ii) The set of all integers between 1 and 2.

1.4.2.Singleton Set

A set which has only one element is called a *singleton set*.

For example,

(i) $A = \{\mathbf{x : 3 < x < 5, x} \in \mathbb{N}\}$ (ii) The set of all even prime numbers.

1.4.3 Finite Set

A set with finite number of elements is called a *finite set*.

Note

An empty set has no elements, so $\varnothing$ is a finite set.

For example,

1. The set of family members.
2. The set of indoor/outdoor games you play.
3. The set of curricular subjects you learn in school.
4. $A = \{x : x$ is a factor of $36\}$

1.4.4 Infinite Set

A set which is not finite is called an *infinite set*.

Thinking Corner

Is the set of natural numbers a finite set?

For example,

(i) $\{5,10,15,...\}$ (ii) The set of all points on a line.

To discuss further about the types of sets, we need to know the cardinality of sets.

Cardinal number of a set : When a set is finite, it is very useful to know how many elements it has. The number of elements in a set is called the Cardinal number of the set.

The cardinal number of a set A is denoted by $n(A)$

Example 1.3

If $A = \{1,2,3,4,5,7,9,11\}$, find $n(A)$.

Solution

$A = \{1,2,3,4,5,7,9,11\}$

Since set A contains 8 elements, $n(A) = 8$.

Thinking Corner

If $A = \{1, b, b, \{4, 2\}, \{x, y, z\}, d, \{d\}\}$, then $n(A)$ is____

1.4.5 Equivalent Sets

Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as $A \approx B$.

If A and B are equivalent sets, then $n(A) = n(B)$

For example,

Consider $A = \{$ ball, bat$\}$ and $B = \{$history, geography$\}$.

Here A is equivalent to B because $n(A) = n(B) = 2$.

Thinking Corner

Let $A=\{x:x$ is a colour in national flag of India$\}$ and $B=\{$Red, Blue, Green$\}$. Are these two sets equivalent?

Example 1.4

Are $P = \{x : -3 \le x \le 0, x \in \mathbb{Z}\}$ and Q = The set of all prime factors of 210, equivalent sets?

Solution

$P = \{-3, -2, -1, 0\}$, The prime factors of 210 are 2,3,5,and 7 and so, $Q = \{2, 3, 5, 7\}$

$n(P) = 4$ and $n(Q) = 4$. Therefore P and Q are equivalent sets.

1.4.6 Equal Sets

Two sets are said to be equal if they contain exactly the same elements, otherwise they are said to be unequal.

In other words, two sets A and B are said to be equal, if

(i) every element of A is also an element of B

(ii) every element of B is also an element of A

Thinking Corner

Are the sets $\varnothing$, $\{0\}$, $\{\varnothing\}$ equal or equivalent?

For example,

Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{4, 2, 3, 1\}$

Since A and B contain exactly the same elements, A and B are equal sets.

Note

(i) If A and B are equal sets, we write $A = B$.

(ii) If A and B are unequal sets, we write $A \neq B$.

A set does not change, if one or more elements of the set are repeated.

For example, if we are given

$A=\{a, b, c\}$ and $B=\{a, a, b, b, b, c\}$ then, we write $B = \{ a, b, c, \}$. Since, every element of A is also an element of B and every element of B is also an element of A, the sets A and B are equal.

Example 1.5

Are $A = \{x : x \in \mathbb{N}, 4 \leq x \leq 8\}$ and

$B = \{ 4, 5, 6, 7, 8\}$ equal sets?

Solution

$A = \{ 4, 5, 6, 7, 8\}$, $B = \{ 4, 5, 6, 7, 8\}$

A and B are equal sets.

Note

Equal sets are equivalent sets but equivalent sets need not be equal sets. For example, if $A = \{ p,q,r,s,t\}$ and $B= \{ 4,5,6,7,8\}$. Here $n(A)=n(B)$, so A and B are equivalent but not equal.

1.4.7 Subset

Let A and B be two sets. If every element of A is also an element of B, then A is called a subset of B. We write $A \subseteq B$.

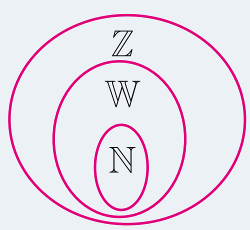
$A \subseteq B$ is read as "A is a subset of B"

Thus $A \subseteq B$, if $a \in A$ implies $a \in B$.

If A is not a subset of B, we write $A \not\subseteq B$

Clearly, if A is a subset of B, then $n(A) \leq n(B)$.

Thinking Corner



Is $\mathbb{W}$ subset of $\mathbb{N}$ or $\mathbb{Z}$?

Since every element of A is also an element of B, the set B must have at least as many elements as A, thus $n(A) \leq n(B)$.

The other way is also true. Suppose that $n(A) > n(B)$, then A has more elements than B, and hence there is at least one element in A that cannot be in B, so A is not a subset of B.

For example,

(i) $\{1\} \subseteq \{1,2,3\}$ (ii) $\{2,4\} \nsubseteq \{1,2,3\}$

Activity-4

Discuss with your friends and give examples of subsets of sets from your daily life situation.

Example1.6

Write all the subsets of $A = \{a, b\}$.

Solution

$A = \{a,b\}$

Subsets of A are $\varnothing$, $\{a\}$, $\{b\}$ and $\{a, b\}$.

Note

(i) If $A \subseteq B$ and $B \subseteq A$, then $A = B$.

In fact this is how we defined equality of sets.

(ii) Empty set is a subset of every set.

This is not easy to see ! Let A be any set. The only way for the empty set to be not a subset of A would be to have an element x in it but with x not in A. But how can x be in the empty set ? That is impossible. So this only way being impossible, the empty set must be a subset of A. (Is your head spinning? Think calmly, explain it to a friend, and you will agree it is alright !)

(iii) Every set is a subset of itself. (Try and argue why.)

1.4.8. Proper Subset

Let A and B be two sets.If A is a subset of B and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.

For example,

If $A=\{1,2,5\}$ and $B=\{1,2,3,4,5\}$ then A is a proper subset of B ie. $A \subset B$.

Example 1.7

Insert the appropriate symbol $\subseteq$ or $\nsubseteq$ in each blank to make a true statement.

(i) $\{10, 20, 30\}$ ____ $\{10, 20, 30, 40\}$ (ii) $\{p, q, r\}$ _____ $\{w, x, y, z\}$

Solution

(i) $\{10, 20, 30\}$ ____ $\{10, 20, 30, 40\}$

Since every element of $\{10, 20, 30\}$ is also an element of $\{10, 20, 30, 40\}$, we get $\{10, 20, 30\} \subseteq \{10, 20, 30, 40\}$.

(ii) $\{p, q, r\}$ _____ $\{w, x, y, z\}$

Since the element p belongs to $\{p, q, r\}$ but does not belong to $\{w, x, y, z\}$, shows that $\{p, q, r\} \nsubseteq \{w, x, y, z\}$.

1.4.9 Power Set

The fun begins when we realise that elements of sets can themselves be sets !

That is not very difficult to imagine: the people in school form a set, that consists of the set of students, the set of teachers, and the set of other staff. The set of students then has many sets as its elements: the set of students in class 1, the set of class 2 children, and so on. So we can easily talk of sets of sets of sets of …. of sets of elements !

Why bother? There is a particular set of sets that is very interesting.

Let A be any set. Form the set consisting of all subsets of A. Let us call it B. What all sets does B contain? For one thing, A is inside it, since A is a subset of A. The empty set is also a subset of A, so it is in B. If x is in A, then the singleton set $\{x\}$ is in B. (This means that B has at least as many elements as A; so $n(A)$ is equal to $n(B)$). For every pair of distinct elements x, y in A, we have $\{x,y\}$ in B. So yes, B has a lot many sets. It is so rich that it gets a very powerful name !

The set of all subsets of a set A is called the power set of 'A'. It is denoted by $P(A)$.

For example,

(i) If $A=\{2, 3\}$, then find the power set of A.

The subsets of A are $\varnothing$, $\{2\},\{3\},\{2,3\}$.

The power set of A,

$P(A) = \{\varnothing ,\{2\},\{3\},\{2,3\}\}$

(ii) If $A = \{\varnothing , \{\varnothing\}\}$, then the power set of A is $\{ \varnothing , \{\varnothing , \{\varnothing\}\}, \{\varnothing\} , \{\{\varnothing\}\} \}$.

An important property.

We already noted that $n(A) \leq n[P(A)]$. But how big is $P(A)$? Think about this a bit, and see whether you come to the following conclusion:

(i) If $n(A) = m$, then $n[P(A)] = 2^m$

(ii) The number of proper subsets of a set A is $n[P(A)]-1 = 2^m-1$.

Example 1.8

Find the number of subsets and the number of proper subsets of a set $X=\{a, b, c, x, y, z\}$.

Solution

Given $X=\{a, b, c, x, y, z\}$.Then, $n(X) =6$

The number of subsets $= n[P(X)] \quad = 2^6 \quad = 64$

The number of proper subsets $= n[P(X)]-1 \quad = 2^6-1$

$= 64 - 1 \quad = 63$

Exercise 1.2

1. Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n+2, n\in \mathbb{W}$ and $x< 15\}$

(iii) $Q = \{y : y = \frac{4}{3n}, n \in \mathbb{N}$ and $2 < n \leq 5\}$

(iv) $R = \{x : x$ is an integers, $x \in \mathbb{Z}$ and $-5 \leq x < 5\}$

(v) S = The set of all leap years between 1882 and 1906.

2. Identify the following sets as finite or infinite.

   (i) X = The set of all districts in Tamilnadu.

   (ii) Y = The set of all straight lines passing through a point.

   (iii) $A = \{ x : x \in \mathbb{Z}$ and $x < 5\}$

   (iv) $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$

3. Which of the following sets are equivalent or unequal or equal sets?

   (i) A = The set of vowels in the English alphabets.

   B = The set of all letters in the word "VOWEL"

   (ii) $C = \{2,3,4,5\}$

   $D = \{ x : x \in \mathbb{W}, 1 < x < 5\}$

   (iii) E = The set of A={ x : x is a letter in the word "LIFE"}

   $F = \{F, I, L, E\}$

   (iv) $G = \{x : x$ is a prime number and $3 < x < 23\}$

   $H = \{x : x$ is a divisor of $18\}$

4. Identify the following sets as null set or singleton set.

   (i) $A = \{x : x \in \mathbb{N}, 1 < x < 2\}$

   (ii) B = The set of all even natural numbers which are not divisible by 2

   (iii) $C = \{0\}$.

   (iv) D = The set of all triangles having four sides.

5. If S = {square, rectangle, circle, rhombus, triangle}, list the elements of the following subset of S.

   (i) The set of shapes which have 4 equal sides.

   (ii) The set of shapes which have no sides.

   (iii) The set of shapes in which the sum of all interior angles is 180°

   (iv) The set of shapes which have 5 sides.

6. If $A = \{a, \{a, b\}\}$, write all the subsets of A.

7. Write down the power set of the following sets.

   (i) $A = \{a, b\}$ (ii) $B = \{1, 2, 3\}$ (iii) $D = \{p, q, r, s\}$ (iv) $E = \varnothing$

8. Find the number of subsets and the number of proper subsets of the following sets.

   (i) $W = \{\text{red, blue, yellow}\}$ (ii) $X = \{ x^2 : x \in \mathbb{N}, x^2 \leq 100\}$.

9. (i) If $n(A) = 4$, find $n[P(A)]$.

   (ii) If $n(A)=0$, find $n[P(A)]$.

   (iii) If $n[P(A)] = 256$, find $n(A)$.

1.5 Set Operations

We started with numbers and very soon we learned arithmetical operations on them. In algebra we learnt expressions and soon started adding and multiplying them as well, writing (x^2+2) $(x-3)$ etc. Now that we know sets, the natural question is, what can we do with sets, what are natural operations on them ?

What can we do with sets ? We can pick an element. But then which element ? There are many in general, and hence "picking an element" is not an operation on a set. But like we did with addition, subtraction etc, we can try and think of operations that combine two given sets to get a new set. How can we do this ?

A simple way is to put the two sets together. This gives us a new set, and exactly one set, so it is an operation. We could pick out exactly the common elements of the two given sets. That's an interesting operation too. We could talk of all the elements not in the given set. But this is problematic: we can list the elements in a set, but which are the elements not in it ? Almost anything. We know that 5734 is in the set of natural numbers, but we also know that my chair is not in it, an elephant is not in it, and so on. How could we ever hope to describe all these elements ? But then when we speak of sets of numbers we are clearly not talking of elephants ! So we should really speak of numbers not in the set of natural numbers. For instance we could implicitly fix the integers and talk of integers not in the set of natural numbers. In general, we call this "fixed" set the universal set (relative to which we speak of what is or not in a given set).

When two or more sets combine together to form one set under the given conditions, then operations on sets can be carried out. We can visualize the relationship between sets and set operations using Venn diagram.

1.5.1 Universal Set

A Universal set is a set which contains all the elements of all the sets under consideration and is usually denoted by U.

For example,

(i) If we discuss about elements in Natural numbers, then the universal set U is the set of all Natural numbers. U=$\{x : x \in \mathbb{N}\}$.

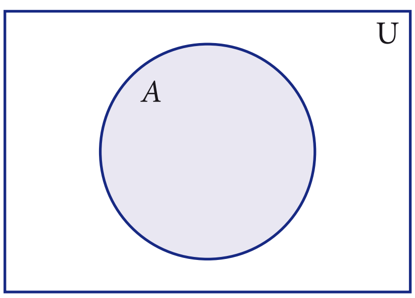
(ii) If A={earth, mars, jupiter}, then the universal set U is the planets of solar system.

1.5.2 Complement of a Set

The Complement of a set A is the set of all elements of U (the universal set) that are not in A.

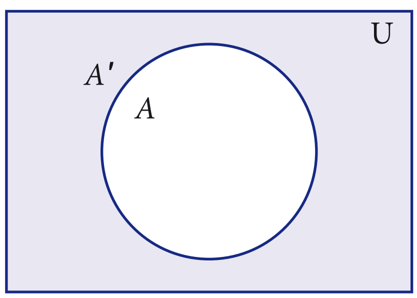
It is denoted by A' or A^c. In symbols $A' = \{x : x \in U, x \notin A\}$

Venn diagram for complement of a set



A (shaded region)

Fig. 1.6



A' (shaded region)

Fig. 1.7

For example,

If U = {all boys in a class} and A= {boys who play Cricket}, then complement of the set A is A'= {boys who do not play Cricket}.

Example 1.9

If U = $\{c, d, e, f, g, h, i, j\}$ and $A = \{c, d, g, j\}$, find A'.

Solution

U = $\{c, d, e, f, g, h, i, j\}$, $A = \{c, d, g, j\}$

$A' = \{e, f, h, i\}$

Note

(i) $(A')' = A$

(ii) $U' = \varnothing$

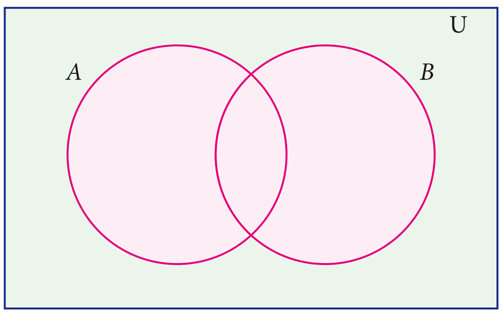
(iii) $\varnothing' = U$

1.5.3 Union of Two Sets

The union of two sets A and B is the set of all elements which are either in A or in B or in both. It is denoted by $A \cup B$ and read as A union B.

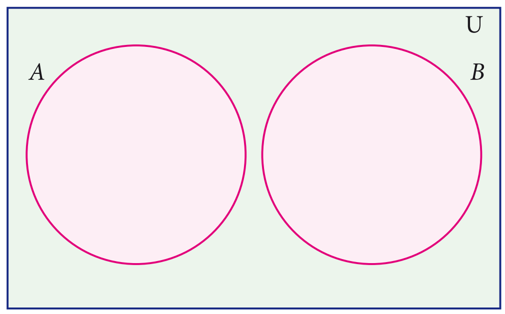
In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

The union of two sets can be represented by Venn diagram as given below



Sets A and B have common elements

Fig. 1.8



Sets A and B are disjoint

Fig. 1.9

For example,

If P ={Asia,Africa, Antarctica, Australia} and Q = {Europe, North America, South America}, then the union set of P and Q is $P \cup Q$ = {Asia, Africa, Antartica, Australia, Europe, North America, South America}.

Note

(i) $A \cup A = A$

(ii) $A \cup \varnothing = A$

(iii) $A \cup U = U$ where A is any subset of universal set U

(iv) $A \subseteq A \cup B$ and $B \subseteq A \cup B$

(v) $A \cup B = B \cup A$ (union of two sets is commutative)

Example 1.10

If $A=\{1, 2, 6\}$ and $B=\{2, 3, 4\}$, find $A \cup B$.

Solution

Given $A=\{1, 2, 6\}$, $B=\{2, 3, 4\}$

$A \cup B=\{1, 2, 3, 4,6\}$.

Example 1.11

If $P=\{m, n\}$ and $Q= \{m, i, j\}$, represent P and Q in Venn diagram and find $P \cup Q$.

Solution

Given $P=\{m, n\}$ and $Q= \{m, i, j\}$

From the diagram,

$P \cup Q=\{n, m, i, j\}$.

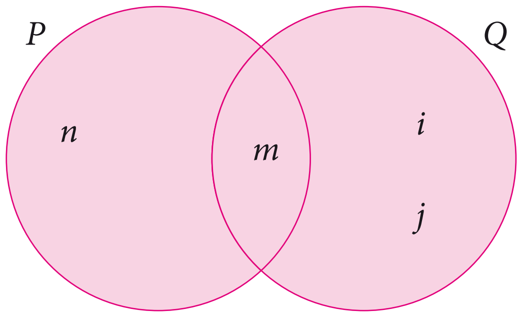


Fig. 1.10

1.5.4 Intersection of TwoSets

The intersection of two sets A and B is the set of all elements common to both A and B. It is denoted by $A \cap B$ and read as A intersection B.

In symbol , $A \cap B=\{x : x \in A \text{ and } x \in B\}$

Intersection of two sets can be represented by a Venn diagram as given below

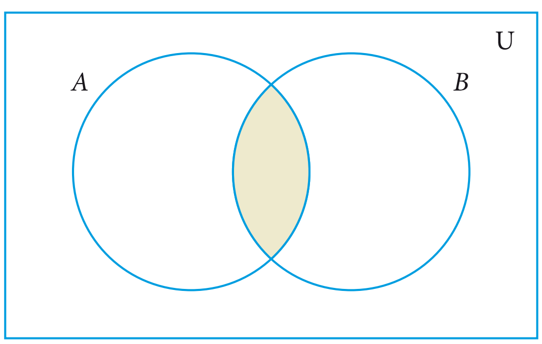


Fig. 1.11

For example,

If $A = \{1,2, 6\}$; $B = \{2, 3, 4\}$, then $A \cap B = \{2\}$ because 2 is common element of the sets A and B.

Note

(i) $A \cap A = A$ (ii) $A \cap \varnothing = \varnothing$

(iii) $A \cap U = A$ where A is any subset of universal set U

(iv) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(v) $A \cap B = B \cap A$ (Intersection of two sets is commutative)

Can we determine $n(A \cap B)$ in terms of $n(A)$ and $n(B)$? This seems to be difficult, but what about $n(A \cup B)$ in terms of $n(A)$ and $n(B)$?

Notice that all the elements in A are in $(A \cup B)$, and all the elements in B also in $A \cup B$

Can we say that $n(A \cup B) = n(A) + n(B)$?

Unfortunately not. Consider an element is common to both A and B? Of course it is in A union B, but is counted both in A and in B and we don't want to count the same element twice in A union B !

So indeed, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. But then it is easy to see that $n(A \cap B) = n(A) + n(B) - n(A \cup B)$. So between the union and the intersection we need to know one to determine the other, given $n(A)$ and $n(B)$.

Example 1.12

Let $A = \{x : x$ is an even natural number and $1 < x \leq 12\}$ and

$B = \{ x : x$ is a multiple of 3, $x \in \mathbb{N}$ and $x \leq 12\}$ be two sets. Find $A \cap B$.

Solution

Here $A = \{2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 9, 12\}$

$A \cap B = \{6, 12\}$

Example 1.13

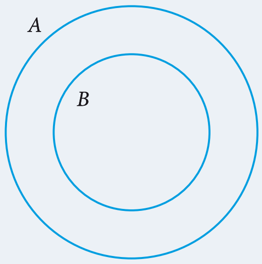
If $A = \{2, 3\}$ and $C = \{\ \}$, find $A \cap C$.

Solution

There is no common element and hence $A \cap C = \{\ \}$

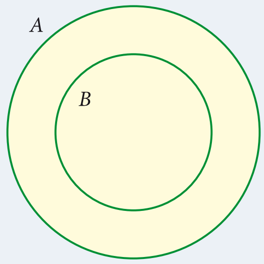
Note

(i) When $B \subseteq A$, the union and intersection of A and B are represented in Venn diagram as follows



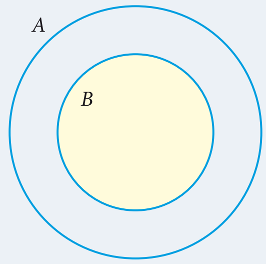
$B \subseteq A$

Fig. 1.12



shaded region is
$A \cup B = A$

Fig. 1.13



shaded region is
$A \cap B = B$

Fig. 1.14

(ii) For any two sets A and B, $A \cup B = A \cap B$. $\therefore A = B$

(iii) Let $n(A) = p$ and $n(B) = q$ then

(a) Minimum of $n(A \cup B) = \max\{p, q\}$

(b) Maximum of $n(A \cup B) = p + q$

(c) Minimum of $n(A \cap B) = 0$

(d) Maximum of $n(A \cap B) = \min\{p, q\}$

1.5.5 Difference of Two Sets

Let A and B be two sets, the difference of sets A and B is the set of all elements which are in A, but not in B. It is denoted by $A - B$ or $A \backslash B$ and read as A difference B.

In symbol, $A - B = \{x : x \in A \text{ and } x \notin B\}$

$$B - A = \{y : y \in B \text{ and } y \notin A\}.$$

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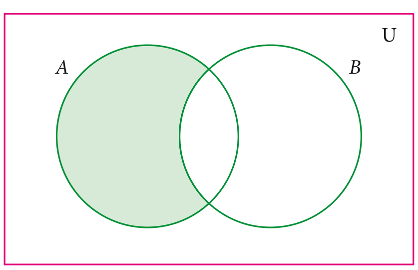
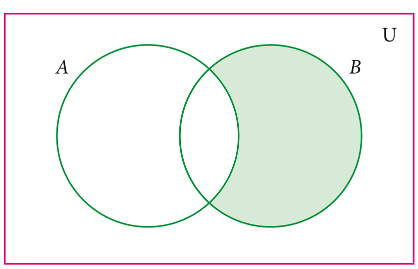
Venn diagram for set difference



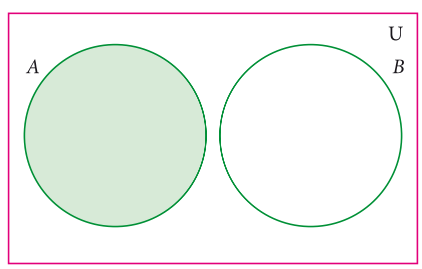
$A - B$

Fig. 1.15



$B - A$

Fig. 1.16



$A - B$

Fig. 1.17

Example 1.14

If $A=\{-3, -2, 1, 4\}$ and $B=\{0, 1, 2, 4\}$, find (i) $A-B$ (ii) $B-A$.

Solution

$A-B = \{-3, -2, 1, 4\} - \{0, 1, 2, 4\} = \{-3, -2\}$

$B-A = \{0, 1, 2, 4\} - \{-3, -2, 1, 4\} = \{0, 2\}$

Note

(i) $A' = U - A$

(ii) $A-B = A \cap B'$

(iii) $A-A = \varnothing$

(iv) $A-\varnothing = A$

(v) $A-B = B-A \Leftrightarrow A=B$

(vi) $A-B = A$ if $A \cap B = \varnothing$

1.5.6 Symmetric Difference of Sets

The symmetric difference of two sets A and B is the set $(A-B)\cup(B-A)$. It is denoted by $A\Delta B$.

$A\Delta B = \{x : x \in A-B \text{ or } x \in B-A\}$

Example 1.15

If $A = \{6, 7, 8, 9\}$ and $B=\{8, 10, 12\}$, find $A\Delta B$.

Solution

$A-B = \{6, 7, 9\}$

$B-A = \{10, 12\}$

$A\Delta B = (A-B)\cup(B-A) = \{6, 7, 9\}\cup\{10,12\}$

$A\Delta B = \{6, 7, 9, 10, 12\}$.

Thinking Corner

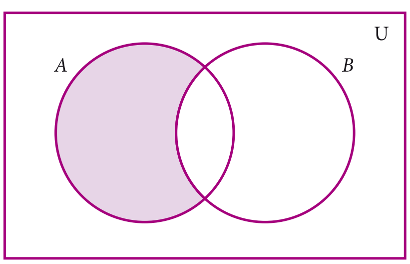
What is $(A-B) \cap (B-A)$?

Example 1.16

Represent $A\Delta B$ through Venn diagram.

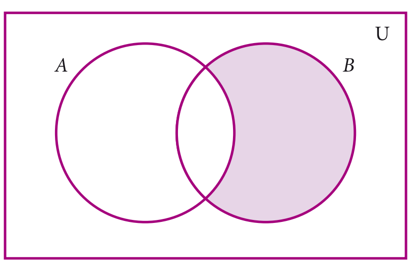
Solution

$A\Delta B = (A-B) \cup (B-A)$



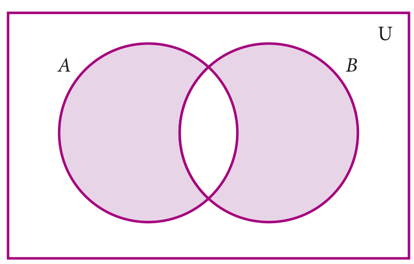
$A-B$

Fig. 1.18



$B-A$

Fig. 1.19



$(A-B) \cup (B-A)$

Fig. 1.20

Note

(i) $A\Delta A=\varnothing$ (ii) $A\Delta B = B\Delta A$

(iii) $A\Delta B=\{x : x\in A\cup B$ and $x\notin A\cap B\}$ (iv) $A\Delta B= (A\cup B) - (A\cap B)$

1.5.7 Disjoint Sets

Two sets A and B are said to be disjoint if they do not have common elements.

In other words, if $A\cap B=\varnothing$, then A and B are said to be disjoint sets.

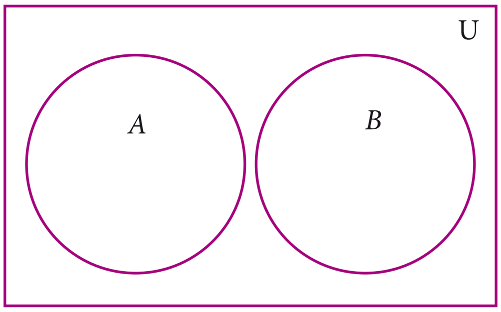


Fig. 1.21

Example 1.17

Verify whether $A=\{20, 22, 23, 24\}$ and $B=\{25, 30, 40, 45\}$ are disjoint sets.

Solution

$$A = \{20,22, 23, 24\}, B=\{25, 30, 40, 45\}$$

$$A\cap B = \{20,22, 23, 24\} \cap \{25, 30, 40, 45\}$$

$$= \{\ \}$$

Since $A\cap B = \varnothing$, A and B are disjoint sets.

Note

If $A\cap B\neq \varnothing$, then A and B are said to be overlapping sets .Thus if two sets have atleast one common element, they are called overlapping sets.

Example 1.18

From the given Venn diagram, write the elements of

(i) A (ii) B (iii) $A-B$ (iv) $B-A$

(v) A' (vi) B' (vii) U

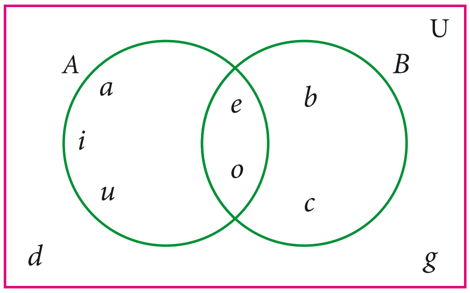


Fig. 1.22

Solution

(i) $A = \{a, e, i, o, u\}$

(ii) $B = \{b, c, e, o\}$

(iii) $A–B = \{a, i, u\}$

(iv) $B–A = \{b, c\}$

(v) $A' = \{b, c, d, g\}$

(vi) $B' = \{a, d, g, i, u\}$

(vii) $U = \{a, b, c, d, e, g, i, o, u\}$

Example 1.19

Draw a Venn diagram similar to one at the side and shade the region representing the following sets

(i) A' (ii) $(A–B)'$ (iii) $(A\cup B)'$

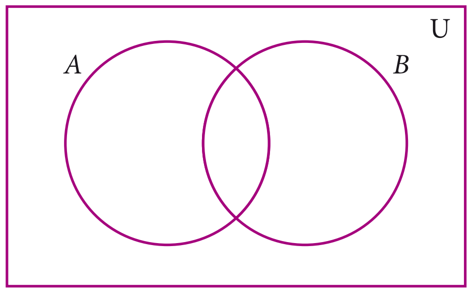


Fig. 1.23

Solution

(i) A'

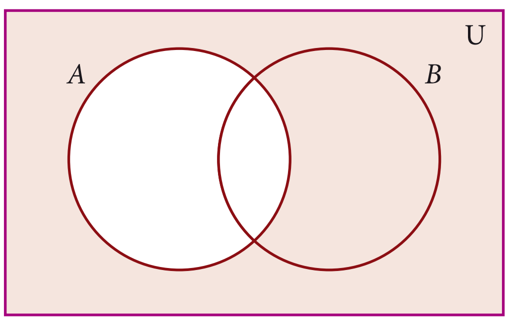


Fig. 1.24 A'

(ii) $(A–B)'$

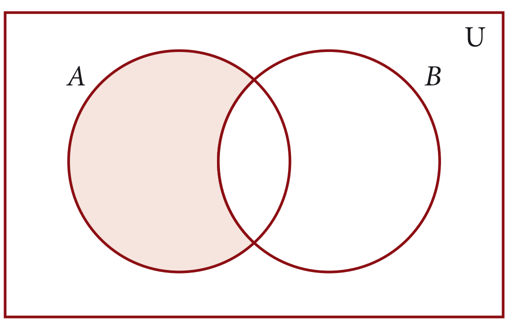


Fig. 1.25 $A–B$

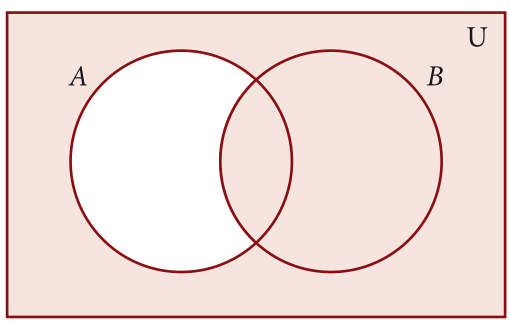


Fig. 1.26 $(A–B)'$

(iii) $(A\cup B)'$

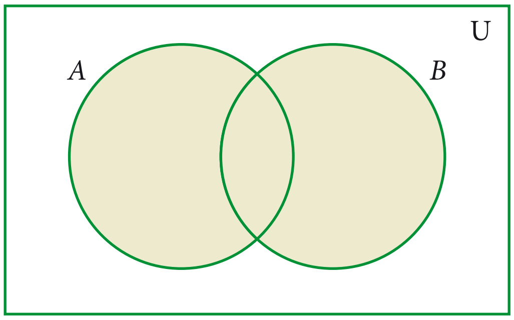


Fig. 1.27 $A\cup B$

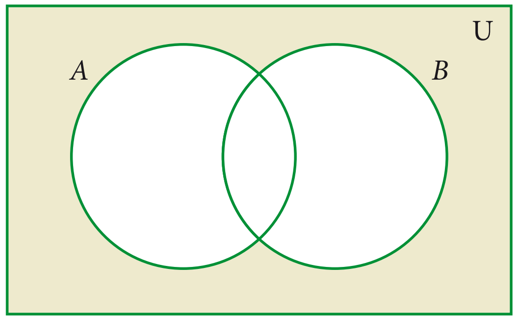


Fig. 1.28 $(A\cup B)'$

Exercise 1.3

1. Using the given Venn diagram, write the elements of

   (i) A (ii) B (iii) $A\cup B$ (iv) $A\cap B$

   (v) $A-B$ (vi) $B-A$ (vii) A' (viii) B'

   (ix) U

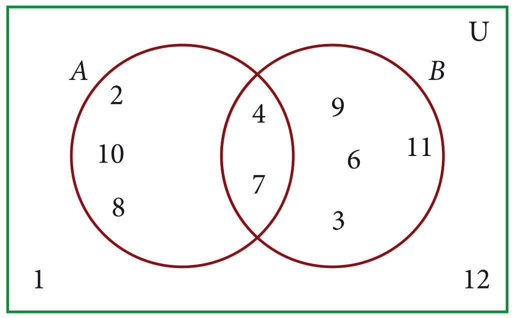


Fig. 1.29

2. Find $A\cup B$, $A\cap B$, $A-B$ and $B-A$ for the following sets.

   (i) $A = \{2, 6, 10, 14\}$ and $B=\{2, 5, 14, 16\}$

   (ii) $A = \{a, b, c, e, u\}$ and $B=\{a, e, i, o, u\}$

   (iii) $A = \{x : x \in N, x \le 10\}$ and $B=\{x : x \in W, x < 6\}$

   (iv) A = Set of all letters in the word "mathematics" and

   B = Set of all letters in the word "geometry"

3. If U=$\{a, b, c, d, e, f, g, h\}$, $A=\{b, d, f, h\}$ and $B=\{a, d, e, h\}$, find the following sets.

   (i) A' (ii) B' (iii) $A'\cup B'$ (iv) $A'\cap B'$ (v) $(A\cup B)'$

   (vi) $(A\cap B)'$ (vii) $(A')'$ (viii) $(B')'$

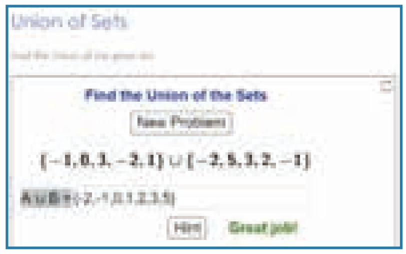
4. Let U=$\{0, 1, 2, 3, 4, 5, 6, 7\}$, $A=\{1, 3, 5, 7\}$ and $B=\{0, 2, 3, 5, 7\}$, find the following sets.

   (i) A' (ii) B' (iii) $A'\cup B'$ (iv) $A'\cap B'$ (v) $(A\cup B)'$

   (vi) $(A\cap B)'$ (vii) $(A')'$ (viii) $(B')'$

ICT Corner

Expected Result is shown in this picture



Step – 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

Step - 2

GeoGebra worksheet "Union of Sets" will appear. You can create new problems by clicking on the box
"NEW PROBLEM"

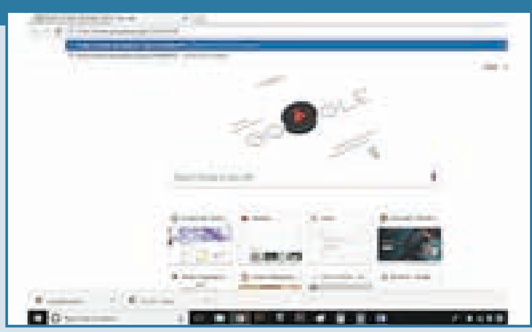
Step-3

Enter your answer by typing the correct numbers in the Question Box and then hit enter. If you have any doubt, you can hit the "HINT" button

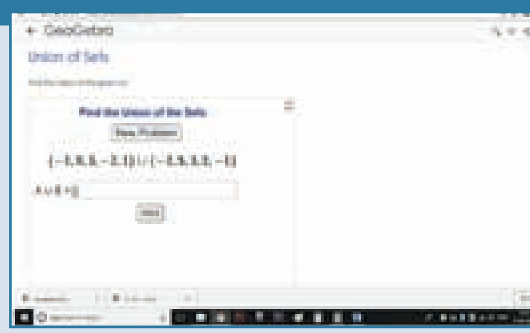
Step-4

If your answer is correct "GREAT JOB" menu will appear. And if your answer is Wrong "Try Again!" menu will appear.
Keep on working new problems until you get 5 consecutive trials as correct.

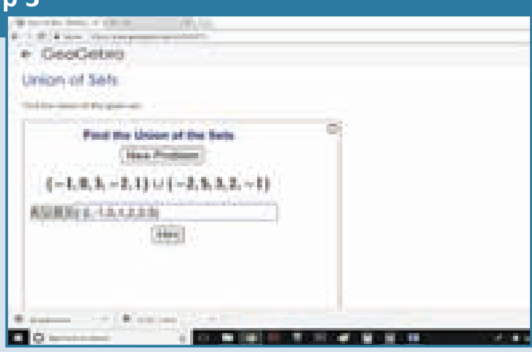
Step 1



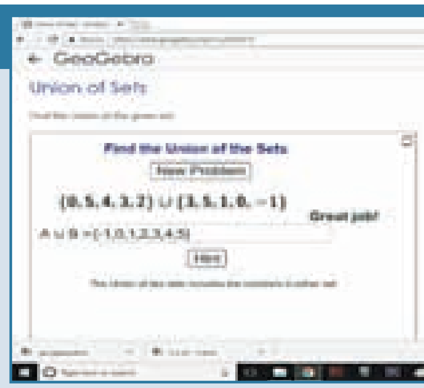
Step 2



Step 3



Step 4



Browse in the link

Union of Sets: https://www.geogebra.org/m/ufxdh47G

5. State which pairs of sets are disjoint or overlapping?

   (i) $A=\{f, i, a, s\}$ and $B=\{a, n, f, h, s\}$

   (ii) $C=\{x : x$ is a prime number, $x > 2\}$ and $D=\{x{:}x$ is an even prime number$\}$

   (iii) $E=\{x : x$ is a factor of $24\}$ and $F=\{x : x$ is a multiple of $3, x < 30\}$

6. Find the symmetric difference between the following sets.

   (i) $P = \{2, 3, 5, 7, 11\}$ and $Q=\{1, 3, 5, 11\}$

   (ii) $R = \{l, m, n, o, p\}$ and $S = \{j, l, n, q\}$

   (iii) $X = \{5, 6, 7\}$ and $Y=\{5, 7, 9, 10\}$

7. Using the set symbols, write down the expressions for the shaded region in the following

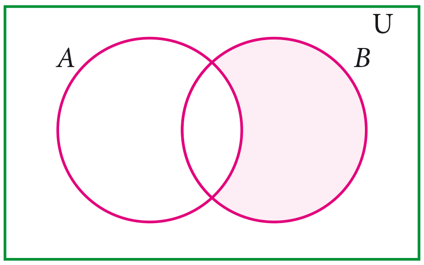


Fig. 1.30

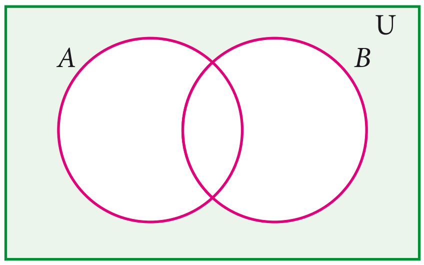


Fig. 1.31

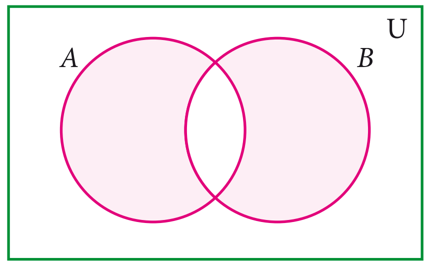


Fig. 1.32

8. Let A and B be two overlapping sets and the universal set be U. Draw appropriate Venn diagram for each of the following,

   (i) $A\cup B$ (ii) $A\cap B$ (iii) $(A\cap B)'$ (iv) $(B-A)'$ (v) $A'\cup B'$ (vi) $A'\cap B'$

   (vii) What do you observe from the Venn diagram (iii) and (v)?

1.6 Cardinality and Practical Problems on Set Operations

We have learnt about the union, intersection, complement and difference of sets.

Now we will go through some practical problems on sets related to everyday life.

Results :

If A and B are two finite sets, then

(i) $n(A\cup B) = n(A)+n(B)- n(A\cap B)$

(ii) $n(A-B) = n(A) - n(A\cap B)$

(iii) $n(B-A) = n(B)- n(A\cap B)$

(iv) $n(A') = n(\text{U}) - n(A)$

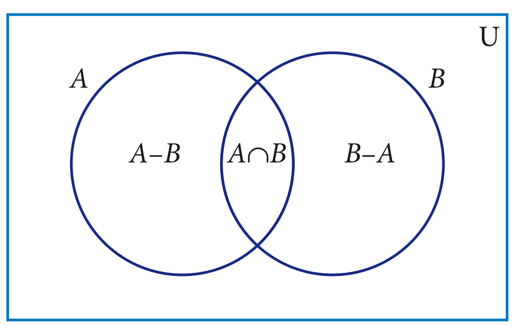


Fig. 1.33

Note

From the above results we may get,

(i) $n(A\cap B) = n(A)+n(B) - n(A\cup B)$ (ii) $n(U) = n(A)+n(A')$

(iii) If A and B are disjoint sets, $n(A\cup B) = n(A)+n(B)$.

Example1.20

From the Venn diagram, verify that

$n(A\cup B) = n(A)+n(B) - n(A\cap B)$

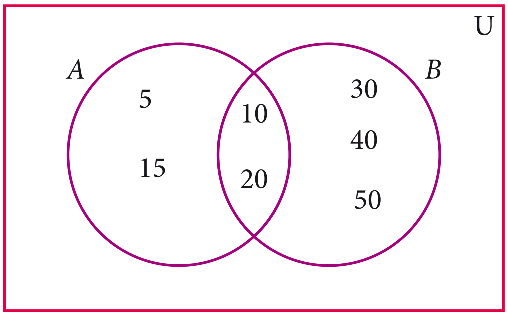


Fig. 1.34

Solution

From the venn diagram,

$A = \{5, 10, 15, 20\}$

$B = \{10, 20, 30, 40, 50,\}$

Then $A\cup B = \{5, 10, 15, 20, 30, 40, 50\}$

$A\cap B = \{10, 20\}$

$n(A) = 4,\ \ n(B) = 5, n(A\cup B) = 7,\ \ n(A\cap B) = 2$

$n(A\cup B) = 7 \quad \rightarrow (1)$

$n(A)+n(B)-n(A\cap B) = 4+5-2$

$=7 \quad \rightarrow (2)$

From (1) and (2), $n(A\cup B) = n(A)+n(B)-n(A\cap B)$.

Example 1.21

If $n(A) = 36$, $n(B) = 10$, $n(A\cup B)=40$, and $n(A')=27$ find $n(U)$ and $n(A\cap B)$.

Solution

$n(A) = 36,\ \ n(B) = 10,\ \ n(A\cup B)=40,\ \ n(A')=27$

(i) $n(U) = n(A)+n(A') = 36+27 = 63$

(ii) $n(A\cap B) = n(A)+n(B)-n(A\cup B) = 36+10-40 = 46-40 = 6$

Activity-5

Fill in the blanks with appropriate cardinal numbers.

S.No.	$n(A)$	$n(B)$	$n(A\cup B)$	$n(A\cap B)$	$n(A-B)$	$n(B-A)$
1	30	45	65			
2	20		55	10		
3	50	65		25		
4	30	43	70			

Example 1.22

Let $A=\{b, d, e, g, h\}$ and $B = \{a, e, c, h\}$. Verify that $n(A-B) = n(A)-n(A\cap B)$.

Solution

$$A = \{b, d, e, g, h\},\ B = \{a, e, c, h\}$$

$$A - B = \{b, d, g\}$$

$$n(A-B) = 3 \qquad \dots (1)$$

$$A \cap B = \{e, h\}$$

$$n(A \cap B) = 2, \qquad n(A) = 5$$

$$n(A) - n(A\cap B) = 5-2$$

$$= 3 \qquad \dots (2)$$

Form (1) and (2) we get $n(A-B) = n(A)-n(A\cap B)$.

Example 1.23

In a school, all students play either Hockey or Cricket or both. 300 play Hockey, 250 play Cricket and 110 play both games. Find

(i) the number of students who play only Hockey.

(ii) the number of students who play only Cricket.

(iii) the total number of students in the School.

Solution:

Let H be the set of all students play Hockey and C be the set of all students play Cricket.

Then $n(H) = 300$, $n(C) = 250$ and $n(H \cap C) = 110$

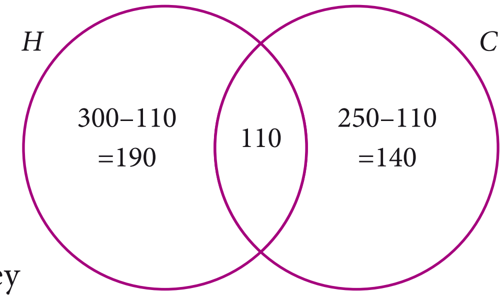


Fig. 1.35

Using Venn diagram,

From the Venn diagram,

(i) The number of students who play only Hockey $= 190$

(ii) The number of students who play only Cricket $= 140$

(iii) The total number of students in the school $= 190+110+140 = 440$

Aliter

(i) The number of students who play only Hockey

$$n(H-C) = n(H) - n(H \cap C)$$
$$= 300 - 110 = 190$$

(ii) The number of students who play only Cricket

$$n(C-H) = n(C) - n(H \cap C)$$
$$= 250 - 110 = 140$$

(iii) The total number of students in the school

$$n(H \cup C) = n(H) + n(C) - n(H \cap C)$$
$$= 300+250 - 110 = 440$$

Example1.24

In a party of 60 people, 35 had Vanilla ice cream, 30 had Chocolate ice cream. All the people had at least one ice cream. Then how many of them had,

(i) both Vanilla and Chocolate ice cream.

(ii) only Vanilla ice cream.

(iii) only Chocolate ice cream.

Solution :

Let V be the set of people who had Vanilla ice cream and C be the set of people who had Chocolate ice cream.

Then $n(V) = 35$, $n(C) = 30$, $n(V \cup C) = 60$,

Let x be the number of people who had both ice creams.

From the Venn diagram

$$35 - x + x + 30 - x = 60$$

$$65 - x = 60$$

$$x = 5$$

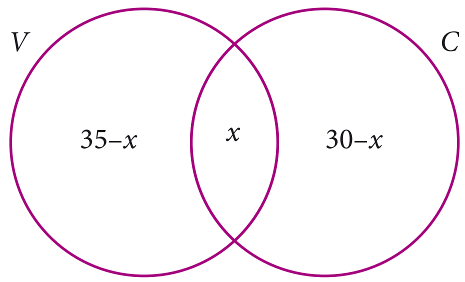


Fig. 1.36

Hence 5 people had both ice creams.

(i) Number of people who had only Vanilla ice cream $= 35 - x$

$= 35 - 5 = 30$

(ii) Number of people who had only Chocolate ice cream $= 30 - x$

$= 30 - 5 = 25$

Aliter

Number of people had both Vanilla and Chocolate ice creams

$$n(V \cap C) = n(V) + n(C) - n(V \cup C)$$

$$= 35 + 30 - 60 = 5$$

Number of people had only Vanilla ice creams

$$n(V - C) = n(V) - n(V \cap C)$$

$$= 35 - 5 = 30$$

Number of people who had only Chocolate ice creams

$$n(C - V) = n(C) - n(V \cap C)$$

$$= 30 - 5 = 25$$

E4AHV7

Exercise 1.4

1. (i) If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.

   (ii) If $n(A) = 300$, $n(A \cup B) = 500$, $n(A \cap B) = 50$ and $n(B') = 350$, find $n(B)$ and $n(U)$.

2. If $U = \{x : x \in \mathbb{N}, x \le 10\}$, $A = \{2,3,4,8,10\}$ and $B = \{1,2,5,8,10\}$, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3. If $U = \{1, 2, 3, \ldots., 10\}$, $P = \{3, 4, 5, 6\}$ and $Q = \{x : x \in \mathbb{N}, x < 5\}$, then verify that $n(Q - P) = n(Q) - n(P \cap Q)$.

4. In a class, all students take part in either music or drama or both. 25 students take part in music, 30 students take part in drama and 8 students take part in both music and drama. Find

   (i) The number of students who take part in only music.

   (ii) The number of students who take part in only drama.

   (iii) The total number of students in the class.

5. In a Mathematics class, 20 children forgot to bring their rulers, 17 children forgot to bring their pencils and 5 children forgot to bring both ruler and pencil. Then find the number of children

   (i) who forgot to bring only pencil. (ii) who forgot to bring only ruler.

   (iii) in the class.

6. In a village of 100 families, 65 families buy Tamil newspapers and 55 families buy English newspapers. Find the number of families who buy

   (i) both Tamil and English newspapers. (ii) Tamil newspapers only.

   (iii) English newspapers only.

7. In a party of 45 people, each one likes tea or coffee or both. 35 people like tea and 20 people like coffee. Find the number of people who

   (i) like both tea and coffee. (ii) do not like Tea.

   (iii) do not like coffee.

8. In an examination 50% of the students passed in Mathematics and 70% of students passed in Science while 10% students failed in both subjects. 300 students passed in atleast one subject. Find the total number of students who appeared in the examination, if they took examination in only two subjects.

9. A and B are two sets such that $n(A-B) = 32 + x$, $n(B-A) = 5x$ and $n(A\cap B) = x$. Illustrate the information by means of a Venn diagram. Given that $n(A) = n(B)$, calculate the value of x.

10. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Exercise 1.5

Multiple Choice Questions

1. Which of the following is correct?

   (a) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$ (b) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$

   (c) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$ (d) $\{7\} \not\subseteq \{1,2,3,4,5,6,7,8,9,10\}$

2. The set $P = \{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a

   (a) Singleton set (b) Power set (c) Null set (d) Subset

3. If $U = \{x \mid x \in \mathbb{N}, x < 10\}$ and $A = \{x \mid x \in \mathbb{N}, 2 \le x < 6\}$ then $(A')'$ is

   (a) $\{1, 6, 7, 8, 9\}$ (b) $\{1, 2, 3, 4\}$ (c) $\{2, 3, 4, 5\}$ (d) $\{\ \}$

4. If $B \subseteq A$ then $n(A\cap B)$ is

   (a) $n(A-B)$ (b) $n(B)$ (c) $n(B-A)$ (d) $n(A)$

5. If $A = \{x, y, z\}$ then the number of non- empty subsets of A is

   (a) 8 (b) 5 (c) 6 (d) 7

6. Which of the following is correct?

   (a) $\varnothing \subseteq \{a, b\}$ (b) $\varnothing \in \{a, b\}$ (c) $\{a\} \in \{a, b\}$ (d) $a \subseteq \{a, b\}$

7. If $A\cup B = A\cap B$, then

   (a) $A \neq B$ (b) $A = B$ (c) $A \subset B$ (d) $B \subset A$

8. If $B - A$ is B, then $A\cap B$ is

   (a) A (b) B (c) U (d) $\varnothing$

9. The shaded region in the adjacent diagram represents

   (a) $(A\cup B)'$ (b) $(A\cap B)'$

   (c) $A'\cap B'$ (d) $A\cap B$

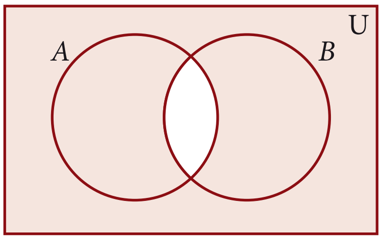


Fig. 1.37

10. From the adjacent diagram $n[P(A\Delta B)]$ is

(*a*) 8 (*b*) 16

(*c*) 32 (*d*) 64

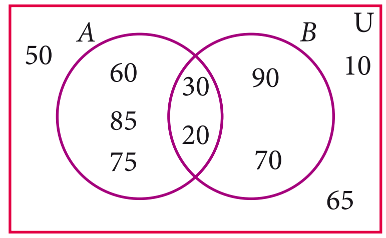


Fig. 1.38

11. If $n(A) = 10$ and $n(B) = 15$, then the minimum and maximum number of elements in $A \cap B$ is

(*a*) (10,15) (*b*) (15,10) (*c*) (10,0) (*d*) (0,10)

12. If $X = \{x : x = 4(n-1),\ n \in \mathbb{N}\}$ and $Y = \{y : y = 3^n - 2n - 1,\ n \in \mathbb{N}\}$, then $X \cup Y$ is

(*a*) $\mathbb{W}$ (*b*) X (*c*) Y (*d*) $\mathbb{N}$

13. Let $A = \{\varnothing\}$ and $B = P(A)$, then $A \cap B$ is

(*a*) $\{\varnothing, \{\varnothing\}\}$ (*b*) $\{\varnothing\}$ (*c*) $\varnothing$ (*d*) $\{0\}$

14. In a class of 50 boys, 35 boys play Carrom and 20 boys play Chess then the number of boys play both games is

(*a*) 5 (*b*) 30 (*c*) 15 (*d*) 10.

Points to remember

1. A set is a well defined collection of objects.
2. Sets are represented in three forms (i) Descriptive form (ii) Set – builder form (iii) Roster form.
3. The number of elements in a set is called the cardinal number of the set.
4. A set consisting of no element is called an empty set .
5. If the number of elements in a set is zero or finite, it is a finite set. Otherwise it is an infinite set.
6. Two finite sets are said to be equivalent if they contain the same number of elements.
7. Two sets are said to be equal when they contain the same elements.

8. If every element of A is also an element of B, then A is called a subset of B.

9. If $A \subseteq B$ and $A \neq B$, then A is a proper subset of B.

10. The power set of the set A is the set of all the subsets of A and it is denoted by $P(A)$.

11. The number of subsets of a set with $\boldsymbol{m}$ elements is $\mathbf{2^m}$.

12. The number of proper subsets of a set with $\boldsymbol{m}$ elements is $\mathbf{2^m - 1}$.

13. The complement A' of the set A contains all the elements of the universal set except that of A.

14. The union of two sets A and B is the set of all elements which are either in A or in *B or in both*.

15. The set of all common elements of the sets A and B is called the intersection of the sets A and B.

16. If $A \cap B = \varnothing$ then A and B are disjoint sets. If $A \cap B \neq \varnothing$ then A and B are overlapping.

17. The difference of two sets A and B is the set of all elements in A but not in B.

18. The symmetric difference of two sets A and B is the union of A-B and B-A.

19. For any two finite sets A and B, we have

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(B - A) = n(B) - n(A \cap B)$

(v) $n(U) = n(A) + n(A')$

Answers

Exercise 1.1

1. (i) set (ii) not a set (iii) Set (iv) not a set

2. (i) {I, N, D, A} (ii) {P, A, R, L, E, O, G, M} (iii) {M, I, S, P}
   (iv) {C, Z, E, H, O, S, L, V, A, K, I}

3. (a) (i) True (ii) True (iii) False (iv) True (v) False (vi) False
   (b) (i) A (ii) C (iii) $\notin$ (iv) $\in$

4. (i) $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ (ii) $B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$
   (iii) $C = \{64, 125\}$ (iv) $D = \{-4, -3, -2, -1, 0, 1, 2\}$

5. (i) $B = \{x : x$ is an Indian player who scored double centuries in One Day International$\}$
   (ii) $C = \left\{x : x = \frac{n}{n+1}, n \in \mathbb{N}\right\}$ (iii) $D = \{x : x$ is a tamil month in a year$\}$
   (iv) $E = \{x : x$ is an odd whole number less than 9$\}$

6. (i) P = The set of English months starting with letter '*J*'
   (ii) Q = The set of Prime numbers between 5 and 31
   (iii) R = The set of natural numbers less than 5
   (iv) S = The set of English consonants

Exercise 1.2

1. (i) $n(M) = 6$ (ii) $n(P) = 5$ (iii) $n(Q) = 3$ (iv) $n(R) = 10$ (v) $n(S) = 5$

2. (i) finite (ii) infinite (iii) infinite (iv) finite

3. (i) Equivalent sets (ii) Unequal sets (iii) Equal sets (iv) Equivalent sets

4. (i) null set (ii) null set (iii) singleton set (iv) null set

5. (i) {square, rhombus} (ii) {circle} (iii) {triangle} (iv) { }

6. (i) { }, {a}, {a, b} {a, {a, b}}

7. (i) {{ }, {a}, {b} {a, b}}

(ii) {{ }, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3 }, {1, 2, 3}}

(iii) {{ }, {p}, {q} {r}, {s}, {p, q}, {p, r}, {p, s}, {q, r}, {q, s}, {r, s}, {p, q, r}, {p, q, s}, {p, r, s}, {q, r, s}, {p, q, r, s}}   (iv) $P(E)$= {{ }}

8. (i) 8, 7   (ii) 1024, 1023

9. (i) 16   (ii) 1   (iii) 8

Exercise 1.3

1. (i) {2, 4, 7, 8, 10}   (ii) {3, 4, 6, 7, 9, 11}   (iii) {2, 3, 4, 6, 7, 8, 9, 10, 11}

(iv) {4, 7}   (v) {2, 8, 10}   (vi) {3, 6, 9, 11}

(vii) {1, 3, 6, 9, 11, 12}   (viii) {1, 2, 8, 10, 12}

(ix) {1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12}

2. (i) {2, 5, 6, 10, 14, 16}, {2, 14}, {6, 10}, {5, 16}

(ii) {a, b, c, e, i, o, u}, {a, e, u}, {b, c}, {i, o}

(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5}, {6, 7, 8, 9, 10,}, {0}

(iv) {m, a, t, h, e, i, c, s, g, o, r, y}, {e, m, t,}, {a, h, i, c, s}, {g, o, r, y}

3. (i) {a, c, e, g} (ii) {b, c, f, g}   (iii) {a, b, c, e, f, g}   (iv) {c, g}   (v) {c, g}

(vi) {a, b, c, e, f, g}   (vii) {b, d, f, h} (viii) {a, d, e, h}

4. (i) {0, 2, 4, 6} (ii) {1, 4, 6}   (iii) {0, 1, 2, 4, 6}   (iv) {4, 6}   (v) {4, 6}

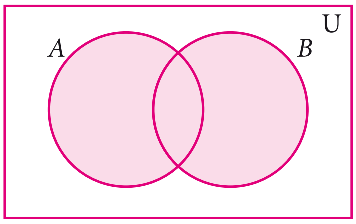
(vi) {0, 1, 2, 4, 6}   (vii) {1, 3, 5, 7} (viii) {0, 2, 3, 5, 7}

5. (i) overlapping   (ii) disjoint   (iii) overlapping

6. (i) {1, 2, 7}   (ii) {m, o, p, q, j}   (iii) {6, 9, 10}

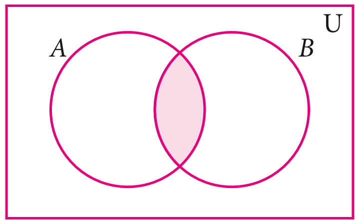
7. (i) $B–A$ (ii) $(A \cup B)'$ (iii) $(A - B) \cup (B - A)$

8. (i)



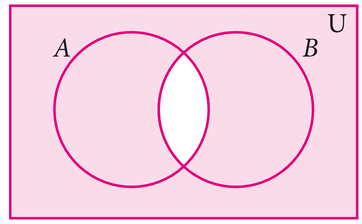
$A \cup B$

(ii)



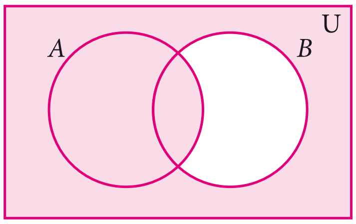
$A \cap B$

(iii)



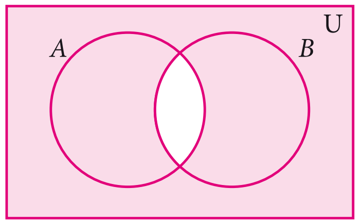
$(A \cap B)'$

(iv)



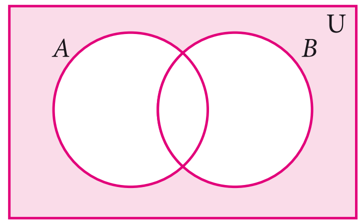
$(B - A)'$

(v)



$A' \cup B'$

(vi)



$A' \cap B'$

(vii) $(A \cap B)' = A' \cup B'$

Exercise 1.4

1. (i) 15, 65 (ii) 250, 600 4. (1) 17 (ii) 22 (iii) 47

5. (i) 12 (ii) 15 (iii) 32

6. (i) 20 (ii) 45 (iii) 35

7. (i) 10 (ii) 10 (iii) 25 8. 1000 9. 8

10. Not correct

Exercise 1.5

1. *b* 2. *a* 3. *c* 4. *b* 5. *d* 6. *a* 7. *b* 8. *d* 9. *b*

10. *c* 11. *d* 12. *b* 13. *b* 14. *a*