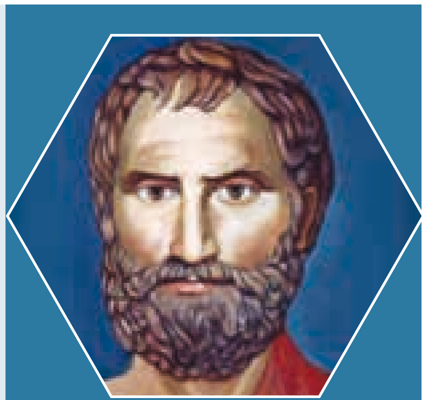
4 GEOMETRY

There is geometry in humming of the strings,
there is music in the spacing of the spheres.

-Pythagoras

E64P7Q

Thales (Pronounced THAYLEES) was born in the Greek city of Miletus. He was known for theoretical and practical understanding of geometry, especially triangles. He used geometry to solve many problems such as calculating the height of pyramids and the distance of ships from the sea shore. He was one of the so-called Seven Sagas or Seven Wise Men of Greece and many regarded him as the first philosopher in the western tradition.



Thales
(Period 620 – 546 BC)

Learning Outcomes

- To understand theorems on linear pairs and vertically opposite angles.
- To understand the angle sum property of triangle.
- To classify quadrilaterals.
- To understand the properties of quadrilaterals and use them in problem solving.
- To construct the Circumcentre of a triangle.
- To construct the Orthocentre of a triangle.

4.1. Introduction

In geometry, we study **shapes**. But what is there to *study* in shapes, you may ask. Think first, what are all the things we do with shapes? We draw shapes, we compare shapes, we *measure* shapes. *What* do we measure in shapes?

Take some shapes like this:

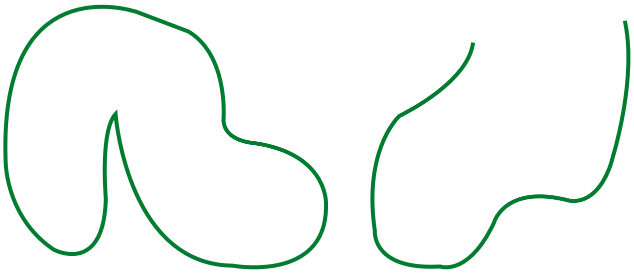


Fig. 4.1

In both of them, there is a *curve* forming the shape: one is a closed curve, enclosing a region, and the other is an open curve. We can use a rope (or a thick string) and measure the **length** of the open curve and the length of the boundary of the region in the case of the closed curve.

Curves are tricky, aren't they? It is so much easier to measure length of straight lines using the scale, isn't it? Consider the two shapes below.

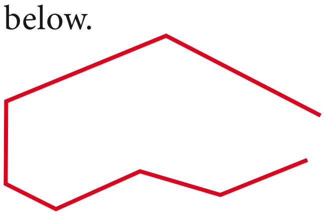


Fig. 4.2

We are going to focus our attention for now only on shapes made using straight lines, and only closed figures. As you will see, there is plenty of interesting things to do already? Fig.4.2 shows an open figure.

We not only want to draw such shapes, we want to compare them, measure them and do much more. For doing so, we want to **describe** them. How would you describe these closed shapes? (See Fig 4.3) They are all made of straight lines, all closed.

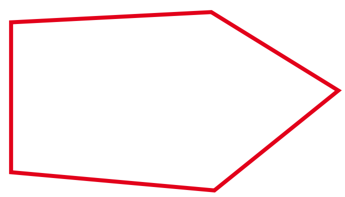


Fig. 4.3

Activity 1

Your friend draws a figure such as this on a piece of paper.

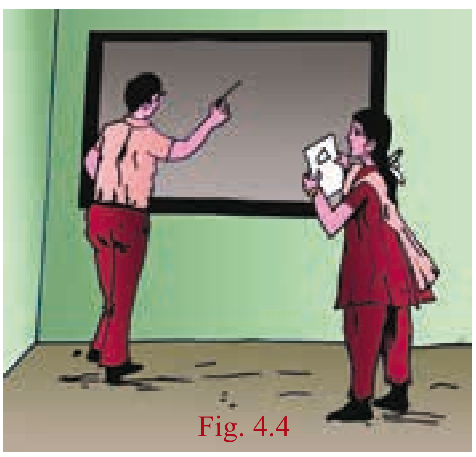


Fig. 4.4

You don't know what it is, and stand at the board.

She has to describe the figure to you so that you draw exactly the **same** figure on the board.

You both know rectangles, so it is easy to describe and draw a rectangle. The rest are not easy.

Can we describe any such shape we can *ever think of*, in a way that another person hearing it can reproduce it **exactly?**

Yes, yes, yes! Now, isn't that exciting?

The answer is so simple that it is breathtaking.

We describe different shapes by their properties.

It is like talking to your friend about someone you saw: "Tall, thin man, big moustache, talks fast, has a deep voice ...."

We will do a similar thing with shapes.

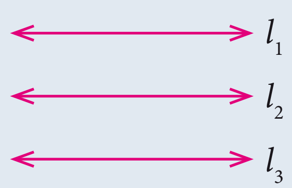
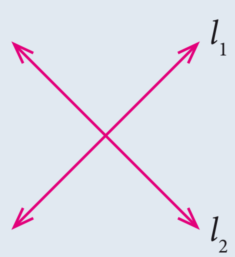
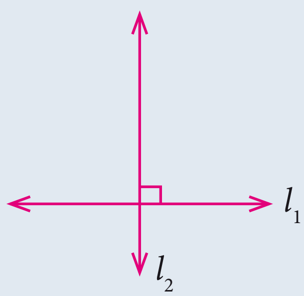
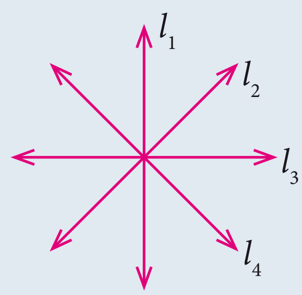
In science we learn that air has certain properties – for instance, it occupies space. Water has the property that it takes the shape of the vessel that contains it, flows from a height to down below. Similarly, triangles, rectangles, circles, and all the shapes we draw, however different they look, have properties that describe them **uniquely.** By the time you learn Geometry well, you can give shape for everything on earth – indeed (***Aadhaar card***), not only on earth but anywhere! But for now, have some patience. Just as we started with small numbers and arithmetical operations on them in primary level classes and now you know it is the same for **all** numbers, similarly we will learn properties of very **simple** shapes now, and slowly but surely we will learn more and when we are done, you will have techniques to describe any shape in 2 dimensions (like what we draw on paper), or in 3 dimensions (like the solids we use in life), or indeed, in any dimensions.

Why should we bother to learn this? One very practical reason is that all science and engineering demands it. We cannot design buildings, or even tables and chairs, or lay out the circuits inside our mobile phones, without a mathematical understanding of shapes. Another important reason is that geometry gives you a new way of looking at the world, at everything in it. You begin to see that all objects, however complicated they look, are made of very simple shapes. You learn to think not only like a mathematician, but also learn to appreciate symmetry and order, as artists do. Yes, geometry helps you become an artist too.

4.2 Geometry Basics –Recall

Draw two lines on a plane. They can be either parallel or intersecting.

Parallel lines	Two or more lines lying in the same plane that never meet.
Intersecting lines	Two lines which meet at a common point.
Perpendicular lines	Two lines which intersect each other at right angle.
Concurrent lines	Three or more lines passing through the same point.

Parallel Lines	Intersecting Lines	Perpendicular Lines	Concurrent Lines
			
$l_1 \parallel l_2 \parallel l_3$		$l_1 \perp l_2$	

4.2.1 Types of Angles

Plumbers measure the angle between connecting pipes to make a good fitting. Wood workers adjust their saw blades to cut wood at the correct angle. Air Traffic Controllers (ATC) use angles to direct planes. Carom and billiards players must know their angles to plan their shots. An angle is formed by two rays that share a common end point provided that the two rays are non-collinear.

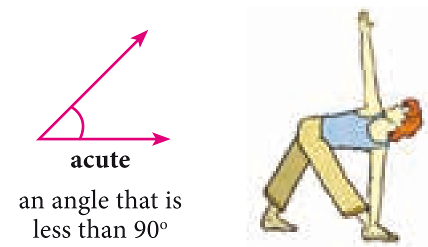


Fig. 4.5

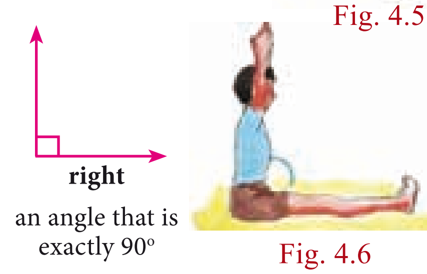
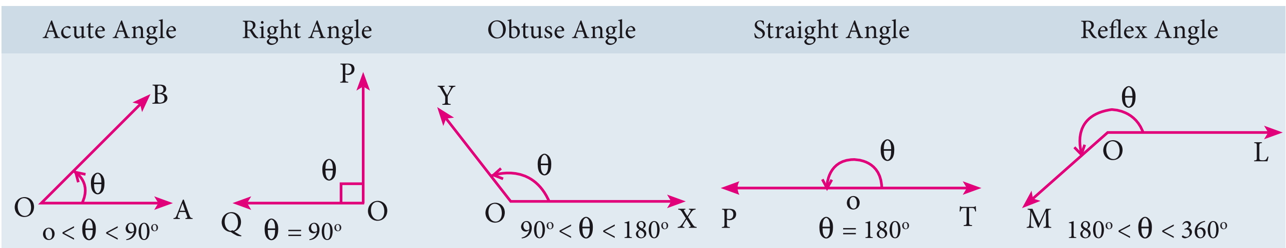


Fig. 4.6



Complementary Angles

Two angles are Complementary if their sum is 90°. For example, if $\angle ABC = 64°$ and $\angle DEF = 26°$, then angles $\angle ABC$ and $\angle DEF$ are complementary to each other because $\angle ABC + \angle DEF = 90°$

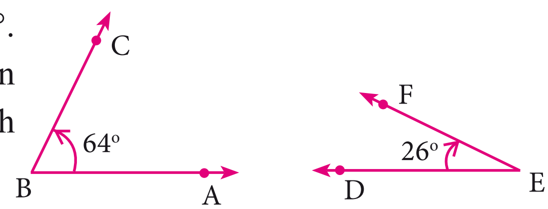


Fig. 4.7

Supplementary Angles

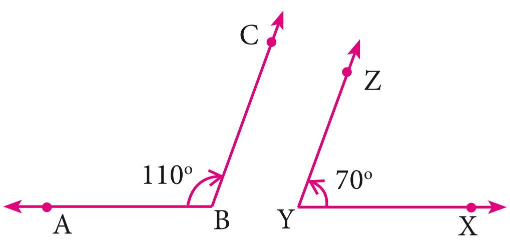


Fig. 4.8

Two angles are Supplementary if their sum is 180°. For example if $\angle ABC = 110°$ and $\angle XYZ = 70°$

Here $\angle ABC + \angle XYZ = 180°$

$\therefore \angle ABC$ and $\angle XYZ$ are supplementary to each other

Adjacent Angles

Two angles are called adjacent angles if

(i) They have a common vertex.

(ii) They have a common arm.

(iii) The common arm lies between the two non-common arms.

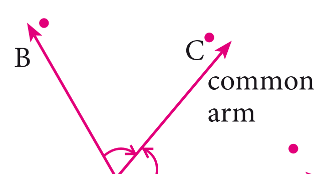


Fig. 4.9

Linear Pair of Angles

If a ray stands on a straight line then the sum of two adjacent angle is 180°. We then say that the angles so formed is a linear pair.

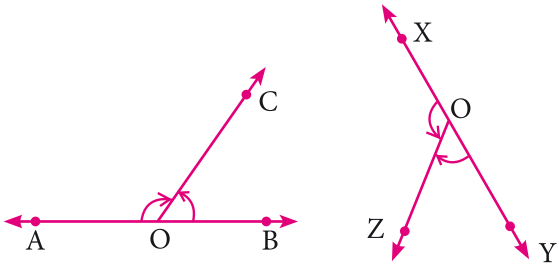


Fig. 4.10

$\angle AOC + \angle BOC = 180°$

$\therefore \angle AOC$ and $\angle BOC$ form a linear pair

$\angle XOZ + \angle YOZ = 180°$

$\angle XOZ$ and $\angle YOZ$ form a linear pair

Vertically Opposite Angles

If two lines intersect each other, then vertically opposite angles are equal.

In this figure $\angle POQ = \angle SOR$

$\angle POS = \angle QOR$

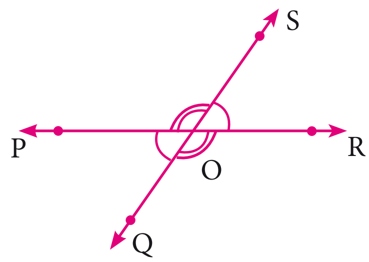


Fig. 4.11

4.2.2 Transversal

A line which intersects two or more lines at a distinct points is called a transversal of those lines.

Case (i) When a transversal intersect two lines, we get eight angles.

In the figure the line l is the transversal for the lines m and n

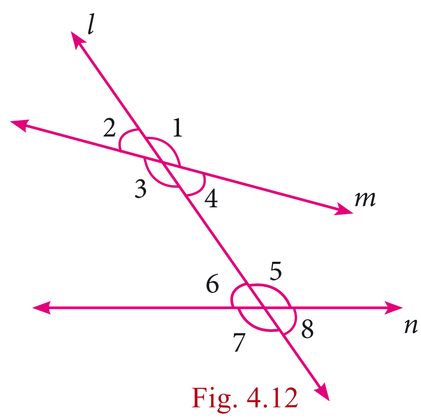


Fig. 4.12

(i) Corresponding Angles: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

(ii) Alternate Interior Angles: $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$

(iii) Alternate Exterior Angles: $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$

(iv) $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$ are interior angles on the same side of the transversal.

(v) $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$ are exterior angles on the same side of the transversal.

Case (ii) If a transversal intersects two parallel lines. The transversal forms different pairs of angles.

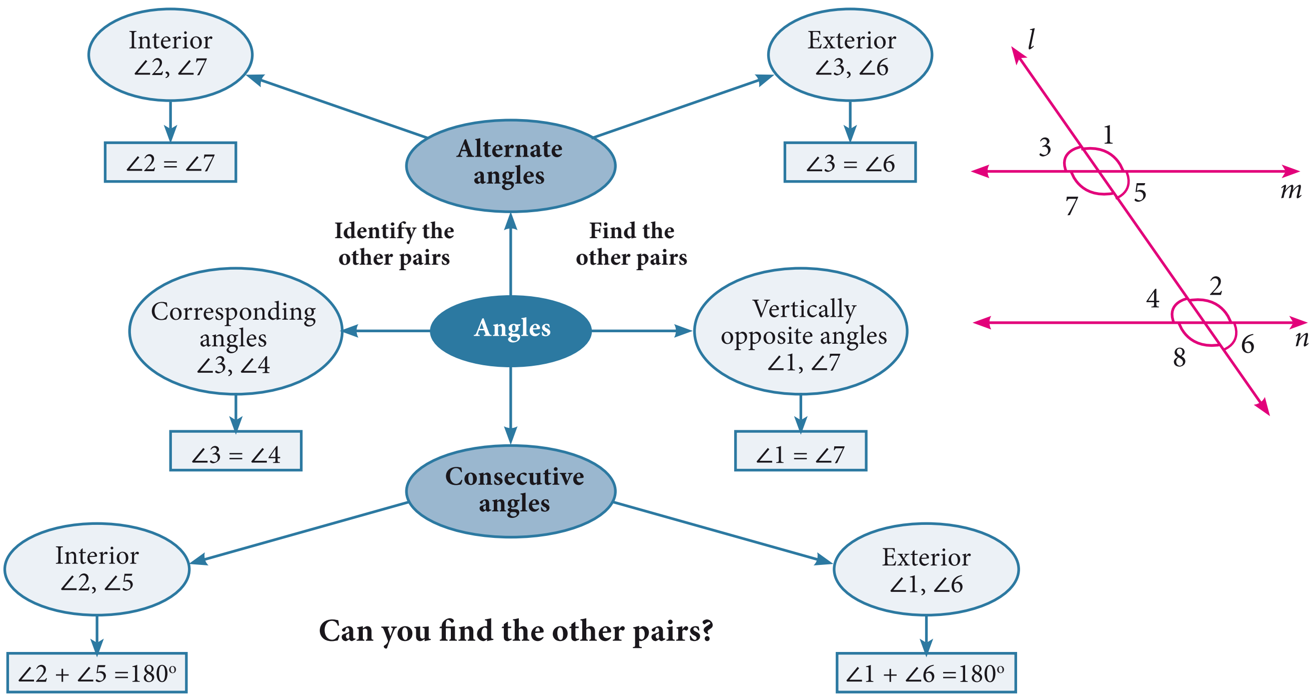


Fig. 4.13

4.2.3 Triangles

Activity 2

Take three different colour sheets; place one over the other and draw a triangle on the top sheet. Cut the sheets to get triangles of different colour which are identical. Mark the vertices and the angles as shown. Place the interior angles ∠1, ∠2 and ∠3 on a straight line, adjacent to each other, without leaving any gap. What can you say about the total measure of the three angles ∠1, ∠2 and ∠3?

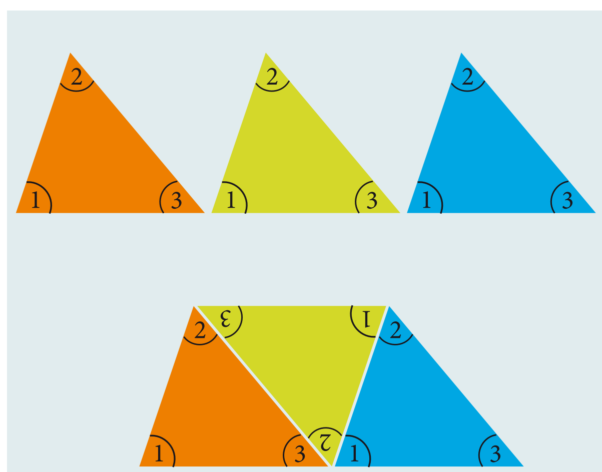


Fig. 4.14

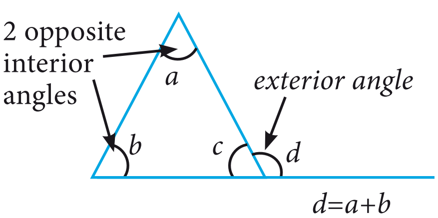


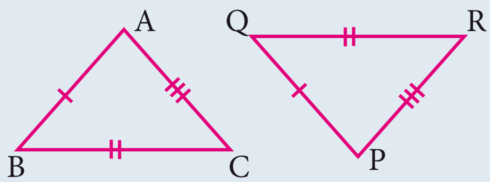
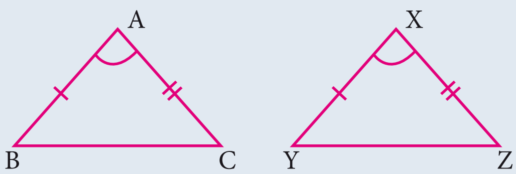
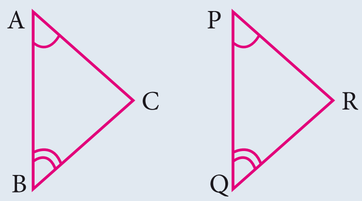
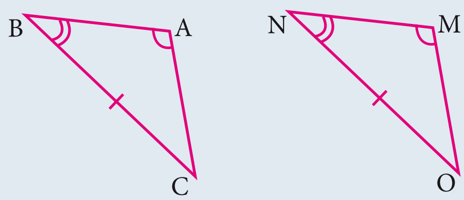
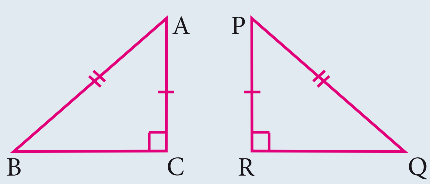
Fig. 4.15

Can you use the same figure to explain the "**Exterior angle property**" of a triangle?

If a side of a triangle is stretched, the exterior angle so formed is equal to the sum of the two remote interior angles.

4.2.4 Congruent Triangles

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle.

Rule	Diagrams	Reason
SSS		$AB = PQ$ $BC = QR$ $AC = PR$ $\Delta ABC \cong \Delta PQR$
SAS		$AB = XY$ $\angle BAC = \angle YXZ$ $AC = XZ$ $\Delta ABC \cong \Delta XYZ$
ASA		$\angle A = \angle P$ $AB = PQ$ $\angle B = \angle Q$ $\Delta ABC \cong \Delta PQR$
AAS		$\angle A = \angle M$ $\angle B = \angle N$ $BC = NO$ $\Delta ABC \cong \Delta MNO$
RHS		$\angle ACB = \angle PRQ = 90°(R)$ $AB =$ PQ hypotenuse (H) $AC = PR$ (S) $\Delta ABC \cong \Delta PQR$

Exercise 4.1

1. Find the complement of the following angles (1°= 60′ minutes, 1′ = 60″ seconds)

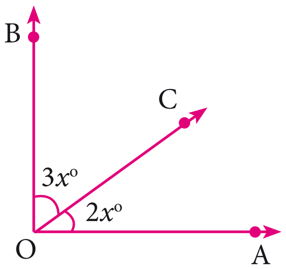
   (i) 70° (ii) 27° (iii) 45° (iv) 62°32′

2. Find the supplement of the following angles.

   (i) 140° (ii) 34° (iii) Right angle (iv) 121°48′

3. Find the value of x

(i)



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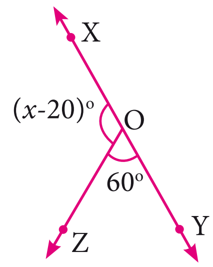

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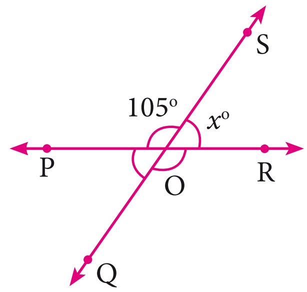

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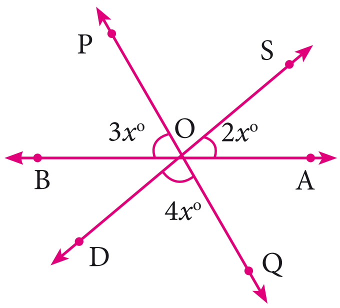

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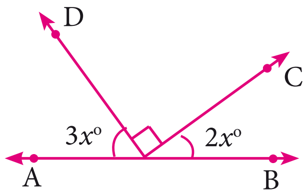
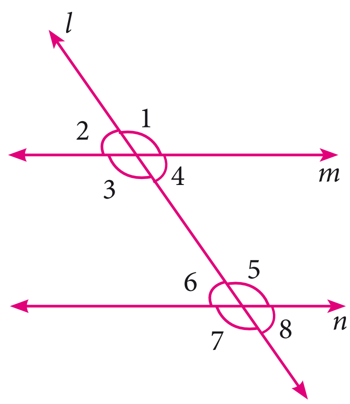


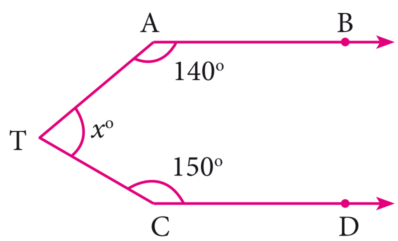
4. Let $m \parallel n$ and l is a transversal

Such that $\angle 1 : \angle 2 = 11 : 7$. Determine all the eight angles.



5. In the figure, AB is parallel to CD, find x

(i)



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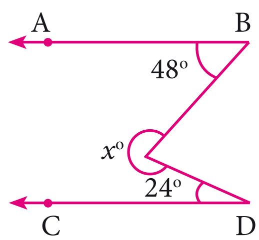

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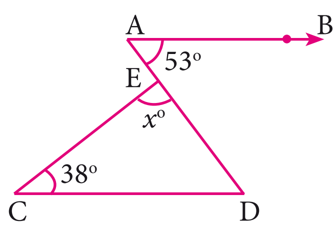
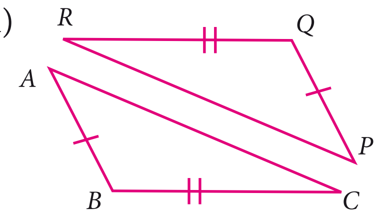


E6DK9D

6. The angles of a triangle are in the ratio 1: 2 : 3, find the measure of each angle of the triangle.

7. Consider the given pairs of triangles and say whether each pair is that of congruent triangles. If the triangles are congruent, say 'how'; if they are not congruent say 'why' and also say if a small modification would make them congruent:

(i)


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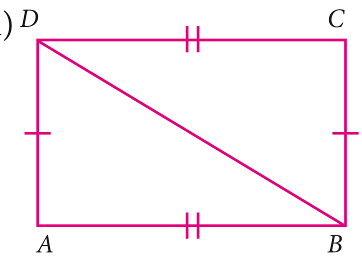

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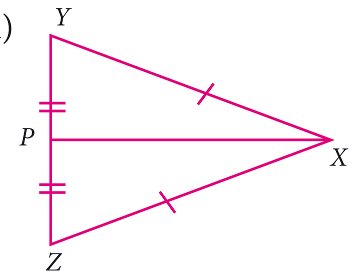

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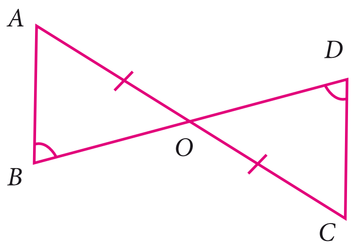

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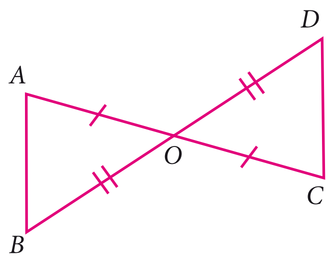

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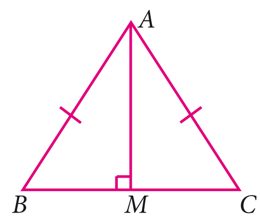

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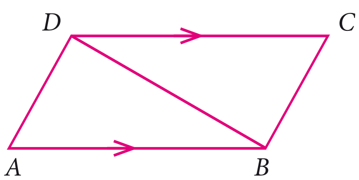

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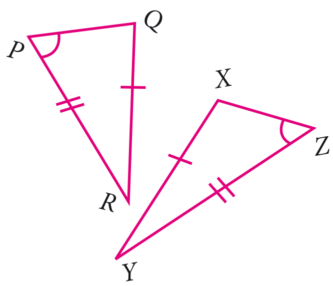

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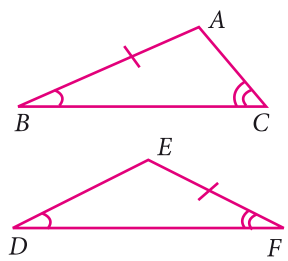
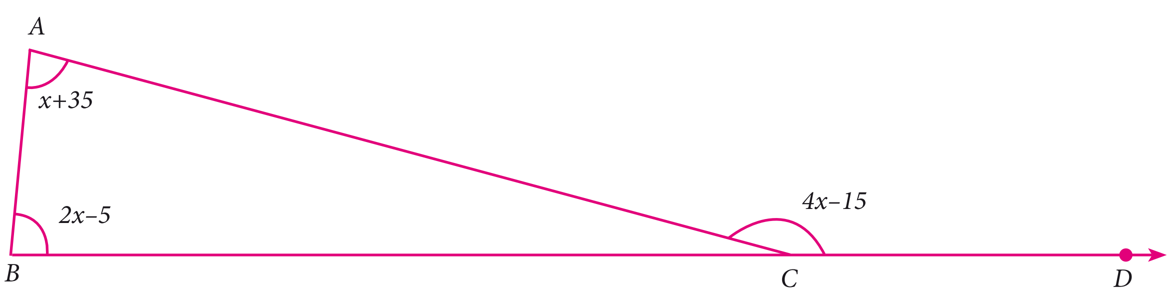


8. ΔABC and ΔDEF are two triangles in which

$AB=DF$, $\angle ACB=70°$, $\angle ABC=60°$; $\angle DEF=70°$ and $\angle EDF=60°$.

Prove that the triangles are congruent.

9. Find all the three angles of the ΔABC



4.3 Quadrilaterals

Activity 3

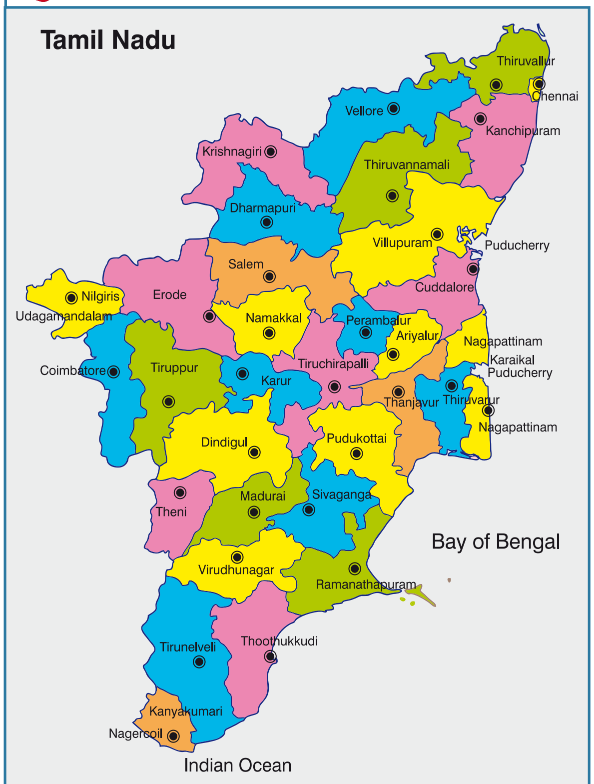


Fig. 4.16

Four Tamil Nadu State Transport buses take the following routes. The first is a one-way journey, and the rest are round trips. Find the places on the map, put points on them and connect them by lines to draw the routes. The places connecting four different routes are given as follows.

(i) Nagercoil, Tirunelveli, Virudhunagar, Madurai

(ii) Sivagangai, Puthukottai, Thanjavur, Dindigul

(iii) Erode, Coimbatore, Dharmapuri, Karur

(iv) Chennai, Cuddalore, Krishnagiri, Vellore

You will get the following shapes.

(i)

Fig. 4.17

(ii)

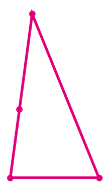


Fig. 4.18

(iii)

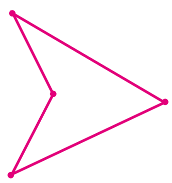


Fig. 4.19

(iv)

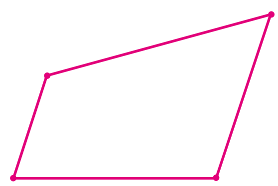


Fig. 4.20

Label the vertices with city names, draw the shapes exactly as they are shown on the map without rotations.

We observe that the first is a single line, the four points are collinear. The other three are closed shapes made of straight lines, of the kind we have seen before. We need names to call such closed shapes, we will call them **polygons** from now on.

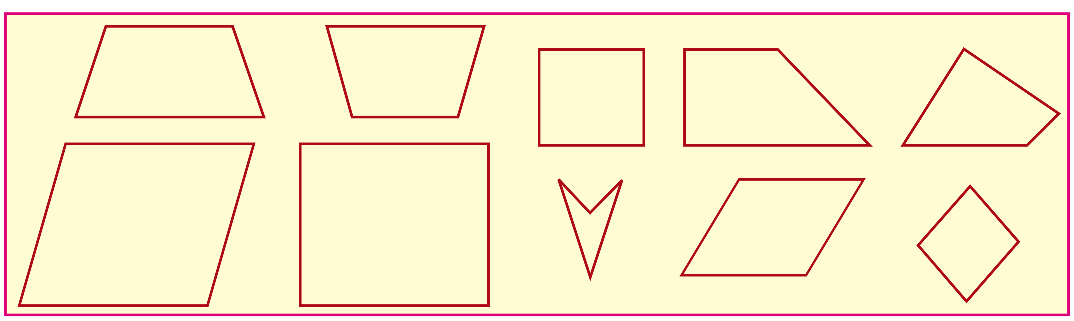


Fig. 4.21

How do polygons look? They have sides, with points at either end. We call these points **vertices** of the polygon. The sides are line segments joining the vertices. The word *poly* stands for many, and a polygon is a many-sided figure.

Note

Concave polygon: *Polygon having any one of the interior angle greater than 180º*

Convex Polygon: *Polygon having each interior angle less than 180º*

(Diagonals should be inside the polygon)

How many sides can a polygon have? One? But that is just a line segment. Two ? But how can you get a closed shape with two sides? Three? Yes, and this is what we know as a triangle. Four sides?

Squares and rectangles are examples of polygons with 4 sides but they are not the only ones. Here are some examples of 4-sided polygons. We call them ***quadrilaterals.***

Thinking Corner

You know *bi*-cycles and *tri*-cycles, don't you ? When we attach these to the front of any word, they stand for 2 (bi) or 3 (tri) of them. Similarly *quadri* stands for 4 of them. We should really speak of quadri-cycles also, but we don't. *Lateral* stands for sideways, thus quadrilateral means a 4-sided figure. You know *trilaterals*; they are also called triangles !

After 4 ? We have: 5 – *penta*, 6 – *hexa*, 7 – *hepta*, 8 – *octa*, 9 – *nano*, 10 – *deca*.

Conventions are made by history. Trigons are called triangles, quadrigons are called quadrilaterals, but then we have pentagons, hexagons, heptagons, octagons, nanogons and decagons. Beyond these, we have 11-gons, 12-gons etc. Perhaps you can draw a 23-gon !

Activity 4

This is a copy of the tangram puzzle. The tangram puzzle consists of 7 geometric pieces which are normally boxed in the shape of a square. The pieces, called 'tans', are used to create different patterns including **animals, people, numbers, geometric shapes** and many more.

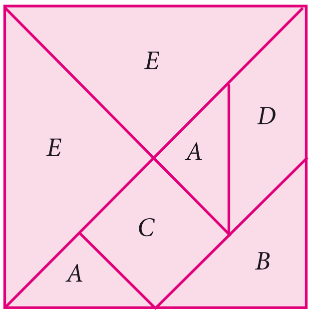


Fig. 4.22

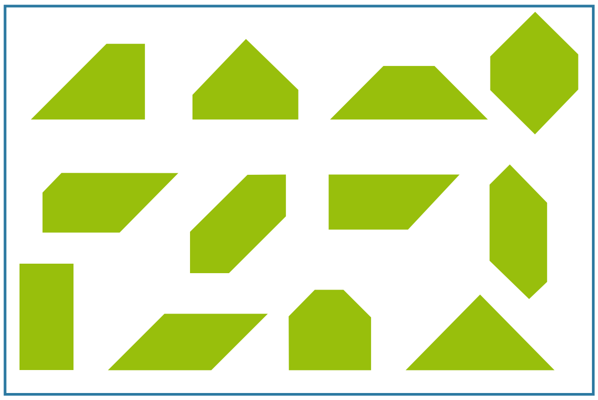


Fig. 4.23

You can make several polygons using the pieces in different ways.

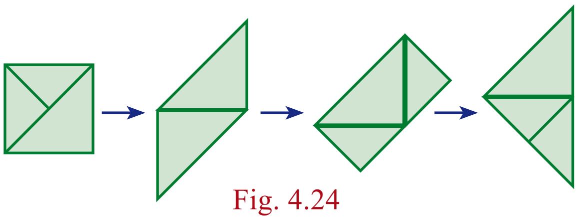


Fig. 4.24

Here are some examples.(See Fig.4.23)

Use three triangular pieces to make a square, a triangle, a rectangle or a parallelogram.

Use 5 of the pieces to make a trapezium.

Use 7 of the pieces to make a hexagon.

Here (See Fig.4.24) is how one can transform a square (made of 3 triangles) into a large triangle. Try to explore similar transformations.

Activity 5

Angle sum for a polygon

Draw any quadrilateral *ABCD*.

Mark a point *P* in its interior.

Join the segments *PA*, *PB*, *PC* and *PD*.

You have 4 triangles now.

How much is the sum of all the angles of the 4 triangles?

How much is the sum of the angles at their vertex, now P?

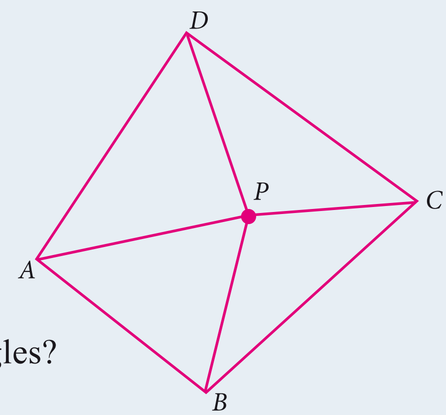


Fig. 4.25

Can you now find the 'angle sum' of the quadrilateral *ABCD*?

Can you extend this idea to any polygon?

Thinking Corner

1. If there is a polygon of n sides ($n \geq 3$), then the sum of all interior angles is $(n-2) \times 180°$

2. For the regular polygon

- Each interior angle is $\frac{(n-2)}{n} \times 180°$
- Each exterior angle is $\frac{360°}{n}$
- The sum of all the exterior angles formed by producing the sides of a convex polygon in the order is 360° .
- If a polygon has 'n' sides , then the number of diagonals of the polygon is $\frac{n(n-3)}{2}$

Activity 6

Draw the following special quadrilaterals on a graph sheet, measure the sides and angles and complete the table to explore the properties of the quadrilaterals with respect to sides and angles.

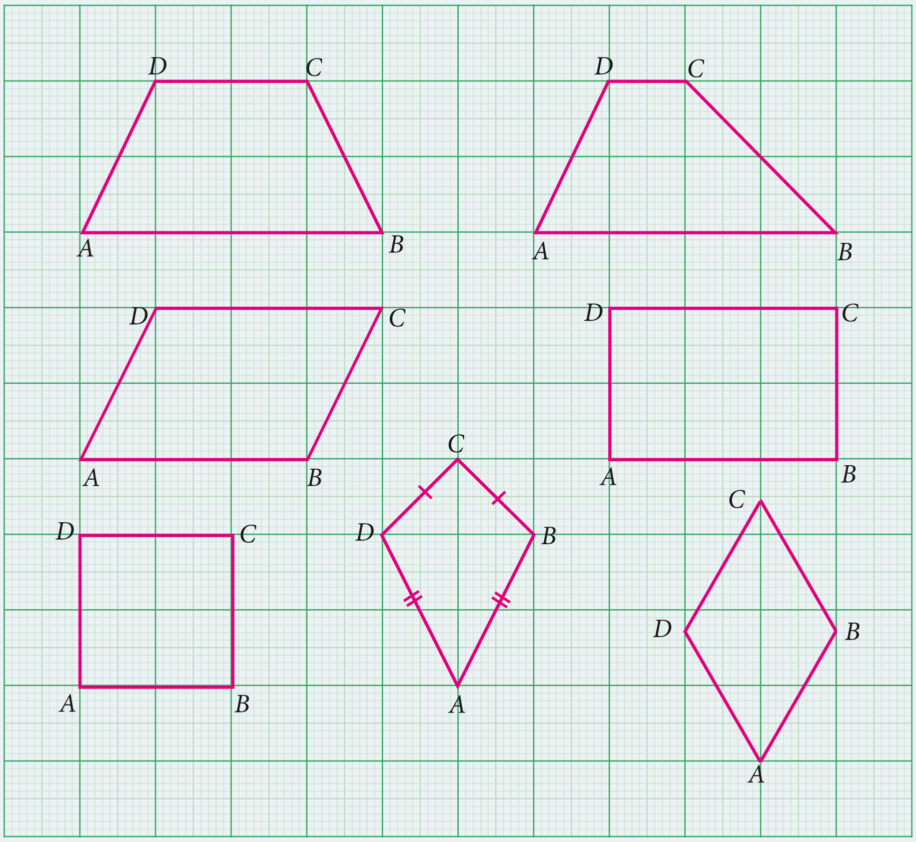


Fig. 4.26

S. No.	Name of the quadrilateral	Length of sides				Measure of angles			
		AB	BC	CD	DA	$\angle A$	$\angle B$	$\angle C$	$\angle D$
1	Trapezium								
2	Isosceles Trapezium								
3	Parallelogram								
4	Rectangle								
5	Rhombus								
6	Square								
7	Kite								

Do you see some patterns in all the data you have recorded? We see many interesting properties, but how do we know whether these are true in general, or happen to hold only for these figures? It is not even clear **what** properties we should look for. The best way to answer this is to go back to what we already know and look at it from this viewpoint. We know *rectangles*, so we can ask what properties rectangles have. Here they are:

- Opposite sides are equal. (1)
- *All* angles are equal, each is 90 degrees. (2)
- Adjacent sides may or may not be equal. (3)

Among these, the last statement really says nothing! (Mathematicians call such statements **trivial**, and they prefer not to write them down.) Note that adjacent sides have a vertex in common, and opposite sides have no vertex in common. A square has all these properties but the third is replaced by; Adjacent sides are equal. (4) Now we can combine (1) and (4) and say that in a square, all sides are equal.

Thus we see that every square is a rectangle but a rectangle need not be a square. This suggests a way of classifying quadrilaterals, of grouping them according to whether some sides are equal or not, some angles are equal or not.

4.3.1 Special Names for Some Quadrilaterals

1. A **parallelogram** is a quadrilateral in which opposite sides are parallel and equal.
2. A **rhombus** is a quadrilateral in which opposite sides are parallel and all sides are equal.
3. A **trapezium** is a quadrilateral in which *one pair of* opposite sides are parallel.

Draw a few parallellograms, a few rhombuses (correctly called rhombii, like cactus and cactii) and a few trapeziums (correctly written trapezia).

Fig. 4.27

The great advantage of listing properties is that we can see the relationships among them immediately.

- Every parallelogram is a trapezium, but not necessarily the other way.
- Every rhombus is a parallelogram, but not necessarily the other way.
- Every rectangle is a parallelogram, but not necessarily the other way.
- Every square is a rhombus and hence every square is a parallelogram as well.

For "not necessarily the other way" mathematicians usually say "the converse is not true". A smart question then is: just *when* is the other way also true? For instance, when is a parallelogram also a rectangle? Any parallelogram in which all angles are also equal is a rectangle. (Do you see why?) Now we can observe many more interesting properties. For instance, we see that a rhombus is a parallelogram in which all **sides** are also equal.

4.3.2 More Special Names

When all sides of a quadrilateral are equal, we call it **equilateral**. When all angles of a quadrilateral are equal, we call it **equiangular**. In triangles, we talked of equilateral triangles as those with all sides equal. Now we can call them equiangular triangles as well!

We thus have:

A rhombus is an equilateral parallelogram.

A rectangle is an equiangular parallelogram.

A square is an equilateral and equiangular parallelogram.

Here are two more special quadrilaterals, called **kite** and **isosceles trapezium**.

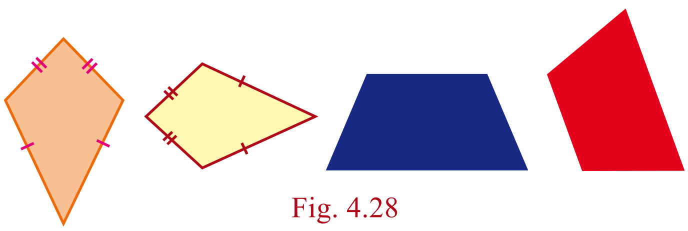


Fig. 4.28

4.3.3 Types of Quadrilaterals

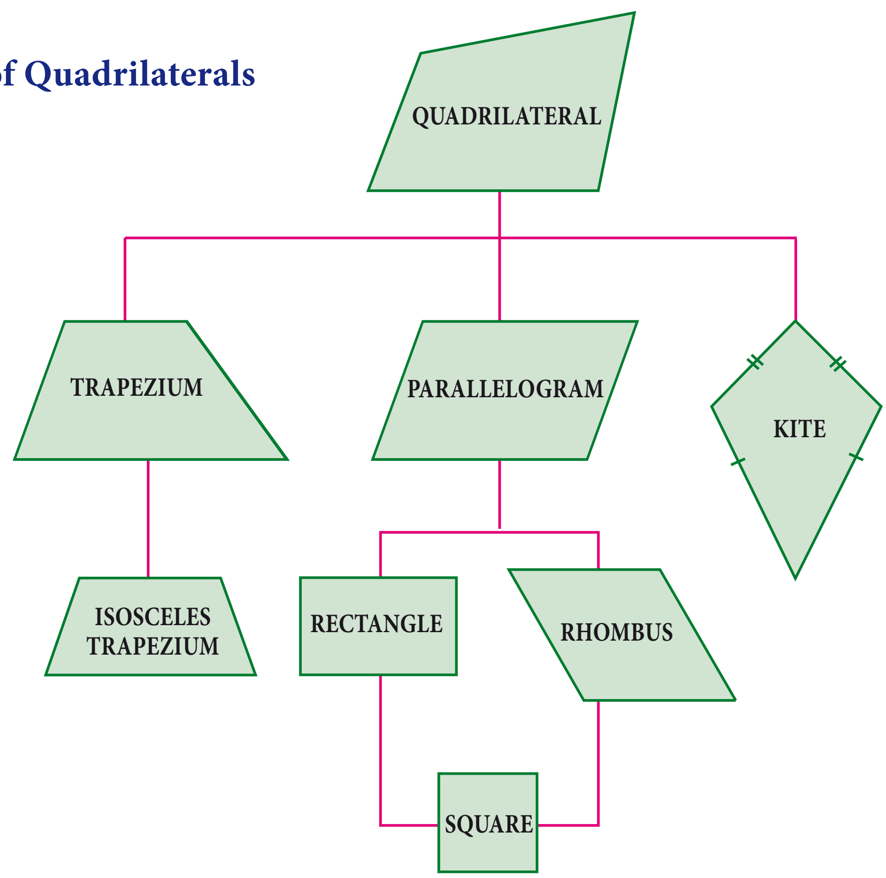


Fig. 4.29

Progress Check

Answer the following question.

(i) Are the opposite angles of a rhombus equal?

(ii) A quadrilateral is a ______________ if a pair of opposite sides are equal and parallel.

(iii) Are the opposite sides of a kite equal?

(iv) Which is an equiangular but not an equilateral parallelogram?

(v) Which is an equilateral but not an equiangular parallelogram?

(vi) Which is an equilateral and equiangular parallelogram?

(vii) __________ is a rectangle, a rhombus and a parallelogram.

Activity 7

Step – 1

Cut out four different quadrilaterals from coloured glazed papers.

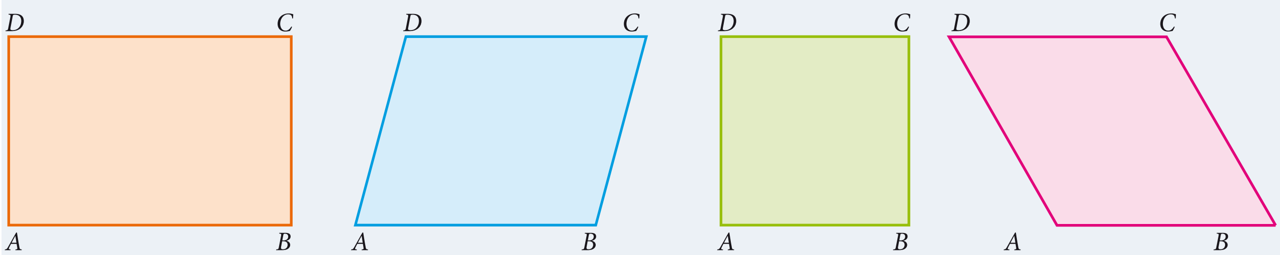


Fig. 4.30

Step – 2

Fold the quadrilaterals along their respective diagonals. Press to make creases. Here, dotted line represent the creases.

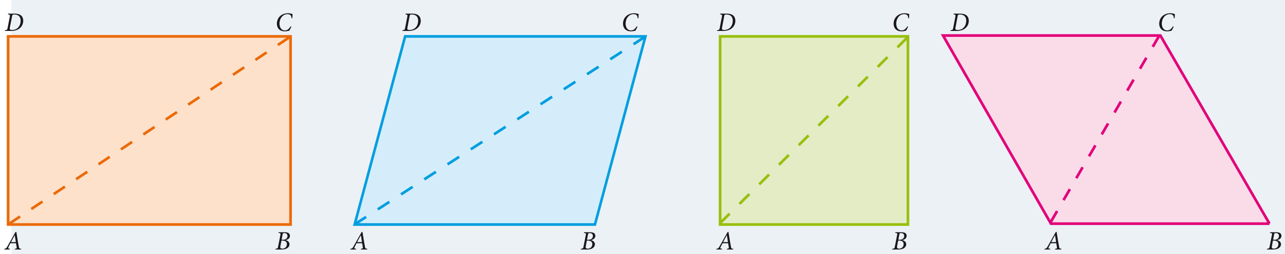


Fig. 4.31

Step – 3

Fold the quadrilaterals along both of their diagonals. Press to make creases.

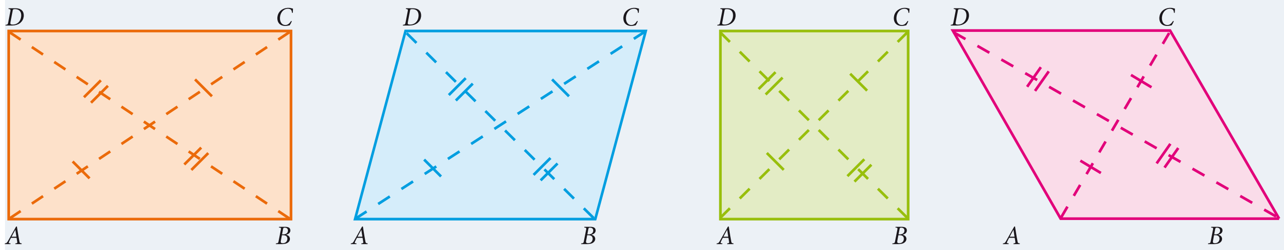


Fig. 4.32

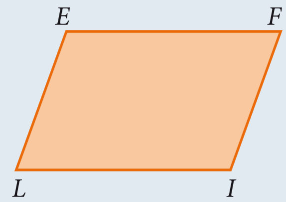
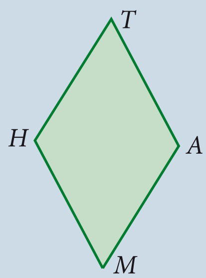
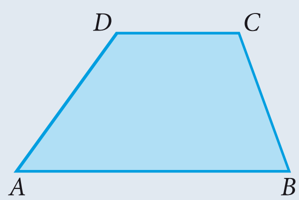
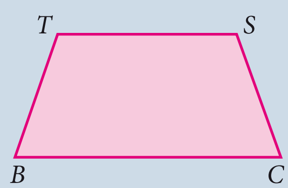
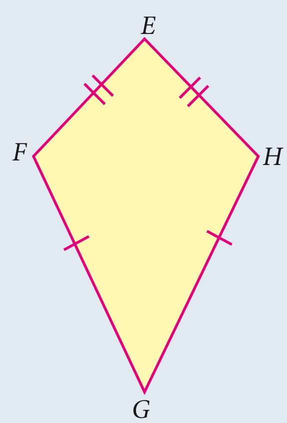
We observe that two imposed triangles are congruent to each other. Measure the lengths of portions of diagonals and angles between the diagonals.

Also do the same for the quadrilaterals such as Trapezium, Isosceles Trapezium and Kite.

From the above activity, measure the lengths of diagonals and angles between the diagonals and record them in the table below:

S. No.	Name of the quadrilateral	Length along diagonals						Measure of angles			
		AC	*BD*	*OA*	*OB*	*OC*	*OD*	$\angle AOB$	$\angle BOC$	$\angle COD$	$\angle DOA$
1	Trapezium										
2	Isosceles Trapezium										
3	Parallelogram										
4	Rectangle										
5	Rhombus										
6	Square										
7	Kite										

4.3.4 Properties of Quadrilaterals

Name	Diagram	Sides	Angles	Diagonals
Parallelogram		Opposite sides are parallel and equal	Opposite angles are equal and sum of any two adjacent angles is 180°	Diagonals bisect each other.
Rhombus		All sides are equal and opposite sides are parallel	Opposite angles are equal and sum of any two adjacent angles is 180°	Diagonals bisect each other at right angle.
Trapezium		One pair of opposite sides are parallel	The angles at the ends of each non-parallel sides are supplementary	Diagonals need not be equal
Isosceles Trapezium		One pair of opposite sides are parallel and non-parallel sides are equal in length.	The angles at the ends of each parallel sides are equal.	Diagonals are of equal length.
Kite		Two pairs of adjacent sides are equal	One pair of opposite angles are equal	1. Diagonals intersect at right angle. 2. Shorter diagonal bisected by longer diagonal 3. Longer diagonal divides the kite into two congruent triangles

Note

(i) A rectangle is an equiangular parallelogram.

(ii) A rhombus is an equilateral parallelogram.

(iii) A square is an equilateral and equiangular parallelogram.

(iv) A square is a rectangle , a rhombus and a parallelogram.

Activity 8

Procedure

(i) Make a parallelogram on a chart/graph paper and cut it.

(ii) Draw diagonal of the parallelogram.

(iii) Cut along the diagonal and obtain two triangles.

(iv) Superimpose one triangle onto the other.

What do you conclude ?

Activity 9

Procedure

Cut off a quadrilateral *ABCD* in a paper with prescribed dimensions. Mark the mid-points *P*, *Q*, *R* and *S* of the sides *AB, BC, CD* and *DA* respectively. By folding the sides appropriately cut off the quadrilateral *PQRS*. Fold the quadrilateral (i) Does *PQ* lie on *SR*? similarly fold the other way and verify *QR* lies on *PS*.

Observation

(i) What do you conclude?

(ii) Name the resulting figure?

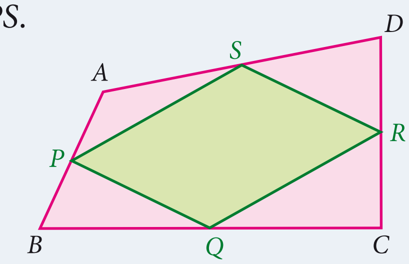


Fig. 4.33

Draw the following quadrilaterals and join the mid points of all its sides. Find the resultant shape and complete the table.

Name of the quadrilateral	Shape obtained by joining the midpoints
Parallelogram	
Rectangle	
Square	
Kite	
Trapezium	
Rhombus	

Progress Check

State the reasons for the following.

(i) A square is a special kind of a rectangle.

(ii) A rhombus is a special kind of a parallelogram.

(iii) A rhombus and a kite have one common property.

(iv) A square and a rhombus have one common property.

What type of quadrilateral is formed when the following pairs of triangles are joined together?

(i) Equilateral triangle.

(ii) Right angled triangle.

(iii) Isosceles triangle.

Activity 10

Complete the given table by placing Yes or No

Quadrilaterals	Opposite sides		All sides Equal	Diagonals			Angles		
	Parallel	Equal		Equal	Perpendicular to each other	Bisect each other	All angles are equal	Opposite angles equal	Adjacent angles are supplementary
Parallelogram									
Rectangle									
Square									
Rhombus									
Trapezium									
Isosceles trapezium									
Kite									

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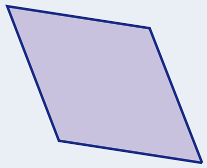
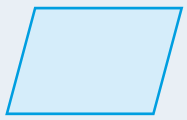
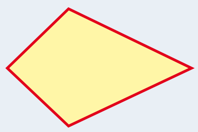
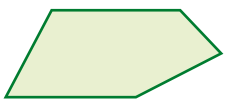
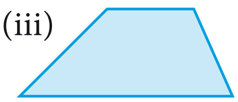
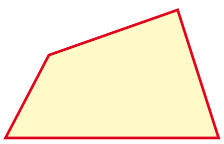
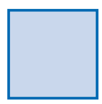
Exercise 4.2

1. Match the name of the shapes with its figure on the right.

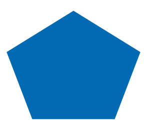
Rhombus	
Kite	
Parallelogram	
Trapezium	
Rectangle	

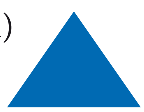
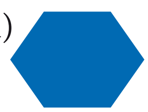
2. Identify which ones are parallelograms and which are not.

(i) 
(ii) 
(iii) 

(iv) 
(v) 
(vi) 

3. Which ones are not quadrilaterals?

(i) 
(ii) 
(iii) 
(iv) 

(v) 
(vi) 
(vii) 
(viii) 

4. Identify which ones are trapeziums and which are not.

(i) 
(ii) 
(iii) 

(iv) 
(v) 
(vi) 

4.3.5 Properties of Parallelogram

We can now embark on an interesting journey. We can tour among lots of quadrilaterals, noting down interesting properties. What properties do we look for, and how do we know they are true?

For instance, opposite sides of a parallelogram are parallel, but are they also **equal**? We could draw any number of parallelograms and verify whether this is true or not. In fact, we see that opposite sides are equal in **all of** them. Can we then conclude that opposite sides are equal in *all* parallelograms? No, because we might later find a parallelogram, one which we had not thought of until then, in which opposite sides are unequal. So, we need an argument, a **proof**.

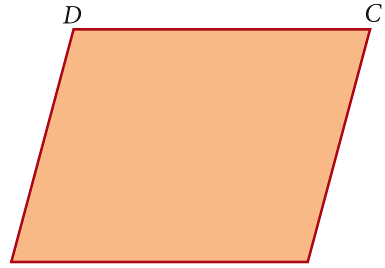


Fig. 4.34

Consider the parallelogram $ABCD$ in the given Fig. 4.34. We believe that $AB = CD$ and $AD = BC$, but how can we be sure? We know triangles and their properties. So we can try and see if we can use that knowledge. But we don't have any triangles in the parallelogram $ABCD$.

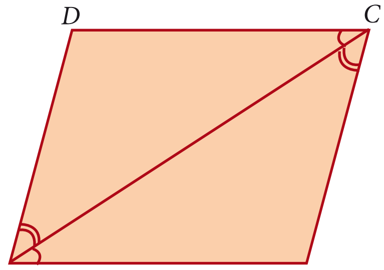


Fig. 4.35

This is easily taken care of by joining AC. (We could equally well have joined BD, but let it be AC for now.) We now have 2 triangles ADC and ABC with a common side AC. If we could somehow prove that these two triangles are congruent, we would get $AB = CD$ and $AD = BC$, which is what we want!

Is there any hope of proving that ΔADC and ΔABC are congruent? There are many criteria for congruence, it is not clear which one is relevant here.

So far we have not used the fact that $ABCD$ is a parallelogram at all. So we need to use the facts that $AB \parallel DC$ and $AD \parallel BC$ to show that ΔADC and ΔABC are congruent. From sides being parallel we have to get to some angles being equal. Do we know any such properties? Yes we do, and that is all about **transversals!**

Now we can see it clearly. $AD \parallel BC$ and AC is a transversal, hence $\angle DAC = \angle BCA$. Similarly, $AB \parallel DC$, AC is a transversal, hence $\angle BAC = \angle DCA$. With AC as common side, the ASA criterion tells us that ΔADC and ΔABC are congruent, just what we needed. From this we can conclude that $AB = CD$ and $AD = BC$.

Thus opposite sides are indeed equal in a parallelogram.

The argument we now constructed is written down as a ***formal proof*** in the following manner.

Theorem 1

In a parallelogram, opposite sides are equal

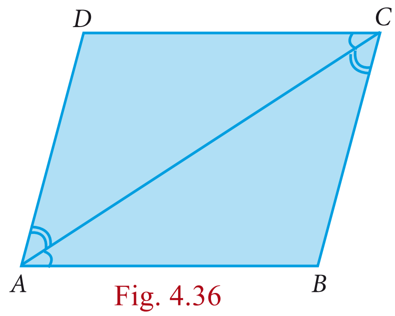


Fig. 4.36

Given $ABCD$ is a parallelogram

To Prove $AB=CD$ and $DA=BC$

Construction Join AC

Proof

Since $ABCD$ is a parallelogram

$AD \parallel BC$ and AC is the transversal

$\angle DAC = \angle BCA$ →(1) (alternate angles are equal)

$AB \parallel DC$ and AC is the transversal

$\angle BAC = \angle DCA$ →(2) (alternate angles are equal)

In ΔADC and ΔCBA

$\angle DAC = \angle BCA$ from (1)

AC is common

$\angle DCA = \angle BAC$ from (2)

$\Delta ADC \cong \Delta CBA$ (By ASA)

Hence $AD = CB$ and $DC = BA$ (Corresponding sides are equal)

Along the way in the proof above, we have proved another property that is worth recording as a theorem.

Theorem 2

A diagonal of a parallelogram divides it into two congruent triangles.

Notice that the proof above established that $\angle DAC = \angle BCA$ and $\angle BAC = \angle DCA$. Hence we also have, in the figure above,

$$\angle BCA + \angle BAC = \angle DCA + \angle DAC$$

But we know that:

$$\angle B + \angle BCA + \angle BAC = 180$$

and $\angle D + \angle DCA + \angle DAC = 180$

Therefore we must have that $\angle B = \angle D$.

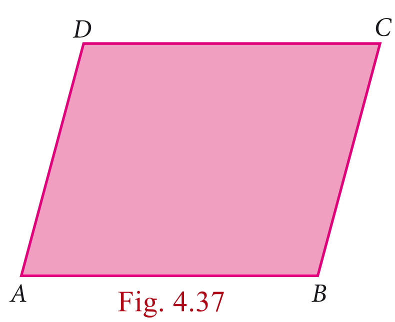


Fig. 4.37

With a little bit of work, proceeding similarly, we could have shown that $\angle A = \angle C$ as well. Thus we have managed to prove the following theorem:

Theorem 3

The opposite angles of a parallelogram are equal.

Now that we see congruence of triangles as a good "strategy", we can look for more triangles. Consider both diagonals AC and DB. We already know that ΔADC and ΔCBA are congruent. By a similar argument we can show that ΔDAB and ΔBCD are congruent as well. Are there more congruent triangles to be found in this figure ?

Yes. The two diagonals intersect at point O. We now see 4 new ΔAOB, ΔBOC, ΔCOD and ΔDOA. Can you see any congruent pairs among them?

Since AB and CD are parallel and equal, one good guess is that ΔAOB and ΔCOD are congruent. We could again try the ASA crierion, in which case we want $\angle OAB = \angle OCD$ and $\angle ABO = \angle CDO$. But the first of these follows from the fact that $\angle CAB = \angle ACD$ (which we already established) and observing that $\angle CAB$ and $\angle OAB$ are the same (and so also $\angle OCD$ and $\angle ACD$). We now use the fact that BD is a transversal to get that $\angle ABD = \angle CDB$, but then $\angle ABD$ is the same as $\angle ABO$, $\angle CDB$ is the same as $\angle CDO$, and we are done.

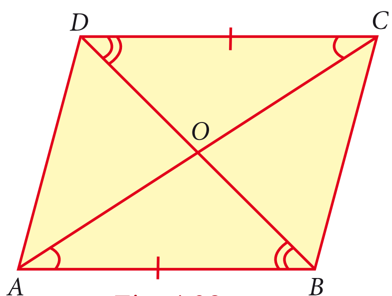


Fig. 4.38

Again, we need to write down the formal proof, and we have another theorem.

Theorem 4

The diagonals of a parallelogram bisect each other.

It is time now to reinforce our concepts on parallelograms. Consider each of the given statements, in the adjacent box, one by one. For each statement, we can conclude that it is a quadrilateral. If the quadrilateral happens to be a parallelogram, what type of parallelogram is it?

- Each pair of its opposite sides are parallel.
- Each pair of opposite sides is equal.
- All of its angles are right angles.
- Its diagonals bisect each other.
- The diagonals are equal.
- The diagonals are perpendicular and equal.
- The diagonals are perpendicular bisectors of each other.
- Each pair of its consecutive angles is supplementary.

Now we begin with lots of interesting properties of parallelograms. Can we try and prove some property relating to two or more parallelograms ? A simple case to try is when two parallelograms share the same base, as in Fig.4.39

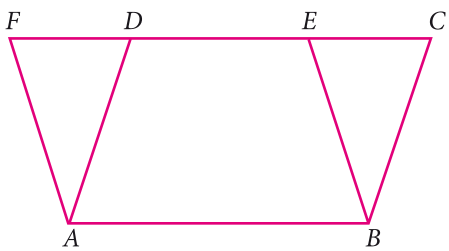


Fig. 4.39

We see parallelograms $ABCD$ and $ABEF$ are on the common base AB. At once we can see a pair of triangles for being congruent ΔADF and ΔBCE. We already have that $AD = BC$ and $AF = BE$. But then since $AD \| BC$ and $AF \| BE$, the angle formed by AD and AF must be the same as the angle formed by BC tand BE. Therefore $\angle DAF = \angle CBE$. Thus ΔADF and ΔBCE are congruent.

That is an interesting observation; can we infer anything more from this ? Yes, we know that congruent triangles have the *same area*. This makes us think about the areas of the parallelograms $ABCD$ and $ABEF$.

Area of $ABCD$ = area of quadrilateral $ABED$ + area of ΔBCE

= area of quadrilateral $ABED$ + area of ΔADF

= area of $ABEF$

Thus we have proved another interesting theorem:

Theorem 5:

Parallelograms on the same base and between the same parallels are equal in area.

In this process, we have also proved other interesting statements. These are called *Corollaries*, which do not need separate detailed proofs.

Corollary 1: Triangles on the same base and between the same parallels are equal in area.

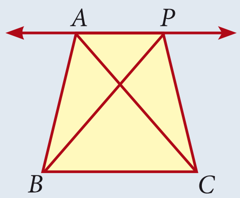
Corollary 2: A rectangle and a parallelogram on the same base and between the same parallels are equal in area.

These statements that we called Theorems and Corollaries, hold for all parallelograms, however large or small, with whatever be the lengths of sides and angles at vertices.

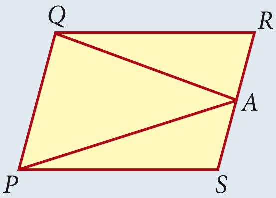
Progress Check

Which of the following figures lie on the same base and between the same parallels? In such a case, write the common base and the two parallel line segments:

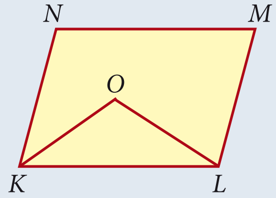
(i)



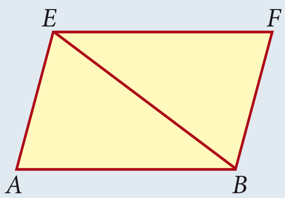
(ii)



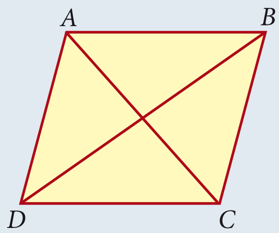
(iii)



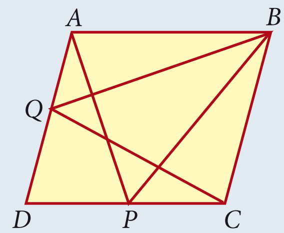
(iv)



(v)



(vi)



We can now apply this knowledge to find out properties of specific quadrilateral that is parallelogram.

Example 4.1

In a parallelogram *ABCD*, the bisectors of the consecutive angles $\angle A$ and $\angle B$ insersect at *P*. Show that $\angle APB = 90^{\circ}$

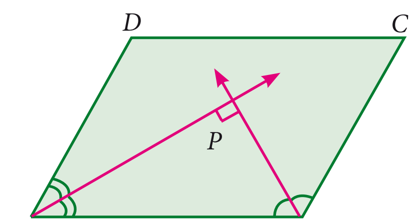


Fig. 4.40

Solution

ABCD is a parallelogram *AP* and *BP* are bisectors of consecutive angles $\angle A$ and $\angle B$.

Since the consecutive angles of a parallelogram are supplementary

$$\angle A + \angle B = 180^{\circ}$$

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{180^{\circ}}{2}$$

$$\implies \angle PAB + \angle PBA = 90^{\circ}$$

In ΔAPB,

$$\angle PAB + \angle APB + \angle PBA = 180^{\circ} \text{ (angle sum property of triangle)}$$

$$\angle APB = 180^{\circ} - [\angle PAB + \angle PBA]$$

$$= 180^{\circ} - 90^{\circ} = 90^{\circ}$$

Hence Proved.

E6HGB1

Example 4.2

In the Fig.4.41 *ABCD* is a parallelogram, *P* and *Q* are the mid-points of sides *AB* and *DC* respectively. Show that *APCQ* is a parallelogram.

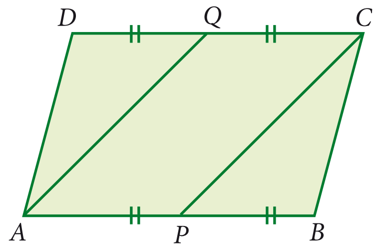


Fig. 4.41

Solution

Since *P* and *Q* are the mid points of

AB and *DC* respectively

Therefore $AP = \frac{1}{2} AB$ and

$QC = \frac{1}{2} DC$ (1)

But $AB = DC$ (Opposite sides of a parallelogram are equal)

$\Longrightarrow \frac{1}{2}AB = \frac{1}{2}DC$

$\Longrightarrow AP = QC$ (2)

Also , $AB \parallel DC$

$\Longrightarrow AP \parallel QC$ (3) [$\because$ *ABCD* is a parallelogram]

Thus , in quadrilateral *APCQ* we have $AP = QC$ and $AP \parallel QC$ [from (2) and (3)]

Hence , quadrilateral *APCQ* is a parallelogram.

Example 4.3

ABCD is a parallelogram Fig.4.42 such that $\angle BAD = 120^\circ$ and *AC* bisects $\angle BAD$ show that *ABCD* is a rhombus.

Solution

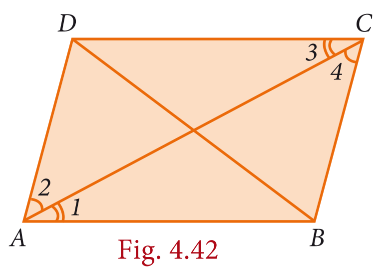


Fig. 4.42

Given $\angle BAD = 120^\circ$ and *AC* bisects $\angle BAD$

$\angle BAC = \frac{1}{2} \times 120^\circ = 60^\circ$

$\angle 1 = \angle 2 = 60^\circ$

$AD \parallel BC$ and *AC* is the traversal

$\angle 2 = \angle 4 = 60^\circ$

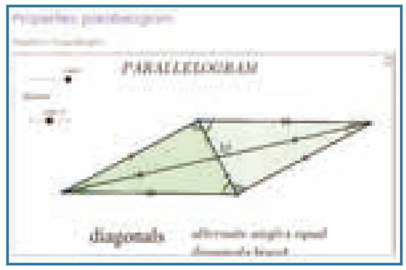
ΔABC is isosceles triangle [$\because \angle 1 = \angle 4 = 60^\circ$]

$\Longrightarrow AB = BC$

Parallelogram *ABCD* is a rhombus.

ICT Corner

Expected Result is shown in this picture



Step – 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

Step - 2

GeoGebra worksheet “Properties: Parallelogram” will appear. There are two sliders named “Rotate” and “Page”

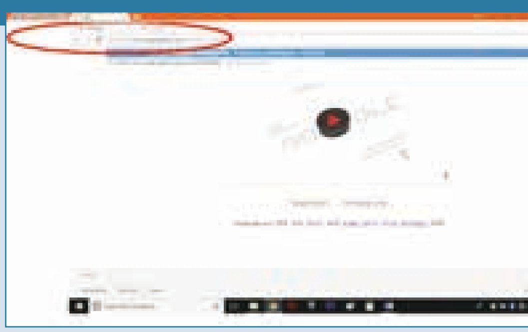
Step-3

Drag the slider named “Rotate “and see that the triangle is doubled as parallelogram.

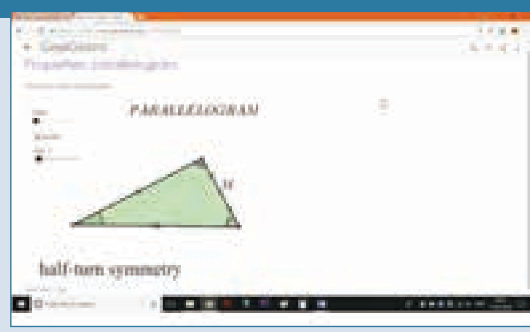
Step-4

Drag the slider named “Page” and you will get three pages in which the Properties are explained.

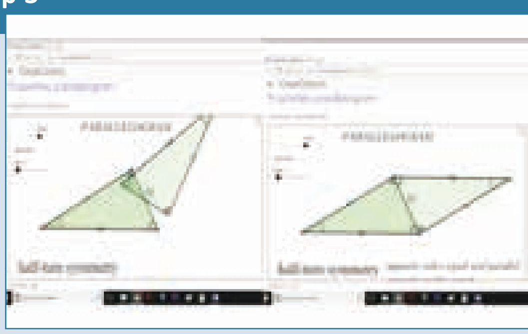
Step 1



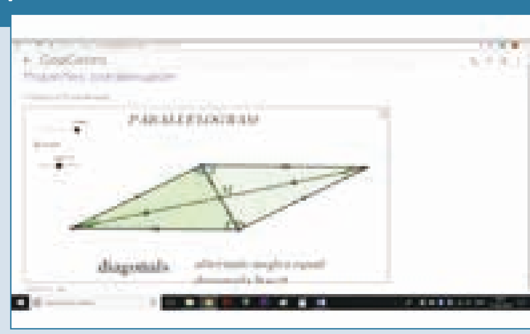
Step 2



Step 3



Step 4



Similarly you can check other worksheets in the Workbook related to your lesson

Browse in the link

Properties: Parallelogram: https://www.geogebra.org/m/m9Q2QpWD

Example 4.4

In a parallelogram $ABCD$, P and Q are the points on line DB such that $PD = BQ$ show that $APCQ$ is a parallelogram

Solution

$ABCD$ is a parallelogram.

$OA = OC$ and

$OB = OD$ ($\because$ Diagonals bisect each other)

now $OB + BQ = OD + DP$

$OQ = OP$ and $OA = OC$

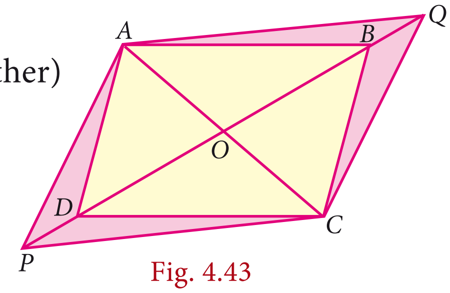


Fig. 4.43

$APCQ$ is a parallelogram.

Exercise 4.3

1. The angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. Find all the angles.

2. In a quadrilateral $ABCD$, $\angle A = 72°$ and $\angle C$ is the supplementary of $\angle A$. The other two angles are $2x–10$ and $x + 4$. Find the value of x and the measure of all the angles.

3. The side of a rhombus is 13 cm and the length of one of the diagonal is 24 cm. Find the length of the other diagonal?

4. $ABCD$ is a rectangle whose diagonals AC and BD intersect at O. If $\angle OAB = 46°$, find $\angle OBC$

5. The lengths of the diagonals of a Rhombus are 12 cm and 16 cm . Find the side of the rhombus.

6. Show that the bisectors of angles of a parallelogram form a rectangle .

7. If a triangle and a parallelogram lie on the same base and between the same parallels, then prove that the area of the triangle is equal to half of the area of parallelogram.

8. The legs of a stool make angle 35 ° with the floor as shown in the figure. Find the angles x and y.

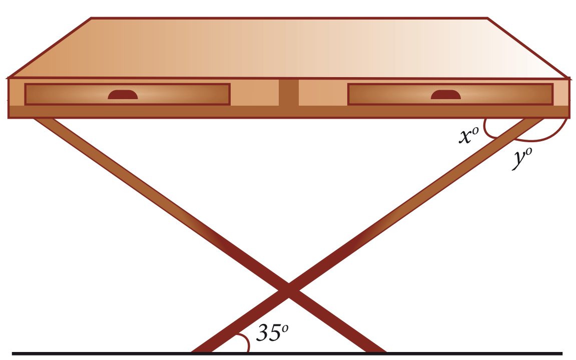


Fig. 4.44

9. Iron rods a, b, c, d, e, and f are making a design in a bridge as shown in the figure. If $a \parallel b$, $c \parallel d$, $e \parallel f$, find the marked angles between

(i) b and c

(ii) d and e

(iii) d and f

(iv) c and f

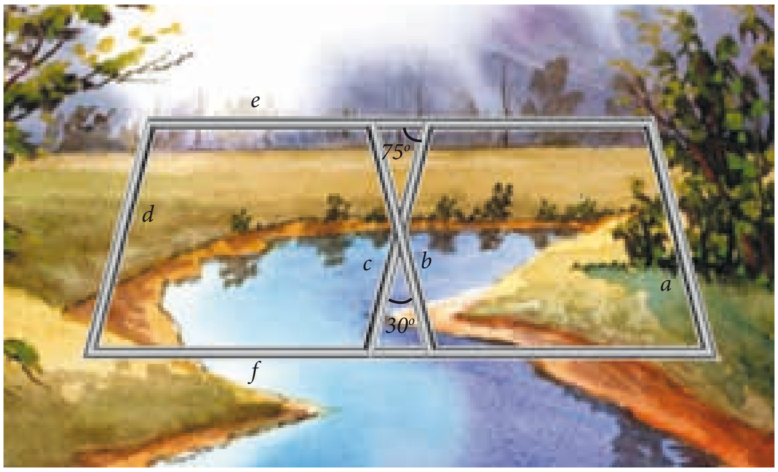


Fig. 4.45

10. In the given Fig. 4.46, $\angle A = 64°$, $\angle ABC = 58°$. If BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively of ΔABC, find $x°$ and $y°$

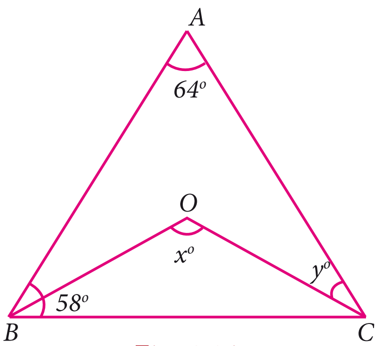


Fig. 4.46

11. Which type of quadrilateral satisfies the following properties?

(i) Both pairs of opposite angles are equal in size.

(ii) Both pairs of opposite sides are equal in length.

(iii) Each diagonal is an angle bisector.

(iv) The diagonals bisect each other.

(v) Each pair of consecutive angles is supplementary.

(vi) The diagonals are equal.

(vii) Can be divided into two congruent triangles.

12. In the given Fig. 4.47, if $AB = 2$, $BC = 6$, $AE = 6$, $BF = 8$, $CE = 7$, and $CF = 7$, compute the ratio of the area of quadrilateral $ABDE$ to the area of ΔCDF.

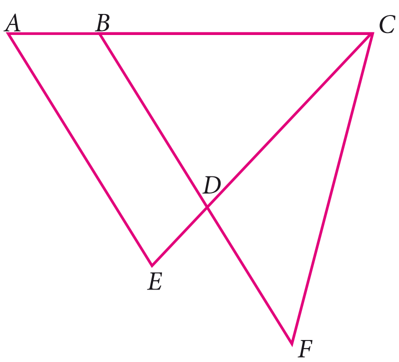


Fig. 4.47

13. In the Fig. 4.48, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?

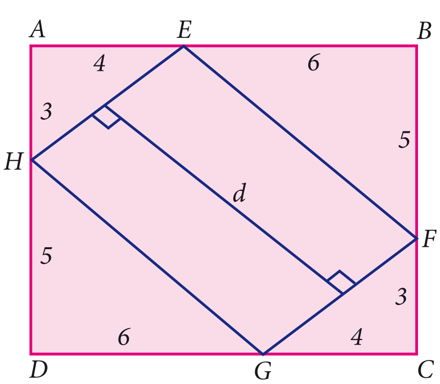


Fig. 4.48

14. *ABCD* is a parallelogram such that *AB* is parallel to *DC* and *DA* parallel to *CB*. The length of side *AB* is 20 cm. *E* is a point between *A* and *B* such that the length of *AE* is 3 cm. *F* is a point between points *D* and *C*. Find the length of *DF* such that the segment *EF* divides the parallelogram in two regions with equal areas.

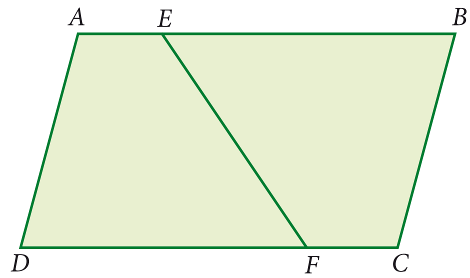


Fig. 4.49

15. In parallelogram *ABCD* of the accompanying diagram, line *DP* is drawn bisecting *BC* at *N* and meeting *AB* (extended) at *P*. From vertex *C*, line *CQ* is drawn bisecting side *AD* at *M* and meeting *AB* (extended) at *Q*. Lines *DP* and *CQ* meet at *O*. Show that the area of triangle *QPO* is $\frac{9}{8}$ of the area of the parallelogram *ABCD*.

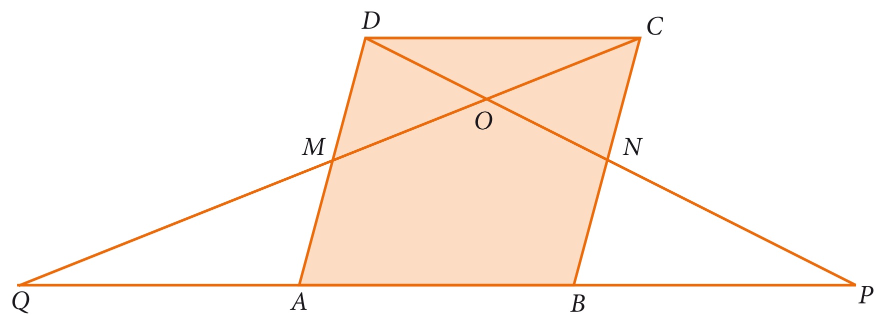


Fig. 4.50

4.4 Constructions

Practical Geometry is the method of applying the rules of Geometry dealt with the properties of Points, Lines and other figures to construct geometrical figures. "Construction" in Geometry means to draw shapes, angles or lines accurately. The geometric constructions have been discussed in detail in Euclid's book 'Elements'. Hence these constructions are also known as Euclidean constructions. These constructions use only compass and straightedge (i.e. ruler). The compass establishes equidistance and the straightedge establishes collinearity. All geometric constructions are based on those two concepts.

It is possible to construct rational and irrational numbers using straightedge and a compass as seen in Chapter II. In 1913 the Indian mathematical Genius, Ramanujan gave a geometrical construction for $355/113 = \pi$. Today with all our accumulated skill in exact measurements. It is a noteworthy feature that lines driven through a mountain meet and make a tunnel.In the earlier classes, we have learnt the construction of angles and triangles with the given measurements.

In this chapter we learn to construct Circumcentre and Orthocentre of a triangle by using concurrent lines.

4.4.1 Construction of the Circumcentre of a Triangle

Circumcentre

The Circumcentre is the point of concurrency of the Perpendicular bisectors of the sides of a triangle.

It is usually denoted by *S*.

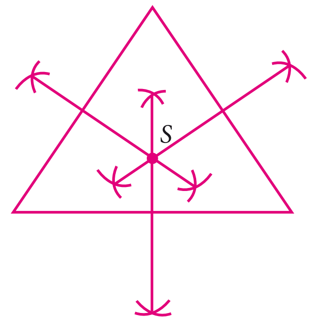


Fig. 4.51

Circumcircle

The circle passing through all the three vertices of the triangle with circumcentre (*S*) as centre is called circumcircle.

Circumradius

The line segment from any vertex of a triangle to the Circumcentre of a given triangle is called circumradius of the circumcircle.

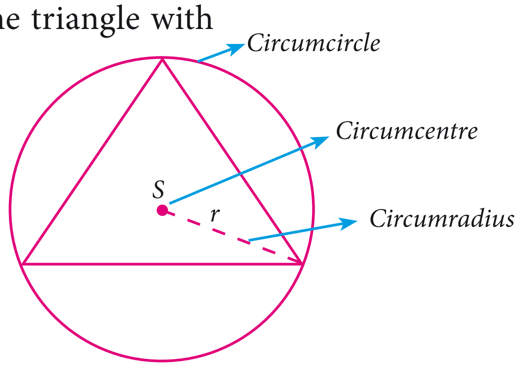


Fig. 4.52

Activity 11

Objective To find the mid-point of a line segment using paper folding

Procedure Make a line segment on a paper by folding it and name it *PQ*. Fold the line segment *PQ* in such a way that *P* falls on *Q* and mark the point of intersection of the line segment and the crease formed by folding the paper as *M*. *M* is the midpoint of *PQ*.

Activity 12

Objective To construct a perpendicular bisector of a line segment using paper folding.

Procedure Make a line segment on a paper by folding it and name it as *PQ*. Fold *PQ* to such a way that *P* falls on *Q* and thereby creating a crease RS. This line *RS* is the perpendicular bisector of *PQ*.

Activity 13

Objective To construct a perpendicular to a line segment from an external point using paper folding.

Procedure Draw a line segment *AB* and mark an external point *P*. Move *B* along *BA* till the fold passes through *P* and crease it along that line. The crease thus formed is the perpendicular to *AB* through the external point *P*.

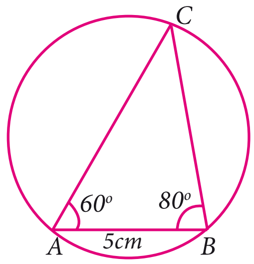
Activity 14

Objective To locate the circumcentre of a triangle using paper folding.

Procedure Using Activity 12, find the perpendicular bisectors for any two sides of the given triangle. The meeting point of these is the circumcentre of the given triangle.

Example 4.5

Construct the circumcentre of the ΔABC with $AB = 5$ cm, $\angle A = 60^0$ and $\angle B = 80^0$. Also draw the circumcircle and find the circumradius of the ΔABC.



Rough Diagram

Solution

Step 1 Draw the ΔABC with the given measurements

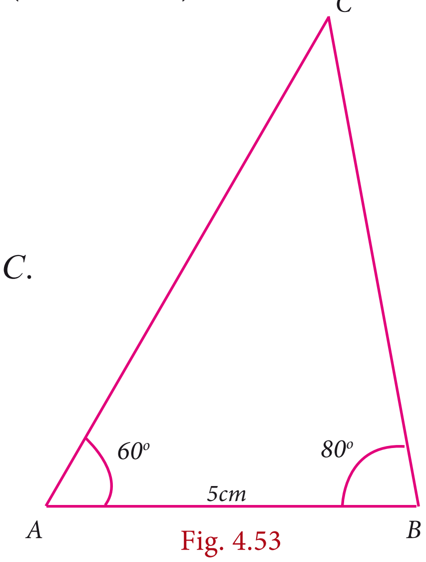


Fig. 4.53

Step 2

Construct the perpendicular bisector of any two sides (AC and BC) and let them meet at S which is the circumcentre.

Step 3

S as centre and $SA = SB = SC$ as radius,

draw the Circumcircle to passes through A,B and C.

Circumradius = 3.9 cm.

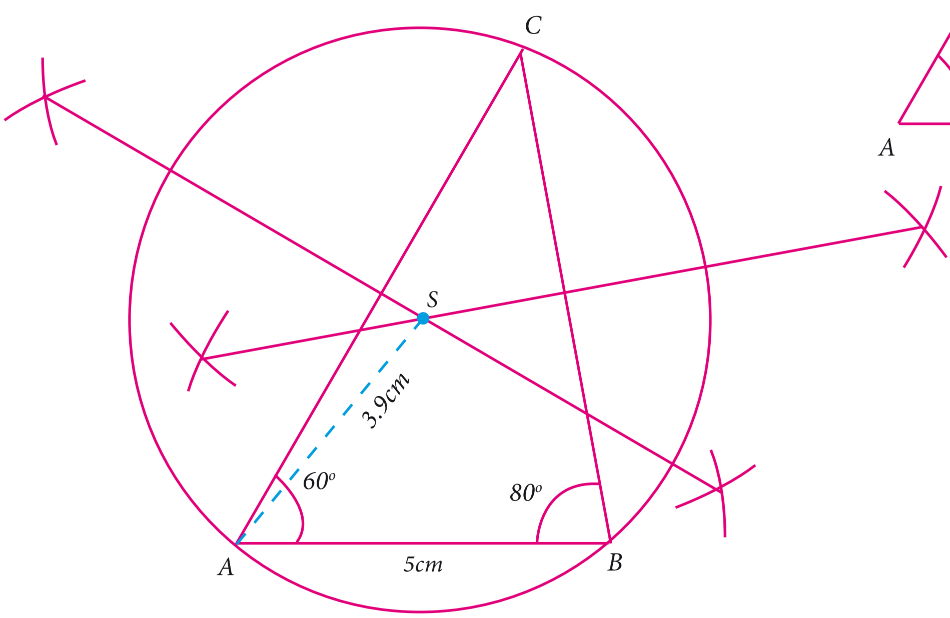


Fig. 4.54

Exercise 4.4

1 Draw the circumcircle for

(i) An equilateral triangle of side 7 cm.

(ii) An isosceles right triangle having 6 cm as the length of the equal sides.

2 Draw a triangle ABC, where $AB = 8$ cm, $BC = 6$ cm and $\angle B = 70^0$ and locate its circumcentre and draw the circumcircle.

3 Construct the right triangle PQR whose perpendicular sides are 4.5 cm and 6 cm. Also locate its circumcentre and draw the circumcircle.

4. Construct ΔABC with $AB = 5$ cm $\angle B = 100^0$ and $BC = 6$ cm. Also locate its circumcentre draw circumcircle.

5. Construct an isosceles triangle PQR where $PQ = PR$ and $\angle Q = 50^0$, $QR = 7$cm. Also draw its circumcircle.

4.4.2 Construction of Orthocentre of a Triangle

Orthocentre

The orthocentre is the point of concurrency of the altitudes of a triangle. Usually it is denoted by H.

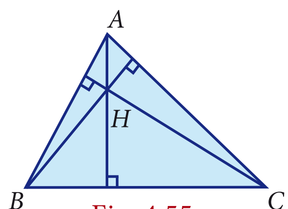


Fig. 4.55

Activity 15

Objective To locate the Orthocentre of a triangle using paper folding.

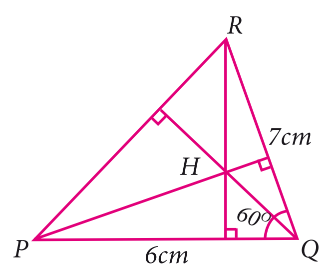
Procedure Using Activity 12 with any two vertices of the triangle as external points, construct the perpendiculars to opposite sides. The point of intersection of the perpendiculars is the Orthocentre of the given triangle.

Example 4.6

Construct ΔPQR whose sides are $PQ = 6$ cm $\angle Q = 60^0$ and $QR = 7$ cm and locate its Orthocentre.

Solution

Step 1 Draw the ΔPQR with the given measurements.



Rough Diagram

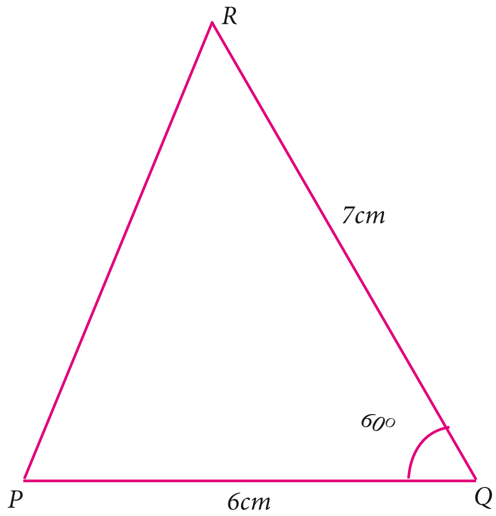


Fig. 4.56

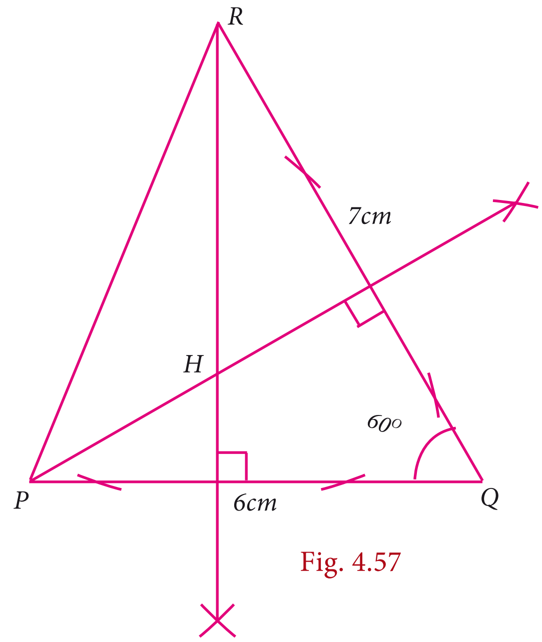


Fig. 4.57

Step 2:

Construct altitudes from any two vertices R and P, to their opposite sides PQ and QR respectively.

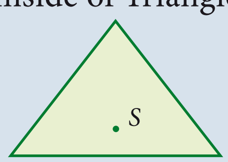
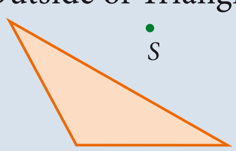
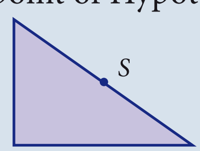
The point of intersection of the altitude H is the Orthocentre of the given ΔPQR.

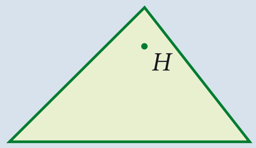
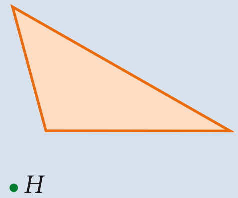
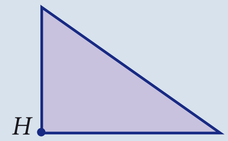
Exercise 4.5

1. Draw ΔPQR with sides PQ = 7 cm, QR = 8 cm and PR = 5 cm and construct its Orthocentre.
2. Draw an equilateral triangle of sides 6.5 cm and locate its Orthocentre.
3. Draw ΔABC, where $AB = 6$ cm, $\angle B = 110^0$ and $BC = 5$ cm and construct its Orthocentre.
4. Draw and locate the Orthocentre of a right triangle PQR where PQ = 4.5 cm, QR = 6 cm and PR = 7.5 cm.
5. Construct an isosceles triangle ABC with $AB = BC$ of sides 7 cm and $\angle B = 70^0$ and locate its Orthocentre.

Note

Where do the Circumcentre and Orthocentre lie in the given triangles.

	Acute Triangle	Obtuse Triangle	Right Triangle
Circumcentre	Inside of Triangle 	Outside of Triangle 	Midpoint of Hypotenuse 

	Inside of Triangle	Outside of Triangle	Vertex at Right Angle
Orthocentre			

Exercise 4.6

Multiple Choice Questions

1. It is not possible to construct a triangle when its sides are

   (a) 8.2 cm, 3.5 cm, 6.5 cm (b) 6.3 cm, 3.1 cm, 3.2 cm

   (c) 7 cm, 8 cm, 10 cm (d) 4 cm, 6 cm, 6 cm

2. The exterior angle of a triangle is equal to the sum of two

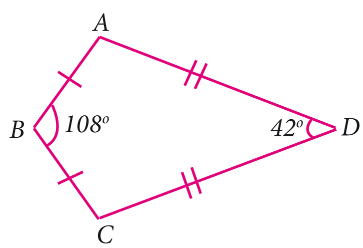
   (a) Exterior angles (b) Interior opposite angles

   (c) Alternate angles (d) Interior angles

3. In the quadrilateral $ABCD$, $AB = BC$ and $AD = DC$ Measure of $\angle BCD$ is

   (a) 150° (b) 30°

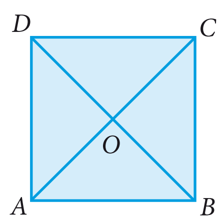
   (c) 105° (d) 72°



4. $ABCD$ is a square, diagonals AC and BD meet at O. The number of pairs of congruent triangles are

   (a) 6 (b) 8

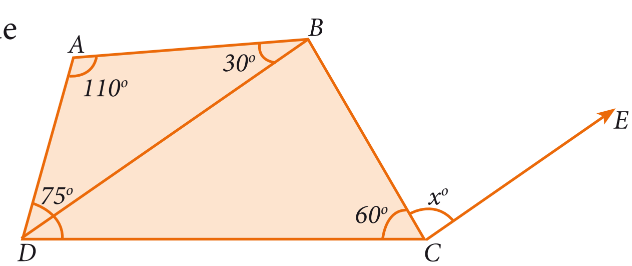
   (c) 4 (d) 12



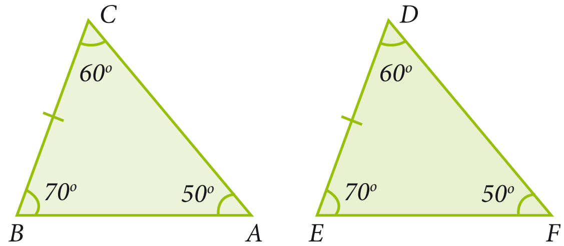
5. In the given figure $CE \parallel DB$ then the value of x^{o} is

   (a) 45° (b) 30°

   (c) 75° (d) 85°



6. The correct statement out of the following is



(a) $\Delta ABC \cong \Delta DEF$ (b) $\Delta ABC \cong \Delta DEF$

(c) $\Delta ABC \cong \Delta FDE$ (d) $\Delta ABC \cong \Delta FED$

7. If the diagonal of a rhombus are equal, then the rhombus is a

(a) Parallelogram but not a rectangle

(b) Rectangle but not a square

(c) Square

(d) Parallelogram but not a square

8. If bisectors of $\angle A$ and $\angle B$ of a quadrilateral $ABCD$ meet at O, then $\angle AOB$ is

(a) $\angle C + \angle D$ (b) $\frac{1}{2}(\angle C + \angle D)$

(c) $\frac{1}{2}\angle C + \frac{1}{3}\angle D$ (d) $\frac{1}{3}\angle C + \frac{1}{2}\angle D$

9. The interior angle made by the side in a parallelogram is 90° then the parallelogram is a

(a) rhombus (b) rectangle

(c) trapezium (d) kite

10. Which of the following statement is correct?

(a) Opposite angles of a parallelogram are not equal.

(b) Adjacent angles of a parallelogram are complementary.

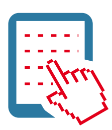
(c) Diagonals of a parallelogram are always equal.

(d) Both pairs of opposite sides of a parallelogram are always equal.

11. The angles of the triangle are $3x-40$, $x+20$ and $2x-10$ then the value of x is

(a) 40 (b) 35 (c) 50 (d) 45

Points to remember



- In a parallelogram the opposite sides are equal.
- In a parallelogram the opposite angles are equal.
- The diagonals of a parallelogram bisect each other.

- The diagonals of a parallelogram divides it into two congruent triangles
- A quadrilateral is a parallelogram if its opposite sides are equal .
- Parallelogram on the same base and between same parallel are equal in area.
- Triangles on the same base and between same parallel are equal in area.
- Parallelogram is a rhombus if its diagonals are perpendicular.
- A diagonal of a parallelogram divides it into two triangles of equal area.
- The circumcentre is the point of concurrency of the Perpendicular bisectors of the sides of a triangle.
- The orthocentre is the point of concurrency of the altitudes of a triangle.

Answers

Exercise 4.1

1. (i) 20° (ii) 63° (iii) 45° (iv) 27°28′
2. (i) 40° (ii)146° (iii) 90° (iv) 58°12′
3. (i) 18° (ii)140° (iii) 75° (iv) 20° (v) 18°
4. $\angle 1 = 110°$, $\angle 2 = 70°$, $\angle 3 = 110°$, $\angle 4 = 70°$, $\angle 5 = 110°$, $\angle 6 = 70°$, $\angle 7 = 110°$ $\angle 8 = 70°$
5. (i) 70° (ii) 288° (iii) 89° 6. 30°, 60°, 90° 9. 80°, 85 °, 15 °

Exercise 4.2

2. (i), (v) and (vi) are parallelograms 3. (iii), (vi) and (viii) are not quadrilaterals
4. (i), (v), and (vi) are trapeziums, (ii), (iii) and (iv) are not trapeziums

Exercise 4.3

1. (i) 40°, 80°, 100°, 140° 2. 62°, 114° , 66° 3. 10cm

4. 44° 5. 10cm

8. 35° , 145° 9. (i) 30° (ii) 105° (iii) 75° (iv) 105° 10. 122° , 29°
11. The quadrilateral which satisfies all the properties is a square 12. Ratios are equal
13. $d = \sqrt{61}$ 14. DF = 17 cm

Exercise 4.6

1. (b) 2. (b) 3. (c) 4. (a) 5. (d) 6. (d) 7. (c) 8. (b) 9. (b) 10. (d) 11. (b)