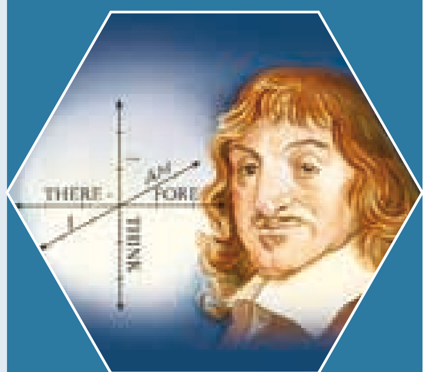
5 COORDINATE GEOMETRY

Geometry is that part of universal mechanics which accurately proposes and demonstrates the art of measuring.

- " Sir Isacc Newton"

The French Mathematician Rene Descartes (pronounced "Day- CART") developed a new branch of Mathematics known as Analytical Geometry or Coordinate Geometry which combined all arithmetic, algebra and geometry of the past ages in a single technique of visualising as points on a graph and equations as geometrical shapes. The fixing of a point position in the plane by assigning two numbers, coordinates, giving its distance from two lines perpendicular to each other, was entirely Descartes' invention.



Rene Descartes
1596-1650

Learning Outcomes

- To understand the Cartesian coordinate system.
- To identify the abscissa, ordinate and coordinates of any given point.
- To find the distance between any two points in the Cartesian plane using formula.

5.1 Mapping the Plane

How do you write your address ? Here is one.

Sarakkalvilai Primary School
135, Sarakkalvilai,
Sarakkalvilai Housing Board Road,
Keezha Sarakkalvilai,
Nagercoil 629002, Kanyakumari Dist.
Tamil Nadu, India.

Fig. 5.1

Somehow, this information is enough for anyone in the world from anywhere to locate the school one studied. Just consider there are crores and crores of buildings on the Earth. But yet, we can use an address system to locate a particular person's studied place, however interior it is.

How is this possible? Let us work out the procedure of locating a particular address. We know the World is divided into countries. One among them is India. Subsequently India is divided into States. Among these States we can locate our State Tamilnadu.

Further going deeper, we find our State is divided into Districts. Districts into taluks, taluks into villages proceeding further in this way, one could easily locate "Sarakkalvilai" among the villages in that Taluk. Further among the roads in that village, 'Housing Board Road' is the specific road which we are interested to explore. Finally we end up the search by the locating Government Primary School building bearing the door number 135 to enable us precisely among the buildings in that road.

In New York city of USA, there is an area called Manhattan. The map shows Avenues run in the North – South direction and the Streets run in the East – West direction. So, if you know that the place you are looking for is on 57th street between 9th and 10th Avenues, you can find it immediately on the map. Similarly it is easy to find a place on 2nd Avenue between 34th and 35th streets. In fact, New Yorkers make it even simpler. From the door number on a street, you can actually calculate which avenues it lies between, and from the door number on an avenue, you can calculate which streets it stands.

Fig. 5.2

All maps do just this for us. They help us in finding our way and locate a place easily by using information of any landmark which is nearer to our search to make us understand whether we are near or far, how far are we, or what is in between etc. We use latitudes (east – west, like streets in Manhattan) and longitudes (north – south, like avenues in Manhattan) to pin point places on Earth. It is interesting to see how using numbers in maps helps us so much.

This idea, of using numbers to map places, comes from geometry. Mathematicians wanted to build maps of planes, solids and shapes of all kinds. Why would they want such maps? When we work with a geometric figure, we want to observe wheather a point lies inside the region or outside or on the boundary. Given two points on the boundary and a point outside, we would like to examine which of the two points on the boundary is closer to the one outside, and how much closer and so on. With solids like cubes, you can imagine how interesting and complicated such questions can be.

Mathematicians asked such questions and answered them only to develop their own understanding of circles, polygons and spheres. But the mathematical tools and techniques were used to find immense applications in the day to day life. Mapping the world using latitudes and longitudes would not have been developed at all in 18th century, if the co-ordinate system had not been developed mathematically in the 17th century.

You already know the map of the real number system is the number line. It extends infinitely on both directions. In between any two points, on a number line, there lies infinite number of points. We are now going to build a map of the plane so that we can discuss about the points on the plane, of the distance between the points etc. We can then draw on the plane all the geometrical shapes we have discussed so far, precisely.

Arithmetic introduced us to the world of numbers and operations on them. Algebra taught us how to work with unknown values and find them using equations. Geometry taught us to describe shapes by their properties. Co-ordinate geometry will teach us how to use numbers and algebraic equations for studying geometry and beautiful integration of many techniques in one place. In a way, that is also great fun as an activity. Can't wait ? Let us plunge in.

5.2 Devising a Coordinate System

You ask your friend to draw a rectangle on a blank sheet of paper, 5 cm by 3 cm. He says, "Sure, but where on this sheet ?" How would you answer him ?

Now look at the picture. How will you describe it to another person?

Fig. 5.3

Let us analyse the given picture (Fig. 5.3). Just like that a particular house is to be pointed out, it is going to be a difficult task. Instead, if any place or an object is fixed for identification then it is easy to identify any other place or object relative to it. For example, you fix the flag and talk about the house to the left of it, the hotel below it, the antenna on the house to right of it etc.

As shown Fig. 5.4 you draw two perpendicular lines in such a way that the flag is pointed out, near the intersection point. Now if you tell your friend the total length and width of the picture frame, keeping the flagpole as landmark, you can also say 2 cm to the right, 3 cm above etc. Since you know directions, you can also say 2 cm east, 3 cm north.

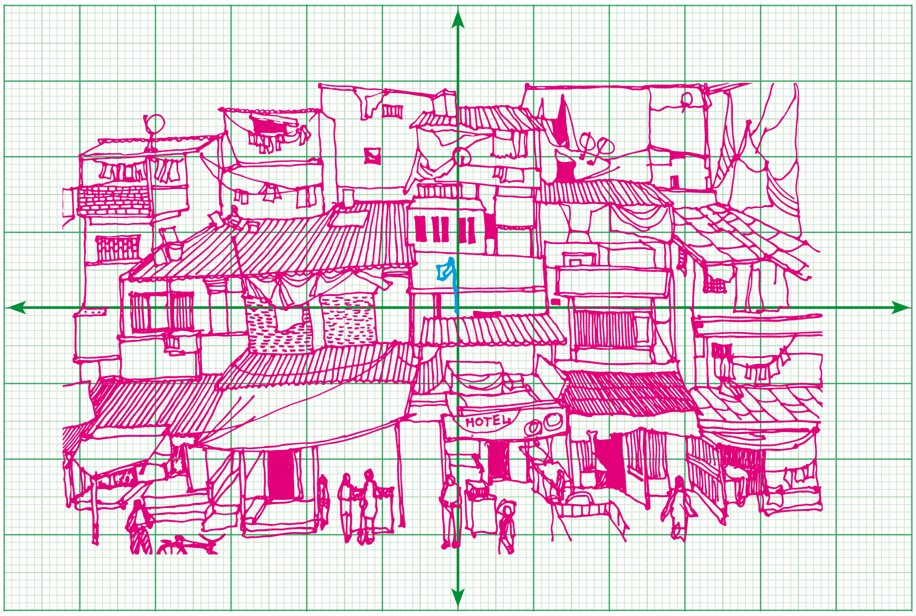


Fig. 5.4

This is what we are going to do. A number line is usually represented as horizontal line on which the positive numbers always lie on the right side of zero, negative on the left side of zero. Now consider another copy of the number line, but drawn vertically: the positive integers are represented above zero and the negative integers are below zero (fig 5.5).

Where do the two number lines meet ? Obviously at zero for both lines. That will be our "flag", the fixed location. We can talk of other numbers relative to it, on both the lines. But now you see that we talk not only of numbers on the two number lines but lots more !

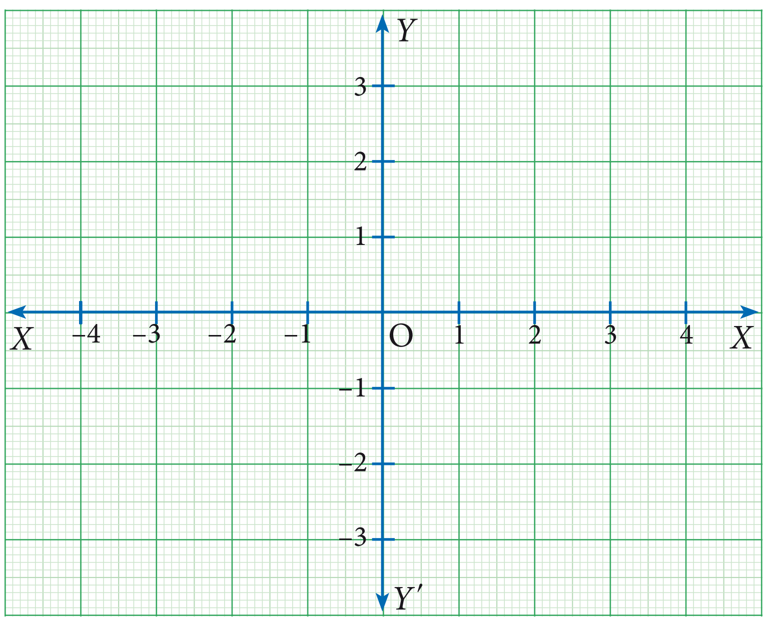


Fig. 5.5

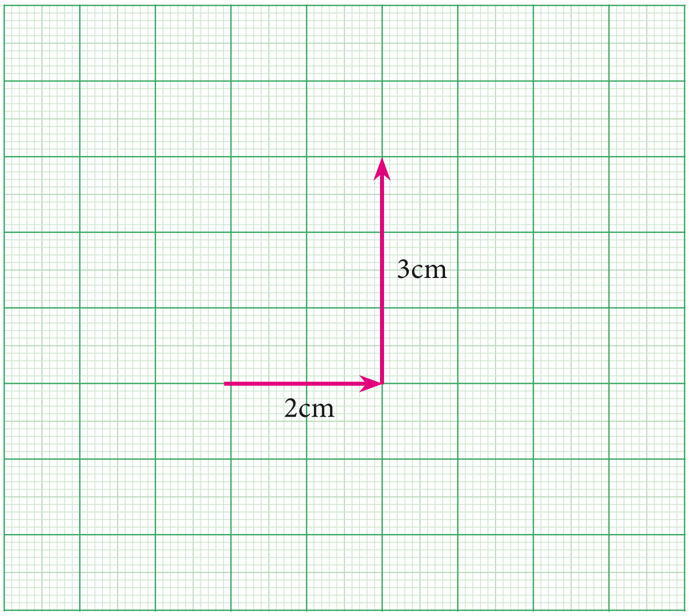


Fig. 5.6

Suppose we go 2 to the right and then 3 to the top. We would call this place ($\rightarrow$ 2, $\uparrow$ 3).

All this vertical, horizontal, up, down etc is all very cumbersome. We simply say (2,3) and understand this as 2 to the right and then 3 up. Notice that we would reach the same place if we first went 3 up and then 2 right, so for us, the instruction (2,3) is not the same as the instruction (3,2). What about (–2,3) ? It would mean 2 left and 3 up. From where ? Always from (0,0). What about the instruction (2,–3)? It would mean 2 right and 3 down. We need names for horizontal and vertical number lines too. We call the horizontal number line the x-axis and the vertical number line the y-axis. To the right we mark it as X, to the left as X', to the top as Y, to the bottom as Y'.

The x-co-ordinate is called the *abscissa* and the y-co-ordinate is called the *ordinate*. We call the meeting point of the axes (0,0) the origin.

Now we can describe any point on a sheet of paper by a pair (x,y). However, what do (1, 2) etc mean on our paper ? We need to choose some convenient unit and represent these numbers. For instance we can choose 1 unit to be 1 cm. Thus (2,3) is the instruction to move 2 cm to the right of (0,0) and then to move 3 cm up. Please remember that the choice of units is arbitrary: if we fix 1 unit to be 2 cm, our figures will be larger, but the relative distances will remain the same.

Note

Whether we place (0,0) at the centre of the sheet, or somewhere else does not matter, (0,0) is always the origin for us, and all "instructions" are relative to that point. We usually denote the origin by the letter 'O'.

In fact, we now have a language to describe all the infinitely many points on the plane, not just our sheet of paper !

Since the x-axis and the y-axis divide the plane into four regions, we call them quadrants. (Remember, quadrilateral has 4 sides, quadrants are 4 regions.) They are usually numbered as I, II, III and IV, with I for upper east side, II for upper west side, III for lower west side and IV for lower east side, thus making an anti-clockwise tour of them all.

Region	Quadrant	Nature of x,y	Signs of the coordinates
XOY	I	$x>0, y>0$	(+,+)
$X'OY$	II	$x<0, y>0$	(–,+)
$X'OY'$	III	$x<0, y<0$	(–,–)
XOY'	IV	$x>0, y<0$	(+,–)

Note

Why this way (anti-clockwise) and not clockwise, or not starting from any of the other quadrants ? It does not matter at all, but it is good to follow some convention, and this is what we have been doing for a few centuries now.

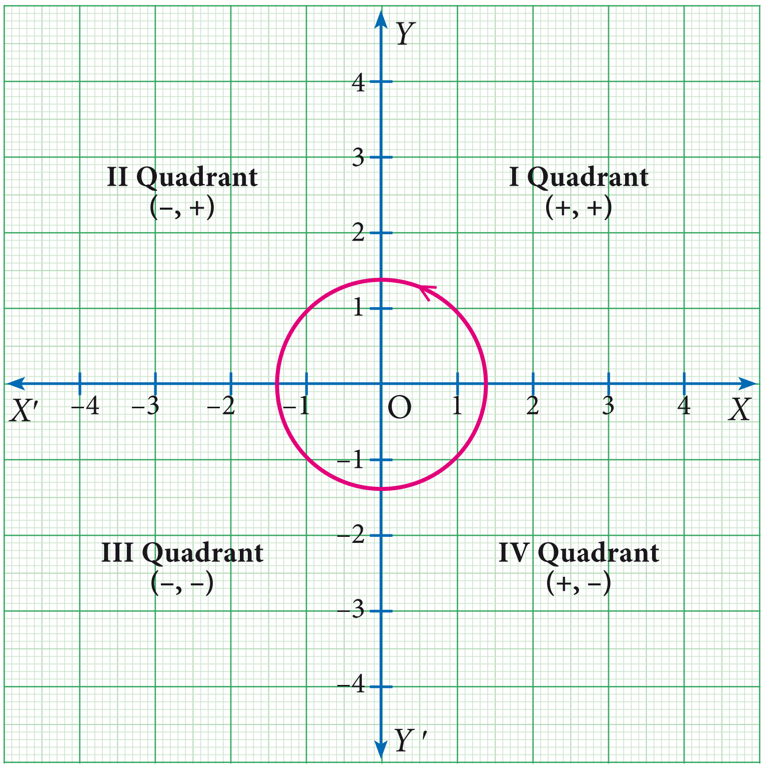


Fig. 5.7

Note

1. *For any point P on the x axis,the value of y coordinate (ordinate) is zero ie. P (x,0).*
2. *For any point Q on the y axis, the value of x coordinate (abscissa) is zero. ie. Q (0, y)*
3. *(x, y) ≠ (y, x) unless x= y*
4. *A plane with the rectangular coordinate system is called the Cartesian plane.*

5.2.1 Plotting Points in Cartesian Coordinate Plane

To plot the points (4, 5) in the Cartesian coordinate plane.

We follow the x – axis until we reach 4 and draw a vertical line at $x = 4$.

Similarly, we follow the y – axis until we reach 5 and draw a horizontal line at $y = 5$.

The intersection of these two lines is the position of (4, 5) in the Cartesian plane.

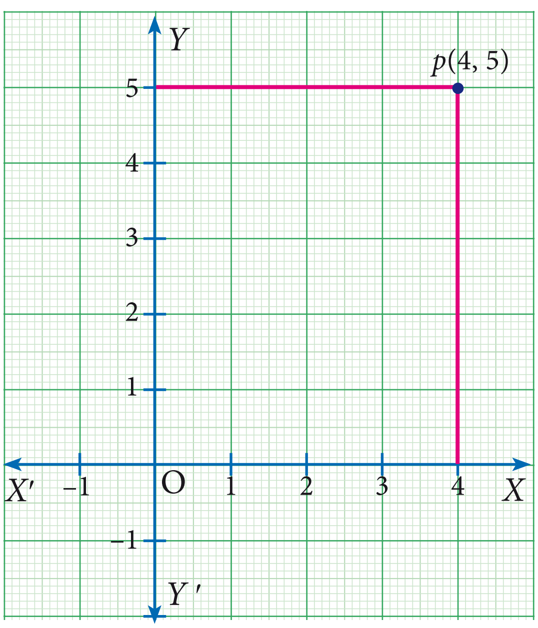


Fig. 5.8

This point is at a distance of 4 units from the y-axis and 5 units from the x-axis. Thus the position of (4, 5) is located in the Cartesian plane.

Activity 1

Observe the given graph and complete the following table:

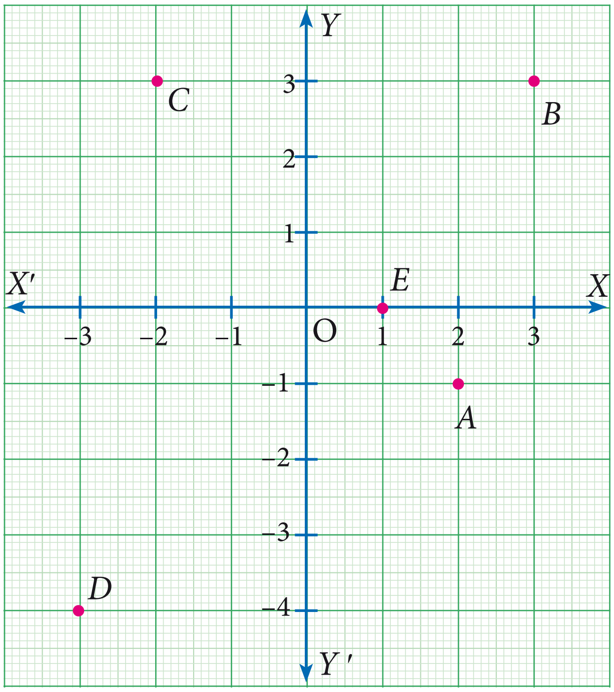


Fig. 5.9

Points	Quadrant	Ordered Pair
A	IV	
B		(3,3)
C		(-2,3)
D	III	

Activity 2

1. Join the points taken in order and name the figure obtained. Name at least any one point which is lying : (i) On x- axis (ii) On y- axis (iii) In each of the Quadrants.

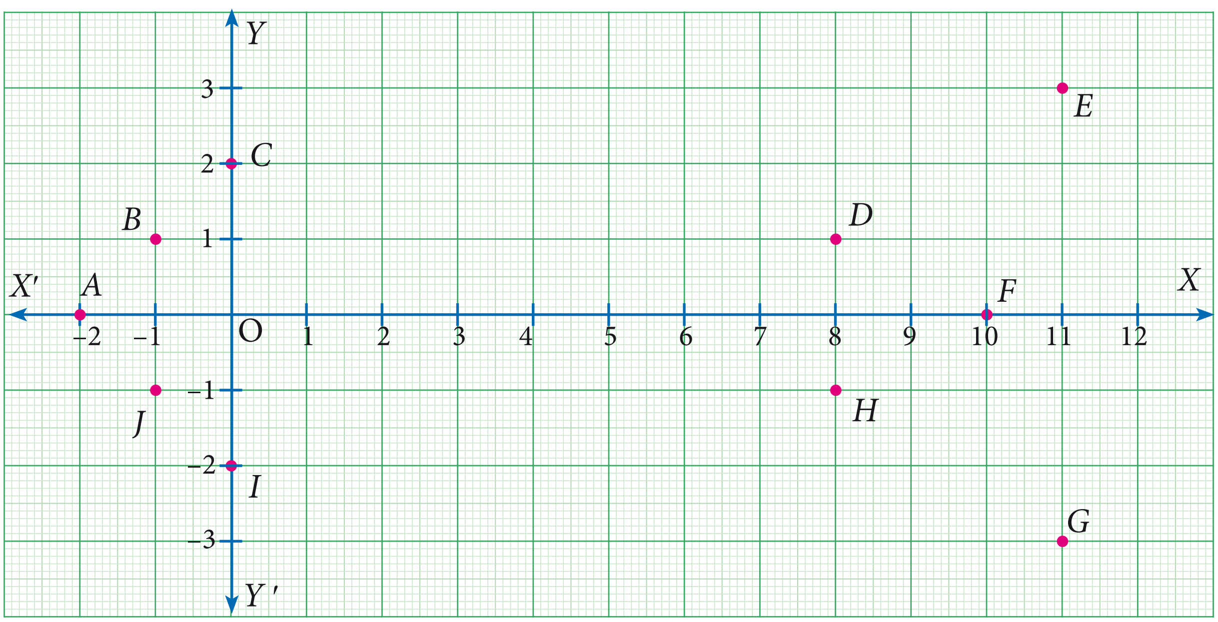


Fig. 5.10

Example 5.1

In which quadrant does the following points lie?

(a) (3,–8) (b) (–1,–3) (c) (2, 5) (d) (–7, 3)

Solution

(a) The x- coordinate is positive and y – coordinate is negative. So, Point(3,–8) lies in the IV quadrant.

(b) The x-coordinate is negative and y – coordinate is negative. So, Point(–1,–3) lies in the III quadrant.

(c) The x-coordinate is positive and y – coordinate is positive. So Point(2,5) lies in the I quadrant.

(d) The x-coordinate is negative and y – coordinate is positive. So, Point(–7,3) lies in the II quadrant

Example 5.2

Plot the points

$A(2,4)$, $B(-3,5)$, $C(-4,-5)$, $D(4,-2)$

in the Cartesian plane.

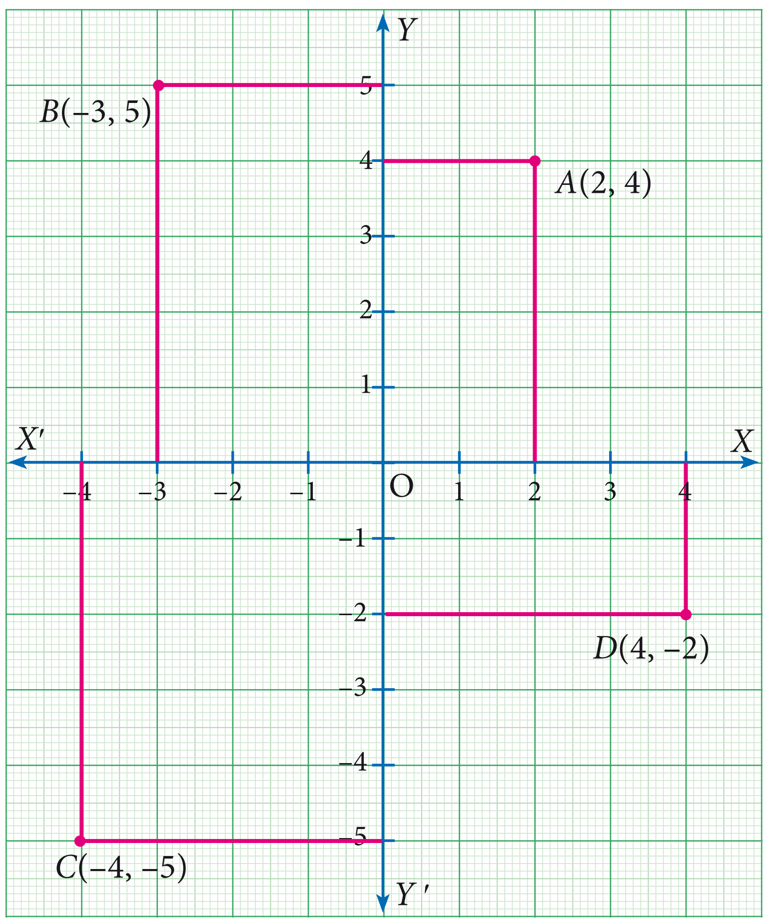


Fig. 5.11

Solution

(i) To plot (2, 4), draw a vertical line at $x = 2$ and draw a horizontal line at $y = 4$. The intersection of these two lines is the position of (2, 4) in the Cartesian plane. Thus, the Point A (2, 4) is located in the I quadrant of Cartesian plane.

(ii) To plot (–3, 5), draw a vertical line at $x = -3$ and draw a horizontal line at $y = 5$. The intersection of these two lines is the position of (–3, 5) in the Cartesian plane. Thus, the Point *B* (–3, 5) is located in the II quadrant of Cartesian plane.

(iii) To plot (–4, –5), draw a vertical line at x =–4 and draw a horizontal line at y = –5. The intersection of these two lines is the position of (-4, 5) in the Cartesian plane. Thus, the Point *C* (–4, –5) is located in the III quadrant of Cartesian plane.

(iv) To plot (4, –2), draw a vertical line at x = 4 and draw a horizontal line at y = –2. The Intersection of these two lines is the position of (4,–2) in the Cartesian plane. Thus, the Point *D* (4,–2) is located in the IV quadrant of Cartesian plane.

Example 5.3

Locate the points

(i) (2,–5) and (–5, 2) (ii) (–3, 4) and (4,–3) in the rectangular coordinate system.

Solution

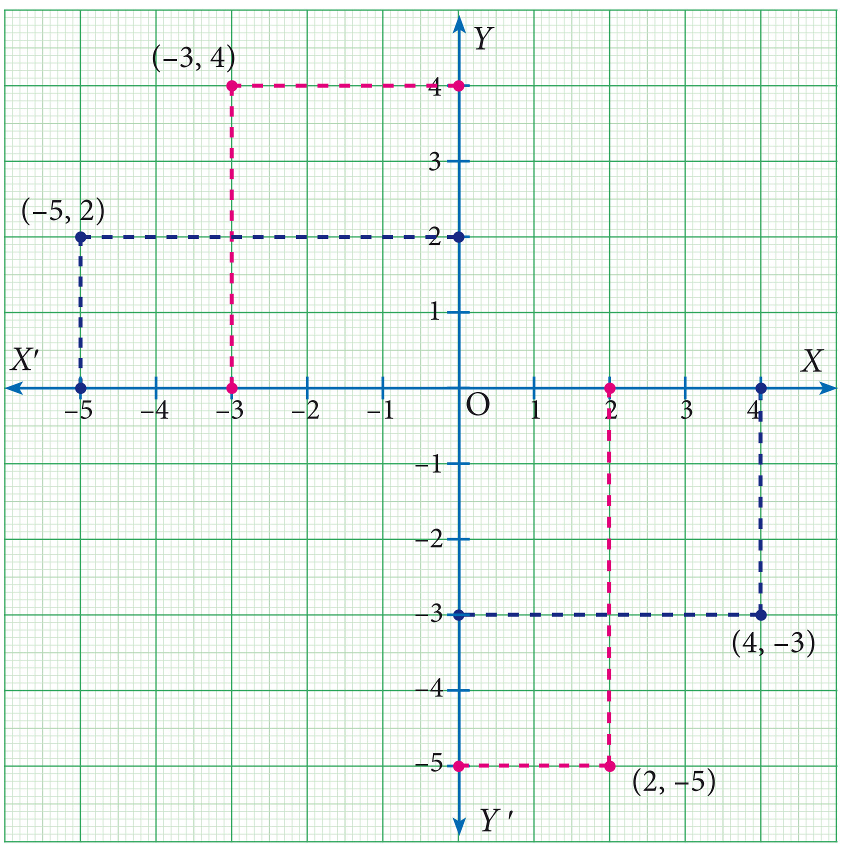


Fig. 5.12

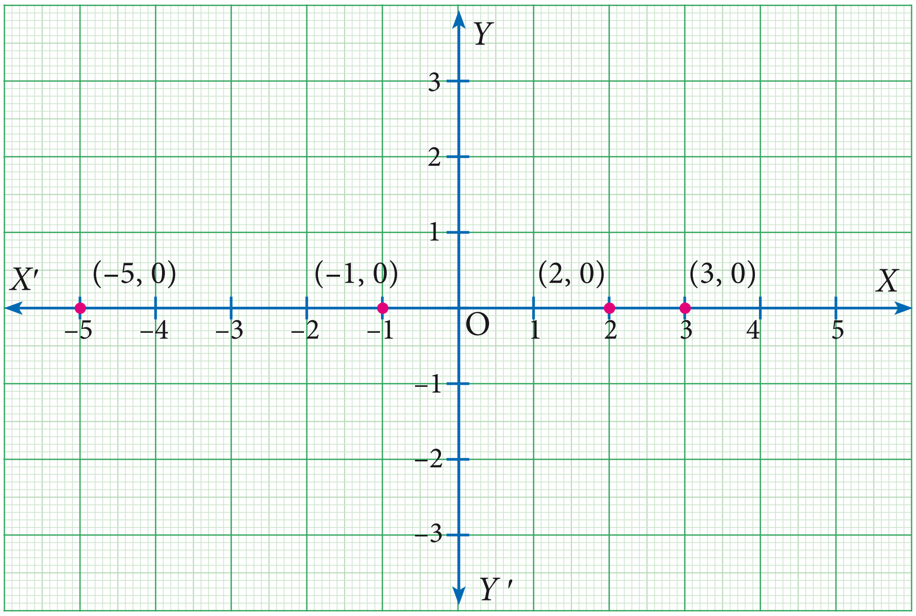
Note

Observe that if we interchange the x and y coordinates of a point, then it will represent a different point in the Cartesian plane. Think! When it will be the same?

Example 5.4

Plot the following points (2, 0), (–5, 0), (3, 0), (–1, 0) in the Cartesian plane. Where do they lie?

Solution



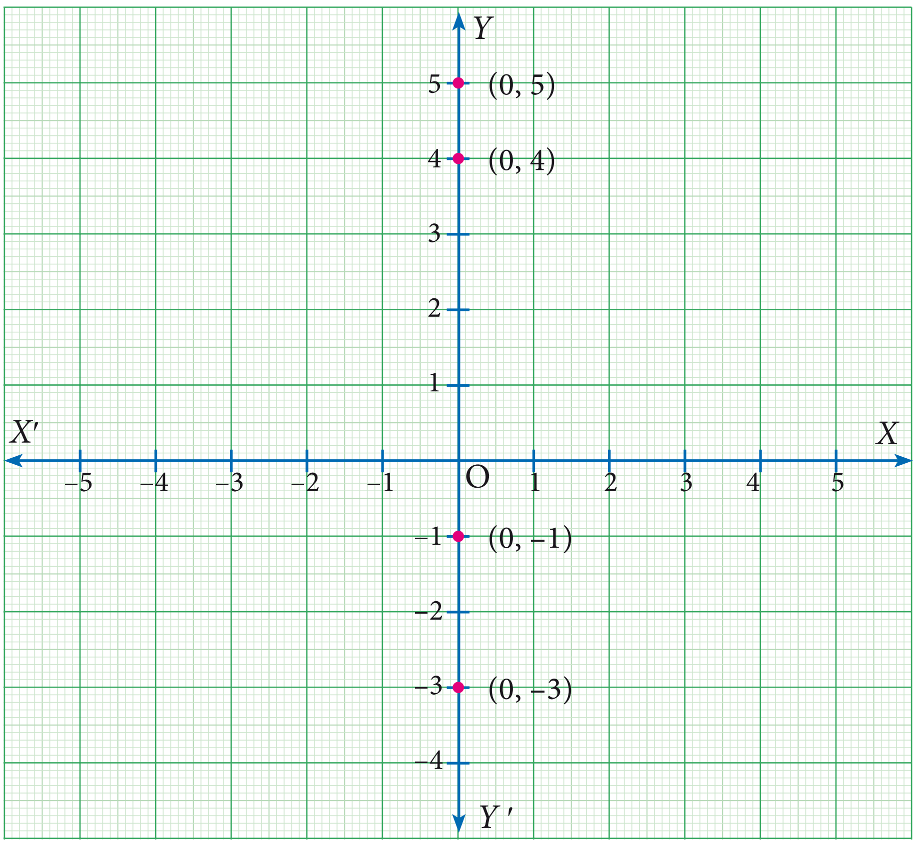
All points lie on x-axis.

Fig. 5.13

Example 5.5

Plot the following points (0,–3),(0,4),(0,–1),(0,5) in the Cartesian plane. Where do they lie?

Solution



All points lie on y-axis.

Fig. 5.14

Example 5.6

Plot the points (–4,3),(–3,3),(–1,3),(0,3),(3,3) in the Cartesian Plane. What can you say about the position of these points?

Solution

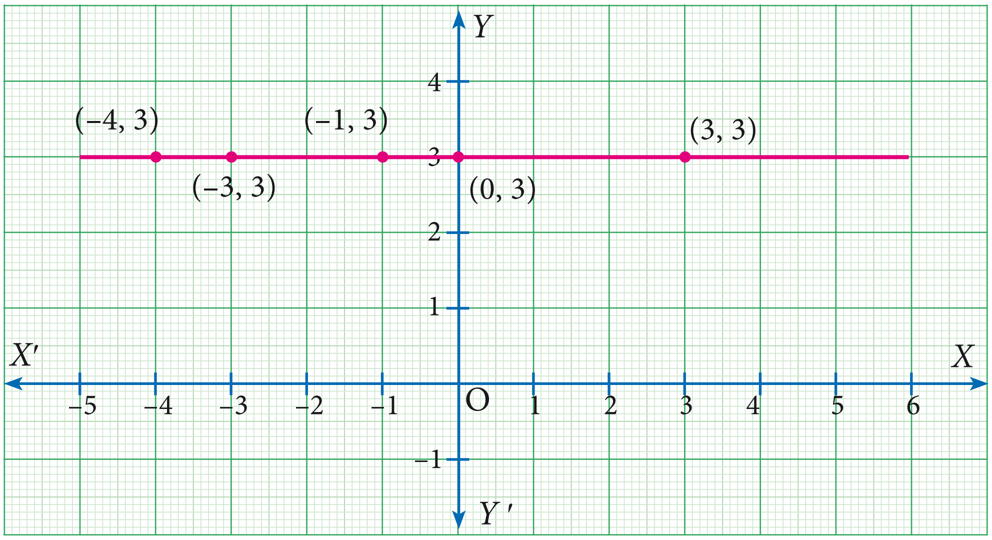


Fig. 5.15

When you join these points, you see that they lie on a line which is parallel to x-axis.

> **Note**
>
> *For points on a line parallel to x-axis, the y- coordinates are equal.*

Example 5.7

Plot the following points $A(2,2)$, $B(-2,2)$, $C(-2,-1)$, $D(2,-1)$ in the Cartesian plane . Discuss the type of the diagram by joining all the points taken in order.

Solution

Point	*A*	*B*	*C*	*D*
Quadrant	I	II	III	IV

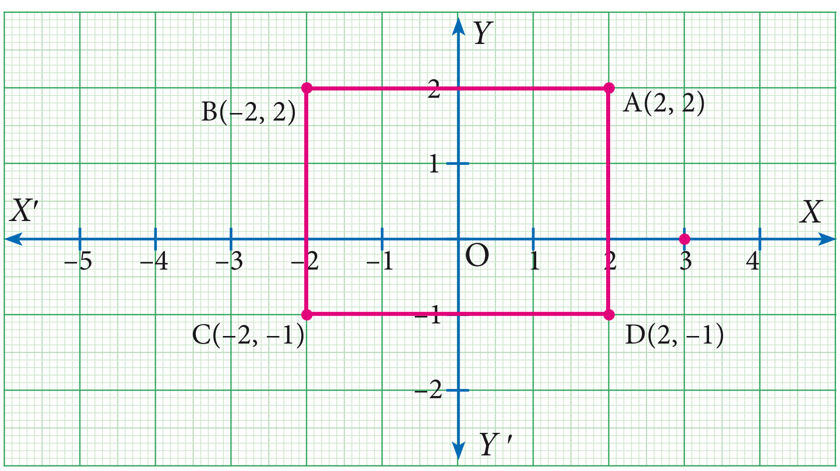


Fig. 5.16

ABCD is a rectangle.

Can you find the length, breath and area of the rectangle?

Exercise 5.1

1. Plot the following points in the coordinate system and identify the quadrants $P(-7,6)$, $Q(7,-2)$, $R(-6,-7)$, $S(3,5)$ and $T(3,9)$

2. Write down the abscissa and ordinate of the following.

   (i) P (ii) Q
   (iii) R (iv) S

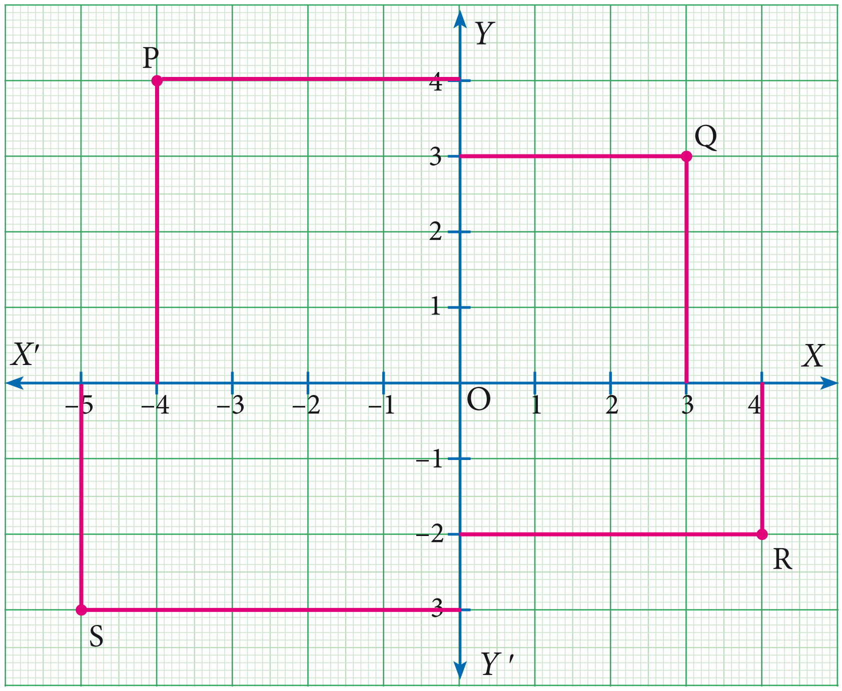


Fig. 5.17

3. Plot the following points in the coordinate plane and join them. What is your conclusion about the resulting figure?

   (i) (–5,3) (–1,3) (0,3) (5,3) (ii) (0,–4) (0,–2) (0,4) (0,5)

4. Plot the following points in the coordinate plane. Join them in order. What type of geometrical shape is formed?

   (i) (0,0) (–4,0) (–4,–4) (0,–4) (ii) (–3,3) (2,3) (–6,–1) (5,–1)

Activity 3

Plot the following points on a graph sheet by taking the scale as 1cm = 1 unit.

Find how far the points are from each other?

A (1, 0) and *D* (4, 0). Find *AD* and also DA.

Is $AD = DA$?

You plot another set of points and verify your Result.

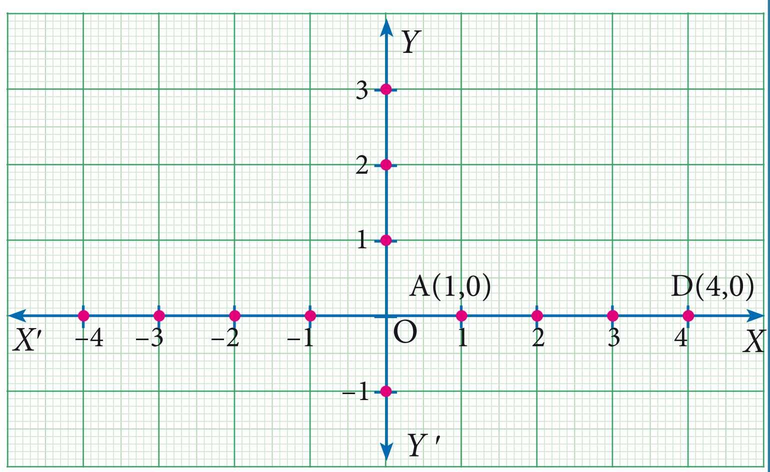


Fig. 5.18

5.3 Distance between any Two Points

Akila and Shanmugam are friends living on the same street in Sathyamangalam. Shanmugam's house is at the intersection of one street with another street on which there is a library. They both study in the same school, and that is not far from Shanmugam's house. Try to draw a picture of their houses, library and school by yourself before looking at the map below.

Consider the school as the origin. (We can do this ! That is the whole point about the co-ordinate language we are using.)

Now fix the scale as 1 unit = 50 metres. Here are some questions for you to answer by studying the given figure (Fig 5.19).

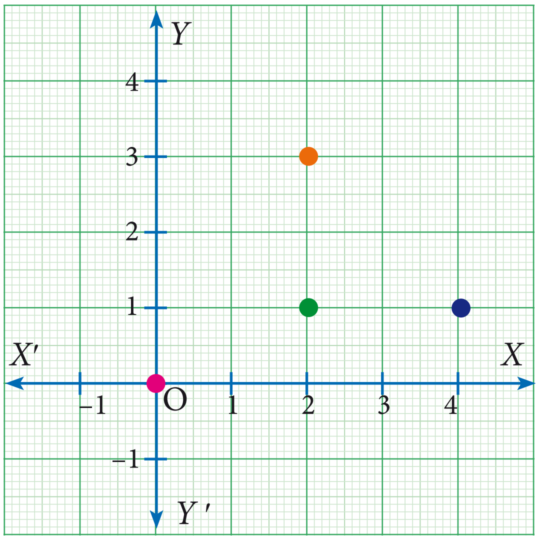


Fig. 5.19

1. How far is Akila's house from Shanmugam's house ?
2. How far is the library from Shanmugam's house?
3. How far is the school from Shanmugam's and Akila's house ?
4. How far is the library from Akila's house ?
5. How far is Shanmugam's house from Akila's house ?

Question 5 is not needed after answering question 1. Obviously, the distance from point A to B is the same as the distance from point B to A, and we usually call it the distance between points A and B. But as mathematicians we are supposed to note down properties as and when we see them, so it is better to note this too: distance (A,B) = distance (B,A). This is true for all points A and B on the plane, so of course question 5 is same as question 1.

Note

The equation distance(A,B)=distance(B,A) is not always obvious. Suppose that the road from A to B is a one-way street on which you cannot go the other way? Then the distance from B to A might be longer ! But we will avoid all these complications and assume that we can go both ways.

What about the other questions ? They are not the same. Since we know that the two houses are on the same street which is running north – south, the y-distance tells us the answer to question 1. Similarly, we know that the library and Shanmugam's house are on the same street running east – west, we can take the x-distance to answer question 2.

Questions 3 and 4 depend on what kind of routes are available. If we assume that the only streets available are parallel to the x and y axes at the points marked 1, 2, 3 etc then we answer these questions by adding the x and y distances. But consider the large field east of Akila's house.

If she can walk across the field, of course she would prefer it. Now there are many ways of going from one place to another, so when we talk of the distance between them, it is not precise. We need some way to fix what we mean. When there are many routes between A and B, we will use distance(A,B) to denote the distance on the shortest route between A and B.

Once we think of distance(A,B) as the "straight line distance" between A and B, there is an elegant way of understanding it for any points A and B on the plane. This is the important reason for using the co-ordinate system at all ! Before that, 2 more questions from our example.

1. With the school as origin, define the coordinates of the two houses, the school and the library.
2. Use the coordinates to give the distance between any one of these and another.

The "straight line distance" is usually called "as the crow flies". This is to mean that we don't worry about any obstacles and routes on the ground, but how we would get from A to B if we could fly. No bird ever flies on straight lines, though.

We can give a systematic answer to this: given any two points $A = (x,y)$ and $B = (x',y')$ on the plane, find distance(A,B). It is easy to derive a formula in terms of the four numbers x, y, x' and y'. This is what we set out to do now

5.3.1 Distance between Two Points on the Coordinate Axes

Points on x – axis: If two points lie on the x- axis, then the distance between them is equal to the difference between the x- coordinates.

Consider two points $A\ (x_1,0)$ and $B\ (x_2,0)$ on the x-axis .

The distance of B from A is

$$AB = OB - OA = x_2-x_1 \text{ if } x_2>x_1 \text{ or}$$
$$= x_1-x_2 \text{ if } x_1>x_2$$
$$AB = |x_2-x_1|$$

(Read as modulus or absolute value of x_2-x_1)

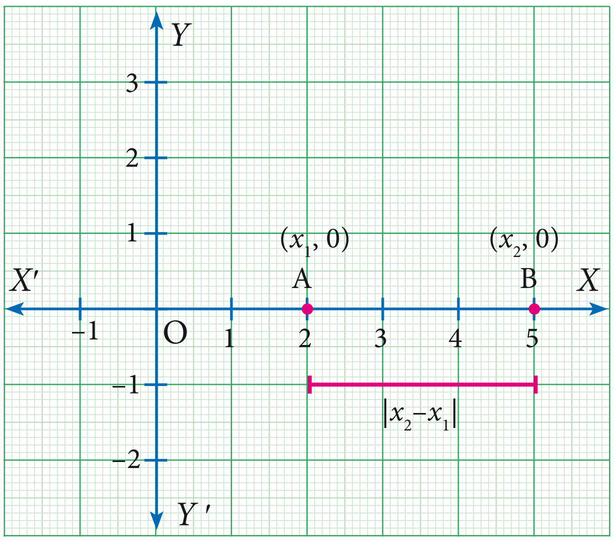


Fig. 5.20

Activity 4

(i) Where do the following points lie?

$P\,(-2, 0)$, $Q\,(2,0)$, and $R(3,0)$.

(ii) Find the distance between the following points using coordinates given in question (i)

(a) P and R (b) Q and R.

Points on x – axis: If two points lie on y-axis then the distance between them is equal to the difference between the y-coordinates.

Consider two points $P(0,y_1)$ and $Q(0,y_2)$

The distance Q from P is

$$PQ = OQ - OP.$$

$$= y_2 - y_1 \text{ if } y_2 > y_1 \text{ or}$$

$$= y_1 - y_2 \text{ if } y_1 > y_2$$

$$PQ = |y_2 - y_1|$$

(Read as modulus or absolute value of $y_2 - y_1$)

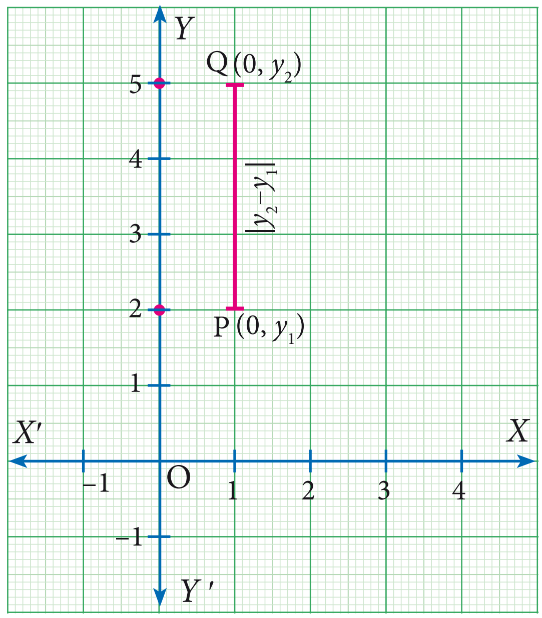


Fig. 5.21

Activity 5

(i). Where do the following points lie?

$P\,(0,-4)$, $Q(0,-1)$, $R(0,3)$, $S(0,6)$.

(ii). Find the distance between the following points using coordinates given in question (i).

(a) P and R (b) Q and S

(iii). Draw the line diagram for the given above points.

5.3.2 Distance Between Two Points Lying on a Line Parallel to Coordinate Axes

Consider the points $A(x_1, y_1)$ and $B(x_2, y_1)$. Since the y - coordinates are equal the points lie on a line parallel to x- axis. From A and B draw AP and BQ perpendicular to x- axis respectively. Observe the given figure (Fig. 5.22), it is obvious that the distance AB is same as the distance PQ

Distance AB = Distance between PQ = $|x_2-x_1|$

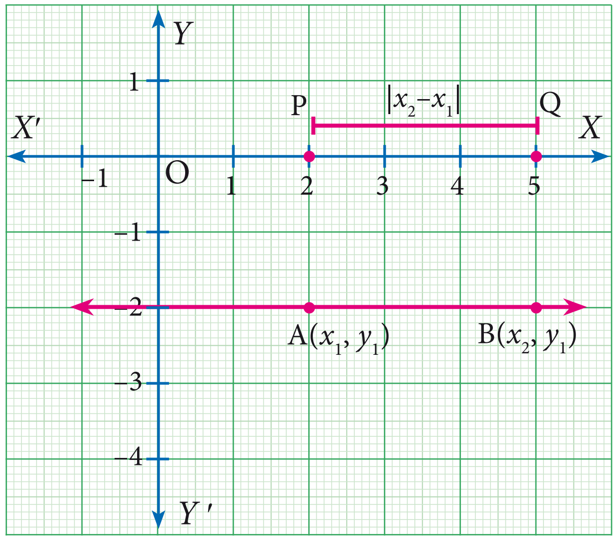


Fig. 5.22

[The difference between x coordinates]

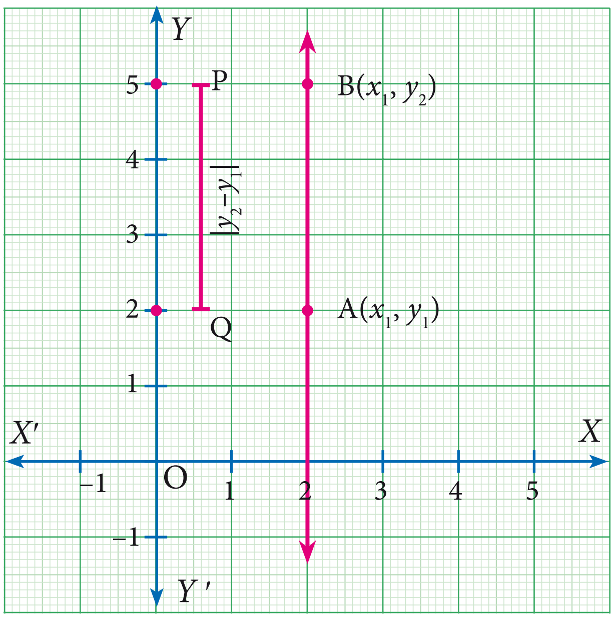


Fig. 5.23

Similarly consider the line joining the two points

$A(x_1, y_1)$ and $B(x_1, y_2)$, parallel to y – axis.

Then the distance between these two points is

$$|y_2 - y_1|$$

[The difference between y coordinates]

Activity 6

Find the distance between the two given points

(i) $A(8,3)$ and $B(-4, 3)$

(ii) $A(3,4)$ and $B(3,8)$

5.3.3 Distance Between the Two Points on a Plane.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the Cartesian plane (or xy – plane), at a distance 'd' apart such that $d = PQ$.

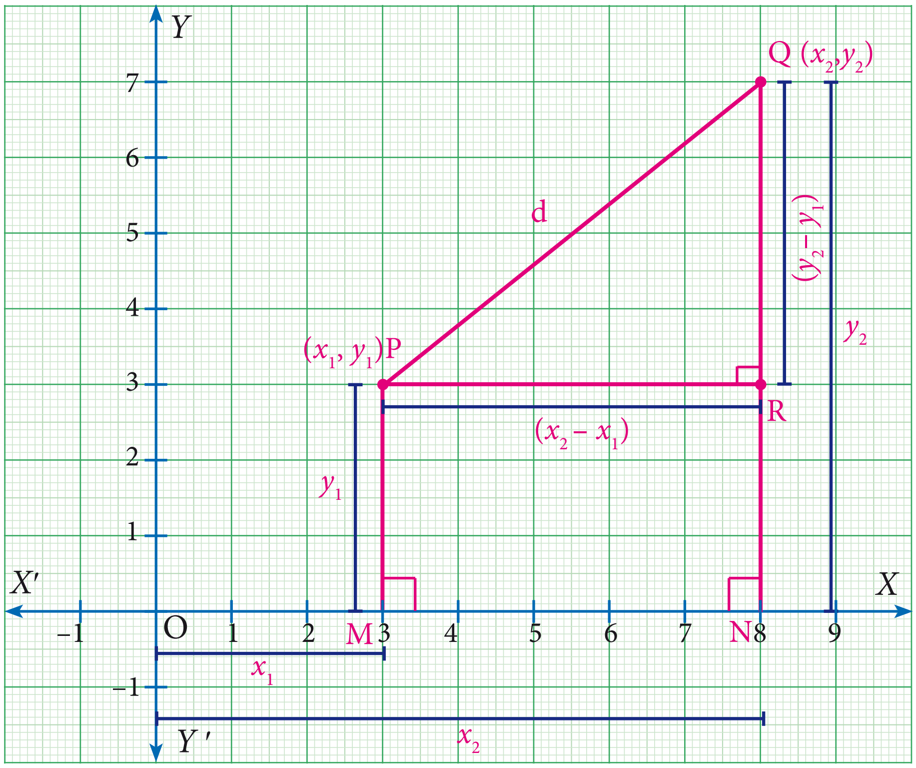


Fig. 5.24

Step 1

By the definition of coordinates,

$OM = x_1 \qquad MP = y_1$

$ON = x_2 \qquad NQ = y_2$

Now $PR = MN$

(Opposite sides of the rectangle $MNRP$)

$= ON - OM$ (Measuring the distance from O)

$= x_2 - x_1$ ……..(1)

And $RQ = NQ - NR$

$= NQ - MP$ (Opposite sides of the rectangle $MNRP$)

$= y_2 - y_1$ ………..(2)

Thinking Corner

A man goes 3 km. towards north and then 4 km. towards each. How far is he away from the initial position?

Step 2

Triangle PQR is right angled at R. ($PR \perp NQ$)

$PQ^2 = PR^2 + RQ^2$ (By Pythagoras theorem)

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Taking positive square root)

E768EB

Note

Distance between two points

1. Given two points $P\,(x_1, y_1)$ and $Q\,(x_2, y_2)$, the distance between these points is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
2. The distance between PQ = The distance between QP
   i.e. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
3. The distance of a point $P\,(x_1, y_1)$ from the origin $O\,(0,0)$ is $OP = \sqrt{x_1^2 + y_1^2}$

5.3.4 Properties of Distances

We have already seen that distance (A,B) = distance (B,A) for any points A, B on the plane. What other properties have you noticed ? In case you have missed them, here are some:

distance $(A,B) = 0$ exactly when A and B denote the identical point: $A = B$.

distance $(A,B) > 0$ for any two distinct points A and B.

Now consider three points A, B and C. If we are given their co-ordinates and we find that their x-co-ordinates are the same then we know that they are collinear, and lie on a line parallel to the y-axis. Similarly, if their y-co-ordinates are the same then we know that they are collinear, and lie on a line parallel to the x-axis. But these are not the only conditions. Points (0,0), (1,1) and (2,2) are collinear as well. Can you think of what relationship should exist between these coordinates for the points to be collinear ?

The distance formula comes to our help here. We know that when A, B and C are the vertices of a triangle, we get,

distance(A,B) + distance(B,C) > distance(A,C) (after renaming the vertices suitably).

When do three points on the plane not form a triangle ? When they are collinear, of course. In fact, we can show that when,

distance(A,B) + distance(B,C) = distance(A,C), the points A, B and C must be collinear.

Similarly, when A, B and C are the vertices of a right angled triangle, $\angle ABC = 90^\circ$ we know that:

distance$(AB)^2$ + distance$(BC)^2$ = distance$(AC)^2$

with appropriate naming of vertices. We can also show that the converse holds: whenever the equality here holds for A, B and C, they must be the vertices of a right angled triangle.

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The following examples illustrate how these properties of distances are useful for answering questions about specific geometric shapes.

Example 5.8

Find the distance between the points (–4, 3), (2,–3).

Solution

The distance between the points (-4, 3), (2,-3) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2+4)^2 + (-3-3)^2}$$

$$= \sqrt{(6^2 + (-6)^2)} = \sqrt{(36+36)}$$

$$= \sqrt{(36 \times 2)}$$

$$= 6\sqrt{2}$$

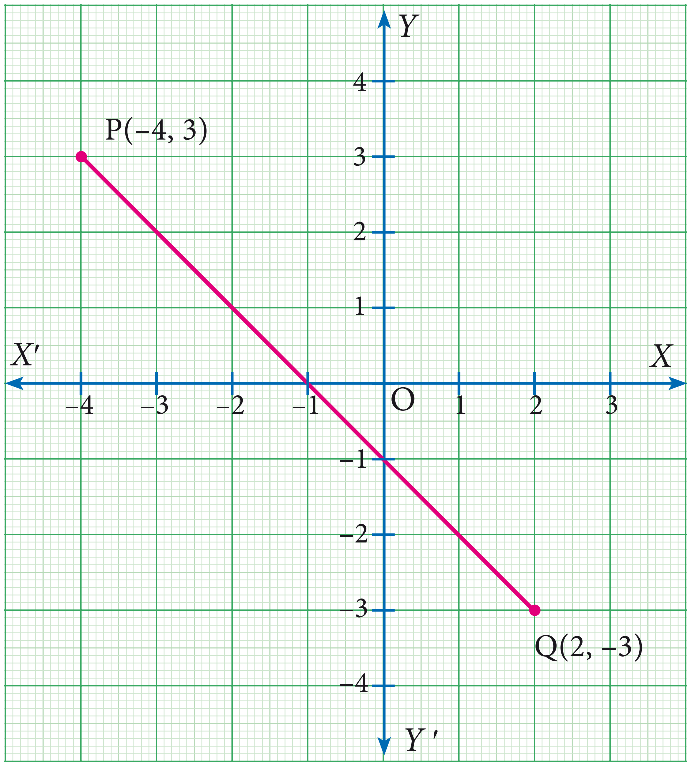


Fig. 5.25

Example 5.9

Show that the following points $A(3,1)$, $B(6,4)$ and $C(8,6)$ lies on a straight line.

Solution

Using the distance formula, we have

$$AB = \sqrt{(6-3)^2 + (4-1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(8-6)^2 + (6-4)^2} = \sqrt{(4+4)} = \sqrt{8} = 2\sqrt{2}$$

$$AC = \sqrt{(8-3)^2 + (6-1)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

$$AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$$

Therefore the points lie on a straight line.

Collinear points

To show the collinearity of three points, we prove that the sum of the distance between two pairs of points is equal to the third pair of points.

In otherwords, points A, B, C are collinear if $AB + BC = AC$

Example 5.10

Show that the points $A(7,10)$, $B(-2,5)$, $C(3,-4)$ are the vertices of a right angled triangle.

Solution

Here $A = (7, 10)$, $B = (-2, 5)$, $C = (3,-4)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2-7)^2 + (5-10)^2}$$

$$= \sqrt{(-9)^2 + (-5)^2}$$

$$= \sqrt{(81+25)}$$

$$= \sqrt{106}$$

$$AB^2 = 106 \qquad \ldots (1)$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-(-2))^2 + (-4-5)^2} = \sqrt{(5)^2 + (-9)^2}$$

$$= \sqrt{25+81} = \sqrt{106}$$

$$BC^2 = 106 \qquad \ldots (2)$$

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{(-4)^2 + (-14)^2}$$

$$= \sqrt{16+196} = \sqrt{212}$$

$$AC^2 = 212 \qquad \ldots (3)$$

Right angled triangle

We know that the sum of the squares of two sides is equal to the square of the third side,which is the hypotenuse of a right angled triangle.

From (1), (2) & (3) we get,

$$AB^2 + BC^2 = 106 + 106 = 212 = AC^2$$

Since $AB^2 + BC^2 = AC^2$

$\therefore$ ΔABC is a right angled triangle, right angled at B.

Example 5.11

Show that the points

$A(-4,-3)$, $B(3,1)$, $C(3,6)$, $D(-4,2)$ taken in that order form the vertices of a parallelogram.

Solution

Let $A(-4, -3)$, $B(3, 1)$, $C(3, 6)$, $D(-4, 2)$ be the four vertices of any quadrilateral $ABCD$. Using the distance formula,

> **Parallelogram**
>
> We know that opposite sides are equal

$$\text{Let} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(3+4)^2 + (1+3)^2} = \sqrt{49+16} = \sqrt{65}$$

$$BC = \sqrt{(3-3)^2 + (6-1)^2}$$

$$= \sqrt{0+25} = \sqrt{25} = 5$$

$$CD = \sqrt{(-4-3)^2 + (2-6)^2}$$

$$= \sqrt{(-7)^2 + (-4)^2} = \sqrt{49+16} = \sqrt{65}$$

$$AD = \sqrt{(-4+4)^2 + (2+3)^2} = \sqrt{(0)^2 + (5)^2} = \sqrt{25} = 5$$

$AB = CD = \sqrt{65}$ and $BC = AD = 5$

Here, the opposite sides are equal. Hence $ABCD$ is a parallelogram.

Example 5.12

Prove that the points $A(3, 5)$, $B(6, 2)$, $C(3,-1)$, and $D(0, 2)$ taken in order are the vertices of a square.

Solution

Let $A(3,5)$, $B(6,2)$, $C(3,-1)$, and $D(0,2)$ be the vertices of any quadrilateral $ABCD$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

> **Square**
>
> We know that four sides are equal and the diagonals are also equal

By using the distance formula we get,

$$AB = \sqrt{(6-3)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(3-6)^2 + (-1-2)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(0-3)^2 + (2+1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(0-3)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

From the above results, we see that $AB=BC=CD=DA= 3\sqrt{2}$

(i.e.) All the four sides are equal.

Further, $A(3, 5)$, $C(3, -1)$

$$\text{Diagonal } AC = \sqrt{(3-3)^2+(-1-5)^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{36} = 6$$

$$\text{Diagonal } BD = \sqrt{(0-6)^2+(2-2)^2} = \sqrt{(-6)^2+(0)^2} = \sqrt{36} = 6$$

From the above we see that $AB = CD = 6$

Hence $ABCD$ is a square.

Example 5.13

If the distance between the points (5,–2), (1, a), is 5 units, find the values of a.

Solution

The two given points are (5,-2), (1, a) and $d = 5$.

By distance formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\sqrt{(1-5)^2+(a+2)^2} = 5$$

$$\sqrt{16+(a+2)^2} = 5$$

$$16 + (a+2)^2 = 25 \text{ (By squaring on both the sides)}$$

$$(a+2)^2 = 25-16$$

$$(a+2)^2 = 9$$

$$(a+2) = \pm 3 \text{ (By taking the square root on both side)}$$

$$a = -2 \pm 3$$

$$a = -2+3 \text{ (or) } a = -2-3$$

$$a = 1 \text{ or } -5.$$

Example 5.14

Calculate the distance between the points A (7, 3) and B which lies on the x-axis whose abscissa is 11.

Solution

Since B is on the x-axis, the y-coordinate of B is 0.

So, the coordinates of the point B is (11, 0)

By the distance formula the distance between the points A (7, 3), B (11, 0) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(11-7)^2+(0-3)^2} = \sqrt{(4)^2+(-3)^2} = \sqrt{16+9} = \sqrt{25}$$

$$= 5$$

Example 5.15

Find the value of 'a' such that $PQ = QR$ where P, Q, and R are the points whose coordinates are (6, –1), (1, 3) and (a, 8) respectively.

Solution

Given P (6, –1), Q (1, 3) and R (a, 8)

$$PQ = \sqrt{(1-6)^2+(3+1)^2} = \sqrt{(-5)^2+(4)^2} = \sqrt{41}$$

$$QR = \sqrt{(a-1)^2+(8-3)^2} = \sqrt{(a-1)^2+(5)^2}$$

Given $PQ = QR$

Therefore $\sqrt{41} = \sqrt{(a-1)^2+(5)^2}$

$41 = (a-1)^2 + 25$ [Squaring both sides]

$(a-1)^2 + 25 = 41$

$(a-1)^2 = 41 - 25$

$(a-1)^2 = 16$

$(a-1) = \pm 4$ [taking square root on both sides]

$a = 1 \pm 4$

$a = 1 + 4$ or $a = 1 - 4$

$a = 5, -3$

Example 5.16

Let $A(2, 2)$, $B(8, -4)$ be two given points in a plane. If a point P lies on the X- axis (in positive side), and divides AB in the ratio 1: 2, then find the coordinates of P.

Solution

Given points are $A(2, 2)$ and $B(8, -4)$ and let $P = (x, 0)$ [P lies on x axis]

By the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AP = \sqrt{(x-2)^2 + (0-2)^2} = \sqrt{x^2 - 4x + 4 + 4} = \sqrt{x^2 - 4x + 8}$$

$$BP = \sqrt{(x-8)^2 + (0+4)^2} = \sqrt{x^2 - 16x + 64 + 16} = \sqrt{x^2 - 16x + 80}$$

Given $\quad AP : PB = 1 : 2$

i.e. $\quad \dfrac{AP}{BP} = \dfrac{1}{2} \quad (\because BP = PB)$

$$\frac{\sqrt{x^2 - 4x + 8}}{\sqrt{x^2 - 16x + 80}} = \frac{1}{2}$$

squaring on both sides,

$$\frac{x^2 - 4x + 8}{x^2 - 16x + 80} = \frac{1}{4}$$

$$4x^2 - 16x + 32 = x^2 - 16x + 80$$

$$3x^2 - 48 = 0$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = \pm 4$$

As the point P lies on x-axis (positive side), its x- coordinate cannot be -4.

Hence the coordinates of P is$(4, 0)$

Example 5.17

Show that (4, 3) is the centre of the circle passing through the points (9, 3), (7,–1), (–1,3). Find the radius.

Solution

Let $P(4, 3)$, $A(9, 3)$, $B(7, -1)$ and $C(-1, 3)$

If P is the centre of the circle which passes through the points A, B, and C, then P is equidistant from A, B and C (i.e.) $PA = PB = PC$

By distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AP = PA = \sqrt{(4-9)^2+(3-3)^2} = \sqrt{(-5)^2+0} = \sqrt{25} = 5$$

$$BP = PB = \sqrt{(4-7)^2+(3+1)^2} = \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$CP = PC = \sqrt{(4+1)^2+(3-3)^2} = \sqrt{(5)^2+0} = \sqrt{25} = 5$$

$$PA = PB = PC$$

Therefore P is the centre of the circle, passing through A, B and C

Radius = PA = 5.

Exercise 5.2

1. Find the distance between the following pairs of points.

   (i) (1, 2) and (4, 3) (ii) (3,4) and (– 7, 2)

   (ii) (a, b) and (c, b) (iv) (3,– 9) and (–2, 3)

2. Determine whether the given set of points in each case are collinear or not.

   (i) (7,–2),(5,1),(3,4) (ii) (–2, –8), (2,–3) (6,2)

   (iii) $(a,-2)$, $(a,3)$, $(a,0)$

3. Show that the following points taken in order form an isosceles triangle.

   (i) A (5,4), B(2,0), C (–2,3) (ii) A (6,–4), B (–2, –4), C (2,10)

4. Show that the following points taken in order form an equilateral triangle in each case.

   (i) $A(2, 2)$, $B(-2, -2)$, $C(-2\sqrt{3}, 2\sqrt{3})$ (ii) $A(\sqrt{3}, 2)$, B (0,1), C(0,3)

5. Show that the following points taken in order form the vertices of a parallelogram.

   (i) $A(-3, 1)$, $B(-6, -7)$, C (3, –9) and $D(6, -1)$

   (ii) A (–7, –3), B(5,10), C(15,8) and D(3, –5)

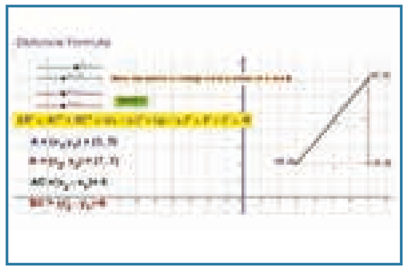
6. Verify that the following points taken in order form the vertices of a rhombus.

   (i) A(3,–2), B (7,6), C (–1,2) and D (–5, –6)

   (ii) A (1,1), B(2,1), C (2,2) and D(1,2)

ICT Corner

Expected Result is shown in this picture



Step – 1

Open the Browser and copy and paste the Link given below (or) by typing the URL given (or) Scan the QR Code.

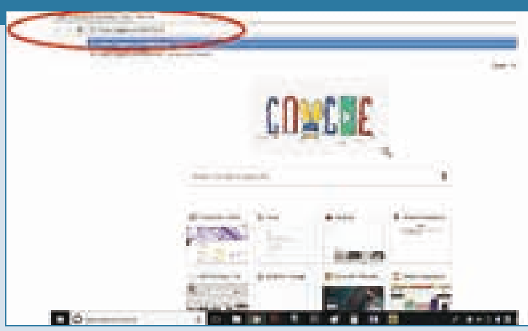
Step - 2

GeoGebra work book called “IX Analytical Geometry” will open. There are several worksheets given. Select the one you want. For example, open” Distance Formula”

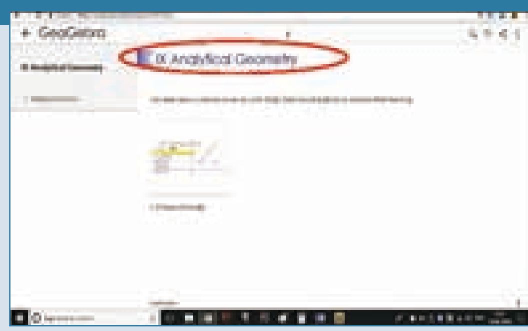
Step-3

Move the sliders x_1, x_2, y_1, y_2 to change the co-ordinates of A and B. Now you calculate the distance AB using theDistance formula in a piece of paper and check your answer

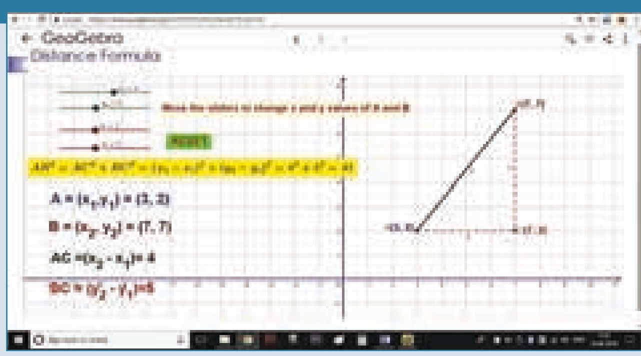
Step 1



Step 2



Step 3



Similarly you can check other worksheets in the Workbook related to your lesson

Browse in the link

Co-Ordinate Geometry: https://ggbm.at/V9HfY4v8

7. If A, B, C are points, $(-1, 1)$, $(1, 3)$ and $(3, a)$ respectively and if $AB = BC$, then find 'a'.

8. The abscissa of a point A is equal to its ordinate, and its distance from the point $B(1, 3)$ is 10 units, What are the coordinates of A?

9. The point (x, y) is equidistant from the points (3,4)and(–5,6). Find a relation between x and y.

10. Let $A(2, 3)$ and $B(2,-4)$ be two points. If P lies on the x-axis, such that $AP = \frac{3}{7}AB$, find the coordinates of P.

11. Find the perimeter of the triangle whose vertices are(3,2), (7,2) and (7, 5).

12. Show that the point (11,2) is the centre of the circle passing through the points (1,2), (3,–4) and (5,–6)

13. The radius of a circle with centre at origin is 30 units. Write the coordinates of the points where the circle intersects the axes. Find the distance between any two such points.

14. Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $C(2, -3y)$. Find the radius of the circle.

Exercise 5.3

Multiple Choice Questions

1. Point (–3,5) lie in the ________ quadrant.

   (a) I (b) II (c) III (d) IV

2. Signs of the abscissa and ordinate of a point in the fourth quadrant are respectively _____

   (a) (+,+) (b) (–, –) (c) (–, +) (d) (+, –)

3. Point (0, –7) lies ________________________________

   (a) on the x-axis (b) in the II quadrant

   (c) on the y-axis (d) in the IV quadrant.

4. Point (–10, 0) lies ________________________________

   (a) on the negative direction of x-axis (b) on the negative direction of y-axis

   (c)in the III quadrant (d) in the IV quadrant

5. If the y-coordinate of a point is zero, then the point always lies ______

(a)in the I quadrant (b) in the II quadrant (c)on x-axis (d) on y-axis

6. The point M lies in the IV quadrant. The coordinates of M is _______

(a) (a,b) (b) $(-a, b)$ (c) $(a, -b)$ (d) $(-a, -b)$

7. The points $(-5, 2)$ and $(2, -5)$ lie in the ________

(a) same quadrant (b) II and III quadrant respectively

(c) II and IV quadrant respectively (d) IV and II quadrant respectively

8. On plotting the points $O(0,0)$, $A(3, -4)$, $B(3, 4)$ and $C(0, 4)$ and joining OA, AB, BC and CO, which of the following figure is obtained?

(a) Square (b) Rectangle (c) Trapezium (d) Rhombus

9. If $P(-1,1)$, $Q(3,-4)$, $R(1, -1)$, $S(-2, -3)$ and $T(-4, 4)$ are plotted on a graph paper, then the points in the fourth quadrant are __________

(a) P and T (b) Q and R (c) only S (d) P and Q

10. The point whose ordinate is 4 and which lies on the y-axis is _______________

(a)$(4, 0)$ (b) $(0, 4)$ (c) $(1, 4)$ (d) $(4, 2)$

11. The distance between the two points $(2, 3)$ and $(1, 4)$ is ______

(a) 2 (b) $\sqrt{56}$ (c) $\sqrt{10}$ (d) $\sqrt{2}$

12. If the points A (2,0), B (-6,0), C $(3, a-3)$ lie on the x-axis then the value of a is _____

(a) 0 (b) 2 (c) 3 (d) -6

13. If $(x+2, 4) = (5, y-2)$, then the coordinates (x,y) are _____

(a) (7, 12) (b) (6, 3) (c) (3, 6) (d) (2, 1)

14. If Q_1, Q_2, Q_3, Q_4 are the quadrants in a Cartesian plane then $Q_2 \cap Q_3$ is ___________

(a) $Q_1 \cup Q_2$ (b) $Q_2 \cup Q_3$ (c) Null set (d) Negative x-axis.

15. The distance between the point $(5, -1)$ and the origin is _________

(a) $\sqrt{24}$ (b) $\sqrt{37}$ (c) $\sqrt{26}$ (d) $\sqrt{17}$

Activity 7

Plot the points $A(-1, 0)$, $B(3, 0)$, $C(3, 4)$, and $D(-1, 4)$ on a graph sheet. Join them to form a rectangle. Draw the mirror image of the diagram in clockwise direction:

(i) about x-axis. (ii) about y-axis.

What is your observation on the coordinates of the mirror image?

Activity 8

Plot the points $A(1, 0)$, $B(-7, 2)$, $C(-3, 7)$ on a graph sheet and join them to form a triangle.

Plot the point $G(-3, 3)$.

Join AG and extend it to intersect BC at D.

Join BG and extend it to intersect AC at E.

What do you infer when you measure the distance between BD and DC and the distance between CE and EA?

Using distance formula find the lengths of CG and GF, where F in on AB.

Write your inference about $AG: GD$, $BG: GE$ and $CG: GF$.

Note: ***G*** is the **centroid** of the triangle and AD, BE and CF are the three medians of the triangle.

Points to remember

- If x_1, x_2 are the x-coordinates of two points on the x-axis then the distance between them is $x_2 - x_1$, if $x_2 > x_1$.
- If y_1, y_2 are the y-coordinates of two points on the y-axis then the distance between them is $|y_1 - y_2|$.
- Distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
- Distance between (x_1, y_1) and the origin $(0, 0)$ is $\sqrt{x_1^2+y_1^2}$

If three or four points are given, in order then to prove that a given figure is:

- Triangle- We prove that the sum of the lengths of any two sides is greater than the length of the third side.
- Isosceles triangle- We prove that the length of any two sides are equal.
- Equilateral triangle- We prove that the length of all the three sides are equal.
- Square - We prove that four sides are equal.
- Rectangle - We prove that opposite sides are equal and the diagonals are also equal.
- Parallelogram (not a rectangle) - We prove that opposite sides are equal but the diagonals are not equal.
- Rhombus (not a square)- We prove that all sides are equal but the diagonals are not equal.

Answers

Exercise 5.1

1. $P(-7,6)$ = II Quadrant; $Q(7,-2)$ = IV Quadrant; $R(-6,-7)$ = III Quadrant; $S(3,5)$ = I Quadrant; and $T(3,9)$ = I Quadrant

2. (i) $P = (-4,4)$ (ii) $Q = (3,3)$ (iii) $R=(4,-2)$ (iv) $S = (-5,-3)$

3. (i) Straight line parallel to x -axis (ii) Straight line which lie on y -axis.

4. (i) Square (ii) Trapezium

Exercise 5.2

1. (i) $\sqrt{10}$ units (ii) $2\sqrt{26}$ units (iii) $c-a$ (iv)13 units

2. (i) Collinear (ii) Collinear (iii) Collinear 7. 5 or 1

8. Coordinates of A (9, 9) or (−5,−5) 9. $y = 4x+9$ 10. Coordinates of $P(2,0)$

11. 12 units 13. $30\sqrt{2}$ 14. $y = 7$, Radius = $\sqrt{793}$, $y = -1$, Radius = 5

Exercise 5.3

1. (b)	2. (d)	3. (c)	4. (a)	5. (c)
6. (c)	7. (c)	8. (c)	9. (b)	10. (b)
11. (d)	12. (c)	13. (c)	14. (c)	15. (c)