CHAPTER 10 - DIRECT AND INVERSE VARIATIONS

Question 1.

In which of the following tables, x and y vary directly:

(i)

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4.5</td>
<td>7.5</td>
<td>12</td>
<td>16.5</td>
</tr>
</tbody>
</table>

Solution:

\[
\frac{x_1}{y_1} = \frac{3}{4.5} = \frac{1}{1.5}
\]

\[
\frac{x_2}{y_2} = \frac{5}{7.5} = \frac{1}{1.5}
\]

\[
\frac{x_3}{y_3} = \frac{8}{12} = \frac{1}{1.5}
\]

\[
\frac{x_4}{y_4} = \frac{11}{16.5} = \frac{1}{1.5}
\]

\[
\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{x_4}{y_4}
\]

Yes, x and y vary directly.

(ii)

<table>
<thead>
<tr>
<th>x</th>
<th>16</th>
<th>30</th>
<th>40</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>32</td>
<td>60</td>
<td>80</td>
<td>84</td>
</tr>
</tbody>
</table>

Solution:

\[
\frac{x_1}{y_1} = \frac{16}{32} = \frac{1}{2}
\]

\[
\frac{x_2}{y_2} = \frac{30}{60} = \frac{1}{2}
\]

\[
\frac{x_3}{y_3} = \frac{40}{80} = \frac{1}{2}
\]

\[
\frac{x_4}{y_4} = \frac{56}{84} = \frac{28}{42} = \frac{14}{21} = \frac{7}{3}
\]
\[ \Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3} = \frac{x_4}{y_4} \]

x and y are not in direct variation.

(iii)

<table>
<thead>
<tr>
<th>x</th>
<th>27</th>
<th>45</th>
<th>54</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>81</td>
<td>180</td>
<td>216</td>
<td>225</td>
</tr>
</tbody>
</table>

Solution:

\[ \frac{x_1}{y_1} = \frac{27}{81} = \frac{3}{9} = \frac{1}{3} \quad \text{(Forming the given data in fractional form)} \]

\[ \frac{x_2}{y_2} = \frac{45}{180} = \frac{15}{60} = \frac{3}{12} \]

\[ \frac{x_3}{y_3} = \frac{54}{216} = \frac{18}{72} = \frac{1}{4} \]

\[ \frac{x_4}{y_4} = \frac{75}{225} = \frac{25}{45} = \frac{5}{9} \]

\[ \Rightarrow \frac{x_1}{y_1} \neq \frac{x_2}{y_2} \neq \frac{x_3}{y_3} \neq \frac{x_4}{y_4} \]

x and y are not in direct variation.

Question 2.

If x and y vary directly, find the values of x, y and z.

<table>
<thead>
<tr>
<th>X</th>
<th>3</th>
<th>x</th>
<th>y</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>36</td>
<td>60</td>
<td>96</td>
<td>z</td>
</tr>
</tbody>
</table>

Solution:

X and y are in direct variation

\[ \therefore \frac{3}{36} = \frac{x}{60} = \frac{y}{96} = \frac{10}{z} \quad \text{(Forming the given data in fractional form)} \]

\[ \Rightarrow \frac{3}{36} = \frac{x}{60} \quad \frac{3}{36} = \frac{y}{96} \quad \frac{3}{36} = \frac{10}{z} \]

\[ x = \frac{3}{36} \times 60, y = \frac{3}{36} \times 96 \]

\[ z = 10 \times \frac{36}{3} \]
\[ x = 5, y = 8, z = 120 \]

<table>
<thead>
<tr>
<th>X</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>36</td>
<td>60</td>
<td>96</td>
<td>120</td>
</tr>
</tbody>
</table>

**Question 3.**

A truck consumes 28 liters of diesel for moving through a distance of 448km. How much distance will it cover in 64 liters of diesel?

**Solution:**

Let the truck cover x km in 64 liters of diesel.

<table>
<thead>
<tr>
<th>Diesel (in liters)</th>
<th>28</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (in km)</td>
<td>448</td>
<td>x</td>
</tr>
</tbody>
</table>

It is the case of direct variation

(Forming the given data in fractional form)

\[ \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{28}{448} = \frac{64}{x} \]

i.e., \[ 28x = 64 \times 448 \]

\[ x = \frac{64 \times 448}{28} = 1024\text{km} \]

**Question 4.**

For 100km, a taxi charges ₹ 1,800. How much will it charge for a journey of 120 km?

**Solution:**

Let a charges of car is ₹ x in 120km

<table>
<thead>
<tr>
<th>Distance in (km)</th>
<th>1800</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxi charges (₹)</td>
<td>100</td>
<td>120</td>
</tr>
</tbody>
</table>

Since it is the case of direct variation

\[ \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{1800}{100} \times \frac{x}{120} \]

\[ 100x = 1800 \times 120 \]

\[ x = \frac{1800 \times 120}{100} = 2160\text{km} \]
Question 5.

If 27 identical articles cost ₹1,890, how many articles can be bought for ₹1,750?

Solution:

Let x number of articles be purchased in ₹ 1750

<table>
<thead>
<tr>
<th>Cost (₹)</th>
<th>1890</th>
<th>1750</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of articles</td>
<td>27</td>
<td>x</td>
</tr>
</tbody>
</table>

Since, it is a case of direct variation

\[ \frac{1890}{27} = \frac{1750}{x} \]

\[ x = \frac{1750 \times 27}{1890} \]

= 25 articles

Question 6.

7kg of rice costs Rs.1,120. How much rice can be bought for Rs.3,680?

Solution:

Rice : Cost : Rice : Cost
7kg : 1120 :: x kg : 3680

\[ x = \frac{7 \times 3680}{1120} = 23 \text{kg} \]

Question 7.

6 note-books cost ₹156, find the cost of 54 such note-books.

Solution:

Notebooks : cost :: notebooks : cost
6 : Rs.156 :: 54 : Rs.x

\[ x = \frac{156 \times 54}{6} = Rs.1404 \]

Question 8.
22 men can dig a 27m long trench in one day. How many men should be employed for digging 135m long trench of the same type in one day?

Solution:

Men : length trench :: men : length of trench

22 : 27m :: x : 135m (Expressing in ratios)

\[ x = \frac{22 \times 135}{27} = 110 \text{ men} \]

Question 9.

If the total weight of 11 identical articles is 77 kg, how many articles of the same type would weigh 224 kg?

Solution:

No. of : weight :: no. of articles : weight

Articles

11 : 77 kg :: x : 224 kg

\[ x = \frac{11 \times 224}{77} = 32 \text{ articles} \]

Question 10.

A train is moving with uniform speed of 120 km per hour.

(i) How far will it travel in 36 minutes?

Solution:

Speed of train in 60 minutes = 120 km

i.e. distance covered in 60 minutes = \( \frac{120}{60} \)

Distance covered in 36 minutes = \( \frac{120 \times 36}{60} \)

= \( 2 \times 36 = 73 \) km

(ii) In how much time will it cover 210 km?

Solution:

If distance covered is 120 km then time taken = 60 minutes
If distance covered is 1 km then time taken = \( \frac{60}{120} \) minutes

If distance covered is 210 km then time taken = \( \frac{60}{120} \times 210 = 105 \) minutes

= 1 hour 45 minutes

**Question 1.**

Check whether \( x \) and \( y \) vary inversely or not.

(i)

<table>
<thead>
<tr>
<th>( x )</th>
<th>4</th>
<th>3</th>
<th>12</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>24</td>
</tr>
</tbody>
</table>

**Solution:**

\( x \) and \( y \) are inversely proportional

Then \( xy \) are equal.

(i) \( xy = 4 \times 6 = 24 \)

\( xy = 3 \times 8 = 24 \)

\( xy = 12 \times 2 = 24 \) (Using the data’s in the table)

\( xy = 1 \times 24 = 24 \)

\( xy \) in each case is equal

\( x \) and \( y \) are inversely proportional

(ii)

<table>
<thead>
<tr>
<th>( x )</th>
<th>30</th>
<th>120</th>
<th>60</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>60</td>
<td>30</td>
<td>30</td>
<td>75</td>
</tr>
</tbody>
</table>

**Solution:**

\( xy = 30 \times 60 = 1800 \)

\( xy = 120 \times 30 = 3600 \)
xy = 60 \times 30 = 1800
xy = 24 \times 75 = 1800

xy in each case is not equal.
x and y are not inversely proportional

Question 2.
If x and y vary inversely, find the values of 1, m and n:

(i)

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>8</th>
<th>2</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>l</td>
<td>m</td>
<td>n</td>
</tr>
</tbody>
</table>

Solution:
\[ \therefore x \text{ and } y \text{ are inversely proportional} \]
\[ \therefore xy \text{ is equal} \]

Now,
\[ xy = 4 \times 4 = 16 \]
\[ 8 \times l = 16 \Rightarrow l = \frac{16}{8} = 2 \]
\[ 2 \times m = 16 \Rightarrow m = \frac{16}{2} = 8 \]
\[ 32 \times n = 16 \Rightarrow n = \frac{16}{32} = 0.5 \]

(ii)

<table>
<thead>
<tr>
<th>x</th>
<th>24</th>
<th>32</th>
<th>m</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>l</td>
<td>12</td>
<td>8</td>
<td>n</td>
</tr>
</tbody>
</table>

Solution:
\[ \therefore x \text{ and } y \text{ are inversely proportional} \]
\[ \therefore xy \text{ is equal} \]
Now,

(ii) \( xy = 32 \times 12 = 384 \)

\( 24 \times l = 384 \Rightarrow l = \frac{384}{24} = 16 \)

\( m \times 8 = 384 \Rightarrow m = \frac{384}{8} = 48 \)

\( 16 \times n = 384 \Rightarrow n = \frac{384}{16} = 24 \)

Question 3.

36 men can do a piece of work in 7 days. How many men will do the same work in 42 days?

Solution:

Men : Days :: Men : Days

36 : 7 :: x : 42

∴ By inverse proportional

\( 36 \times 7 = x \times 42 \)

\( \Rightarrow x = \frac{36\times7}{42} = 6 \text{ men} \)

Question 4.

12 pipes, all of the same size, fill a tank in 42 minutes. How long will it take to fill the same tank, if 21 pipes of the same size are used?

Solution:

Pipes : Time :: Pipes : Time

12 : 2x :: 21 : 42

∴ By inverse proportion

\( 12 \times 42 = 21 \times x \)

\( \Rightarrow x = \frac{12\times42}{21} = 24 \text{ minutes} \)

Question 5.
In a fort 150 men had provisions for 45 days. After 10 days, 25 men left the fort. How long would the food last at the same rate?

Solution:

After 10 days: For 150 men, provision will last \((45-10)\) days = 35 days

\[\Rightarrow \text{For 1 man, the provisions will last} = 150 \times 35 \text{ days}\]

And for \((150-25) = 125\) men, the provisions will last for

\[\frac{150 \times 35}{125} = 42 \text{ days}\]