Question 1

Evaluate:

(i) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

Solution:

$= \left(\frac{1}{3} \times \frac{1}{9}\right) \div \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{27} \div \frac{1}{9}$ (Expressing the equation in fractional form)

$= \frac{1}{27} \times \frac{9}{1} = \frac{1}{3}$

(ii) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

Solution:

$= \left(\frac{1}{3} \times \frac{1}{4}\right) \div \frac{1}{6}$

$= \frac{1}{12} \div \frac{1}{6}$ (Expressing the equation in fractional form)

$= \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$

(iii) $(2^{-1} + 3^{-1})^3$

Solution:

$= \left(\frac{1}{2} + \frac{1}{3}\right)^3 = \left(\frac{1\times3}{2\times3} + \frac{1\times2}{3\times2}\right)^3$

$= \left(\frac{3+2}{6}\right)^3 = \left(\frac{5}{6}\right)^3$ (Expressing the equation in fractional form)

$= \frac{5\times5\times5}{6\times6\times6} = \frac{125}{216}$

(iv) $(3^{-1} \div 4^{-1})^2$

Solution:

$= \left(\frac{1}{3} \div \frac{1}{4}\right)^2$ (Expressing the equation in fractional form)

$= \left(\frac{1}{3} \times \frac{4}{1}\right)^2 = \left(\frac{4}{3}\right)^2$ (Expressing the equation in mixed fraction)

$= \frac{16}{9} = 1\frac{7}{9}$

(v) $\left(2^2 + 3^2\right) \times \left(\frac{1}{2}\right)^2$

Solution:

$= (2 \times 2) + (3 \times 3) \times \left(\frac{1}{2} \times \frac{1}{2}\right)$

$= 4 + 9 \times \frac{1}{4} = \frac{13}{4} = 3\frac{1}{4}$ (Simplifying the given equation)

(vi) $\left(5^2 - 3^2\right) \times \left(\frac{2}{3}\right)^{-3}$

Solution:

$= (5 \times 5) - (3 \times 3) \times \left(\frac{3}{2}\right)^3$

$= 25 - 9 \times \left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}\right)$ (Simplifying the given equation)

$= 16 \times \frac{27}{8} = 54$

(vii) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] + \left(\frac{1}{6}\right)^{-3}$

Solution:

$= \left[\left(\frac{4}{1}\right)^3 - \left(\frac{3}{1}\right)^3\right] \div \left(\frac{6}{1}\right)^3$

$= \left(\frac{4}{1} \times \frac{4}{1} \times \frac{4}{1} - \frac{3}{1} \times \frac{3}{1} \times \frac{3}{1}\right) \div \left(\frac{6}{1}\right)^3$

$= 64 - 27 \times \left(\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}\right)$ (Simplifying the given equation)

$= 37 \times \frac{1}{216} = \frac{37}{216}$

(viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

Solution:

$\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-2\times 2} = \left(-\frac{3}{4}\right)^{-4}$

$= \left(\frac{4}{3}\right)^4 = \frac{4\times4\times4\times4}{3\times3\times3\times3}$

$= \frac{256}{81} = 3\frac{13}{81}$ (Simplifying the given equation)

(ix) $\left(\left(\frac{3}{5}\right)^{-2}\right)^{-2}$

Solution:

$\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2} = \left(\frac{3}{5}\right)^{-2x(-2)} = \left(\frac{3}{5}\right)^4$

$= \frac{3\times3\times3\times3}{5\times5\times5\times5} = \frac{81}{625}$ (Simplifying the given equation)

(x) $(5^{-1} \times 3^{-1}) + 6^{-1}$

Solution:

$= \left(\frac{1}{5} \times \frac{1}{3}\right) + \frac{1}{6}$

$= \frac{1}{15} \div \frac{1}{6}$ (Simplifying the given equation)

$= \frac{1}{15} \times \frac{6}{1} = \frac{2}{5}$

Question 2

$1125 = 3^m \times 5^n$; find m and n

Solution:

$1125 = 3^2 \times 5^3$

The factors of 1125 are 3×3×5×5×5

3	1125
3	375
5	125
5	25
5	5
	1

$\therefore 1125 = 3 \times 3 \times 5 \times 5 \times 5$

Now comparing, $3^2 \times 5^3 = 3^m \times 5^n$

$\therefore m = 2\ n = 3$

Question 3

Find x, if $9 \times 3^x = (27)^{2x-3}$

Solution:

$9 \times 3^x = (27)^{2x-3}$

$3^2 \times 3^x = (3 \times 3 \times 3)^{2x-3}$ (Simplifying the given equation)

$\Rightarrow 3^{x+2} = (3)^{3(2x-3)}$

$\Rightarrow 3^{x+2} = (3)^{6x-9}$

Since, bases are same, compare them,

x+2=6x-9

6x-x=9+2

$\Rightarrow 5x = 11$

$\Rightarrow x = \frac{11}{2}$(Shifting the terms)

$\Rightarrow x = 2\frac{1}{5}$

Exercise 2(B)

Question 1.

Compute:

(i) $\mathbf{1^8 \times 3^0 \times 5^3 \times 2^2}$

Solution:

$1^8 \times 3^0 \times 5^3 \times 2^2$

$= 1 \times 1 \times 5 \times 5 \times 5 \times 2 \times 2$

$= 125 \times 4$ (Simplifying the given equation)

$= 500$

(ii) $\left(4^7\right)^2 \times \left(4^{-3}\right)^4$

Solution:

(ii) $(4^7)^2 \times (4^{-3})^4$

$= 4^{14} \times 4^{-12}$

$= 4^{14-12} = 4^2$(Simplifying the given equation)

$= 4 \times 4 = 16$

(iii) $\left(2^{-9} \div 2^{-11}\right)^3$

Solution:

$= (2^{-9+11})^3$

$= (2^2)^3 = 2^6$ (Simplifying the given equation)

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(iv) $\left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2}$

$\left(\frac{2}{3}\right)^{-4} \times \left(\frac{27}{8}\right)^{-2} = \left(\frac{2}{3}\right)^{-4} \times \left(\frac{3^3}{2^3}\right)^{-2}$

$= \frac{2^{-4}}{3^{-4}} \times \frac{3^{-6}}{2^{-6}} = \frac{2^{-4}}{2^{-6}} \times \frac{3^{-6}}{3^{-4}}$

$= 2^{-4+6} \times \frac{1}{3^{-4+6}} = \frac{2^2}{3^2} = \frac{4}{9}$

(v) $\left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$

$\left(\frac{56}{28}\right)^0 \div \left(\frac{2}{5}\right)^3 \times \frac{16}{25}$

$= 1 \div \frac{2^3}{5^3} \times \frac{2\times2\times2\times2}{5\times5}$

$\left[\because \left(\frac{56}{28}\right)^0 = 1\right]$

$= 1 \times \frac{5^3}{2^3} \times \frac{2^4}{5^2} = 5^{3-2} \times 2^{4-3}$

$= 5^1 \times 2^1 = 10$

(vi) $(\mathbf{12})^{-2} \times \mathbf{3^3}$

$= (2 \times 2 \times 3)^{-2} \times 3^3$

$= (2^2 \times 3)^{-2} \times 3^3$

$= 2^{-2\times2} \times 3^{-2} \times 3^3$

$= 2^{-4} \times 3^{-2+3} \times 3^3$

$= 2^{-4} \times 3^1$

$= \frac{3}{2^4} = \frac{3}{2\times2\times2\times2} = \frac{3}{16}$

(vii) $(-\mathbf{5})^4 \times (-\mathbf{5})^6 \div (-\mathbf{5})^9$

$= (-5)^4 \times (-5)^6 \times \frac{1}{(-5)^9}$

$= (-5)^{4+6-9}$

$= (-5)^1 = -5$

(viii) $\left(-\frac{1}{3}\right)^4 \div \left(-\frac{1}{3}\right)^8 \times \left(-\frac{1}{3}\right)^5$

$= \left(-\frac{1}{3}\right)^4 \times \frac{1}{\left(-\frac{1}{3}\right)^8} \times \left(-\frac{1}{3}\right)^5$

$= \left(-\frac{1}{3}\right)^{4+5-8} = \left(-\frac{1}{3}\right)^{9-8}$

$= -\frac{1}{3}$

(ix) $\mathbf{9^0 \times 4^{-1} \div 2^{-4}}$

$9^0 \times 4^{-1} \div 2^{-4} = 1 \times \frac{1}{4^1} \times \frac{1}{2^{-4}}$

$= 1 \times \frac{1}{4} \times 2^4 = 1 \times \frac{1}{2^2} \times 2^4$

$= 2^{4-2} = 2^2 = 4$

(x) $(625)^{-\frac{3}{4}}$

$(625)^{-\frac{3}{4}} = (5 \times 5 \times 5 \times 5)^{-\frac{3}{4}}$

$= (5^4)^{-\frac{3}{4}} = 5^{4 \times -\frac{3}{4}}$

$= 5^{-3} = \frac{1}{5^3}$

$= \frac{1}{5 \times 5 \times 5}$

$= \frac{1}{125}$

(xi) $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

$\left(\frac{27}{64}\right)^{-\frac{2}{3}} = \left[\frac{(3^3)}{(4^3)}\right]^{-\frac{2}{3}}$

$= \frac{3^{3 \times -\frac{2}{3}}}{4^{3 \times -\frac{2}{3}}} = \frac{3^{-2}}{4^{-2}}$

$= \frac{4^2}{3^2} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9} = 1\frac{7}{9}$

(xii) $\left(\frac{1}{32}\right)^{-\frac{2}{5}}$

$\left(\frac{1}{32}\right)^{-\frac{2}{5}} = \left(\frac{1}{2 \times 2 \times 2 \times 2 \times 2}\right)^{\frac{2}{5}}$

$= \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} = \frac{1}{2^{5 \times -\frac{2}{5}}}$

$= \frac{1}{2^{-2}} = 2^2 = 4$

(xiii) $(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}}$

$(125)^{-\frac{2}{3}} \div (8)^{\frac{2}{3}} = (5^3)^{-\frac{2}{3}} \div (2^3)^{\frac{2}{3}}$

$= 5^{-\frac{2}{3} \times 3} \div 2^{3 \times \frac{2}{3}}$

$= 5^{-2} \div 2^2 = \frac{1}{5^2} \times \frac{1}{2^2}$

$= \frac{1}{25} \times \frac{1}{4} = \frac{1}{100}$

(xiv) $(\mathbf{243})^{\frac{2}{5}} \div (\mathbf{32})^{-\frac{2}{5}}$

$= (3 \times 3 \times 3 \times 3 \times 3)^{\frac{2}{5}} \div (2 \times 2 \times 2 \times 2 \times 2)^{-\frac{2}{5}}$

$= (3^5)^{\frac{2}{5}} \div (2^5)^{-\frac{2}{5}}$

$= 3^{5\times\frac{2}{5}} \div 2^{-\frac{2}{5}\times 5} = 3^2 \div 2^{-2}$

$= 3^2 \times \frac{1}{2^{-2}} = 3^2 \times 2^{+2}$

$= 3 \times 3 \times 2 \times 2 = 36$

(xv) $(-\mathbf{3})^{\mathbf{4}} - (\sqrt[4]{\mathbf{3}})^{\mathbf{0}} \times (-\mathbf{2})^{\mathbf{5}} \div (\mathbf{64})^{\frac{2}{3}}$

$= (-3 \times -3 \times -3 \times -3) - 1 \times -2 \times -2 \times -2 \times -2 \times -2 \div (2^6)^{\frac{2}{3}}$

Note: $(\sqrt[4]{3})^0 = 1$

$= 3^4 + 2^5 \div 2^{6\times\frac{2}{3}}$

$= 3^4 + 2^5 \div 2^4 = 3^4 + \frac{2^5}{2^4}$

$= 3^4 + 2^{5-4} = 3^4 + 2 = 3 \times 3 \times 3 \times 3 + 2$

$= 81 + 2 = 83$

(xvi) $(\mathbf{27})^{\frac{2}{3}} \div \left(\frac{\mathbf{81}}{\mathbf{16}}\right)^{-\frac{1}{4}}$

$(27)^{\frac{2}{3}} \div \left(\frac{81}{16}\right)^{-\frac{1}{4}} = (3^3)^{\frac{2}{3}} \div \left(\frac{3^4}{2^4}\right)^{-\frac{1}{4}}$

$= 3^{3\times\frac{2}{3}} \div \frac{3^{-\frac{1}{4}\times 4}}{2^{-\frac{1}{4}\times 4}} = 3^2 \div \frac{3^{-1}}{2^{-1}}$

$= 3^2 \times \frac{2^{-1}}{3^{-1}}$

$= 3^{2+1} \times 2^{-1} = 3^3 \times \frac{1}{2^{+1}}$

$= \frac{3\times3\times3}{2} = \frac{27}{2} = 13\frac{1}{2}$

Question 2

Simplify:

(i) $8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$

Solution:

$= (2^3)^{\frac{4}{3}} + (5^2)^{\frac{3}{2}} - \left(\frac{1}{3^3}\right)^{-\frac{2}{3}}$

$= 2^{3\times\frac{4}{3}} + 5^{2\times\frac{3}{2}} - \frac{1}{3^{3\times\left(\frac{-2}{3}\right)}}$

$= 2^4 + 5^3 - \frac{1}{3^{-2}}$

$= 16 + 125 - 3^2$

=141-9=132

(ii) $\left[(64)^{-2}\right]^{-3} \div \left[\left\{(-8)^2\right\}^3\right]^2$

Solution

$= (2^6)^{-2\times-3} \div (-8)^{2\times3\times2}$

$= 2^{6\times(6)} \div (-8)^{12}$

$= 2^{+36} \div (-8)^{12}$

$= 2^{+36} \div [(-2)^3]^{12} = 2^{36} \div (-2)^{36}$

$= \frac{2^{36}}{(-2)^{36}} = \frac{2^{36}}{2^{36}}$ $\quad (\because 36 \text{ is even})$

$= 2^{36-36} = 2^0 = 1$ $\quad (\because a^0 = 1)$

(iii) $\left(2^{-3} - 2^{-4}\right)\left(2^{-3} + 2^{-4}\right)$

Solution

$= (2^{-3})^2 - (2^{-4})^2$

$\{\because (a-b)(a+b) = a^2 - b^2\}$

$= 2^{-6} - 2^{-8} = \frac{1}{2^6} - \frac{1}{2^8}$

$= \frac{1}{64} - \frac{1}{256}$

$= \frac{4-1}{256} = \frac{3}{256}$

Question 3.

Evaluate:

(i)$(-5)^0$

Solution:

$(-5)^0 = 1 \ (\because a^0 = 1)$

(ii)$8^0 + 4^0 + 2^0$

Solution:

$8^0 + 4^0 + 2^0 = 1 + 1 + 1 = 3 \ (\because a^0 = 1)$

(iii)$(8 + 4 + 2)^0$

Solution:

$(8 + 4 + 2)^0 = (14)^0 = 1 \ (\because a^0 = 1)$

(iv) $4x^0$

Solution:

$4x^0 = 4 \times 1 = 4$

(v)$(4x)^0$

Solution:

$(4x)^0 = 1$

(vi) $\left[\left(10^3\right)^0\right]^5$

Solution:

$[(10^3)^0]^5 = 10^{3\times0\times5} = 10^0 = 1$

(vii) $\left(7x^0\right)^2$

Solution:

$(7x^0)^2 = 7^2 \times x^{0\times2} = 49 \times 1 = 49$

(viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

Solution:

$9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

$= 1 + \frac{1}{9} - \frac{1}{9^2} + (3^2)^{\frac{1}{2}} - (3^2)^{-\frac{1}{2}}$

$= 1 + \frac{1}{9} - \frac{1}{81} + 3^{2\times\frac{1}{2}} - 3^{2\times\left(-\frac{1}{2}\right)}$

$= 1 + \frac{1}{9} - \frac{1}{81} + 3 - 3^{-1}$

$= 1 + \frac{1}{9} - \frac{1}{81} + \frac{3}{1} - \frac{1}{3}$

$= \frac{81+9-1+243-27}{81} = \frac{333-28}{81}$

$= \frac{305}{81} = 3\frac{62}{81}$

Question 4.

Simplify:

(i) $\frac{a^5b^2}{a^2b^{-3}}$

Solution:

$\frac{a^5b^2}{a^2b^{-3}} = a^{5-2} \cdot b^{2+3} = a^3b^5$

(ii) $\mathbf{15y^8 \div 3y^3}$

Solution:

$15y^8 \div 3y^3 = \frac{15y^8}{3y^3}$

$= 5\, y^{\{8-3\}}$

$= 5y^5$

(iii) $\boldsymbol{x^{10}y^6 \div x^3\, y^{-2}}$

Solution:

$x^{10}y^6 \div x^3y^{-2} = \frac{x^{10}y^6}{x^3y^{-2}}$

$= x^{10-3} \cdot y^{6+2}$

$= x^7y^8$

(iv) $\mathbf{5z^{16} \div 15z^{-11}}$

Solution:

$5z^{16} \div 15z^{-11} = \frac{5z^{16}}{15z^{-11}}$

$= \frac{5}{15} z^{16+11}$

$= \frac{1}{3} z^{27}$

(v) $\boldsymbol{(36x^2)^{\frac{1}{2}}}$

Solution:

$(36x^2)^{\frac{1}{2}} = (36)^{\frac{1}{2}} \cdot x^{2\times\frac{1}{2}}$

$= (6 \times 6)^{\frac{1}{2}} \cdot x = (6^2)^{\frac{1}{2}} \cdot x$

$= 6x$

(vi) $\left(\mathbf{125x^{-3}}\right)^{\frac{1}{3}}$

Solution:

$(125x^{-3})^{\frac{1}{3}} = (125)^{\frac{1}{3}} x^{-3\times\frac{1}{3}}$

$= (5 \times 5 \times 5)^{\frac{1}{3}} x^{-1}$

$(5^3)^{\frac{1}{3}} \cdot x^{-1} = 5x^{-1}$

$= \frac{5}{x} = 5x^{-1}$

(vii) $\left(\mathbf{2x^2y^{-3}}\right)^{-2}$

Solution:

$(2x^2y^{-3})^{-2} = 2^{-2} x^{2\times-2} \cdot y^{-3x-2}$

$= \frac{1}{2^2} x^{-4} \cdot y^6$

$= \frac{1}{4} \times \frac{y^6}{x^4}$

$= \frac{y^6}{4x^4} = \frac{1}{4} \cdot y^6 x^{-4}$

(viii) $\left(\mathbf{27x^{-3}y^6}\right)^{\frac{2}{3}}$

Solution:

$(27x^{-3}y^6)^{\frac{2}{3}} = (27)^{\frac{2}{3}} \cdot x^{-3\times\frac{2}{3}} \cdot y^{6\times\frac{2}{3}}$

$= (3 \times 3 \times 3)^{\frac{2}{3}} x^{-2} \cdot y^4$

$= \left[(3 \times 3 \times 3)^{\frac{1}{3}}\right]^2 x^{-2} \cdot y^4$

$= 3^2 x^{-2} y^4$

$= 9x^{-2}y^4$

$= \frac{9y^4}{x^2} = 9x^{-2}y^4$

(ix) $\left(-2x^{\frac{2}{3}}y^{-\frac{3}{2}}\right)^6$

Solution:

$= (-2)^6\, x^{\frac{2}{3}\times 6} y^{-\frac{3}{2}\times 6}$

$= 64x^4y^{-9}$

$= \frac{64x^4}{y^9}$

$= 64x^4y^{-9}$

Question 5.

Simplify:

$\left(x^{a+b}\right)^{a-b} \cdot \left(x^{b+c}\right)^{b-c} \cdot \left(x^{c+a}\right)^{c-a}$

Solution:

$\left(x^{a+b}\right)^{a-b} \cdot \left(x^{b+c}\right)^{b-c} \cdot \left(x^{c+a}\right)^{c-a}$

$= x^{(a+b)(a-b)} x^{(b+c)(b-c)} x^{(c+a)(c-a)}$

$= x^{a^2-b^2} x^{b^2-c^2} x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= x^0$

$= 1$

Question 6.

Simplify:

(i) $\sqrt[5]{x^{20}y^{-10}z^{5}} \div \frac{x^3}{y^3}$

Solution:

$\sqrt[5]{x^{20}y^{-10}z^{5}} \div \frac{x^3}{y^3}$

$= (x^{20}y^{-10}z^{5})^{\frac{1}{5}} \div \frac{x^3}{y^3}$

$x^{20\times\frac{1}{5}} \cdot y^{-10\times\frac{1}{5}} \cdot z^{5\times\frac{1}{5}} \div \frac{x^3}{y^3}$

$= x^4 \cdot y^{-2} \cdot z^1 \times \frac{y^3}{x^3}$

$= x^{4-3} \cdot y^{-2+3} \cdot z^1$

= xyz

(ii) $\left(\frac{256a^{16}}{81b^4}\right)^{\frac{-3}{4}}$

Solution:

$\left[\frac{256a^{16}}{81b^4}\right]^{-\frac{3}{4}} = \left[\frac{4^4a^{16}}{3^4b^4}\right]^{\frac{-3}{4}}$

Where $256 = 4 \times 4 \times 4 \times 4 = 4^4$

$81 = 3 \times 3 \times 3 \times 3 = 3^4$

$= \frac{4^{4\times\frac{-3}{4}} \cdot a^{16\times\frac{-3}{4}}}{3^{4\times\frac{-3}{4}} b^{4\times\frac{-3}{4}}}$

$= \frac{4^{-3} \cdot a^{-12}}{3^{-3} \cdot b^{-3}}$

$= \frac{3^3 b^3}{4^3 a^{12}}$

$= \frac{27b^3}{64a^{12}}$

$= \frac{27}{64} \cdot a^{-12}b^3$

Question 7

(i) $\left(a^{-2}\right)^{-2} \cdot (ab)^{-3}$

Solution:

$\left(a^{-2}\right)^{-2} \cdot (ab)^{-3}$

$= (a^{-2\times-2}.b^{-2}) \cdot (a^{-3} \cdot b^{-3})$

$= a^{+4} \cdot b^{-2} \cdot a^{-3} \cdot b^{-3}$

$= a^{4-3} \cdot b^{-2-3}$

$= ab^{-5}$

$= \frac{a}{b^5}$

(ii) $(x^n y^{-m})^4 \times \left(x^3 y^{-2}\right)^{-n}$

Solution:

$(x^n y^{-m})^4 \times (x^3 y^{-2})^{-n} = x^{4n} y^{-4m} \times x^{-3n} y^{2n}$

$= x^{4n-3n} \cdot y^{-4m+2n}$

$= x^n y^{-4m+2n}$

(iii) $\left(\frac{125a^{-3}}{y^6}\right)^{\frac{-1}{3}}$

Solution:

$\left[\frac{125a^{-3}}{y^6}\right]^{-\frac{1}{3}} = \left[\frac{5^3a^{-3}}{y^6}\right]^{-\frac{1}{3}}$

Where $125 = 5 \times 5 \times 5 = 5^3$

$= \frac{5^{3\times\frac{-1}{3}} \cdot a^{-3\times\frac{-1}{3}}}{y^{6\times\frac{-1}{3}}}$

$= \frac{5^{-1} \cdot a^{1}}{y^{-2}}$

$= \frac{a \cdot y^2}{5}$

(iv) $\left(\frac{32x^{-5}}{243y^{-5}}\right)^{\frac{-1}{5}}$

Solution:

$\left[\frac{32x^{-5}}{243y^{-5}}\right]^{\frac{-1}{5}} = \left[\frac{2^5x^{-5}}{3^5y^{-5}}\right]^{\frac{-1}{5}}$

Where $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

$= \frac{2^{5\times\frac{-1}{5}} \cdot x^{-5\times\frac{-1}{5}}}{3^{5\times\frac{-1}{5}} y^{-5\times\frac{-1}{5}}}$

$= \frac{2^{-1}x^{+1}}{3^{-1}y^{+1}}$

$= \frac{3x}{2y}$

(v) $(a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$

Solution:

$(a^{-2}b)^{\frac{1}{2}} \times (ab^{-3})^{\frac{1}{3}}$

$= \left(a^{-2\times\frac{1}{2}} \cdot b^{\frac{1}{2}}\right) \times \left(a^{\frac{1}{3}} b^{-3\times\frac{1}{3}}\right)$

$= a^{-1}b^{\frac{1}{2}} \times a^{\frac{1}{3}}b^{-1}$

$= a^{-1+\frac{1}{3}} b^{\frac{1}{2}-1}$

$= a^{-\frac{2}{3}} b^{-\frac{1}{2}}$

$= \frac{1}{a^{\frac{2}{3}} b^{\frac{1}{2}}}$

(vi) $(xy)^{m-n} \cdot (yz)^{n-l} \cdot (zx)^{l-m}$

Solution:

$(xy)^{m-n} \cdot (yz)^{n-l} \cdot (xz)^{l-m}$

$= x^{m-n} \cdot y^{m-n} \cdot y^{n-l} \cdot z^{n-l} x^{l-m} \cdot z^{l-m}$

$= x^{m-n+l-m} \cdot y^{m-n+n-l} \cdot z^{n-l+l-m}$

$= x^{l-n} \cdot y^{m-l} \cdot z^{n-m}$

Question 8.

Show that:

$\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a} = 1$

Solution:

L.H.S. = $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b-c} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c-a}$

$= \left(x^{a+b}\right)^{a-b} \cdot \left(x^{b+c}\right)^{b-c} \cdot (x^{c+a})^{c-a}$

$= x^{(a+b)(a-b)} x^{(b+c)(b-c)} x^{(c+a)(c-a)}$

$= x^{a^2-b^2} x^{b^2-c^2} x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= x^0$

=1 = R.H.S

Question 9.

Evaluate:

$$\frac{x^{5+n}(x^2)^{3n+1}}{x^{7n-2}}$$

Solution:

$$\frac{x^{5+n}\times(x^2)^{3n+1}}{x^{7n-2}}$$

$$=\frac{x^{5+n}\times x^{2(3n+1)}}{x^{7n-2}}$$

$$=\frac{x^{5+n}\times x^{6n+2}}{x^{7n-2}}$$

$$=x^{5+n+6n+2-7n+2}$$

$$=x^9$$

Question 10 .

Evaluate:

$$\frac{a^{2n+1}\times a^{(2n+1)(2n-1)}}{a^{n(4n-1)}\times(a^2)^{2n+3}}$$

Solution:

$$\frac{a^{2n+1}\times a^{(2n+1)(2n-1)}}{a^{n(4n-1)}\times(a^2)^{2n+3}}$$

$$=\frac{a^{2n+1}\times a^{(2n)^2-(1)^2}}{a^{4n^2-n}\times a^{2(2n+3)}}$$

$$=\frac{a^{2n+1}\times a^{4n^2-1}}{a^{4n^2-n}\times a^{4n+6}}$$

$$=a^{2n+1+4n^2-1-4n^2+n-4n-6}$$

$$=a^{-n-6}$$

$$=a^{-(n+6)}$$

$$=\frac{1}{a^{n+6}}$$

Question 11.

$(m+n)^{-1}(m^{-1}+n^{-1}) = (mn)^{-1}$

Solution:

L.H.S. $= (m+n)^{-1}(m^{-1}+n^{-1})$

$= \frac{1}{m+n}\left(\frac{1}{m}+\frac{1}{n}\right) = \frac{1}{m+n}\cdot\frac{n+m}{mn} = \frac{1}{mn}$

$= (mn)^{-1}$

=R.H.S.

Hence proved.

Question 12 .

Prove that:

(i) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = \mathbf{1}$

Solution:

$\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

L.H.S $= \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$= \left(x^{a-b}\right)^{\frac{1}{ab}}\left(x^{b-c}\right)^{\frac{1}{bc}}\left(x^{c-a}\right)^{\frac{1}{ca}}$

$= x^{\frac{a-b}{ab}}\cdot x^{\frac{b-c}{bc}}\cdot x^{\frac{c-a}{ca}}$

$= x^{\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c-a}{ca}}$

$= x^{\frac{ac-bc+ab-ac+bc-ab}{abc}}$

$= x^0 = 1$ = R.H.S.

(ii) $\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$

Solution:

$\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$

L.H.S. $=\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}$

$=\frac{1}{x^{a-a}+x^{a-b}}+\frac{1}{x^{b-b}+x^{b-a}}$

$=\frac{1}{x^{a}x^{-a}+x^{a}x^{-b}}+\frac{1}{x^{b}x^{-b}+x^{b}x^{-a}}$

$=\frac{1}{x^{a}(x^{-a}+x^{-b})}+\frac{1}{x^{b}(x^{-b}+x^{-a})}$

$=\frac{1}{(x^{-a}+x^{-b})}\left[\frac{1}{x^{a}}+\frac{1}{x^{b}}\right]$

$=\frac{1}{x^{-a}+x^{-b}}[x^{-a}+x^{-b}]$

=1=R.H.S