Exercise 1.3

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1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$

Solution:

$$
\begin{array}{r|l}
 & 00.36 \\
\hline
100 & 360- \\
 & 300 \\
\hline
 & 600- \\
 & 600 \\
\hline
 & 0
\end{array}
$$

= 0.36 (Terminating)

(ii) $\frac{1}{11}$

Solution:

$$
\begin{array}{r|l}
 & 0.0909\ldots \\
\hline
11 & 1 \\
 & 0 \\
\hline
 & 10 \\
 & 0 \\
\hline
 & 100 \\
 & 99 \\
\hline
 & 10 \\
 & 0 \\
\hline
 & 100 \\
 & 99 \\
\hline
 & 1
\end{array}
$$

= 0.0909… = $0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$4\frac{1}{8} = \frac{33}{8}$

```
      4.125
   8 | 33
     | 32
     |  10
     |   8
     |   20
     |   16
     |    40
     |    40
     |     0
```

= 4.125 (Terminating)

(iv) $\frac{3}{13}$

Solution:

```
      0.230769
  13 | 30
     | 26
     |  40
     |  39
     |   10
     |    0
     |   100
     |    91
     |     90
     |     78
     |     120
     |     117
     |       3
```

= 0.230769… = $0.\overline{230769}$ (Non terminating and repeating)

(v) $\frac{2}{11}$

Solution:

```
      0.18
  11 | 2
     | 0
     | 20
     | 11
     |  90
     |  88
     |   2
```

= 0.181818181818… = $0.\overline{18}$ (Non terminating and repeating)

(vi) $\frac{329}{400}$

Solution:

$$
\begin{array}{r|l}
 & 0.8225 \\
\hline
400 & 329 \\
 & 0 \\
\hline
 & 3290 \\
 & 3200 \\
\hline
 & 900 \\
 & 800 \\
\hline
 & 1000 \\
 & 800 \\
\hline
 & 2000 \\
 & 2000 \\
\hline
 & 0
\end{array}
$$

= 0.8225 (Terminating)

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Solution:

$\frac{1}{7} = 0.\overline{142857}$

$\therefore 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\bar{6}$

Solution:

$0.\bar{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = \frac{2}{3}$

(ii) $0.4\overline{7}$

Solution:

$0.\overline{47} =$
$0.4777\ldots$
$= \frac{4}{10} + \frac{0.777}{10}$

Assume that $x = 0.777\ldots$
Then, $10x = 7.777\ldots$
$10x = 7 + x$
$x = \frac{7}{9}$

$\frac{4}{10} + \frac{0.777\ldots}{10} = \frac{4}{10} + \frac{7}{90}$ ($\because x = \frac{7}{9}$ and $x = 0.777\ldots \Rightarrow \frac{0.777\ldots}{10} = \frac{7}{9\times10} = \frac{7}{90}$)

$= \frac{36}{90} + \frac{7}{90} = \frac{43}{90}$

(iii) $0.\overline{001}$

Solution:
$0.\overline{001} = 0.001001\ldots$
Assume that $x = 0.001001\ldots$
Then, $1000x = 1.001001\ldots$
$1000x = 1 + x$
$999x = 1$
$x = \frac{1}{999}$

4. Express 0.99999.... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:
Assume that $x = 0.9999\ldots$ ——— Eq. (a)
Multiplying both sides by 10,
$10x = 9.9999\ldots$ ——— Eq. (b)
Eq.(b) – Eq.(a), we get
$10x = 9.9999\ldots$ —
$x = 0.9999\ldots$
$9x = 9$
$x = 1$

The difference between 1 and 0.999999 is 0.000001 which is negligible.
Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solution:
$\frac{1}{17}$

Dividing 1 by 17:

$$
\begin{array}{r|l}
 & 0.0588235294117647 \\
\hline
17 & 1 \\
 & 0 \\
\hline
 & 10 \\
 & 0 \\
\hline
 & 100 \\
 & 85 \\
\hline
 & 150 \\
 & 136 \\
\hline
 & 140 \\
 & 136 \\
\hline
 & 40 \\
 & 34 \\
\hline
 & 60 \\
 & 51 \\
\hline
 & 90 \\
 & 85 \\
\hline
 & 50 \\
 & 34 \\
\hline
 & 160 \\
 & 153 \\
\hline
 & 70 \\
 & 68 \\
\hline
 & 20 \\
 & 17 \\
\hline
 & 30 \\
 & 17 \\
\hline
 & 130 \\
 & 119 \\
\hline
 & 110 \\
 & 102 \\
\hline
 & 80 \\
 & 68 \\
\hline
 & 120 \\
 & 119 \\
\hline
 & 1
\end{array}
$$

$\frac{1}{17} = \overline{0.0588235294117647}$

∴There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:

$\frac{1}{2}$ = 0. 5, denominator q = 2^1

$\frac{7}{8}$ = 0. 875, denominator q = 2^3

$\frac{4}{5}$ = 0. 8, denominator q = 5^1

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

We know that all irrational numbers are non-terminating non-recurring. ∴ three numbers with decimal expansions that are non-terminating non-recurring are:

a) $\sqrt{3}$ = 1.732050807568

b) $\sqrt{26}$ = 5.099019513592

c) $\sqrt{101} = 10.04987562112$

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solution:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

∴Three different irrational numbers are:

a) 0.73073007300073000073...

b) 0.75075007300075000075…

c) 0.76076007600076000076...

9. Classify the following numbers as rational or irrational according to their type:

(i) $\sqrt{23}$

Solution:
$\sqrt{23} = 4.79583152331\ldots$

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii) $\sqrt{225}$

Solution:

$\sqrt{225} = 15 = 15/1$

Since the number can be represented in $\frac{p}{q}$ form, it is a rational number.

(iii) 0.3796

Solution:
Since the number, 0.3796, is terminating, it is a rational number.

(iv) 7.478478

Solution:
The number, 7.478478, is non-terminating but recurring, it is a rational number.

(v) 1.101001000100001…

Solution:
Since the number, 1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.