Exercise: 10.5 **(Page No: 184)**

1. In Fig. 10.36, A,B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.

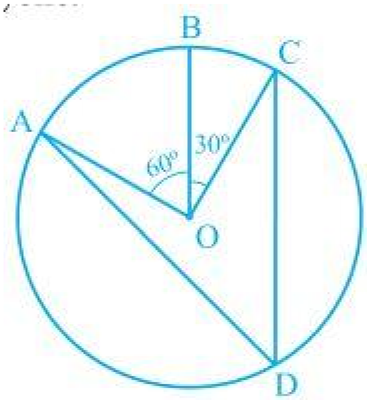


Fig. 10.36

Solution:

It is given that,

∠AOC = ∠AOB + ∠BOC

So, ∠AOC = 60° + 30°

∴ ∠AOC = 90°

It is known that an angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

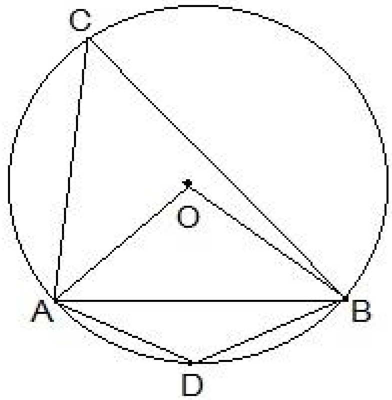
So,

∠ADC = 1/2 ∠AOC

= 1/2 × 90° = 45°

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



Here, the chord AB is equal to the radius of the circle. In the above diagram, OA and OB are the two radii of the circle.

Now, consider the ΔOAB. Here,

AB = OA = OB = radius of the circle.

So, it can be said that ΔOAB has all equal sides and thus, it is an equilateral triangle.

∴ $\angle AOC = 60°$

And, $\angle ACB = 1/2\ \angle AOB$

So, $\angle ACB = 1/2 \times 60° = 30°$

Now, since ACBD is a cyclic quadrilateral,

$\angle ADB + \angle ACB = 180°$ (Since they are the opposite angles of a cyclic quadrilateral)

So, $\angle ADB = 180° - 30° = 150°$

So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc are 150° and 30° respectively.

3. In Fig. 10.37, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.

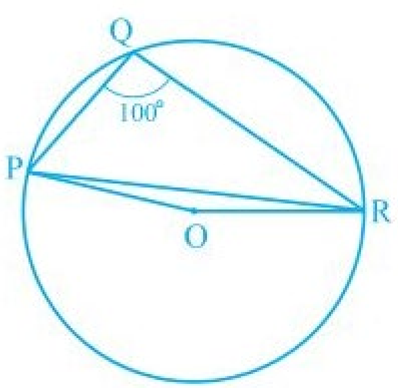


Fig. 10.37

Solution:

Since angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

So, the reflex $\angle POR = 2 \times \angle PQR$

We know the values of angle PQR as 100°

So, $\angle POR = 2 \times 100^\circ = 200^\circ$

$\therefore \angle POR = 360^\circ - 200^\circ = 160^\circ$

Now, in ΔOPR,

OP and OR are the radii of the circle

So, OP = OR

Also, $\angle OPR = \angle ORP$

Now, we know sum of the angles in a triangle is equal to 180 degrees

So,

$\angle POR + \angle OPR + \angle ORP = 180^\circ$

=> $\angle OPR + \angle OPR = 180^\circ - 160^\circ$

As $\angle OPR = \angle ORP$

=> $2\angle OPR = 20^\circ$

Thus, $\angle OPR = 10^\circ$

4. In Fig. 10.38, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.

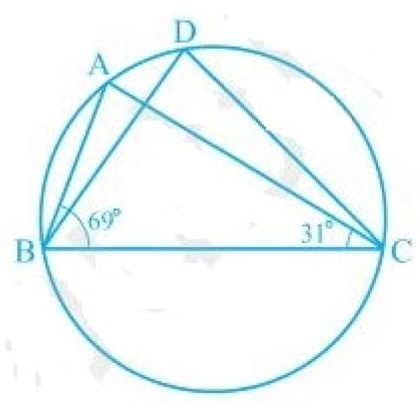


Fig. 10.38

Solution:

We know that angles in the segment of the circle are equal so,

$\angle BAC = \angle BDC$

Now in the In ΔABC, sum of all the interior angles will be 180°

So, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

Now, by putting the values,

$\angle BAC = 180^\circ - 69^\circ - 31^\circ$

So, $\angle BAC = 80^\circ$

5. In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.

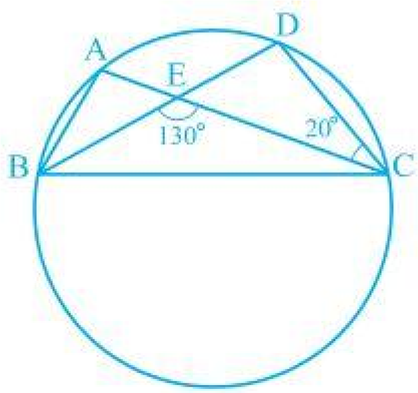


Fig. 10.39

Solution:

We know that the angles in the segment of the circle are equal.

So,

$\angle BAC = \angle CDE$

Now, by using the exterior angles property of the triangle In ΔCDE we get,

$\angle CEB = \angle CDE + \angle DCE$

We know that $\angle DCE$ is equal to 20°

So, $\angle CDE = 110^\circ$

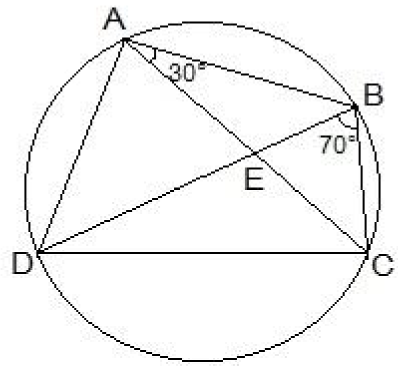
$\angle BAC$ and $\angle CDE$ are equal

$\therefore \angle BAC = 110^\circ$

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Solution:

Consider the following diagram.



Consider the chord CD,

We know that angles in the same segment are equal.

So, $\angle CBD = \angle CAD$

$\therefore \angle CAD = 70°$

Now, $\angle BAD$ will be equal to the sum of angles BAC and CAD.

So, $\angle BAD = \angle BAC + \angle CAD$

$= 30° + 70°$

$\therefore \angle BAD = 100°$

We know that the opposite angles of a cyclic quadrilateral sums up to 180 degrees.

So,

$\angle BCD + \angle BAD = 180°$

It is known that $\angle BAD = 100°$

So, $\angle BCD = 80°$

Now consider the ΔABC.

Here, it is given that AB = BC

Also, $\angle BCA = \angle CAB$ (They are the angles opposite to equal sides of a triangle)

$\angle BCA = 30°$

also, $\angle BCD = 80°$

$\angle BCA + \angle ACD = 80°$

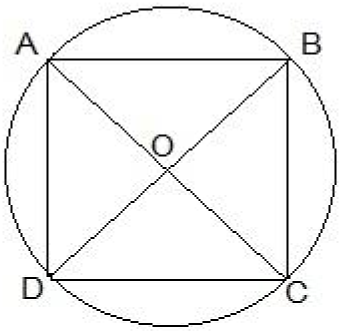
So, $\angle ACD = 50°$ and,

$\angle ECD = 50°$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Draw a cyclic quadrilateral ABCD inside a circle with center O such that its diagonal AC and BD are two diameters of the circle.



We know that the angles in the semi-circle are equal.

So, $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ$

So, as each internal angle is 90°, it can be said that the quadrilateral ABCD is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:

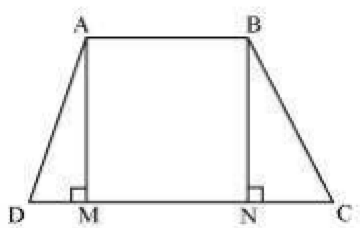
Construction:

Consider a trapezium ABCD with AB | |CD and BC = AD.

Draw AM CD and BN CD.

In ΔAMD and ΔBNC,

The diagram will look as follows:



In ΔAMD and ΔBNC,

AD = BC (Given)

AMD = BNC (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

ΔAMD ΔBNC (RHS congruence rule)

ADC = BCD (CPCT) ... (1)

BAD and ADC are on the same side of transversal AD.

BAD + ADC = 180° ... (2)

BAD + BCD = 180° [Using equation (1)]

This equation shows that the opposite angles are supplementary.

Therefore, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that ∠ACP = ∠QCD.

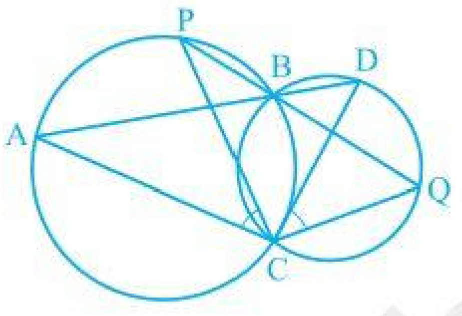


Fig. 10.40

Solution:

Construction:

Join the chords AP and DQ.

For chord AP, we know that angles in the same segment are equal.

So, ∠PBA = ∠ACP -- (i)

Similarly for chord DQ,

∠DBQ = ∠QCD -- (ii)

It is known that ABD and PBQ are two line segments which are intersecting at B.

At B, the vertically opposite angles will be equal.

∴ ∠PBA = ∠DBQ -- (iii)

From equation (i), equation (ii) and equation (iii) we get,

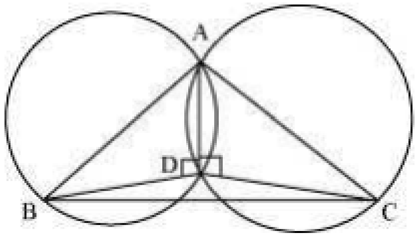
∠ACP = ∠QCD

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution:

First draw a triangle ABC and then two circles having diameter as AB and AC respectively.

We will have to now prove that D lies on BC and BDC is a straight line.



Proof:

We know that angle in the semi-circle are equal

So, $\angle ADB = \angle ADC = 90°$

Hence, $\angle ADB + \angle ADC = 180°$

∴ $\angle BDC$ is straight line.

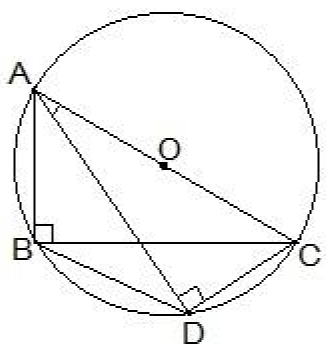
So, it can be said that D lies on the line BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution:

We know that AC is the common hypotenuse and $\angle B = \angle D = 90°$.

Now, it has to be proven that $\angle CAD = \angle CBD$



Since, $\angle ABC$ and $\angle ADC$ are $90°$, it can be said that They lie in the semi circle.

So, triangles ABC and ADC are in the semi circle and the points A, B, C and D are concyclic.

Hence, CD is the chord of the circle with center O.

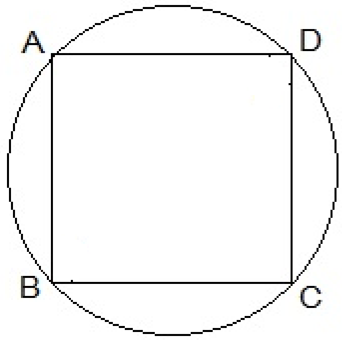
We know that the angles which are in the same segment of the circle are equal.

$\therefore \angle CAD = \angle CBD$

12. Prove that a cyclic parallelogram is a rectangle.

Solution:

It is given that ABCD is a cyclic parallelogram and we will have to prove that ABCD is a rectangle.



Proof:

Let ABCD be a cyclic parallelogram.

A + C = 180° (Opposite angles of a cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal.

A = C and B = D

From equation (1),

A + C = 180°

A + A = 180°

2 A = 180°

A = 90°

Parallelogram ABCD has one of its interior angles as 90°.

Thus, ABCD is a rectangle.