

Exercise 2.3

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1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$

Solution:

$$x+1=0$$

$$\Rightarrow x=-1$$

∴ Remainder:

$$\begin{aligned} p(-1) &= (-1)^3 + 3(-1)^2 + 3(-1) + 1 \\ &= -1 + 3 - 3 + 1 \\ &= 0 \end{aligned}$$

(ii) $x - \frac{1}{2}$

Solution:

$$x - \frac{1}{2} = 0$$

$$\Rightarrow x = \frac{1}{2}$$

∴ Remainder:

$$\begin{aligned} p\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 \\ &= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 \\ &= \frac{27}{8} \end{aligned}$$

(iii) x

Solution:

$$x=0$$

∴ Remainder:

$$\begin{aligned} p(0) &= (0)^3 + 3(0)^2 + 3(0) + 1 \\ &= 1 \end{aligned}$$

(iv) $x+\pi$

Solution:

$$x+\pi=0$$

$$\Rightarrow x=-\pi$$

∴ Remainder:

$$\begin{aligned} p(0) &= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 \\ &= -\pi^3 + 3\pi^2 - 3\pi + 1 \end{aligned}$$

(v) $5+2x$

Solution:

$$5+2x=0$$

$$\Rightarrow 2x=-5$$

$$\Rightarrow x = -\frac{5}{2}$$

∴ Remainder:

$$\begin{aligned} \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1 &= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 \\ &= -\frac{27}{8} \end{aligned}$$

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Solution:

$$\text{Let } p(x) = x^3 - ax^2 + 6x - a$$

$$x - a = 0$$

$$\therefore x = a$$

Remainder:

$$\begin{aligned} p(a) &= (a)^3 - a(a^2) + 6(a) - a \\ &= a^3 - a^3 + 6a - a = 5a \end{aligned}$$

3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Solution:

$$7 + 3x = 0$$

$$\Rightarrow 3x = -7 \text{ only if } 7 + 3x \text{ divides } 3x^3 + 7x \text{ leaving no remainder.}$$

$$\Rightarrow x = \frac{-7}{3}$$

\therefore Remainder:

$$\begin{aligned} 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) &= -\frac{343}{9} + \frac{-49}{3} \\ &= \frac{-343 - (49)3}{9} \\ &= \frac{-343 - 147}{9} \\ &= \frac{-490}{9} \neq 0 \end{aligned}$$

$\therefore 7 + 3x$ is not a factor of $3x^3 + 7x$