Exercise 2.4

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x)= x^3+x^2+x+1$

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1$
$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let $p(x)= x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$
$=1-1+1-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let $p(x)= x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)4+3(-1)3+3(-1)2+(-1)+1$
$=1-3+3-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x)= x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+ \sqrt{2}$
$=-1-1+2+\sqrt{2}+\sqrt{2}$
$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2–2x–1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2–2(-1)–1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii)$p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3x2+3x+1, g(x) = x + 2

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii)$p(x)=x^3–4x^2+x+6$, $g(x) = x – 3$

Solution:

$p(x)= x^3–4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$
$\Rightarrow 1+1+k=0$
$\Rightarrow 2+k=0$
$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0
$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0$
$\Rightarrow 2+k+\sqrt{2}=0$
$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0
By Factor Theorem
$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$
$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0
By Factor Theorem
$\Rightarrow k(1)^2-3(1)+k=0$
$\Rightarrow k-3+k=0$
$\Rightarrow 2k-3=0$
$\Rightarrow k=\frac{3}{2}$

4. **Factorize:**

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-7 and product=1×12=12
We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4=12]

$$12x^2-7x+1=12x^2-4x-3x+1$$
$$=4x(3x-1)-1(3x-1)$$
$$=(4x-1)(3x-1)$$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=2×3=6
We get 6 and 1 as the numbers [6+1=7 and 6×1=6]

$2x^2+7x+3 = 2x^2+6x+1x+3$
$= 2x(x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii) $\mathbf{6x^2+5x-6}$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=5 and product=6×-6= -36
We get -4 and 9 as the numbers [-4+9=5 and -4×9=-36]

$6x^2+5x-6=6x^2+9x-4x-6$
$=3x(2x+3)-2(2x+3)$
$=(2x+3)(3x-2)$

(iv) $\mathbf{3x^2-x-4}$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-1 and product=3×-4= -12
We get -4 and 3 as the numbers [-4+3=-1 and -4×3=-12]

$3x^2-x-4=3x^2-x-4$
$=3x^2-4x+3x-4$
$=x(3x-4)+1(3x-4)$
$=(3x-4)(x+1)$

5. Factorize:

(i) $\mathbf{x^3-2x^2-x+2}$

Solution:

Let $p(x)=x^3-2x^2-x+2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
p(1) = 0
So, (x+1) is factor of p(x)
Now,
$p(x)= x^3-2x^2-x+2$
$p(-1)=(-1)^3-2(-1)^2-(-1)+2$
$=-1-1+1+2$
$=0$
Therefore, (x+1) is the factor of p(x)

$$\begin{array}{r|l} & x^2-3x+2 \\ \hline x+1 & x^3-2x^2-x+2 \\ & x^3+x^2 \\ & - \quad - \\ \hline & -3x^2-x+2 \\ & -3x^2-3x \\ & + \quad\quad + \\ \hline & 2x+2 \\ & 2x+2 \\ & - \quad - \\ \hline & 0 \end{array}$$

Now, Dividend = Divisor × Quotient + Remainder

$$\begin{aligned}(x+1)(x^2-3x+2) &= (x+1)(x^2-x-2x+2) \\ &= (x+1)(x(x-1)-2(x-1)) \\ &= (x+1)(x-1)(x+2)\end{aligned}$$

(ii) x^3-3x^2-9x-5

Solution:

Let $p(x) = x^3-3x^2-9x-5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, (x-5) is factor of p(x)

Now,

$p(x) = x^3-3x^2-9x-5$

$$\begin{aligned}p(5) &= (5)^3-3(5)^2-9(5)-5 \\ &= 125-75-45-5 \\ &= 0\end{aligned}$$

Therefore, (x-5) is the factor of p(x)

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
x\text{-}5 & x^3-3x^2-9x-5 \\
 & x^3-5x^2 \\
 & -\quad + \\
\hline
 & \quad 2x^2-9x-5 \\
 & \quad 2x^2-10x \\
 & \quad -\quad + \\
\hline
 & \qquad x-5 \\
 & \qquad x-5 \\
 & \qquad -\quad + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$
$= (x-5)(x(x+1)+1(x+1))$
$= (x-5)(x+1)(x+1)$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x) = x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$=-1+13-32+20$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$\begin{array}{r|l} & x^2+12x+20 \\ \hline x+1 & x^3+13x^2+32x+20 \\ & x^3+x^2 \\ & - \quad - \\ \hline & 12x^2+32x+20 \\ & 12x^2+12x \\ & - \quad - \\ \hline & 20x+20 \\ & 20x+20 \\ & - \quad - \\ \hline & 0 \end{array}$$

Now, Dividend = Divisor × Quotient + Remainder

$$\begin{aligned}(x+1)(x^2+12x+20) &= (x+1)(x^2+2x+10x+20) \\ &= (x-5)x(x+2)+10(x+2) \\ &= (x-5)(x+2)(x+10)\end{aligned}$$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1) = -2$ are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

$$\begin{aligned}p(y) &= 2y^3+y^2-2y-1 \\ p(1) &= 2(1)^3+(1)^2-2(1)-1 \\ &= 2+1-2 \\ &= 0\end{aligned}$$

Therefore, $(y-1)$ is the factor of $p(y)$

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y\text{-}1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
 & \qquad - \ + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$