Exercise: 7.1 **(Page No: 118)**

1. In quadrilateral ACBD, AC = AD and AB bisect ∠A (see Fig. 7.16). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?

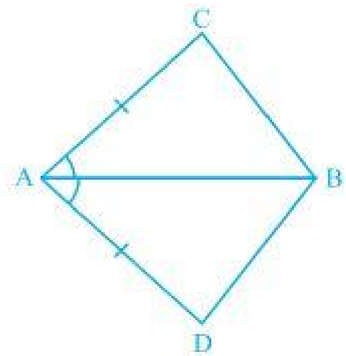

Fig. 7.16

Solution:

It is given that AC and AD are equal i.e. AC=AD and the line segment AB bisects ∠A.

We will have to now prove that the two triangles ABC and ABD are similar i.e. **ΔABC ≅ ΔABD**

Proof:

Consider the triangles ΔABC and ΔABD,

(i) AC = AD (It is given in the question)

(ii) AB = AB (Common)

(iii) ∠CAB = ∠DAB (Since AB is the bisector of angle A)

So, by **SAS congruency criterion**, ΔABC ≅ ΔABD.

For the 2nd part of the question, BC and BD are of equal lengths.

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig. 7.17). Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.

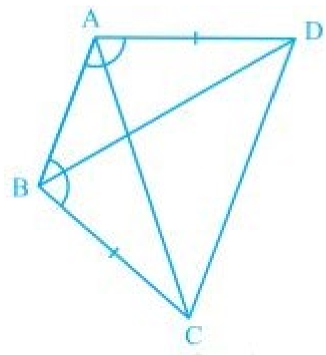

Fig. 7.17

Solution:

The given parameters from the questions are ∠DAB = ∠CBA and AD = BC.

(i) ΔABD and ΔBAC are similar by SAS congruency as

AB = BA (It is the common arm)

∠DAB = ∠CBA and AD = BC (These are given in the question)

So, triangles ABD and BAC are similar i.e. ΔABD ≅ ΔBAC. (Hence proved).

(ii) It is now known that ΔABD ≅ ΔBAC so,

BD = AC (by the rule of CPCT).

(iii) Since ΔABD ≅ ΔBAC so,

Angles ∠ABD = ∠BAC (by the rule of CPCT).

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

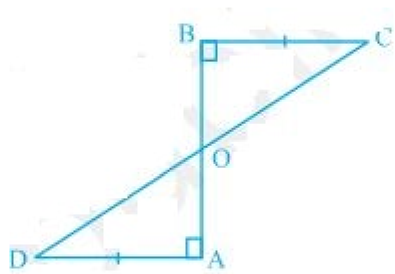

Fig. 7.18

Solution:

It is given that AD and BC are two equal perpendiculars to AB.

We will have to prove that **CD is the bisector of AB**

Proof:

Triangles ΔAOD and ΔBOC are similar by AAS congruency since:

(i) ∠A = ∠B (They are perpendiculars)
(ii) AD = BC (As given in the question)
(iii) ∠AOD = ∠BOC (They are vertically opposite angles)
∴ ΔAOD ≅ ΔBOC.
So, AO = OB (by the rule of CPCT).
Thus, CD bisects AB (Hence proved).

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that ΔABC ≅ ΔCDA.

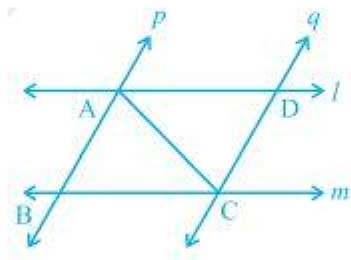

Fig. 7.19

Solution:
It is given that p ∥ q and l ∥ m
To prove:
Triangles ABC and CDA are similar i.e. ΔABC ≅ ΔCDA
Proof:
Consider the ΔABC and ΔCDA,
(i) ∠BCA = ∠DAC and ∠BAC = ∠DCA Since they are alternate interior angles
(ii) AC = CA as it is the common arm
So, by **ASA congruency criterion** ΔABC ≅ ΔCDA.

5. Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A (see Fig. 7.20). Show that:
(i) ΔAPB ≅ ΔAQB
(ii) BP = BQ or B is equidistant from the arms of ∠A.

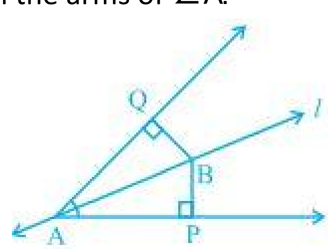

Fig. 7.20

Solution:

It is given that the line "l" is the bisector of angle ∠A and the line segments BP and BQ are perpendiculars drawn from l.

(i) ΔAPB and ΔAQB are similar by AAS congruency because:

∠P = ∠Q (They are the two right angles)

AB = AB (It is the common arm)

∠BAP = ∠BAQ (As line l is the bisector of angle A)

So, ΔAPB ≅ ΔAQB.

(ii) By the rule of CPCT, BP = BQ. So, it can be said the point B is equidistant from the arms of ∠A.

6. In Fig. 7.21, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.

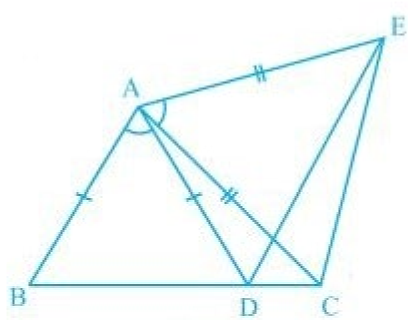

Fig. 7.21

Solution:

It is given in the question that AB = AD, AC = AE, and ∠BAD = ∠EAC

To proof:

The line segment BC and DE are similar i.e. BC = DE

Proof:

We know that ∠BAD = ∠EAC

Now, by adding ∠DAC on both sides we get,

∠BAD + ∠DAC = ∠EAC + ∠DAC

This implies, ∠BAC = ∠EAD

Now, ΔABC and ΔADE are similar by SAS congruency since:

(i) AC = AE (As given in the question)

(ii) ∠BAC = ∠EAD

(iii) AB = AD (It is also given in the question)

∴ Triangles ABC and ADE are similar i.e. ΔABC ≅ ΔADE.

So, by the rule of CPCT, it can be said that BC = DE.

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Fig. 7.22). Show that

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE

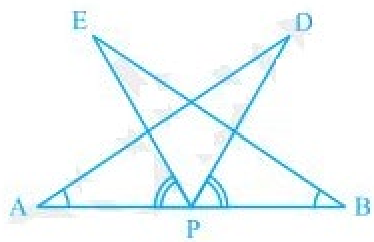

Fig. 7.22

Answer

In the question, it is given that P is the mid-point of line segment AB. Also, ∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) It is given that ∠EPA = ∠DPB

Now, add ∠DPE om both sides,

∠EPA + ∠DPE = ∠DPB + ∠DPE

This implies that angles DPA and EPB are equal i.e. ∠DPA = ∠EPB

Now, consider the triangles DAP and EBP.

∠DPA = ∠EPB

AP = BP (Since P is the mid-point of the line segement AB)

∠BAD = ∠ABE (As given in the question)

So, by **ASA congruency**, ΔDAP ≅ **ΔEBP.**

(ii) By the rule of CPCT, AD = BE.

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle.

(iii) ΔDBC ≅ ΔACB

(iv) CM = 1/2 AB

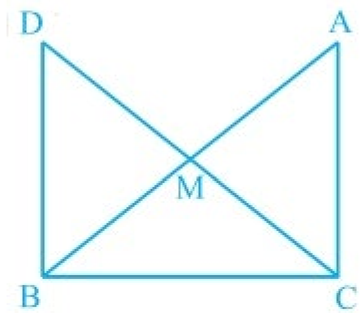

Fig. 7.23

Solution:

It is given that M is the mid-point of the line segment AB, $\angle C = 90°$, and DM = CM

(i) Consider the triangles ΔAMC and ΔBMD:

AM = BM (Since M is the mid-point)

CM = DM (Given in the question)

$\angle CMA = \angle DMB$ (They are vertically opposite angles)

So, by **SAS congruency criterion**, ΔAMC ≅ ΔBMD.

(ii) $\angle ACM = \angle BDM$ (by CPCT)

∴ AC ∥ BD as alternate interior angles are equal.

Now, $\angle ACB + \angle DBC = 180°$ (Since they are co-interiors angles)

⇒ $90° + \angle B = 180°$

∴ $\angle DBC = 90°$

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

$\angle ACB = \angle DBC$ (They are right angles)

DB = AC (by CPCT)

So, ΔDBC ≅ ΔACB by **SAS congruency**.

(iv) DC = AB (Since ΔDBC ≅ ΔACB)

⇒ DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM + AM

Hence, CM + CM = AB

⇒ CM = (½) AB