Exercise: 7.3 **(Page No: 128)**

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ ΔACD

(ii) ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.

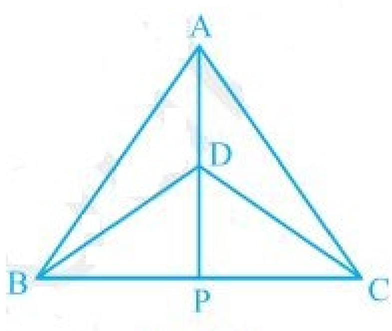


Fig. 7.39

Solution:

In the above question, it is given that ΔABC and ΔDBC are two isosceles triangles.

(i) ΔABD and ΔACD are similar by SSS congruency because:

AD = AD (It is the common arm)

AB = AC (Since ΔABC is isosceles)

BD = CD (Since ΔDBC is isosceles)

∴ ΔABD ≅ ΔACD.

(ii) ΔABP and ΔACP are similar as:

AP = AP (It is the common side)

∠PAB = ∠PAC (by CPCT since ΔABD ≅ ΔACD)

AB = AC (Since ΔABC is isosceles)

So, ΔABP ≅ ΔACP by SAS congruency condition.

(iii) ∠PAB = ∠PAC by CPCT as ΔABD ≅ ΔACD.

AP bisects $\angle A$ --- (i)

Also, ΔBPD and ΔCPD are similar by SSS congruency as

PD = PD (It is the common side)

BD = CD (Since ΔDBC is isosceles.)

BP = CP (by CPCT as ΔABP ≅ ΔACP)

So, ΔBPD ≅ ΔCPD.

Thus, $\angle BDP = \angle CDP$ by CPCT. --- (ii)

Now by comparing (i) and (ii) it can be said that AP bisects $\angle A$ as well as $\angle D$.

(iv) $\angle BPD = \angle CPD$ (by CPCT as ΔBPD ≅ ΔCPD)

and BP = CP --- (i)

also,

$\angle BPD + \angle CPD = 180°$ (Since BC is a straight line.)

$\Rightarrow 2\angle BPD = 180°$

$\Rightarrow \angle BPD = 90°$ ---(ii)

Now, from equations (i) and (ii), it can be said that

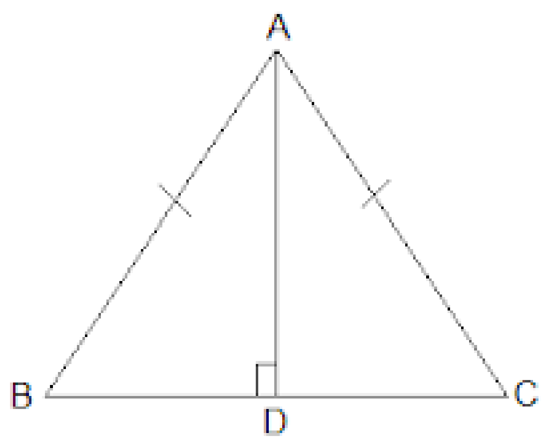
AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects $\angle A$.

Solution:

It is given that AD is an altitude and AB = AC. The diagram is as follows:



(i) In ΔABD and ΔACD,

$\angle ADB = \angle ADC = 90°$

AB = AC (It is given in the question)
AD = AD (Common arm)
∴ △ABD ≅ △ACD by RHS congruence condition.
Now, by the rule of CPCT,
BD = CD.
So, AD bisects BC

(ii) Again by the rule of CPCT, ∠BAD = ∠CAD
Hence, AD bisects ∠A.

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Fig. 7.40). Show that:
(i) ΔABM ≅ ΔPQN
(ii) ΔABC ≅ ΔPQR

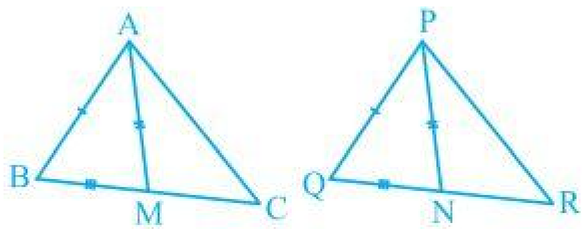


Fig. 7.40

Solution:
Given parameters are:
AB = PQ,
BC = QR and
AM = PN

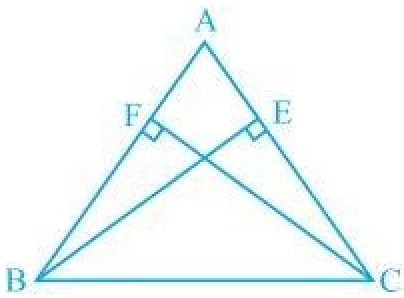
(i) 1/2 BC = BM and 1/2QR = QN (Since AM and PN are medians)
Also, BC = QR
So, 1/2 BC = 1/2QR
⇒ BM = QN
In ΔABM and ΔPQN,
AM = PN and AB = PQ (As given in the question)
BM = QN (Already proved)
∴ △ABM ≅ △PQN by SSS congruency.

(ii) In ΔABC and ΔPQR,
AB = PQ and BC = QR (As given in the question)

$\angle ABC = \angle PQR$ (by CPCT)

So, $\Delta ABC \cong \Delta PQR$ by SAS congruency.

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Solution:

It is known that BE and CF are two equal altitudes.

Now, in ΔBEC and ΔCFB,

$\angle BEC = \angle CFB = 90°$ (Same Altitudes)

BC = CB (Common side)

BE = CF (Common side)

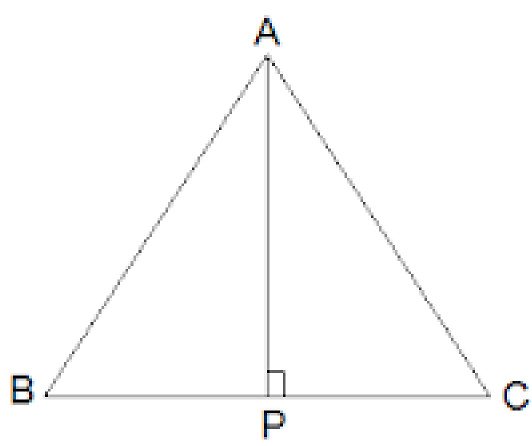
So, $\Delta BEC \cong \Delta CFB$ by RHS congruence criterion.

Also, $\angle C = \angle B$ (by CPCT)

Therefore, AB = AC as sides opposite to the equal angles is always equal.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that $\angle B = \angle C$.

Solution:



In the question, it is given that AB = AC

Now, ΔABP and ΔACP are similar by RHS congruency as

$\angle APB = \angle APC = 90°$ (AP is altitude)

AB = AC (Given in the question)

AP = AP (Common side)

So, ΔABP ≅ ΔACP.

$\therefore \angle B = \angle C$ (by CPCT)