Exercise 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that :
(i) SR || AC and SR = 1/2 AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.

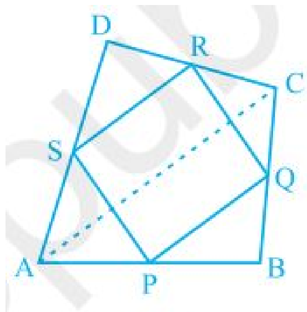


Fig. 8.29

Solution:

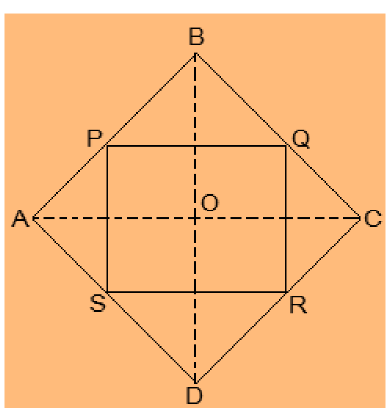
(i) In ΔDAC,
R is the mid point of DC and S is the mid point of DA.
Thus by mid point theorem, SR || AC and SR = 1/2 AC

(ii) In ΔBAC,
P is the mid point of AB and Q is the mid point of BC.
Thus by mid point theorem, PQ || AC and PQ = 1/2 AC
also, SR = 1/2 AC
∴, PQ = SR

(iii) SR || AC ---------------------- from question (i)
and, PQ || AC ---------------------- from question (ii)
⇒ SR || PQ - from (i) and (ii)
also, PQ = SR
∴, PQRS is a parallelogram.

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:



Given in the question,

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

To Prove,

PQRS is a rectangle.

Construction,

Join AC and BD.

Proof,

In ΔDRS and ΔBPQ,

DS = BQ (Halves of the opposite sides of the rhombus)

∠SDR = ∠QBP (Opposite angles of the rhombus)

DR = BP (Halves of the opposite sides of the rhombus)

∴, ΔDRS ≅ ΔBPQ [SAS congruency]

RS = PQ [CPCT]--------------------- (i)

In ΔQCR and ΔSAP,

RC = PA (Halves of the opposite sides of the rhombus)

∠RCQ = ∠PAS (Opposite angles of the rhombus)

CQ = AS (Halves of the opposite sides of the rhombus)

∴, ΔQCR ≅ ΔSAP [SAS congruency]

RQ = SP [CPCT]--------------------- (ii)

Now,

In ΔCDB,

R and Q are the mid points of CD and BC respectively.

⇒ QR || BD

also,

P and S are the mid points of AD and AB respectively.

⇒ PS || BD

⇒ QR || PS

∴, PQRS is a parallelogram.

also, ∠PQR = 90°

Now,

In PQRS,

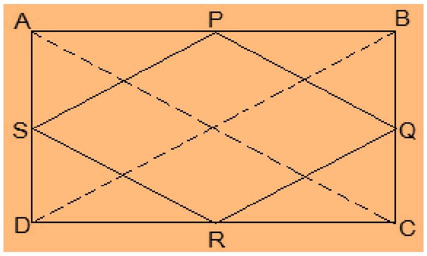
RS = PQ and RQ = SP from (i) and (ii)

∠Q = 90°

∴, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution:



Given in the question,

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.

Construction,

Join AC and BD.

To Prove,

PQRS is a rhombus.

Proof,

In ΔABC

P and Q are the mid-points of AB and BC respectively

∴, PQ || AC and PQ = $\frac{1}{2}$AC (Midpoint theorem) --- (i)

In ΔADC,

SR || AC and SR = $\frac{1}{2}$ AC (Midpoint theorem) --- (ii)

So, PQ || SR and PQ = SR

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

∴PS || QR and PS = QR (Opposite sides of parallelogram) --- (iii)

Now,

In ΔBCD,

Q and R are mid points of side BC and CD respectively.

∴, QR || BD and QR = $\frac{1}{2}$BD (Midpoint theorem) --- (iv)

AC = BD (Diagonals of a rectangle are equal) --- (v)

From equations (i), (ii), (iii), (iv) and (v),

PQ = QR = SR = PS

So, PQRS is a rhombus.

Hence Proved

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

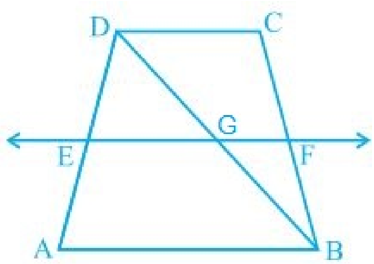


Fig. 8.30

Solution:

Given that,

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD.

To prove,

F is the mid-point of BC.

Proof,

BD intersected EF at G.

In ΔBAD,

E is the mid point of AD and also EG || AB.

Thus, G is the mid point of BD (Converse of mid point theorem)

Now,
In ΔBDC,
G is the mid point of BD and also GF || AB || DC.
Thus, F is the mid point of BC (Converse of mid point theorem)

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.

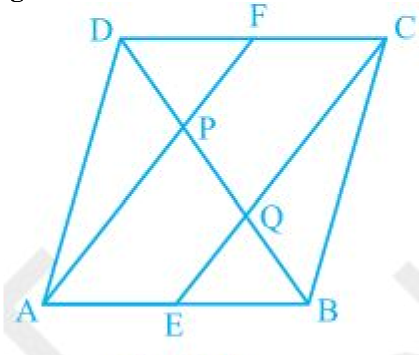


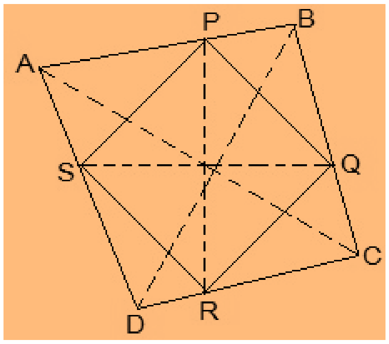
Fig. 8.31

Solution:

Given that,
ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.
To show,
AF and EC trisect the diagonal BD.
Proof,
ABCD is a parallelogram
∴, AB || CD
also, AE || FC
Now,
AB = CD (Opposite sides of parallelogram ABCD)
$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$CD
⇒ AE = FC (E and F are midpoints of side AB and CD)
AECF is a parallelogram (AE and CF are parallel and equal to each other)
AF || EC (Opposite sides of a parallelogram)
Now,
In ΔDQC,
F is mid point of side DC and FP || CQ (as AF || EC).
P is the mid-point of DQ (Converse of mid-point theorem)
⇒ DP = PQ --- (i)
Similarly,
In APB,
E is mid point of side AB and EQ || AP (as AF || EC).
Q is the mid-point of PB (Converse of mid-point theorem)
⇒ PQ = QB --- (ii)
From equations (i) and (i),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.
Hence Proved.

6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:



Let ABCD be a quadrilateral and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.

Now,

In ΔACD,

R and S are the mid points of CD and DA respectively.

∴, SR || AC.

Similarly we can show that,

PQ || AC

PS || BD

QR || BD

∴, PQRS is parallelogram.

PR and QS are the diagonals of the parallelogram PQRS. So, they will bisect each other.

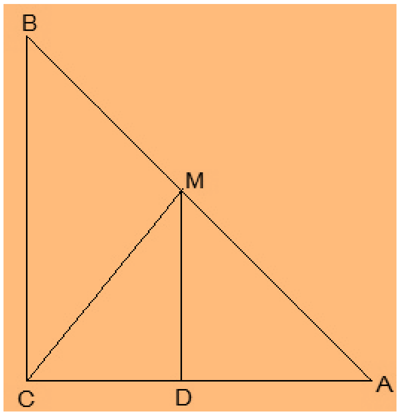
7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = $\frac{1}{2}$AB

Solution:



(i) In ΔACB,
M is the mid point of AB and MD || BC
$\therefore$, D is the mid point of AC (Converse of mid point theorem)

(ii) $\angle ACB = \angle ADM$ (Corresponding angles)
also, $\angle ACB = 90°$
$\therefore$, $\angle ADM = 90°$ and $MD \perp AC$

(iii) In ΔAMD and ΔCMD,
$AD = CD$ (D is the midpoint of side AC)
$\angle ADM = \angle CDM$ (Each 90°)
$DM = DM$ (common)
$\therefore$, $\Delta AMD \cong \Delta CMD$ [SAS congruency]
$AM = CM$ [CPCT]
also, $AM = 1/2\ AB$ (M is mid point of AB)
Hence, $CM = MA = 1/2\ AB$