

**Solutions  
of  
Waves & Thermodynamics**

**Lesson 14<sup>th</sup> to 19<sup>th</sup>**

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# 14. Wave Motion

## Introductory Exercise 14.1

1. A function,  $f$  can represent wave equation, if it satisfy

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

For,  $y = a \sin t$ ,

$$\frac{\partial^2 y}{\partial t^2} = a \sin t$$

but,  $\frac{\partial^2 y}{\partial x^2} = 0$

So,  $y$  do not represent wave equation.

2.  $y(x, t) = ae^{(bx - ct)^2} = ae^{(kx - vt)^2}$

$$k = b \text{ and } e = v = \frac{c}{b}$$

3.  $y(x, t) = \frac{1}{1 + (4x - t)^2}$  represent the given pulse, where,

$$y(x, 0) = \frac{1}{1 + k^2 x^2} = \frac{1}{1 + x^2}$$

$$k = 1$$

$$\text{and } y(x, z) = \frac{1}{1 + (x - 2)^2} = \frac{1}{1 + (x - 1)^2}$$

$$v = \frac{1/2}{1} = 0.5 \text{ m/s}$$

4.  $y = \frac{10}{5 + (x - 2t)^2} = \frac{a}{b + (kx - vt)^2}$

$$\text{Amplitude, } y_{\max} = \frac{a}{b} = \frac{10}{5} = 2 \text{ m}$$

$$\text{and } k = 1; \quad v = 2$$

$v = \frac{2}{k} = 2 \text{ m/s}$  and is travelling in  $(-)$   $x$  direction.

5.  $y = \frac{10}{(kx - t)^2 + 2}$

$$y(x, 0) = \frac{10}{k^2 x^2 + 2} = \frac{10}{x^2 + 2} \quad k = 1$$

$$vk = 2 \text{ m/s} \quad 1 \text{ m} \cdot 1 = 2 \text{ rad/s}$$

$$y = \frac{10}{(x - 2t)^2 + 2}$$

## Introductory Exercise 14.2

1.  $y(x, t) = 0.02 \sin \frac{x}{0.05} - \frac{t}{0.01} \text{ m}$

$$A \sin(kx - vt) \text{ m} \quad 2 \cos(4 - 30) = 2 \cos 34 = 2(0.85) = 1.7 \text{ m/s}$$

(a)  $v = \frac{0.05}{0.01} \text{ m/s} = 5 \text{ m/s}$

(b)  $v_p = \frac{y}{t} = A \cos(kx - vt)$

$$v_p(0.2, 0.3) = 0.02 \frac{1}{0.01}$$

2. Yes,  $(v_p)_{\max} = A \cdot Ak = (Ak)v$

3.  $4 \text{ cm}, v = 40 \text{ cm/s}$  (given)

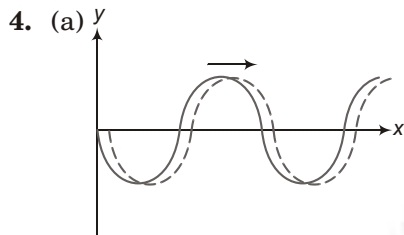
(a)  $\frac{v}{4 \text{ cm}} = \frac{40 \text{ cm/s}}{4 \text{ cm}} = 10 \text{ Hz}$

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$$(b) \quad \frac{2}{4 \text{ cm}} \cdot 2.5 \text{ cm} = \frac{5}{4} \text{ rad}$$

$$(c) \quad t = \frac{T}{2} = \frac{1}{2} \cdot \frac{1}{10} = \frac{1}{20} \text{ s}$$

$$(d) \quad v_p = \frac{(v_p)_{\max}}{A} = \frac{2 \text{ cm}}{2 \text{ cm}} = 1 \text{ s}^{-1} = 1 \text{ cm/s} = 1.26 \text{ cm/s}$$



$$y = A \sin \left( 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right) \right)$$

$$0.05 \sin \left( 2\pi \left( \frac{t}{0.4} - \frac{x}{0.4} \right) \right)$$

$$0.05 \sin (60\pi t - 5\pi x)$$

$$(b) \quad y(0.25, 0.15)$$

$$0.05 \sin (60\pi \cdot 0.15 - 5\pi \cdot 0.25)$$

$$0.05 \sin (9\pi - 1.25\pi)$$

$$0.05 \sin (7.75\pi) = 0.05 \sin (1.75\pi)$$

$$0.0354 \text{ m} = 3.54 \text{ cm}$$

$$(c) \quad t = \frac{T}{2} = \frac{0.25}{60}$$

$$\frac{1}{240} \text{ s} = 4.2 \text{ ms}$$

### Introductory Exercise 14.3

$$1. \quad v = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{500 \cdot 2}{0.06}} = \frac{100\sqrt{5}}{\sqrt{3}} = 129.1 \text{ m/s}$$

$$2. \quad v = \sqrt{\frac{T}{A}} = \sqrt{\frac{0.98}{9.8 \cdot 10^3 \cdot 10^{-6}}} = 10 \text{ m/s}$$

### Introductory Exercise 14.4

$$1. \quad I = \frac{P}{4r^2} = \frac{1 \text{ W}}{4(1 \text{ m})^2} = \frac{1}{4} \text{ W/m}^2$$

$$I = \frac{1}{r} \text{ and as } I \propto A^2$$

$$2. \quad \text{For line source, } I = \frac{1}{2rl}$$

$$A = \frac{1}{\sqrt{r}}$$

## AIEEE Corner

### ■ Subjective Questions (Level 1)

1.  $y(x, t) = 6.50 \text{ mm} \cos 2$

$$\frac{x}{28.0 \text{ cm}} - \frac{t}{0.0360 \text{ s}}$$

$$A \cos 2 \frac{x}{T} - \frac{t}{T}$$

$$A = 6.50 \text{ mm}, \quad \frac{2\pi}{T} = 27.78 \text{ Hz}$$

$$v = \frac{28.0 \text{ cm}}{0.036 \text{ s}} = 778 \text{ cm/s} = 7.78 \text{ m/s}$$

The wave is travelling along ( )ve x-axis.

2.  $y = 5 \sin 30 \pi t - \frac{x}{240}$

$$5 \sin 30 \pi t - \frac{x}{8} = A \sin ( 2\pi t - kx )$$

(a)  $y(2, 0) = 5 \sin 3\pi = 0$

$$5 \sin \frac{\pi}{4} = \frac{5}{\sqrt{2}} = 3.535 \text{ cm}$$

(b)  $\frac{2\pi}{k} = \frac{2}{8} = 16 \text{ cm}$

(c)  $v = \frac{30}{k} = \frac{30}{8} = 240 \text{ cm/s}$

(d)  $\frac{30}{2} = 15 \text{ Hz}$

3.  $y = 3 \text{ cm} \sin(3.14 \text{ cm}^{-1}x - 314 \text{ s}^{-1}t)$

$$3 \text{ cm} \sin( \pi x - 100 \pi t )$$

$$A \sin(kx - \omega t)$$

(a)  $(v_p)_{\max} = A = 3 \text{ cm} = 0.03 \text{ m}$   
 $300 \text{ cm/s} = 3 \text{ m/s} = 9.4 \text{ m/s}$

(b)  $a = \omega^2 y = (100 \pi \text{ s}^{-1})^2 \cdot 3 \text{ cm} = 300 \sin( 105 \pi ) = 0$

4. (a)  $x = \frac{v}{2} t = \frac{350}{500} t = \frac{7}{10} t$

$$\frac{7}{60} - \frac{7}{50} = -0.166 \text{ m}$$

(b)  $\frac{2}{T} t = 2 \pi t = 500 \cdot 10^{-3}$

5.  $y(x, t) = \frac{180}{(kx - t)^2 + 3}$

$$y(x, 0) = \frac{6}{k^2 x^2 + 3} = \frac{6}{x^2 + 3}$$

$$k = 1 \text{ m}^{-1}$$

$$v = \frac{4.5 \text{ m/s}}{1 \text{ m}^{-1}} = 4.5 \text{ rad/s}$$

$$y(x, t) = \frac{6}{(x - 4.5t)^2 + 3}$$

6.  $y = 1.0 \sin \frac{x}{2.0} - \frac{t}{0.01}$

$$1.0 \sin 2 \frac{x}{4.0} - \frac{t}{0.02}$$

$$A \sin 2 \frac{x}{T} - \frac{t}{T}$$

(a)  $A = 1.0 \text{ mm}, \quad T = 4.0 \text{ cm}, \quad T = 0.02 \text{ s}$

(b)  $v_p = \frac{y}{t} = A \cos 2 \frac{x}{T} - \frac{t}{T}$

$$\frac{2}{T} A \cos 2 \frac{x}{T} - \frac{t}{T}$$

$$\frac{2}{0.02 \text{ s}} \cdot 1.0 \text{ mm} \cos 2 \frac{x}{4.0} - \frac{t}{0.02 \text{ s}}$$

$$\frac{100 \text{ m/s}}{10} \cos \frac{x}{2.0 \text{ cm}} - \frac{t}{0.01 \text{ s}}$$

$$v_p(1.0 \text{ cm}, 0.01 \text{ s})$$

$$\frac{10 \text{ m/s}}{10} \cos \frac{1}{2} = \frac{0.01}{2} = 0.01$$

$$\frac{10 \text{ m/s}}{10} \cos \frac{0}{2} = 0 \text{ m/s}$$

(c)  $v_p(3.0, 0.01)$

$$\frac{10 \text{ m/s}}{10} \cos \frac{3}{2} = 1 = 0 \text{ m/s}$$

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$$v_p(5.0 \text{ cm}, 0.01\text{s}) = \frac{0}{10} \text{ m/s} \cos \frac{5}{2} 1$$

$$v_p(7.0 \text{ cm}, 0.01\text{s}) = \frac{0}{10} \text{ m/s} \cos \frac{7}{2} 1$$

(d)  $v_p(1.0 \text{ cm}, 0.011\text{s}) = \frac{0}{10} \text{ m/s}$

$$\cos \frac{1}{2} \frac{0.011}{0.01} = \frac{1}{10} \cos \frac{1}{12} 1.1$$

$$\frac{1}{10} \cos 0.6 = \frac{1}{10} \cos \frac{3}{5} 9.7 \text{ cm/s}$$

$$v_p(1.0 \text{ cm}, 0.012\text{s}) = \frac{0}{10} \text{ m/s} \cos \frac{1}{2} \frac{0.012}{0.01}$$

$$\frac{1}{10} \cos (0.5 \cdot 1.2)$$

$$\frac{1}{10} \cos 0.7 = 18.5 \text{ cm/s}$$

$$v_p(1.0 \text{ cm}, 0.013 \text{ s}) = \frac{0}{10} \text{ m/s}$$

$$\cos \frac{1}{2} \frac{0.013}{0.01} = \frac{1}{10} \cos 0.8$$

$$25.4 \text{ cm/s}$$

7. (a)  $k = \frac{2}{40 \text{ cm}} = \frac{1}{20} \text{ cm}^{-1}$

$$T = \frac{1}{8} \text{ s} = 0.125 \text{ s}$$

$$v = \frac{2}{8 \text{ s}} = \frac{16}{40 \text{ cm}} = \frac{2}{5} \text{ rad/s} = 50.26 \text{ rad/s}$$

(b)  $y(x, t) = A \cos(kx - \omega t)$   
 $15.0 \text{ cm} \cos(0.157x - 50.3t)$

8.  $A = 0.06 \text{ m}$  and  $2.5 = 20 \text{ cm}$

$$\frac{20}{2.5} \text{ cm} = 8 \text{ cm}$$

$$v = \frac{300 \text{ m/s}}{8 \text{ cm}} = 3750 \text{ Hz}$$

$$y = A \sin(kx - \omega t) = 0.06 \text{ m}$$

$$\sin \frac{2}{0.08} x - 2 \cdot 3750 t$$

$$0.06 \text{ m} \sin(78.5 \text{ m}^{-1} x - 23561.9 \text{ s}^{-1} t)$$

9. (a)  $v = \frac{8.00 \text{ m/s}}{0.32 \text{ m}} = 25 \text{ Hz}$

$$T = \frac{1}{15} \text{ s} = 0.043 \text{ Hz}$$

$$k = \frac{2}{0.32 \text{ m}} = 19.63 \text{ rad/m}$$

(b)  $y = A \cos(kx - \omega t) = A \cos 2 \frac{x}{0.32 \text{ m}} - \frac{t}{0.04 \text{ s}}$

$$0.07 \text{ m} \cos 2 \frac{x}{0.32 \text{ m}} - \frac{t}{0.04 \text{ s}}$$

(c)  $y = 0.07 \text{ m} \cos 2 \frac{0.36}{0.32} \frac{0.15}{0.04}$

$$0.07 \text{ m} \cos 2 \frac{9}{8} \frac{30}{8}$$

$$0.07 \text{ m} \cos \frac{39}{4}$$

$$0.07 \text{ m} \cos 10 \frac{t}{4}$$

$$0.07 \text{ m} \cos \frac{t}{4} = 0.0495 \text{ m}$$

(d)  $t = \frac{T}{2} = \frac{1/4}{2} = \frac{1}{8} \text{ s} = 0.015 \text{ s}$

10.  $v = \sqrt{\frac{T}{A}} = \sqrt{\frac{Mg}{A}}$

$$\sqrt{\frac{2 \cdot 9.8}{8920 \cdot 3.14 \cdot (1.2 \cdot 10^3)^2}}$$

$$\sqrt{\frac{2 \cdot 9.8 \cdot 10^4}{89.2 \cdot 3.14 \cdot 1.44}} = 22 \text{ m/s}$$

11.  $\frac{\sqrt{T}}{2} = \frac{\sqrt{M}}{1}$

$$\frac{2}{1} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

$$\sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

$$\frac{2}{2} = \frac{2}{1} = 0.12 \text{ m}$$

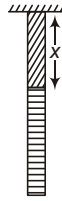
12.  $T(x) = (L - x)g, v(x) = \sqrt{\frac{T(x)}{\mu}}$

$$\frac{\sqrt{g(L-x)}}{dx} dt;$$

Let,  $L - x = y$

$$\frac{dx}{dy} = -1$$

$$t = \frac{1}{\sqrt{g}} \int_0^L \frac{dy}{\sqrt{y}} = \frac{2\sqrt{L}}{\sqrt{g}}$$



15.

$$t = \sqrt{\frac{2}{g}} \cdot 2\sqrt{l_0}$$

$$t = \sqrt{\frac{8l_0}{g}}$$

$$\frac{dm}{dx} = kx$$

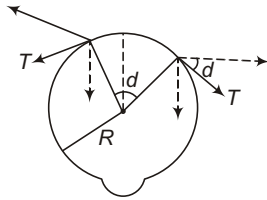
$$M = \int_0^L kx dx = \frac{1}{2}kL^2$$

$$k = \frac{2M}{L^2}$$

$$v(x) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{kx}} = \sqrt{\frac{TL^2}{2Mx}} \frac{dx}{dt}$$

$$t = \int_0^L \frac{L}{\sqrt{TL^2}} \sqrt{\frac{2Mx}{TL^2}} dx = \sqrt{\frac{2M}{TL^2}} \int_0^L \frac{1}{\sqrt{x}} dx$$

13. (a)  $dm = \frac{2R}{L} 2T \sin d$



$$R2d = \frac{2R}{L} 2T \sin d$$

Wave speed,  $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{2R^2/L}} = R \sqrt{\frac{T}{2R^2}}$

(b) Kink remains stationary when rope and kink moves in opposite sense i.e., if rope is rotating anticlockwise then kink has to move clockwise.

14.  $x$  is being measured from lower end of the string

$$m(x) = \int_0^x \mu dx = \frac{1}{2} \mu x^2$$

$$v(x) = \sqrt{\frac{T(x)}{\mu}} = \sqrt{\frac{m(x)g}{\mu}}$$

$$= \sqrt{\frac{\frac{1}{2} \mu x^2 g}{\mu}} = \sqrt{\frac{1}{2} gx}$$

$$l = \int_0^l \frac{dx}{\sqrt{\frac{1}{2} gx}} = \int_0^t dt$$

16. (a)  $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\mu}}$

$$v = \sqrt{\frac{1.5 \cdot 9.8}{0.055}} = 16.3 \text{ m/s}$$

(b)  $\frac{v}{\lambda} = \frac{16.3 \text{ m/s}}{120 \text{ s}} = 0.136 \text{ m}$

(c)  $v \propto \sqrt{T} \propto \sqrt{M}$  i.e., if  $M$  is doubled both speed and wavelength increases by a factor of  $\sqrt{2}$ .

17.  $E = I A t = \frac{1}{2} \mu a^2 v A t$

$$E = \frac{1}{2} \mu a^2 (A) (v) t$$

$$E = \frac{1}{2} \mu a^2 \cdot l$$

$$E = \frac{1}{2} \mu a^2 m$$

$$E = \frac{1}{2} (3.14)^2 (120)^2 (0.16 \cdot 10^{-3})^2 \cdot 80 \cdot 10^{-3}$$

$$E = 582 \cdot 10^6 \text{ J} = 582 \text{ J} = 0.58 \text{ mJ}$$

18.  $P = \frac{E}{t} = I A v = \frac{1}{2} \mu a^2 v A = \frac{1}{2} \mu a^2 v$

$$P = \frac{1}{2} \mu a^2 \sqrt{T}$$

$$P = \frac{1}{2} (3.14)^2 (60)^2$$

$$= \frac{1}{2} (6 \cdot 10^2)^2 \sqrt{80 \cdot 5 \cdot 10^2}$$

$$P = 4(3.14 \cdot 60 \cdot 0.06)^2 = 511.6 \text{ W}$$

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$$19. P = IA = 2^2 \cdot 2^2 \cdot a^2 \cdot \sqrt{T}$$

$$= 2 \cdot (3.14)^2 \cdot (200)^2 \cdot 10^6 \cdot \sqrt{60 \cdot 6 \cdot 10^3}$$

$$= 8 \cdot (3.14)^2 \cdot 10^2 \cdot 6 \cdot 10^1 \text{ W}$$

$$= 0.474 \text{ W}$$

$$E = Pt = \frac{P \cdot l}{v}$$

$$= \frac{0.474 \cdot 2}{\sqrt{\frac{60}{6 \cdot 10^3}}} = \frac{0.474 \cdot 2}{100} \text{ J} = 9.48 \text{ mJ}$$

$$20. P = 2^2 v^2 a^2 \cdot vA = 2^2 v^2 a^2 \cdot v; v = \sqrt{\frac{T}{\mu}}$$

$$= 2^2 \cdot 2^2 \cdot a^2 \cdot \frac{T}{v^2} \cdot v = 2^2 \cdot 2^2 \cdot a^2 \cdot \frac{T}{v}$$

$$= \frac{2 \cdot (3.14)^2 \cdot (100)^2 \cdot (0.5 \cdot 10^{-3})^2 \cdot 100}{2 \cdot (3.14)^2 \cdot 10^4 \cdot 0.25 \cdot 10^6}$$

$$= 4.93 \cdot 10^{-2} \text{ W} = 49 \text{ mW}$$

■ Objective Questions (Level 1)

1.  $\frac{150}{60} = 2.5 \text{ rad/s}$ ,  $A = 0.04 \text{ m}$  and  $\frac{1}{4}$

$$y = A \sin(\omega t) = 0.04 \sin 2.5 t \quad \frac{1}{4}$$

2.  $600$ ,  $v = 300$ ,  $k = \frac{2}{v}$

$$y = A \sin(\omega t - kx)$$

$$= 0.04 \sin(600 t - 2x)$$

$$y(0.75, 0.01) = 0.04 \sin 600 \cdot 0.01 - 2 \cdot \frac{3}{4}$$

$$= 0.04 \sin 6 - \frac{3}{2}$$

$$= 0.04 \sin 4 - \frac{3}{2} = 0.04 \text{ m}$$

3.  $y(x, t) = \frac{1}{2} \sin(3kx - \omega t)^2$

$$y(x, 0) = \frac{1}{2} \sin^2(3kx) = \frac{1}{2} \sin^2(3x^2)$$

$$y(x, 2) = \frac{1}{2} \sin^2(3(x-2)^2) = \frac{1}{2} \sin^2(3(x-2)^2)$$

$$1 = v = \frac{1}{k} = 1 \text{ m/s}$$

4.  $y = A \sin(\omega t - kx) = \frac{A}{2}$

$$\omega t - kx = \frac{\pi}{6}$$

$$\frac{2\pi}{T} \cdot \frac{T}{6} - \frac{\pi}{6} = kx$$

5.  $\frac{v}{25 \text{ Hz}} = \frac{300 \text{ m/s}}{25} = 12 \text{ m}$

$$\frac{2}{12} x = \frac{2}{12m} (16 - 10) \text{ m}$$

6.  $y = 0.02 \sin(x - 30t) = A \sin(kx - \omega t)$

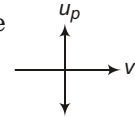
$$k = 1, \omega = 30$$

$$v = \frac{\omega}{k} = 30 \text{ m/s} = \sqrt{\frac{T}{\mu}}$$

$$T = v^2 \cdot \mu = 1.3 \cdot 10^4 = 900 = 0.117 \text{ N}$$

7.  $v_p = \frac{y}{t} = \frac{y}{x} \cdot \frac{x}{t} = v \cdot \frac{y}{x}$  slope  $v$

In transverse wave  $u_p$  they are



perpendicular i.e.,  $\frac{\pi}{2}$ . In longitudinal

wave  $u_p$  they are either at 0 or  $\pi$  so,  $0, \frac{\pi}{2}$  and  $\pi$  are the possible angles

between  $v_p$  and  $v$ .

8.  $2 = 200 \text{ rad/s}$ ,

$$k = \frac{2\pi}{v} = \frac{2\pi}{200} \sqrt{\frac{3.5 \cdot 10^3}{35}}$$

$$y = A \cos(200t - 2x) \quad \text{2 rad/m}$$

$$\frac{y}{x} = 2A \sin(200t - 2x)$$

When,  $y = 0$

$$\sin(200t - 2x) = 0$$

$$\sin(200t - 2x) = 1$$

$$2A = \frac{1}{20} \quad A = \frac{1}{40} = 0.025 \text{ m}$$

$$y = 0.025 \cos(200t - 2x)$$

9.  $\frac{2}{T} = \frac{2}{0.25} = 8 \text{ rad/s;}$

$$k = \frac{8}{v} = \frac{8}{48} = \frac{1}{6} \text{ rad/cm}$$

$$y = A \sin(8t - \frac{1}{6}x)$$

$$A \sin 8t = 1 \quad \frac{1}{6} = \frac{67}{6}$$

$$A \sin \frac{1}{6} = A \sin 30 \quad \frac{A}{2} = 3 \text{ cm}$$

$$A = 6 \text{ cm}$$

10.  $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{A_A} \frac{A_B}{T_B}} = \sqrt{\frac{T_A d_B^2}{T_B d_A^2}}$

$$\frac{d_B}{d_A} \sqrt{\frac{T_A}{T_B}} = \frac{d_B}{d_B/2} \sqrt{\frac{T_B/2}{T_B}}$$

$$2 \frac{1}{\sqrt{2}} = \sqrt{2}$$

11.  $E = A^2 v^2$  for  $E$  to constant,  $A$  constant

$$\frac{A_A}{A_B} = \frac{A_B}{4A_A} \quad \frac{A_A^4}{A_B^4} = \frac{A_B}{A_A}$$

12.  $k = 1 \text{ rad/m; } v = 4 \text{ m/s}$

$$y = \frac{vk}{6} \frac{4 \text{ rad/s}}{3} \frac{6}{(kx - t)^2 - 3} \frac{6}{(x - 4t)^2 - 3}$$

13.  $v_l = \sqrt{\frac{Y}{\rho}}$  and  $v_t = \sqrt{\frac{Y}{\rho} \frac{l}{l}}$   $v_l = \sqrt{\frac{l}{\rho}}$

$$\sqrt{\frac{l}{\rho}} = \frac{v_l}{v_t} = 10 \quad \frac{l}{\rho} = \frac{1}{100}$$

$$\text{Stress} = Y \frac{l}{l} = E \frac{l}{l} = \frac{E}{100}$$

14.  $A = 4 \text{ m, } \frac{1}{5}, k = \frac{1}{9}, \frac{1}{6}$

$$v = \frac{1/5}{k} = \frac{9}{5} \text{ m/s}$$

$$\frac{2}{k} = \frac{2}{9} = 18 \text{ m}$$

$$\frac{1}{2} = \frac{1/5}{2} = \frac{1}{10} \text{ Hz}$$

15.  $10$  and  $k = 0.1$

$$\frac{2}{k} = \frac{2}{0.1} = 20 \text{ m}$$

$$\frac{2}{x} = \frac{2}{20} = 10$$

16.  $y = \frac{2}{(2x - 6.2t)^2} \frac{2}{20}$

$$A = \frac{2}{20} = 0.1 \text{ m, } k = 2 \text{ rad/m}$$

and

$$6.2 \text{ rad/s}$$

$$v = \frac{6.2}{k} = \frac{6.2}{2} = 3.1 \text{ m/s}$$

$$\frac{6.2}{2} = 3.1 \text{ Hz}$$

$$\frac{2}{k} = \frac{2}{2} = 1 \text{ m}$$

17.  $I = 2^2 \frac{1}{2} A^2 v = \frac{1}{2} A^2 v$

$$u = \frac{E}{V} = \frac{IST}{V} = \frac{2^2 A^2 v St}{V}$$

$$2^2 A^2 \frac{1}{2} A^2$$

$$P = \frac{E}{t} = I.S = 2^2 A^2 v S$$

$$\frac{1}{2} A^2 v S$$

$$E = Pt = P \frac{E}{t} = IS = I \frac{P}{S}$$

18.  $y = A \sin(x - t)$

$$y(x, 0) = A \sin(x) \quad y = 0 \text{ for } x = 0 \text{ and } 1$$

$$a = \frac{2}{2} y = A \sin(x)$$

$$a = \frac{2}{2} A \text{ at } x = \frac{1}{2} \text{ and } \frac{3}{2}$$



$$v_p = A \cos(x) \quad v_p = 0 \text{ for } x = \frac{1}{2} \text{ and } \frac{3}{2}$$

So all the above options are correct.

19.  $y = A \sin \frac{2}{a}(x - bt) = A \sin(kx - \omega t)$

$$k = \frac{2}{a}, \quad \omega = \frac{2b}{a}$$

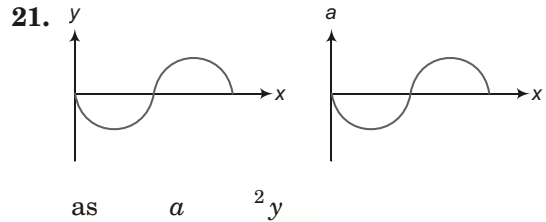
$$v = \frac{\omega}{k} = \frac{2b/a}{2/a} = b$$

$$\frac{2}{k} = \frac{2}{2/a} = a$$

20.  $y = A \sin 2\pi \left( \frac{x}{a} - \frac{t}{T} \right) = A \sin 2\pi \left( \frac{x}{a} - \frac{t}{T} \right)$

$$a, T, b$$

$$v = \frac{a}{T} = \frac{a}{b}$$



## JEE Corner

### ■ Assertion and Reason

- For propagation of transverse waves medium require tension which is possible due to modulus of rigidity. And in gases there is no such Young's modulus or surface tension. So the reason given is correct explanation.
- Surface tension of water plays the role of modulus of rigidity and that is why transverse waves can travel on liquid surface.
- Both the waves are travelling in same direction with a phase difference of  $\pi$ . So reason is false.
- $v = f\lambda$  is constant for a particular medium so if frequency is doubled wavelength becomes half, and speed remains constant. Thus assertion is false.
- Sound is mechanical wave which requires material medium for propagation and as on moon there is no atmosphere, sound cannot travel.
- Angular wave number,  $k = \frac{2\pi}{\lambda}$  while wave number,  $n = \frac{1}{\lambda}$  which is defined as the number of waves per unit length.
- Electromagnetic wave are non-mechanical, they travel depending upon electric and magnetic properties of medium. They can travel in medium as well as an vacuum. So reason is false.
- As speed,  $v = \sqrt{\frac{T}{\mu}} = \frac{1}{\sqrt{\mu}}$  in second string is more (by looking) so  $v$  will be less. Thus reason is true explanation of assertion.
- At point A both  $v_p$  and  $a$  is zero i.e., K.E. and P.E. are minimum while at B both  $v_p$  and  $a$  are maximum i.e., both K.E. and P.E. are maximum. Thus both assertion and reason are true but not correct explanation.
- If P is moving downward then it shows that the wave is travelling in ( ) ve x direction. So assertion is false.

11.  $A = 2a \cos \frac{\omega t}{2}$ , for  $A = a \cos \frac{\omega t}{2} + \frac{1}{2} a \cos \frac{\omega t}{3}$

$\frac{2}{3} \times \frac{360}{3} = 120$

Assertion is true but the reason is false.

■ Match the Columns

1.  $y = a \sin(bt - cx) = A \sin(\omega t - kx)$

(a)	$v = \frac{b}{k} \frac{a}{c}$	r
(b)	$(v_p)_{\max} = A \omega$	s
(c)	$\frac{b}{2} \frac{a}{2}$	p
(d)	$\frac{2}{k} \frac{2}{c}$	s

2.  $y = 4 \text{ cm} \sin(\omega t - 2x)$

$v_p = 4 \text{ cm/s} \cos(\omega t - 2x)$   
 $a = 4^2 \text{ cm/s}^2 \sin(\omega t - 2x)$

(a)	$v_p(0, t) = 4 \text{ cm/s} \cos \omega t$ 4 for $\cos \omega t = 1$ or $\omega t = n\pi$ , $n = 0, 1, 2, 3,$	q, r
(b)	$a(0, t) = 4^2 \text{ cm/s}^2 \sin \omega t$ 4 <sup>2</sup> for $\sin \omega t = 1$ or $\omega t = (2n - 1)\frac{\pi}{2}$ $\omega t = n\pi$ , $n = \frac{1}{2}, 0.5, 2.5,$	p, s
(c)	$v_p(0.5, t) = 4 \text{ cm/s} \cos(\omega t - 2)$ 4 for $\cos(\omega t - 2) = 1$ or $\omega t - 2 = n\pi$ , $n = 0, 1, 2, 3,$	q, r
(d)	$a(0.5, t) = 4^2 \text{ cm/s}^2 \sin(\omega t - 2)$ 4 <sup>2</sup> for $\sin(\omega t - 2) = 1$ or $\omega t - 2 = (2n - 1)\frac{\pi}{2}$ $\omega t = n\pi$ , $n = \frac{1}{2}, 0.5, 1.5, 2.5,$	p

3.  $y = A \sin(\omega t - kx)$  at  $t = 0$

$y = A \sin kx$   
 $v_p = A \cos kx$  and  $a = -\omega^2 y$

(a)	$v_p = A \cos kx$	s
(b)	$a_A = (-\omega^2) v_e$ as $y_A$ is negative	p

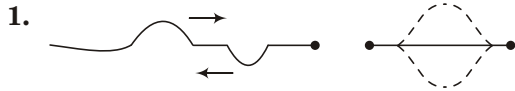
(c)	$v_B = A \omega$	s
(d)	$a_B = 0$ ve $y_0 = 0$	r

4. (a)  $u = \frac{E}{V} \frac{IST}{V} = \frac{2^2 \cdot 2^2 \cdot A^2 \cdot vst}{2^2 \cdot 2^2 \cdot A^2 \cdot \frac{1}{2} \cdot 2^2 \cdot A^2}$   
 $[u] = \frac{[ML^2T^{-2}]}{[L^3]} \frac{[ML^1T^{-2}]}{[ML^1T^{-2}]} = s$   
 (b)  $P = \frac{E}{t} \frac{IS t}{t} = IS = 2^2 \cdot 2^2 \cdot A^2 \cdot vs$   
 $\frac{1}{2} \cdot 2^2 \cdot A^2 \cdot vs = \frac{1}{2} \cdot 2^2 \cdot A^2 \cdot s \cdot v = q$   
 $[P] = \frac{E}{t} \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}] = p$   
 (c)  $I = \frac{E}{St} \frac{[ML^2T^{-2}]}{[L^2T]} = [MT^{-3}] \frac{[ML^0T^{-3}]}{[L^2T]} = s$   
 (d)  $\frac{1}{2} \frac{[L^1]}{[L^1]} \frac{[M^0L^1T^0]}{[M^0L^1T^0]} = s$

5.	(a)	$y = A \sin(\omega t - kx)$	p
		$v_p = A \cos(\omega t - kx)$	r
		$a = -\omega^2 A \sin(\omega t - kx)$	
	(b)	$y = A \sin(kx - \omega t)$	p
		$v_p = A \cos(kx - \omega t)$	
		$a = -\omega^2 A \sin(kx - \omega t)$	
	(c)	$y = A \cos(\omega t - kx)$	q
		$v_p = A \sin(\omega t - kx)$	
		$a = -\omega^2 A \cos(\omega t - kx)$	s
	(d)	$y = A \cos(kx - \omega t)$	p
		$v_p = A \sin(kx - \omega t)$	
		$a = -\omega^2 A \cos(kx - \omega t)$	d s

# 15. Superposition of Waves

## Introductory Exercise 15.1



When displacement of all the particles is momentarily zero, then there is no elastic potential energy stored in the string and as the speed is maximum at mean position, so entire energy is purely kinetic.

2. (a)  $v = \sqrt{\frac{T}{\mu}}$

$$\frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{1}{0.25}} = \frac{1}{\sqrt{0.25}} = \frac{1}{0.5} = 2$$

(b)  $a_t = \frac{2v_2}{v_1 + v_2} a_i = \frac{2 \times 20}{10 + 20} a_i = \frac{4}{3} a_i$

and  $a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i = \frac{20 - 10}{20 + 10} a_i = \frac{1}{3} a_i$

3. (a) For fixed end, a phase change of  $\pi$  takes place in reflected wave and direction becomes opposite.

as  $Y_i = 0.3 \cos(2x - 40t)$   
 $Y_r = 0.3 \cos(2x + 40t)$

- (b) For free end, there is no change in phase for reflected wave and direction becomes opposite.

as  $Y_i = 0.3 \cos(2x - 40t)$   
 $Y_r = 0.3 \cos(2x - 40t)$

4.  $v_1 = \frac{50}{k_1} = 25 \text{ m/s}$  and  $v_2 = 50 \text{ m/s}$

$a_t = \frac{2v_2}{v_1 + v_2} a_i = \frac{2 \times 50}{25 + 50} a_i$

$= \frac{4}{3} \times 2 \times 10^3 \text{ m} = \frac{8}{3} \text{ mm}$

$a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i = \frac{50 - 25}{50 + 25} a_i$   
 $= \frac{1}{3} \times 2 \times 10^3 \text{ m} = \frac{2}{3} \text{ mm}$

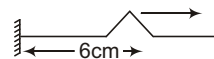
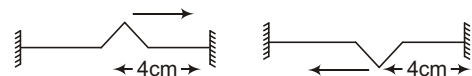
- as  $v_2 > v_1$  the boundary is rarer and there is no phase change.

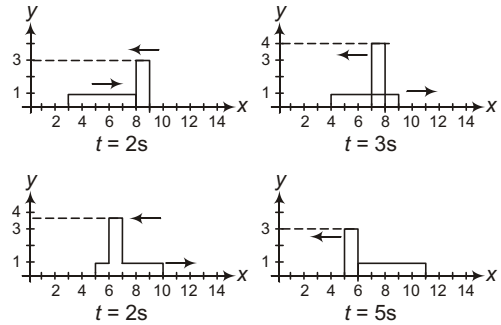
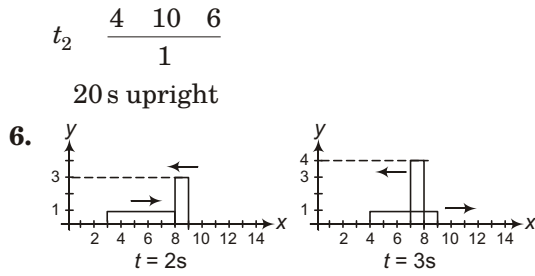
$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{50}$

$y_r = \frac{2}{3} \times 10^3 \cos(0.2x - 50t)$

and  $y_t = \frac{8}{3} \times 10^3 \cos(1.0x - 50t)$

5.  $t_1 = \frac{2 \times 40 \text{ cm}}{1 \text{ cm/s}} = 8 \text{ s}$ , inverted





### Introductory Exercise 15.2

1.  $y = 5 \sin \frac{x}{3} \cos 40 t = 2a \sin kx \cos t$

$a = \frac{5}{2} = 2.5 \text{ cm}, k = \frac{1}{3} \text{ cm}^{-1}, \omega = 40 \text{ s}^{-1}$

$v = \frac{\omega}{k} = \frac{40}{\frac{1}{3}} = 120 \text{ cm/s}$

$x = \frac{1}{2} \cdot \frac{2}{k} = \frac{1}{2} \cdot \frac{2}{\frac{1}{3}} = 3 \text{ cm}$

$v_P = \frac{dy}{dt} = 200 \sin \frac{x}{3} \sin 40 t$

$v_P = 1.5 \cdot \frac{9}{8} \cdot 200 \sin \frac{3}{2} \sin 40 t$   
 $= 200 \sin \frac{3}{2} \sin (45 t)$

$200 \cdot 1 = 200$   
 $0 \text{ cm/s}$

2. Two waves with different amplitudes can produce partial stationary waves with amplitude of antinodes being  $a_1 + a_2$  and amplitude of nodes being  $a_1 - a_2$ . As here node is not stationary that is why energy is also transported through nodes.

3. (a)  $\frac{0}{2} = 2 \text{ m}$        $4 \text{ m},$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{4 \cdot 10^{-2}}} = \frac{10^2}{2} = 50 \text{ m/s}$

$\frac{v}{\lambda} = \frac{50}{4} = 12.5 \text{ Hz}$

and is fundamental tone or first harmonic.

$y = 0.1 \sin \frac{2}{x} \sin 2 t$

$0.1 \sin \frac{2}{4} x \sin 2 t = 12.5 t$

$0.1 \sin \frac{2}{2} x \sin 25 t$

(b)  $3 \cdot \frac{2}{2} = 2 \text{ m}$        $\frac{4}{3} \text{ m}$  and  $v = 50 \text{ m/s}$

$\frac{v}{\lambda} = \frac{50}{\frac{4}{3}} = 37.5 \text{ Hz}$  and is 2nd

overtone or 3rd harmonic.

$y = 0.04 \sin \frac{2}{4/3} x \sin 2 t = 37.5 t$

$0.04 \sin \frac{3}{2} x \sin 75 t$

4.  $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}} = \sqrt{\frac{400 \cdot 4}{160 \cdot 10^{-3}}}$   
 $= \sqrt{\frac{1600}{16 \cdot 10^{-2}}} = 10^2 = 100 \text{ m/s}$

(a)  $\frac{0}{4} = l = 0$        $4l = 16 \text{ m}$

## 12 | Superposition of Waves

- $\frac{3}{4} l = \frac{4l}{3} = \frac{16}{3} \text{ m} = 5.33 \text{ m}$   
 and  $\frac{5}{4} l = \frac{4l}{5} = \frac{16}{5} \text{ m} = 3.2 \text{ m}$
- (b)  $\frac{v}{16} = 6.25 \text{ Hz}$   
 $\frac{v}{16/3} = 18.75 \text{ Hz}$   
 $\frac{v}{16/5} = 31.25 \text{ Hz}$
5.  $l = \frac{0.54}{2} n = 0.27 n$   
 and  $l = \frac{0.48}{2} (n - 1)$   
 $0.24 (n - 1)$   
 $0.27 n - 0.24 n = 0.24$   
 $0.03 n = 0.24$   $n = 8$
- (a) These are 8th and 9th harmonic  
 (b)  $l = 0.27 n = 0.27 \times 8 = 2.16 \text{ m}$   
 (c)  $\frac{l}{2} = 2l = 4.32 \text{ m}$
6.  $5 \times 2 = 10 = 54 \text{ Hz}$   
 $3 \times 6 = 18 = 18 \text{ Hz}$

7.  $\frac{1}{2l} \sqrt{\frac{F}{M}}$   
 $\frac{2}{1} \sqrt{\frac{F_2}{F_1}}$   
 $\sqrt{\frac{M}{2.2}} = \frac{260}{220} = \frac{13}{11}$   
 $\frac{M}{2.2} = \frac{169}{121} \Rightarrow M = \frac{48}{121} \times 2.2 = 0.873 \text{ kg}$
8.  $n = 250 \text{ Hz}$  and  $(n - 1) = 300 \text{ Hz}$   
 $n = 550 \text{ Hz}$   
 and  $n = 5$  So these are 5th and 6th harmonics.  
 $\frac{1}{2l} \sqrt{\frac{F}{M}}$   
 $F = 4l^2 v_0^2 = 4 \times 50^2 \times \frac{36 \times 10^3}{1} = 360 \text{ N}$

## AIEEE Corner

### ■ Subjective Question (Level 1)

1.  $A = \sqrt{A_1^2 + A_1^2 + 2A_1 A_1 \cos 90}$   
 $A = \sqrt{2} A_1 = 4\sqrt{2} \text{ cm} = 5.66 \text{ cm}$
2.  $v_2 = 2v_1$   
 $A_r = \frac{v_2}{v_2} \frac{v_1}{v_1} A = \frac{v_1}{3v_1} A = \frac{1}{3} A$   
 $A_t = \frac{2v_2}{v_2} \frac{v_1}{v_1} A = \frac{4v_1}{3v_1} A = \frac{4}{3} A$   
 $\frac{I_r}{I_i} = \frac{A_r^2}{A^2} = \frac{1}{9}$   
 and  $\frac{I_t}{I_i} = 1 - \frac{1}{9} = \frac{8}{9}$

$$A = \sqrt{10^2 + 20^2 + 2 \times 10 \times 20 \cos \frac{\pi}{3}}$$

$$= \sqrt{100 + 400 + 200} = \sqrt{700} = 10\sqrt{7}$$

26.46 units

$$\tan^{-1} \frac{20 \sin \frac{\pi}{3}}{10 + 20 \cos \frac{\pi}{3}}$$

$$= \tan^{-1} \frac{\sqrt{3}}{2} = 0.714 \text{ rad}$$

Phase =  $5x - 25t + 0.714 \text{ rad}$ .

4.  $y_1 = 1 \text{ cm} \sin ( \pi x - 50 \pi t )$

$$y_2 = 1.5 \text{ cm} \sin \left( \frac{9}{2} \text{ cm}^{-1} x - 100 \text{ s}^{-1} t \right)$$

$$y_1 = (4.5, 5 \times 10^{-3}) \quad 1 \text{ cm} \sin \left( 4.5 \frac{250}{1000} \right)$$

$$1 \sin \frac{9}{2} \frac{-}{4}$$

$$1 \sin \frac{17}{4}$$

$$1 \text{ cm} \sin \left( 4 \frac{-}{4} \right)$$

$$1 \sin \frac{1}{4} \frac{1}{\sqrt{2}} \text{ cm and}$$

$$y_2 = (4.5, 5 \times 10^{-3}) \quad 1.5 \text{ cm} \sin \left( \frac{9}{4} \frac{500}{1000} \right)$$

$$1.5 \text{ cm} \sin \left( \frac{9}{4} \frac{-}{2} \right)$$

$$= 1.5 \sin \frac{5}{4}$$

$$1.5 \sin \frac{-}{4}$$

$$1.5 \sin \frac{-}{4}$$

$$\frac{1.5}{\sqrt{2}} \text{ cm}$$

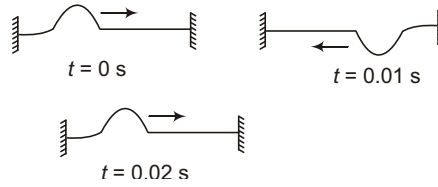
$$y = y_1 + y_2 = \frac{1}{\sqrt{2}} \frac{1.5}{\sqrt{2}} + \frac{0.5}{\sqrt{2}} \frac{1}{2\sqrt{2}} \text{ cm}$$

$$5. \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{16\text{N}}{0.4 \times 10^{-3} \times 10^2 \text{ kg/N}}} = \sqrt{\frac{16 \times 10^2}{4}} = 20 \text{ m/s}$$

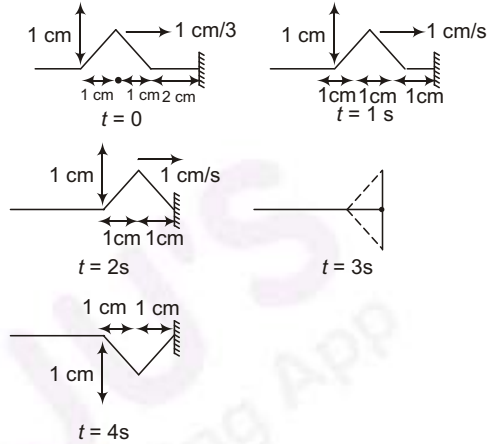
(a) For same shape, time,

$$t = \frac{2l}{v} = \frac{2 \times 0.2}{20} \text{ s} = 0.02 \text{ s}$$

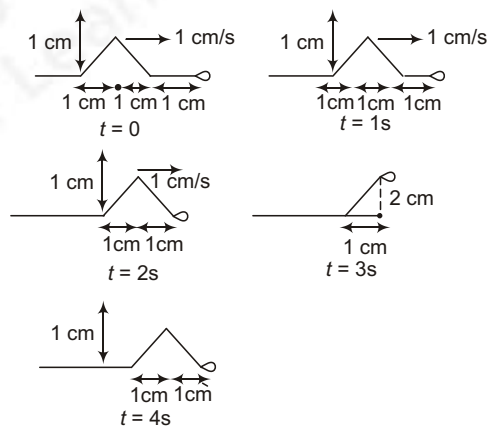
(b)



6. (a)



(b)



$$7. \quad y = 1.5 \sin(0.4x) \cos(200t)$$

$$\frac{2A}{k} = \frac{2}{0.4} = 5 \text{ m} = 15.7 \text{ m}$$

$$\frac{2}{2} \frac{200}{2} = 100 \text{ Hz} = 31.8 \text{ Hz}$$

$$v = \frac{200}{0.4} = 500 \text{ m/s}$$

## 14 | Superposition of Waves

8.  $y = y_1 + y_2 = 3 \text{ cm} \sin(x - 0.6t) + 3 \text{ cm} \sin(x + 0.6t)$

$6 \text{ cm} \sin x \cos 0.6t = R \cos 0.6t$   
where,  $R = 6 \text{ cm} \sin x$ .

(a)  $R(0.25) = 6 \text{ cm} \sin \frac{1}{4}$

$\frac{6}{\sqrt{2}} = 3\sqrt{2} \text{ cm} = 4.24 \text{ cm}$

(b)  $R(0.50) = 6 \text{ cm} \sin \frac{1}{2} = 6 \text{ cm}$

(c)  $R(1.50) = 6 \text{ cm} \sin \frac{3}{2} = 6 \text{ cm}$

$|R| = 6 \text{ cm}$

(d) For antinodes,  $R = 6 \text{ cm}$

$\sin x = 1 \Rightarrow x = (2n - 1)\frac{\pi}{2}$

or  $x = n \frac{\pi}{2} = 0.5 \text{ cm}, 1.5 \text{ cm}, 2.5 \text{ cm}$

9.  $\frac{2}{4} = \frac{2}{\lambda/2} = 4 \text{ cm}$

(a) Distance between successive antinodes  $= \frac{\lambda}{2} = 2 \text{ cm}$

(b)  $R(x) = 2A \sin kx = 2 \text{ cm} \sin \frac{x}{2} = 0.5$

$2 \sin \frac{x}{4}$

$\frac{2}{\sqrt{2}} = \sqrt{2} \text{ cm}$

10.  $n \frac{n-1}{2l} \sqrt{\frac{T}{\mu}} = \frac{n-1}{2 \cdot 20} \sqrt{\frac{20}{9 \cdot 10^{-3}}}$

$\frac{n-1}{60} = \frac{100\sqrt{2}}{3} = \frac{5\sqrt{2}}{9} = \frac{5\sqrt{2}}{9} (n-1)$

$0.786(n-1)$

$0.786 \text{ Hz}$

$1.57 \text{ Hz}, 2.36 \text{ Hz}, 3.14 \text{ Hz}$

11. (a)  $T = \frac{v^2}{g} = \frac{2^2}{2}$

$\frac{1.2 \cdot 10^3}{0.7} = (220)^2 = (1.4)^2$

$162.6 \text{ N}$

(b)  $2 \cdot 3 = 0 \cdot 3 = 220 \text{ Hz} = 660 \text{ Hz}$

12.  $n \frac{n-1}{2l} \sqrt{\frac{T}{\mu}} = \frac{n-1}{2 \cdot 0.6} \sqrt{\frac{50}{0.01}}$   
 $\frac{50\sqrt{2}}{1.2} (n-1) = 58.93 (n-1) \text{ Hz}$

$n = 20,000 \text{ Hz} \quad n = 338$

$338 \cdot 339 = 58.93 \cdot 19975.8 \text{ Hz}$

$19.976 \text{ kHz}$

13.  $n_0 = 420 \text{ Hz}$  and  $(n-1)_0 = 490 \text{ Hz}$

$0 = 70 \text{ Hz}$  and  $n = 6$

$0 = \frac{1}{2l} \sqrt{\frac{T}{\mu}} = l \frac{1}{2 \cdot 0} \sqrt{\frac{T}{\mu}} = \sqrt{\frac{450}{2 \cdot 70}}$

$\frac{300}{140} = 2.143 \text{ m}$

14.  $\frac{v}{\lambda} = \frac{400 \text{ m/s}}{800 \text{ Hz}} = \frac{1}{2} \text{ m}$ ,

$l = 4 \frac{\pi}{2} = 2 \pi = 1 \text{ m}$

(a)  $4_0 = 400 \text{ Hz} \quad 0 = 100 \text{ Hz}$

(b)  $7_0 = 700 \text{ Hz}$

16.  $0 = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

$0 = \frac{1}{l}$

$1 : 2 : 3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$

$l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$

$6 : 3 : 2 = 6x : 3x : 2x$

$6x = 3x = 2x = 1 \text{ m}$

$x = \frac{1}{11} \text{ m}$

position of first bridge  $6x = \frac{6}{11} \text{ m}$

and position of second bridge

$6x = 3x = 9x = \frac{9}{11} \text{ m}$

From the same end or  $1 \frac{9}{11} = \frac{2}{11} \text{ m}$

from other end.

$$17. \quad \frac{v}{2l} = \frac{v}{2 \times 124} = \frac{v}{248} = \frac{90}{186} = \frac{60}{124}$$

Thus length of the vibrating string has to be 60 cm.

$$18. \quad \frac{v}{2} = 15 \text{ cm} \quad 30 \text{ cm},$$

$$R_{\max} = 2A = 0.85 \text{ cm},$$

$$T = 0.075 \text{ s}$$

$$(a) \quad y = 2A \sin kx \sin \omega t$$

$$0.85 \text{ cm} \sin \frac{2}{0.3 \text{ m}} x \sin \frac{2}{0.075 \text{ s}} t$$

$$(b) \quad v = \frac{2}{k} = \frac{2}{2/0.3} = \frac{0.3}{0.075} = 4 \text{ m/s}$$

$$(c) \quad \frac{30}{4} = 7.5 \text{ cm}$$

$$R(7.5 \text{ cm}) = 2A \sin kx$$

$$0.85 \sin \frac{2}{30} \times 7.5 = 10.5$$

$$R(115 \text{ cm}) = 0.85 \sin \frac{21}{30}$$

$$= 0.85 \sin (0.7) = 0.85 (126) = 0.688 \text{ cm}$$

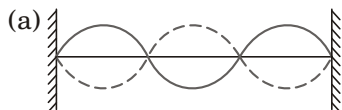
$$19. \quad \frac{v}{2l} = \frac{48}{2 \times 1.5} = 16 \text{ Hz}$$

and  $\frac{v}{2l} = \frac{3}{2} = 48 \text{ Hz}$  and  $\frac{v}{2} = \frac{48}{2} = 1 \text{ m}$

and  $\frac{v}{3} = \frac{4}{3} = \frac{64 \text{ Hz}}{64} = \frac{3}{4} = 0.75 \text{ m}$

$$20. \quad y = 5.60 \text{ cm} \sin (0.340 \text{ rad/cm } x) \sin (50.0 \text{ rad/s } t)$$

$$2A \sin (kx) \sin (\omega t)$$



$$(b) \quad 2A = 5.60 \text{ cm} \quad A = 2.80 \text{ cm}$$

$$(c) \quad \frac{1}{2} = \frac{3}{2} = \frac{2}{k} = \frac{3}{k}$$

$$\frac{0.0340}{2} \text{ cm} = 277.2 \text{ cm}$$

$$(d) \quad \frac{2}{k} = \frac{2}{0.0340} \text{ cm} = 184.8 \text{ cm}$$

$$\frac{50}{2} = 7.96 \text{ Hz}$$

$$T = \frac{1}{7.96} \text{ s} = 0.216 \text{ s}$$

$$v = 7.96 \text{ Hz} \times 184.8 \text{ cm} = 1470 \text{ cm/s}$$

$$(e) \quad (v_p)_{\max} = \frac{R_{\max}}{5.60 \text{ cm}} = \frac{2A}{50 \text{ rad/s}} = 280 \text{ cm/s}$$

$$(f) \quad \text{for eight harmonic,}$$

$$\frac{8}{2} = \frac{l}{4} = \frac{277.2}{4} = 69.3 \text{ cm}$$

$$k = \frac{2}{69.3} = 0.0289 \text{ rad/cm}$$

$$v = 8 v_0 = \frac{8}{3} v = \frac{8}{3} \times 7.96 \text{ Hz}$$

$$21.22 \text{ Hz}$$

$$2 \times 133.4 \text{ rad/s}$$

$$y = 5.60 \text{ cm} \sin (0.0907 \text{ rad/s } x) \sin (133 \text{ rad/s } t)$$

$$21. \quad (a) \quad \frac{v}{2l} = \frac{60}{2 \times 0.8} = 37.5 \text{ Hz}$$

$$96 \text{ m/s}$$

$$(b) \quad T = \frac{40}{80} = \frac{10^3}{10^2} = (96)^2$$

$$\frac{96^2}{20} = 460.8 \text{ N}$$

$$(c) \quad (v_p)_{\max} = \frac{R_{\max}}{0.3 \text{ cm}} = 2$$

$$60 \text{ rad/s} \quad 113 \text{ cm/s} \quad 1.13 \text{ m/s}$$

$$a_{\max} = \frac{2R_{\max}}{0.3 \text{ cm}} = (120)^2 = 0.3 \text{ cm/s}^2$$

$$426.4 \text{ m/s}^2$$

### Objective Questions (Level 1)



16 | Superposition of Waves

$$1. \frac{2}{1} \sqrt{\frac{T_2}{T_1}} \quad \frac{3}{2} \sqrt{\frac{T}{T}}$$

$$\frac{9T}{5T} = \frac{4(T)}{10T} = \frac{2.5}{2}$$

$$2. \frac{n}{2l} \sqrt{\frac{T}{r^2}} = \frac{n}{2l} \sqrt{\frac{T}{r^2}} \quad \text{constant.}$$

$$\frac{n}{2lr} \sqrt{\frac{T}{ld}} = \frac{n}{2lr} \sqrt{\frac{T}{ld}}$$

$$n \propto \frac{ld}{\sqrt{T}}$$

$$\frac{n_1}{n_2} = \frac{l_1}{l_2} \frac{d_1}{d_2} \sqrt{\frac{T_2}{T_1}}$$

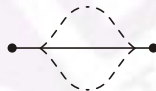
$$\frac{1}{2} \frac{1}{3} \sqrt{2}$$

$$\frac{1}{3\sqrt{2}} \quad 1:3\sqrt{2}$$

$$\text{or } \frac{n_2}{n_1} = \frac{1}{3\sqrt{2}}$$

$$3. f = \frac{1}{l} \left[ \frac{1}{f_0} + \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \right]$$

4. During overlapping the displacement of particles is zero while velocity is maximum. So the entire energy is purely kinetic.



$$5. y(x, y) = y_1 + y_2 = a \cos(kx - \omega t) + a \cos(kx + \omega t)$$

$2a \sin kx \sin \omega t$  is necessary for a node at  $x = 0$ . Thus,

$$y_2 = 2a \sin kx \sin \omega t = a \cos(kx - \omega t) - a \cos(kx + \omega t)$$

$$= a [\cos(kx - \omega t) - \cos(kx + \omega t)]$$

$$= a \cos(kx - \omega t)$$

6. In transverse stationary wave, longitudinal strain is maximum at node. While in longitudinal stationary wave at displacement node pressure and density are maximum. So all are correct.

7. In stationary wave all particles errors the mean position simultaneously and are at their maximum displacement simultaneously at different instant at this time all of them are at rest. So all are correct.

8. Maximum displacement

$$y_{\max} = 3A = \frac{A}{\sqrt{\frac{Y}{l}}} = \frac{2A}{\sqrt{\frac{Y}{l}}} = \frac{4A}{\sqrt{\frac{Y}{l}}}$$

$$\frac{v_t}{v_l} = \frac{v_t}{v_l} \sqrt{\frac{Y}{l}} = \frac{1}{\sqrt{\frac{Y}{l}}}$$

$$10. f_n = \frac{n}{2l} \sqrt{\frac{100}{0.01}} = 50(n-1)$$

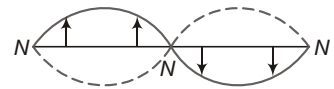
50 Hz, 100 Hz, 150 Hz

$$n_n = \frac{2n-1}{4l} \sqrt{\frac{100}{0.01}} = 25(2m-1)$$

25 Hz, 75 Hz, 125 Hz

$$n_2 = \frac{75 \text{ Hz} \left( \frac{f_1}{2} + \frac{f_2}{2} \right)}{50 \text{ Hz} + 100 \text{ Hz}} = 75 \text{ Hz}$$

11. In stationary waves all particles perform SHM such that they are at their positive and negative extremes



one time each in a time period, where they come to rest. Particles between two successive nodes are in phase while beside node are in opposite phase. So all the particles cannot be at positive extreme simultaneously.

12. The question is wrong, string has to be fix at one end and free at other. Then  $(2n-1) \times 90 \text{ Hz}$ ,  $(2n-3) \times 50 \text{ Hz}$  and  $(2n-5) \times 210 \text{ Hz}$

$2 \times 60 \text{ Hz}$  or  $3 \times 30 \text{ Hz}$  and  $n = 1$   
 i.e., vibrations are 3rd, 5th and 7th harmonic.

$$v = \frac{2l}{\lambda} \times f = \frac{1.6 \text{ m}}{30 \text{ Hz}} \times 48 \text{ m/s}$$

13.  $y = y_1 + y_2 + y_3 = 12 \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right)$

$$= 6 \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right) + 4 \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right)$$

$$R = \sqrt{6^2 + 4^2 + 2 \times 6 \times 4 \cos \frac{2\pi}{3}}$$

$$= \sqrt{100} = 10 \text{ mm}$$

14.  $(2n + 1) \times 105 \text{ Hz}$

and  $(2n + 3) \times 175 \text{ Hz}$

$$2 \times 70 \text{ Hz}$$

$$35 \text{ Hz}$$

15.  $l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 1 : \frac{1}{2} : \frac{1}{3}$

$$12 : 4 : 3$$

$$12x \quad 4x \quad 3x \quad 114 \text{ cm}$$

$$x = \frac{114}{19} \text{ cm} = 6 \text{ cm}$$

$$l_1 = 12x = 72 \text{ cm}, l_2 = 4x = 24 \text{ cm},$$

$$l_3 = 3x = 18 \text{ cm}$$

16.  $f = \frac{1}{\sqrt{T}}$

$$\frac{f/2}{f} = \sqrt{\frac{V_1 g}{V_2 g}} = \sqrt{1 - \frac{\rho_2}{\rho_1}} = \frac{1}{2}$$

$$1 - \frac{\rho_2}{\rho_1} = \frac{1}{4} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{3}{4}$$

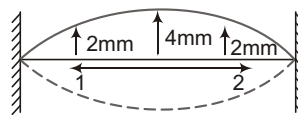
$$\frac{f/3}{f} = \sqrt{\frac{V_1 g}{V_2 g}} = \sqrt{1 - \frac{\rho_2}{\rho_1}} = \frac{1}{3}$$

$$1 - \frac{\rho_2}{\rho_1} = \frac{8}{9}$$

$$\frac{1}{2} = \frac{3/4}{8/9} = \frac{27}{32} \Rightarrow \frac{\rho_2}{\rho_1} = \frac{32}{27} = 1.18$$

where  $\rho_1$  is density of water and  $\rho_2$  is density of the other liquid.

17.  $R = 2A \sin Kx = 4 \text{ mm} \sin \frac{2\pi}{3} x$



$$4 \text{ mm} \sin \frac{2\pi}{3} x$$

$$2 \text{ mm} + 4 \text{ mm} \sin \frac{2\pi}{3} x$$

$$\frac{2\pi}{3} x$$

$$x = 0.5 \text{ m}$$

Thus points 1 and 2 are at 0.5 m from their nearest boundary. So separation between them is

$$1.5 \text{ m} - 2 \times 0.5 \text{ m} = 0.5 \text{ m}$$

18.  $y = A \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t \right)$

$$A \sin \left( 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right)$$

$$A \sin \left( 6\pi \left( \frac{x}{3} - \frac{t}{6} \right) \right)$$

$$y(3, t) = A \sin \left( 6\pi \left( \frac{3}{3} - \frac{t}{6} \right) \right)$$

$$A \sin \left( 6\pi \left( 1 - \frac{t}{6} \right) \right)$$

$$6\pi \left( 1 - \frac{t}{6} \right) = 6\pi - \pi t$$

$$t = \frac{11}{12} \text{ s}$$

19.  $\frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda} \times \frac{3}{2} = \pi$

and  $\frac{2\pi}{T} t = \frac{2\pi}{T} \times \frac{5}{2} = \pi$

20.  $n = 400 \text{ Hz}, (n + 1) = 450 \text{ Hz}$

$$50 \text{ Hz and } n = 8$$

$$\frac{1}{2l} \sqrt{\frac{T}{\mu}} = \frac{1}{2} \sqrt{\frac{T}{\mu}}$$

$$\frac{1}{2} \sqrt{\frac{490}{0.1}} = \frac{70}{100} \times 0.7 \text{ m}$$

21.  $3 \times \frac{2}{2} = 1 \text{ m}, \frac{2}{3} \text{ m}$

$$v = 300 \text{ Hz} \times \frac{2}{3} \text{ m} = 200 \text{ m/s}$$

22.  $l = \frac{1}{2}, \frac{2}{2}, \frac{3}{2}$

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$$1 \quad 2l, \quad 2 \quad \frac{2l}{2}, \quad 3 \quad \frac{2l}{3}$$

$$1 : 2 : 3 \quad 2l : \frac{2l}{2} : \frac{2l}{3}$$

$$1 : \frac{1}{2} : \frac{1}{3}$$

23.  $\frac{2}{vT} x \quad \frac{2}{300 \cdot 0.04} \quad (16 \quad 10)$

$$\frac{2}{12} \quad 6$$

24.  $v = \frac{30}{k} = 30 \text{ m/s}$

$$\sqrt{\frac{T}{A}} \quad \sqrt{\frac{T}{A}}$$

$$T = Av^2 = 8000 \cdot 10^6 = 900 \cdot 7.2 \text{ N}$$

25.  $5_0 = 480 \text{ Hz}, \quad 2_0 = \frac{2}{5} \cdot 480 \text{ Hz}$

$$192 \text{ Hz}$$

26.  $\frac{I_r}{I_i} = 0.64 = \left(\frac{A_r}{A_i}\right)^2$

$$\frac{A_r}{A_i} = 0.8 \quad A_r = 0.8 A_i$$

$$A_r \frac{v_2}{v_2} \frac{v_1}{v_1} A_i = 0.8 A_i \quad \frac{4}{5} A_i$$

$$5v_2 = 5v_1 \quad (4v_2 = 4v_1)$$

$$v_2 = 9v_1, \quad \frac{1}{9} v_1$$

For,  $v_2 > v_1$  the boundary is rarer and there will not be any change in phase of reflected wave and for  $v_2 < v_1$  a phase change of  $180^\circ$  takes place.

27.  $Y_r = 0.8 A \sin(kx - \omega t + 30^\circ + 180^\circ)$

$$A \frac{1}{2l} \sqrt{\frac{T}{d^2}} \quad \frac{1}{ld} \sqrt{\frac{T}{d^2}}$$

$$B \frac{1}{2} \frac{1}{2l} \sqrt{\frac{2T}{4d^2}}$$

$$\frac{1}{4ld} \sqrt{\frac{T}{d^2}} \quad \frac{1}{4} A$$

Third overtone of  $B = 4 B = A$

■ Passage (Q 28 to 30)

$$I_r = (100\% - 36\%) I_i = 64\% I_i = 0.64 I_i$$

$$\frac{A_r}{A_i} = \sqrt{\frac{I_r}{I_i}} = \sqrt{0.64} = 0.8 \quad \frac{v_2}{v_2} \frac{v_1}{v_1}$$

$$0.8 v_2 = 0.8 v_1 \quad (v_2 = v_1)$$

$$0.2 v_2 = 1.8 v_1$$

$$v_2 = 9v_1$$

for rarer boundary  
or  $1.8 v_2 = 0.8 v_1 \quad v_2 = \frac{1}{9} v_1$

for denser boundary

28.  $A_r = 0.8 A$

29.  $Y = A \sin(ax - bt) - \frac{0.8}{2} A \sin(ax - bt)$

$$A \sin(ax - bt) - \frac{0.8}{2} A \sin(ax - bt)$$

$$A \sin(ax - bt) - \frac{0.8}{2} A \sin(ax - bt)$$

$$A \cos(ax - bt) - 0.8 A \cos(ax - bt)$$

$$A \cos(ax - bt) - A \sin(ax - bt)$$

$$0.8 A \cos(ax - bt)$$

$$0.8 A \sin(ax - bt)$$

$$0.2 A \cos(ax - bt) - 1.8 A \sin(ax - bt)$$

$$0.2 A \cos(ax - bt) - 0.2 A \sin(ax - bt)$$

$$1.6 A \sin(ax - bt)$$

$$0.2 A \cos(ax - bt)$$

$$1.6 A \sin(ax - bt)$$

$$cA \cos(ax - bt) - 1.6 A \sin(ax - bt)$$

$$e = 0.2$$

30. For antinodes,  $\sin(ax - bt) = 1$

$$ax - (2n - 1) \frac{\pi}{2}$$

$$x = (2n - 1) \frac{\pi}{2a}, \quad \frac{3\pi}{2a}, \quad \frac{5\pi}{2a}$$

- So for second antinode,  $x = \frac{3}{2a}$
31.  $\frac{0}{0} \frac{15}{0} \sqrt{\frac{1.021}{1}} \quad 1.1$   
 $\frac{15}{2} \frac{0.1}{1} \frac{0}{\sqrt{T_2}} \frac{0}{\sqrt{T_1}} \frac{150 \text{ Hz}}{\sqrt{1.21}} \quad 1.1$   
 $\frac{2}{1} \frac{0}{2} \quad 110\% \text{ of } v_1$   
 $\frac{0}{2l}$  which do not change  
 So, (a), (c) and (d) are correct.

32. For interference, sources must be coherent their frequency has to be equal and phase different has to be constant. So, (a) and (d) are correct.
33. Stationary waves are formed due to superposition (**here use of the term 'interference' is literary and not scientific because interference is a different phenomenon than stationary waves**) of waves having same amplitude, same frequency and travelling opposite direction. Here nodes are the points who always remain at rest. Total energy is always conserved.
34. A medium is said to be rarer if speed of wave in it is higher. And as frequency is

constant, wavelength increases while frequency is constant, wavelength increases while phase do not change during change in medium.

35.  $y = A \sin kx \cos t - 2a \sin kx \cos t$   
 $a = \frac{A}{2}$ , third overtone means fourth harmonic and wire oscillate with four loops.  
 $l = 4 \frac{2}{2} = 2 \frac{2}{k} = \frac{4}{k}$   
 and stationary wave do not propagate.
36. For stationary waves, frequency and amplitude has to be same and direction has to be opposite with constant phase difference.

It is satisfied in (b) and (d) only.

37.  $y = y_1 + y_2 = 2A \cos kx \sin t + R \sin t$   
 $R = 2A \cos kx$  so at  $x = 0$  there is antinode.  
 $\cos kx = 1$   
 $kx = n\pi, x = \frac{n\pi}{k} = 0, \frac{\pi}{4}, \frac{2\pi}{4}, \dots$   
 are antinodes.

## JEE Corner

### ■ Assertion and Reason

1.  $y_1 = y_2 = A \sin(t - kx) + A \cos(t - kx)$   
 $A \sin(t - kx) = A \sin \frac{t - kx}{2} \cos \frac{t - kx}{2}$   
 $2A \sin \frac{t - kx}{2} \cos \frac{t - kx}{2}$   
 $\cos \frac{t - kx}{2} = \cot \frac{kx}{2}$

$$2A \sin kx = \frac{4}{4} \cos t = \frac{4}{4}$$

$$R \cos t = \frac{4}{4}$$

where,  $R = 2A \sin kx = \frac{4}{4}$  ;  
 $R(0) = 2A \sin \frac{4}{4} = A\sqrt{2}$

So, at  $x = 0$ , node is not present, i.e., Assertion is false.

2. In stationary waves only nodes are at rest and not other particles. It is so

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called as energy is not transmitted, thus assertion is false.

3. In rarer medium speed of wave is higher and as

$$A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$$A_r = A_i$$

so reason is correct explanation to assertion.

4. In second overtone or third harmonic there are three loops or three antinodes or four nodes. And length of the string,  $l = 3 \frac{\lambda}{2}$  so, assertion and reason are both true.



5. As speed of wave is constant in stretched wire, and  $v \propto f$ , so with increase in frequency, wavelength decreases. So reason is correct explanation of assertion.
6. In stationary waves, amplitude of nodes is zero and it is possible only when superposing waves has same amplitude. But it is not the only condition, there has to be same frequency, opposite direction of propagation and constant phase difference. So assertion is not completely true.
7. Energy lying between conservative node and antinode is constant where it moves to and fro between node and antinode.
8.  $\frac{I_{\max}}{I_{\min}} = \frac{25}{1} = \frac{5^2}{1^2} = \frac{A_1^2 + A_2^2}{A_1^2 - A_2^2}$
- $$\frac{5(A_1^2 + A_2^2)}{4A_1^2 - 6A_2^2} = \frac{A_1^2 + A_2^2}{A_1^2 - A_2^2} = 3 : 2$$

Thus reason is the correct explanation of assertion.

9.  $y = A \sin \frac{\pi}{2} x + A \sin \frac{\pi}{2} x$

$$A \sin \frac{\pi}{2} x + A \sin \frac{\pi}{2} x = 2A \sin \frac{\pi}{2} x$$

$$A \cos \frac{\pi}{2} x + A \sin \frac{\pi}{2} x = A \cos \frac{\pi}{2} x + A \sin \frac{\pi}{2} x$$

$$R = A \quad I_r = I_i$$

Assertion and reason are both true but reason do not explain assertion.

10. For two coherent sources phase difference has to be constant and that constant be same at all points as (t). Different light sources can never be coherent. So phase difference must be same, thus assertion is false.

■ Match the Columns

1.  $v_1 = \sqrt{\frac{T}{\mu}}$  and  $v_2 = \sqrt{\frac{T}{9\mu}} = \frac{v_1}{3}$

$$\frac{v_1}{v_2} = 3$$

$$A_r = \frac{v_1 - v_2}{v_1 + v_2} A_i = \frac{2/3}{4/3} A_i = \frac{1}{2} A_i$$

$$\text{and } A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$$= \frac{2 \cdot \frac{v_1}{3}}{v_1 + \frac{v_1}{3}} A_i = \frac{1}{2} A_i$$

(a)  $\frac{A_1}{A_2} = \frac{A_r}{A_t} = \frac{1/2 A_i}{1/2 A_i} = 1$  q

(b)  $\frac{v_1}{v_2} = 3$  r

(c)  $\frac{I_r}{I_i} = \frac{A_r^2}{A_i^2} = \frac{1}{4}$  and  $\frac{I_t}{I_i} = 1 - \frac{1}{4} = \frac{3}{4}$  s

$$\frac{I_t}{I_i} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\frac{I_1}{I_2} = \frac{I_r}{I_t} = \frac{I_r/I_i}{I_t/I_i} = \frac{1/4}{3/4} = \frac{1}{3}$$

(d)  $P = IS = 2^2 \cdot 2^2 \cdot A^2 v$

$$= \frac{1}{2} \cdot 2^2 \cdot A^2 \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2} \cdot 2^2 \cdot A^2 \sqrt{T}$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{2} A_1^2 \sqrt{T_1}}{\frac{1}{2} A_2^2 \sqrt{T_2}} = \frac{A_1^2 \sqrt{\frac{1}{2}}}{A_2^2 \sqrt{\frac{1}{9}}} = \frac{3}{1} = 3$$

2. (a)  $\frac{2}{4} \frac{2l}{5v} \frac{3}{5} = \frac{3}{5} r$

(b) Number of nodes in 3rd harmonic is 4 and in Fifth harmonic 6, so,  $\frac{4}{6} = \frac{2}{3} p$

(c) Number of antnodes in 3rd harmonic is 3 and in fifth harmonic 5,  $\frac{3}{5} = \frac{3}{5} r$

(d)  $\frac{2}{4} \frac{4}{2} \frac{5}{3} = \frac{5}{3} s$

3. In denser medium speed of wave is lesser and in rarer medium it is greater.

(a) When wave goes from denser to rarer medium its speed increases  $p$

(b) As frequency do not change with change in medium then with

increase in speed wavelength increases  $p$

(c) As  $v_t = v_i$  then  $A_t = A_i = p$

(d) Frequency remains unchanged  $r$

4.  $R = \frac{\sqrt{A^2 + A^2}}{2} = \frac{A\sqrt{2}}{2} = 2A \cos \frac{60}{2}$

(a)  $R(60) = 2A \cos \frac{60}{2}$

$2A \cos 30 = 2A \frac{\sqrt{3}}{2} = A\sqrt{3} = s$

(b)  $R(120) = 2A \cos 120/2 = 2A \cos 60 = 2A \frac{1}{2} = A = s$

(c)  $R(90) = 2A \cos 90/2 = 2A \cos 45 = A\sqrt{2}$

(d)  $R(0) = 2A \cos (0/2) = 2A$   
 $I_R = 2A^2 = 4I_i = r$

5.  $\frac{2}{3} \frac{3}{0} = \frac{210 \text{ Hz}}{70 \text{ Hz}} = 3$

(a)  $\frac{0}{70 \text{ Hz}} = s$

(b)  $\frac{2}{3} \frac{3}{0} = \frac{210 \text{ Hz}}{70 \text{ Hz}} = p$

(c)  $\frac{3}{4} \frac{4}{0} = \frac{280 \text{ Hz}}{70 \text{ Hz}} = r$

(d)  $\frac{1}{2} \frac{2}{0} = \frac{140 \text{ Hz}}{70 \text{ Hz}} = s$

# 16. Sound Waves

## Introductory Exercise 16.1

1.  $P_0 = S_0 k B$

$$B = \frac{P_0}{S_0 k} = \frac{P_0}{2 S_0} = \frac{1.4 \times 10^5 \text{ N/m}^2}{2 \times 3.14 \times 5.5 \times 10^{-6}}$$

2.  $v_{\text{max}} = \frac{1450 \text{ m/s}}{20 \text{ Hz}} = 72.5 \text{ m}$   
 $v_{\text{min}} = \frac{1450 \text{ m/s}}{20000 \text{ Hz}} = 7.25 \text{ cm}$

3. Pressure wave and displacement wave has a phase difference of  $\frac{\pi}{2}$ , so,

(a) When pressure is maximum, displacement is minimum *i.e.*, zero.

(b)  $S_0 = \frac{P_0}{k B} = \frac{P_0}{2 \frac{P_0}{v^2}} = \frac{P_0 v^2}{2 P_0} = \frac{v^2}{2}$   
 $S_0 = \frac{1.29 \times 340^2}{2} = 3.63 \times 10^6 \text{ m}$

4.  $S_0 = \frac{P_0}{k B} = \frac{P_0}{2 \frac{P_0}{v}} = \frac{P_0 v}{2 P_0} = \frac{v}{2}$   
 $S_0 = \frac{12 \times 8.18}{2 \times 1.29 \times (2700)^2} = 1.04 \times 10^{-5} \text{ m}$

## Introductory Exercise 16.2

1.  $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{4T_1}{T_1}} = 2$   
 $T_2 = 4T_1 = 4 \times 273 \text{ K} = 1092 \text{ K}$   
 $T_2 = 819 \text{ C}$

2.  $v_t = v_0 \left(1 + \frac{t}{273}\right)^{1/2} = v_0 \left(1 + \frac{t}{546}\right)$   
 $v_{30} = v_3 = v_0 \left(1 + \frac{30}{546}\right) = v_0 \left(1 + \frac{3}{54.6}\right)$   
 $v_0 = \frac{33}{54.6} = 332 \times \frac{33}{546} = 20.06 \text{ m/s}$

3.  $v = 250 \times 8 = 2000 \text{ m/s}$   
 $B = v^2 = 900 \times (2000)^2 = 3.6 \times 10^8 \text{ N/m}$   
 $B = 3.6 \times 10^9 \text{ Pa}$

4.  $v = \sqrt{\frac{Rt}{M}} = \sqrt{\frac{7 \times 8.314 \times 273}{32 \times 10^3}} = 315 \text{ m/s}$

### Introductory Exercise 16.3

$$\begin{aligned}
 1. \quad P_0 &= S_0 k B \quad 2 \quad v S_0 \\
 &= 2 \cdot 3.14 \cdot 300 \cdot 1.2 \cdot 344 \cdot 6 \cdot 10^{-6} \\
 &= 4.67 \text{ Pa} \\
 I &= \frac{P_0^2}{2 v} = \frac{(4.67)^2}{2 \cdot 1.2 \cdot 344} \\
 &= 2.64 \cdot 10^{-2} \text{ W/m}^2 \\
 L &= 10 \log \frac{I}{I_0} = 10 \log \frac{2.64 \cdot 10^{-2}}{10^{-12}} \\
 &= 104 \text{ dB}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad 2L &= L + 10 \log \frac{I}{I_0} + 10 \log \frac{I}{I_0} \\
 &= 10 \log ( ) + 9 \text{ dB} \\
 \log &= 0.9, \quad 10^{0.9} = 7.9
 \end{aligned}$$

$$\begin{aligned}
 3. \quad I &= \frac{1}{r^2} \quad I = \frac{k}{r^2} \\
 L_F &= L_M + 10 \log \frac{I_F}{I_M} \\
 &= 10 \log \frac{r_M^2}{r_F^2}
 \end{aligned}$$

$$20 \log \frac{3}{0.3} = 20 \text{ dB}$$

$$4. \quad (a) \quad I = \frac{P_0^2}{2 v}; I_{\max} = \frac{(28)^2}{2 \cdot 1.29 \cdot 345}$$

$$= 0.881 \text{ W/m}^2$$

$$L_{\max} = 10 \log \frac{0.881}{10^{-12}} = 119.45 \text{ dB}$$

$$I_{\min} = \frac{(2 \cdot 10^{-5})^2}{2 \cdot 1.29 \cdot 345}$$

$$= 4.49 \cdot 10^{-13} \text{ W/m}^2$$

$$L_{\min} = 10 \log \frac{4.49 \cdot 10^{-13}}{10^{-12}} = -1.43 \text{ dB}$$

$$= 3.48 \text{ dB}$$

$$(b) \quad S_0 = \frac{P_0}{k B} = \frac{P_0}{2 v}$$

$$(S_0)_{\max} = \frac{28}{2 \cdot 3.14 \cdot 500 \cdot 1.29 \cdot 345} = 2 \cdot 10^{-5} \text{ m}$$

$$(S_0)_{\min} = \frac{2 \cdot 10^{-5}}{2 \cdot 3.14 \cdot 500 \cdot 1.29 \cdot 345} = 1.43 \cdot 10^{-11} \text{ m}$$

### Introductory Exercise 16.4

$$\begin{aligned}
 1. \quad (2n - 1) \frac{\lambda}{2} &= 12 \text{ cm} \\
 \text{and } (2n - 1) \frac{\lambda}{2} &= 36 \text{ cm} \\
 \frac{36}{12} &= \frac{2n - 1}{2n - 1} \cdot \frac{24 \text{ cm}}{12 \text{ cm}} \\
 \frac{v}{0.24 \text{ m}} &= \frac{330 \text{ m/s}}{1375 \text{ Hz}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad x &= \frac{2}{v} = \frac{2}{6} = \frac{1}{3} \\
 &= \frac{350}{6 \cdot 500} = 0.117 \text{ m} = 11.7 \text{ cm} \\
 \frac{2}{T} &= t \cdot 2 = t \cdot 2 = 500 \cdot 10^3 \\
 \text{rad} &= 180
 \end{aligned}$$

$$3. \quad x_1 = 2 \sqrt{H^2 - \frac{d^2}{4}} \quad d = n$$

and

$$x_2 = 2 \sqrt{(H - h)^2 - \frac{d^2}{4}} \quad d = n \cdot \frac{1}{2}$$

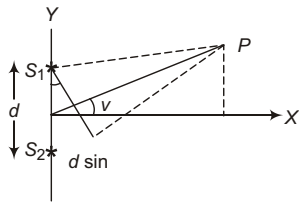
$$\frac{2}{2} = 2 \sqrt{H^2 - \frac{d^2}{4}} = 2 \sqrt{H^2 - \frac{d^2}{4}}$$

$$\text{or } 4 \sqrt{H^2 - \frac{d^2}{4}} = 4 \sqrt{H^2 - \frac{d^2}{4}}$$

$$2 \sqrt{4(H - h)^2 - d^2} = 2 \sqrt{4H^2 - d^2}$$

$$4. \quad x_p = d \sin \left( n \cdot \frac{1}{2} \right) \quad \text{for minima}$$





(a)  $d \sin \frac{\nu}{2}$  for first minima

$$\sin^{-1} \frac{\nu}{2d} = \sin^{-1} \frac{v}{2d}$$

$$\sin^{-1} \frac{340}{2 \cdot 600} = \frac{340}{1200}$$

$$\sin^{-1}(0.142) = 0.142 \text{ rad} = 8.14^\circ$$

(b) For, first maxima  $d \sin \theta = \lambda$

$$\sin^{-1} \frac{\lambda}{d} = \sin^{-1} \frac{340}{1200}$$

$$16.46^\circ$$

(c)  $x_{\max} = \frac{d}{n}$

$$\frac{2 \cdot 600}{340} = 3.53$$

$n = 3$  maxima.

5. (a) For coherent speakers in phase,

$$I_R = 4I_0 \cos^2 \frac{\nu}{2}$$

$$I_R = 4I_0 \cos^2 \frac{0}{2} = 4I_0$$

(b) For incoherent sources,

$$I_R = I_1 + I_2 = I_0 + I_0 = 2I_0$$

(c) For coherent speakers with a phase difference  $180^\circ$ .

$$I_R = 4I_0 \cos^2 \frac{180}{2} = 0$$

6.  $60 \text{ dB} = 10 \log \frac{I_0}{10^{-12}}$

$$\frac{10^6}{10^{-12}} = I_0 = 10^6 \text{ W/m}^2$$

$$\frac{2}{340} x = \frac{2}{v} x$$

$$\frac{2}{340} (11.8) = \frac{2}{3} x$$

(a)  $I_R = 4I_0 \cos^2 \frac{\nu}{2} = 4I_0 \cos^2 \frac{3}{2} = 0$

(b)  $I_R = 4I_0 \cos^2 \frac{4}{2} = 4I_0$

$$L_R = 10 \log \frac{4 \cdot 10^6}{10^{-12}} = 10 \log 4 \cdot 10^6 \text{ dB}$$

$$10 \log 4 + 60 \text{ dB} = 2 \log 2 + 60 \text{ dB} = 66 \text{ dB}$$

(e)  $\frac{60 \text{ dB}}{2} x = \frac{6 \text{ dB}}{2} \frac{85}{340} (11.8)$

$$I_R = 4I_0 \cos^2 \frac{3}{4} = 4I_0 \cos^2 \frac{\pi}{4} = 2I_0$$

$$L_R = 10 \log \frac{2 \cdot 10^6}{10^{-12}} = 63 \text{ dB}$$

7. (a)  $I_1 = \frac{10^3}{4 \cdot 2^2} = \frac{10^3}{16}$

$$19.9 \cdot 10^6 \text{ W/m}^2$$

$$199 \text{ W/m}^2$$

$$I_2 = \frac{10^3}{4 \cdot 3^2} = \frac{10^3}{36}$$

$$8.84 \cdot 10^6 \text{ W/m}^2$$

$$88.4 \text{ W/m}^2$$

(b)  $(I_P)_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (4.46 + 2.97)^2 = 55.27 \text{ W/m}^2$

(c)  $(I_P)_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = (4.46 - 2.97)^2 = 2.22 \text{ W/m}^2$

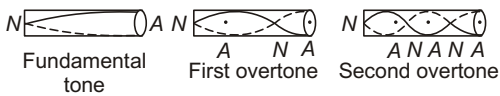
(d)  $I_P = I_1 + I_2 = 28.7 \text{ W/m}^2$

### Introductory Exercise 16.5

1. (a)  $0 \frac{v}{4l_c} l_c \frac{v}{4} \frac{345 \text{ m/s}}{220 \text{ Hz}}$

(b)  $\frac{3v}{2l_0} 5_0$   
 $l_0 \frac{3v}{10} \frac{3}{10} \frac{345}{220} 0.470 \text{ m}$

2. (a)



$d_A \ l \ 0.8 \text{ m}$   
 $\frac{0.8}{5} \text{ m}, \frac{2.4^3}{4} \text{ m}, 0.8 \text{ m}$   
 $d_A \ \frac{l}{5}, \frac{3l}{5}, l$

(b)



$d_A \ 0_n$   
 $d_A \ 0, \frac{2l}{3}, 0.533 \text{ m}$   
 $d_A \ 0, \frac{2l}{5}, \frac{4l}{5} \text{ m}, 0.32 \text{ m}, 0.64 \text{ m}$

3.  $\begin{array}{r|l} 2 & 400, 560 \\ 0 & \\ \hline 4 & 20, 28 \end{array}$

HCF of the two shows, 80 and the values, 400 Hz and 560 Hz are odd multiples of 80. These conservative

harmonics are odd, which can be seen in closed organ pipe only.

(b) These are 5th and 7th harmonic.

(c)  $0 \frac{v}{4l_c}$   
 $l_c \frac{v}{4} \frac{344}{80} 1.075 \text{ m}$

4.  $v \ 1000 \ 2 \ 6.77 \ 10^2 \text{ m/s}$

$\frac{135.4 \text{ m/s}}{\sqrt{\frac{RT}{M}}} \ r \ \frac{Mv^2}{RT}$   
 $n \ \frac{127 \ 10^3 \ (135.4)^2}{8.314 \ 400} \ 0.7 \ n$

As  $1 \ r \ 2$

$n \ 2 \ r \ 0.7 \ 2 \ 1.4 \ \frac{7}{5} \text{ diatonic}$

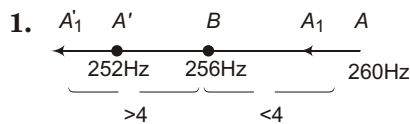
5.

$\frac{(2n-1)v}{4l_1} \ \frac{(2n-3)v}{4l_2}$

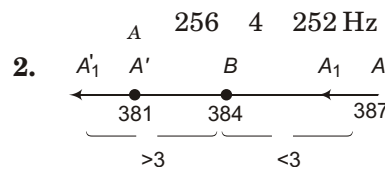
$\frac{2n-3}{2n-1} \ \frac{l_2}{l_1} \ \frac{100}{60} \ \frac{5}{3}$

$v \ \frac{4l_1}{2n-1} \ \frac{n}{4} \ \frac{1}{0.6} \ \frac{440}{3} \ 352 \text{ m/s}$

### Introductory Exercise 16.6



$A \ 252 \text{ Hz}$   
 $A \ (256 \ 4) \text{ Hz}$   
 and  $A \ n \ (256 \ 6) \text{ Hz}$   
 $256 \ 4 \ n \ 256 \ 6$   
 $4 \mp 6 \ n \ n \ 4 \ 6 \ 2$



$A \ 387 \text{ Hz}$   
 $A \ (384 \ 3) \text{ Hz}$   
 and  $A \ n \ 384 \ \text{m, m} \ 3$   
 $384 \ 3 \ n \ 385 \ \text{m}$

$$\frac{3}{2} \frac{v}{v_s} = \frac{1}{1 \mp \frac{v_s}{v}}$$

$$1 \mp \frac{v_s}{v} = \frac{1}{\frac{3}{2} \frac{v}{v_s}}$$

$$1 \mp \frac{v_s}{v} = \frac{2}{3} \frac{v_s}{v}$$

$$\frac{v_s}{v} = \frac{2}{3} \frac{v_s}{v}$$

When observer is moving,

$$1 \mp \frac{v_0}{v} = \frac{1}{1 \mp \frac{v_s}{v}}$$

$$1 \mp \frac{v_0}{v} = \frac{1}{1 \mp \frac{v_s}{v}}$$

So, it can be seen that,  $v_0$  and  $v_s$  are equal if  $u = v$ .

2.  $\frac{340}{200} = 1.7 \text{ m}$
- (a)  $uT = 1.7 \text{ m} \cdot \frac{80}{200} = 1.7 \text{ m}$
- (b)  $\frac{0.4 \text{ m} \cdot 1.3 \text{ m}}{340 \text{ m/s}} = 1.3 \text{ m}$   
 $262 \text{ Hz}$

$$\frac{T_A}{T_B} = 1.02$$

$$4. \quad 256 \cdot 4 \cdot \frac{v}{2 \cdot 0.25}$$

and  $256 \cdot \frac{v}{2 \cdot (0.25 - x)}$

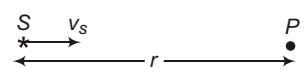
$$\frac{256}{252} = \frac{2 \cdot 0.25}{2 \cdot (0.25 - x)} \cdot \frac{1}{1 - 4x}$$

$$256 \cdot 4 \cdot 256x = 252$$

$$4 \cdot 4 \cdot 256x = 252$$

$$x = \frac{1}{256} \text{ m} = \frac{100}{256} \text{ cm} = 0.4 \text{ cm}$$

### Introductory Exercise 16.7

1. When source is moving,
- $$\frac{v}{v - v_s} = \frac{1}{1 \mp \frac{v_s}{v}}$$
- $$1 \mp \frac{v_s}{v} = \frac{1}{\frac{v}{v - v_s}}$$
- $$1 \mp \frac{v_s}{v} = \frac{v - v_s}{v}$$
- When observer is moving,
- $$1 \mp \frac{v_0}{v} = \frac{1}{1 \mp \frac{v_s}{v}}$$
- $$1 \mp \frac{v_0}{v} = \frac{1}{1 \mp \frac{v_s}{v}}$$
- So, it can be seen that,  $v_0$  and  $v_s$  are equal if  $u = v$ .
2.  $\frac{340}{200} = 1.7 \text{ m}$
- (a)  $uT = 1.7 \text{ m} \cdot \frac{80}{200} = 1.7 \text{ m}$
- (b)  $\frac{0.4 \text{ m} \cdot 1.3 \text{ m}}{340 \text{ m/s}} = 1.3 \text{ m}$   
 $262 \text{ Hz}$
3. For doppler effect there has to be relative motion between source and receiver, but as they are at rest relative to each other that's why there is no shift in wavelength and frequency.
4.  $\frac{v}{500} = \frac{344}{500} = 0.688 \text{ m}$
- (a) front  $uT = 0.688 \cdot \frac{30}{500} = 0.688 \cdot 0.060 = 0.628 \text{ m}$
- (b) behind  $uT = 0.688 \cdot 0.060 = 0.748 \text{ m}$
- (c) front  $\frac{344}{0.628} = 547.8 \text{ Hz}$
- (d) behind  $\frac{344}{0.748} = 459.9 \text{ Hz}$
5.  $\frac{v}{v - w} \frac{v_0}{v_s} = \frac{340}{340} \cdot \frac{5}{5} \cdot \frac{20}{10} = 300 \text{ Hz}$
- $$\frac{315}{345} = 300 \text{ Hz} \cdot 273.9 \text{ Hz}$$
6. 

## AIEEE Corner

### ■ Subjective Questions (Level 1)

$$1. \quad d \quad d_1 \quad d_2 \quad v \quad \frac{t_1}{2} \quad v \quad \frac{t_2}{2}$$

$$\frac{v}{2} (t_1 \quad t_2) \quad \frac{332}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

$$332 \quad 2 \quad 664 \text{ m}$$

The time for third echo is,

$$t \quad t_1 \quad t_2 \quad \frac{3}{2} \quad \frac{5}{2} \quad 4 \text{ s}$$

$$2. \quad v \quad \sqrt{\frac{RT}{M}} \quad \sqrt{\frac{7}{5} \frac{8.314}{2} \frac{300}{10^3}}$$

$$\sqrt{21 \frac{8.314}{10^4}} \quad 1321 \text{ m/s}$$

$$3. \quad v \quad \sqrt{\frac{P}{\frac{5}{3} \frac{76}{10^2} \frac{13.6}{10^3} \frac{9.8}{0.179}}}$$

$$\sqrt{\frac{5 \frac{76}{3} \frac{136}{0.179} \frac{9.8}{0.179}}{3 \frac{0.179}{0.179}}} \quad 971 \text{ m/s}$$

$$4. \quad (a) \quad B \quad v^2 \quad 2^2$$

$$1300 \quad 16 \quad 10^4 \quad 64$$

$$1.33 \quad 10^{10} \text{ N/m}^2$$

$$(b) \quad Y \quad v^2 \quad \frac{l^2}{t^2} \quad \frac{6400}{(3.9 \cdot 10^4)^2} \quad \frac{(15)^2}{(10^4)^2}$$

$$9.47 \quad 10^{10} \text{ Pa}$$

$$5. \quad v_t \quad \sqrt{\frac{l}{l} v_l} \quad \frac{l}{l} \quad \frac{v_t}{v_l} \quad \frac{F}{A} \quad Y \quad \frac{l}{l}$$

$$Y \quad \frac{v_t}{t_l} \quad \frac{Y}{30} \quad \frac{Y}{900}$$

$$6. \quad M_{\text{mix}} \quad \frac{2 \cdot 2 \cdot 1 \cdot 14}{2 \cdot 1} \quad 6 \text{ m/mole}$$

$$\frac{v_{\text{mix}}}{v_{\text{H}_2}} \quad \sqrt{\frac{M_{\text{H}_2}}{M_{\text{mix}}}} \quad \sqrt{\frac{2}{6}} \quad \frac{1}{\sqrt{3}}$$

$$v_{\text{mix}} \quad \frac{1}{\sqrt{3}} v_{\text{H}_2} \quad \frac{1}{\sqrt{3}} v_0 \quad \sqrt{\frac{T_2}{T_1}} \quad \frac{v_0}{\sqrt{3}} \quad \sqrt{\frac{300}{273}}$$

$$\frac{v_0}{\sqrt{2.73}} \quad \frac{1300}{\sqrt{2.73}} \quad 787 \text{ m/s}$$

$$7. \quad L_1 \quad 10 \log \frac{10^6}{10^{12}} \quad 60 \log 10 \quad 60 \text{ dB}$$

$$L_2 \quad 10 \log \frac{10^9}{10^{12}} \quad 30 \log 10 \quad 30 \text{ dB}$$

$$L_1 \quad 2L_2$$

$$8. \quad 100 \text{ dB} \quad 10 \log \frac{I}{I_0} \text{ dB}$$

$$I \quad 10^{10} I_0 \quad 10^2 \text{ W/m}^2$$

$$P \quad 4 r^2 I \quad 4 \quad (40)^2 \quad 10^2$$

$$64 \text{ W} \quad 201 \text{ W}$$

$$9. \quad (a) \quad 60 \text{ dB} \quad 10 \log \frac{I}{I_0} \text{ dB}$$

$$I \quad 10^6 I_0 \quad 10^6 \text{ W/m}^2$$

$$(b) \quad P \quad AI \quad 120 \quad 10^4 \quad 10^6 \text{ W}$$

$$1.2 \quad 10^8 \text{ watt}$$

$$10. \quad (a) \quad L \quad 13 \text{ dB} \quad 10 \log \frac{I_2}{I_1} \text{ dB}$$

$$I_2 \quad 10^{1.3} I_1 \quad 20 I_1$$

(b) As with doubling the intensity, loudness increases by 3 dB irrespective of the initial intensity.

$$11. \quad I \quad \frac{P}{4 r^2} \quad \frac{5}{4 (20)^2} \quad \frac{5}{4 \cdot 400}$$

$$\frac{1}{320} \text{ W/m}^2 \quad 9.95 \quad 10^4 \text{ W/m}^2$$

$$(b) \quad I \quad 2^2 \cdot 2 a^2 \quad v \quad a \quad \frac{1}{\sqrt{2}} \frac{I}{v}$$

$$\frac{1}{300} \sqrt{\frac{1}{320 \cdot 2 \cdot 129 \cdot 330}}$$

$$\frac{1}{300} \frac{1}{10^{12}} \sqrt{\frac{1}{85.5}}$$

$$1.15 \cdot 10^6 \text{ m}$$

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12. 60 dB  $10 \log \frac{I}{10^{-12}}$  dB

$$I = 10^6 \text{ W/m}^2 \text{ and } a = \frac{1}{\sqrt{2}} \sqrt{\frac{T}{v}}$$

$$\frac{1}{800} \sqrt{\frac{10^6}{2 \cdot 1.29 \cdot 330}} = 13.6 \cdot 10^{-9} \text{ m}$$

13. 102 dB  $10 \log \frac{I}{I_0}$  dB

$$I = 10^{10.2} I_0 = 10^{10.2} \cdot 10^{-12} = 10^{1.8} \text{ W/m}^2$$

$$P = 4 \pi r^2 I = 4 \cdot 3.14 \cdot (20)^2 \cdot 10^{1.8} = 80 \text{ W}$$

14.  $I = 2^2 v^2 a^2 = 4 \cdot (3.14)^2 \cdot (300)^2 \cdot (0.2 \cdot 10^{-3})^2 = 1.29 \cdot 330 \text{ W/m}^2 = 30.25 \text{ W/m}^2$

$$L = 10 \log \frac{I}{I_0} \text{ dB} = 10 \log \frac{30.25}{10^{-12}} \text{ dB} = 134.8 \text{ dB}$$

15. (a)  $v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.18 \cdot 10^9}{10^3}} = 1.48 \cdot 10^3 \text{ m/s}$

$$A_w = \frac{1}{\sqrt{2}} \sqrt{\frac{I}{\rho v}} = \frac{1}{\sqrt{2}} \sqrt{\frac{I}{\rho v}}$$

$$\frac{1}{3400} \sqrt{\frac{3 \cdot 10^6}{2 \cdot 10^3 \cdot 1.48 \cdot 10^3}} = \frac{9.44 \cdot 10^{11} \text{ m}}{1.48 \cdot 10^3} = 0.43 \text{ m}$$

(b)  $v_a = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.4 \cdot 10^5}{1.2}} = 341.6 \text{ m/s}$

$$A_a = \frac{1}{3400} \sqrt{\frac{3 \cdot 10^6}{2 \cdot 1.2 \cdot 341.6}} = \frac{5.66 \cdot 10^9 \text{ m}}{341.6} = 0.1 \text{ m}$$

(c)  $A_a = A_w; \frac{A_a}{A_w} = \frac{5.66 \cdot 10^9}{9.44 \cdot 10^{11}} = 60$

As bulk modulus of water is much larger than air, such that displacement of particles of medium becomes less.

16.  $I = \frac{p_0^2}{2 \rho v} = \frac{(6 \cdot 10^5)^2}{2 \cdot 1.29 \cdot 343} \text{ W/m}^2 = 4 \cdot 10^{12} \text{ W}$

$$L = 10 \log \frac{I}{I_0} = 10 \log \frac{4 \cdot 10^{12}}{10^{-12}} = 20 \log 2 = 6 \text{ dB}$$

17.  $v = \frac{v}{2l} = 594 \text{ Hz}$

$$c = \frac{v}{4l} = \frac{594}{2} = 297 \text{ Hz}$$

18.  $f_n = \frac{(n-1)v}{2l} = (n-1) \frac{344}{2 \cdot 0.45} = (n-1) \cdot 382.2 \text{ Hz}$

382.2 Hz, 764.4 Hz, 1146.7 Hz, (2n-1)v

19.  $f_n = \frac{v}{4l} = \frac{344}{2 \cdot 0.45} = (2n-1) \cdot 191.1 \text{ Hz}$

191.1 Hz, 573.3 Hz, 955.6 Hz

20.  $v = \frac{v}{4l} = \frac{4 \cdot 0.15 \cdot 500}{300} = 300 \text{ m/s}$

$$f_n = \frac{v}{2l} = \frac{300}{2 \cdot 0.6} = 250 \text{ Hz}$$

21.  $y = A \cos kx \cos \omega t = A \cos \frac{2}{1.6} x \cos 2 \frac{330}{1.6} t$

$$y = A \cos 3.93x \cos 1296 t$$

$$\frac{2n-1}{4} \cdot \frac{1}{0.5} = \frac{2n-1}{4} \cdot \frac{3}{0.84} = \frac{2n-1}{1} \cdot \frac{84}{50} = 1.68$$

$$n = \frac{3 \cdot 1.68 \cdot 2n \cdot 0.68}{4lv} = \frac{1 \text{ as } n \text{ is an integer}}{4 \cdot 0.5 \cdot 512} \text{ m/s}$$

$$v = \frac{341.3 \text{ m/s}}{2n \cdot 5} = \frac{4l}{2n \cdot 5} v = \frac{7}{4 \cdot 512} \cdot 341.3$$

22.  $c = \frac{v}{4l} = \frac{340}{4 \cdot 1} = 85 \text{ Hz}$

$s = \frac{v}{0.4} \sqrt{\frac{F}{\dots}} = 85$

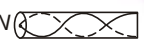
$F = (85 \cdot 0.4)^2 = (34)^2 = \frac{4 \cdot 10^3}{0.4} = 11.65 \text{ N}$

23.  $c = \frac{v}{4(l - e)} = v \cdot 4 \cdot (l - l)$

$4 \cdot (l - 0.3d) = 4 \cdot 480(0.16 - 0.3 \cdot 0.05) = 336 \text{ m/s}$

24. (a)  $e = \frac{(2n - 1) \cdot 4l}{440} = \frac{5 \cdot 330}{4l} = \frac{15}{16} \text{ m}$

$l = \frac{5 \cdot 330}{4 \cdot 440} = \frac{15}{16} \text{ m}$

(b)   $A = \frac{5}{4} l = \frac{15}{16} \text{ m}$

$\frac{15}{16} = \frac{4}{5} \cdot \frac{3}{4} \text{ m}$

$p = p_0 \cos kx = p_0 \cos \frac{2}{3/4} = \frac{15}{32}$

$p_0 \cos \frac{15}{12} = p_0 \cos \frac{5}{4} = \frac{p_0}{\sqrt{2}}$

(c) At open end there is pressure node, so,  $p_{\max} = p_{\min} = p_0$

(d) At closed end there is pressure antinode, such that,  $p_{\max} = p_0$  and  $p_{\min} = p_0$

25. (a)  $c = \frac{v}{4l_c} = \frac{345}{4 \cdot 220} = 0.392 \text{ m}$

(b)  $l_0 = \frac{5}{4} l_c, l_0 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{4}{5} l_c = \frac{6}{5} l_c = \frac{6}{5} \cdot 0.392 \text{ m} = 0.47 \text{ m}$

26.  $s = c = \frac{v_s}{2 \cdot 0.8 l_c} = \frac{v_s}{4 l_c} = \frac{v_s}{v_a} = \frac{1.6}{4} = 0.4$

27. (a)  $s = \frac{v}{300} = \frac{17}{15} \text{ m} = 1.13 \text{ m}$

(b)  $a = \frac{v_s T}{300} = \frac{30}{30} = 1.03 \text{ m}$

$b = \frac{v_s T}{300} = \frac{37}{30} = 1.23 \text{ m}$

28.  $\frac{1}{2l} \sqrt{\frac{F}{\dots}} = \frac{1}{2} \frac{F}{F} = \frac{F}{F} = 2 = 2 \cdot \frac{15}{440} = \frac{3}{440} = 0.68\%$

$438.5 \text{ Hz or } 441.5 \text{ Hz}$

29.  $v = 0.32 \text{ m/s};$

$vT = 0.32 \cdot 1.6 \text{ m} = 0.512 \text{ m.}$

$a = \frac{v_s T}{1.6} = \frac{0.12}{1.6} = 0.245 \text{ m/s}$

$b = \frac{v_s T}{0.392} = \frac{0.512 \text{ m}}{0.392} = 1.6 \text{ m.}$

30. (a)  $a = \frac{v \cdot v_0}{v \cdot v_s} = \frac{340 \cdot 18}{240 \cdot 30} = 262 \text{ Hz}$

$\frac{358}{310} = 262 \text{ Hz} \quad 302.5 \text{ Hz}$

(b)  $r = \frac{v \cdot v_0}{v \cdot v_s} = \frac{340 \cdot 18}{340 \cdot 30} = 262$

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$$\frac{322}{370} \quad 262 \text{ Hz} \quad 228 \text{ Hz}$$

31.  $\frac{v}{v} \frac{v}{v_s} \sim \frac{2v_s}{v} \quad v \frac{v}{2v_s}$

$$\frac{340}{2} \frac{4}{1} \quad 680 \text{ Hz}$$

32.  $l_c \frac{v}{4} \frac{330}{4} \quad 110 \quad 2.2; \quad c \frac{v}{4l_c}$

$$l_c \frac{3}{4} \text{ m}; \quad 0 \frac{2v}{2l_0}$$

$$l_0 \frac{2v}{2} \frac{2}{2(330 \quad 2.2)}$$

0.993 m or 1.007 m

33.  $\frac{7}{2}$  and  $\frac{P}{Q}$  as beat  
frequency increases waxing of  $P$ .

$$Q \frac{5}{v} \frac{v}{v_s} \quad Q \frac{332}{327} \frac{5}{Q}$$

$$Q \frac{327 \text{ Hz}}{327} \text{ and } \frac{7}{2} \quad 323.5 \text{ Hz}$$

When  $Q$  gives 5 beats with its own echo.

OR

$$P \quad Q \quad \frac{7}{2} \quad q \quad 5 \quad \frac{332}{327} \quad Q \quad 5$$

$$5 \quad \frac{7}{2} \quad \frac{5}{327} \quad Q$$

$$Q \quad \frac{327 \quad 1.5}{5} \quad 98.1 \text{ Hz}$$

$$P \quad 98.1 \quad 2.5 \quad 94.6 \text{ Hz}$$

When  $P$  gives 5 beats with the echo of  $Q$ .

34.  $\frac{v}{v} \frac{v}{v_s} \sim \frac{2v_s}{v} \quad \frac{2v_s}{v_s^2} \sim \frac{2v_s}{v}$

$$v_s \frac{v}{2} \frac{340}{2} \quad 680$$

$$v_s \quad \frac{1}{2} \text{ m/s}$$

35.  $(2n - 1) \frac{1}{2} \quad 11.5 \text{ cm}$

$$(2n - 3) \frac{1}{2} \quad 34.5 \text{ cm}$$

$$\frac{2n - 3}{2n - 1} \frac{34.5}{11.5} \quad 3 \quad 4n - 0 \quad n - 0$$

$$\frac{11.5 \text{ cm}}{2} \quad 23 \text{ cm}$$

$$\frac{v}{0.23 \text{ m}} \quad \frac{331.2 \text{ m/s}}{0.23 \text{ m}} \quad 1440 \text{ Hz}$$

36.  $\frac{v}{220} \quad 1.5 \text{ m}$

$$x \quad S_2 P \quad S_1 P \quad 3 \quad \frac{3}{4} \quad \frac{9}{4} \text{ m}$$

$$\frac{3}{2} \quad \frac{3}{2} \quad \frac{3}{2} \quad (2n - 1) \frac{1}{2}$$

Here,  $S_1 P \quad \frac{3}{4} \quad \frac{1}{2}$

$$1 \quad \frac{2}{2} S_1 P \quad \frac{2}{2}$$

and  $S_2 P \quad 3 \quad 2 \quad \frac{3}{2} \quad 2$

$$2 \quad \frac{2}{2} \quad 2 \quad 4$$

Destructive interference will take place at  $P$ .

$$P_P \quad P_{\min} \quad (\sqrt{P_1} \quad \sqrt{P_2})^2$$

$$(\sqrt{1.8 \quad 10^3} \quad \sqrt{1.2 \quad 10^3})^2$$

$$0.6 \quad 10^3 (\sqrt{3} \quad \sqrt{2})^2$$

$$0.6 \quad 10^3 \quad 0.1 \quad 6 \quad 10^5 \text{ W}$$

37.  $x \quad 2 \sqrt{2^2 \quad \frac{x^2}{2}} \quad x \quad n \quad 1$

$$\frac{360 \text{ m/s}}{360 \text{ Hz}} \quad 1 \text{ m}$$

$$2 \sqrt{4 \quad \frac{x^2}{4}} \quad 1 \quad x$$

or  $4 \quad 4 \quad \frac{x^2}{4} \quad 1 \quad 2x \quad x^2$

$$16 \quad 1 \quad 2x$$

$$x \quad 7.5 \text{ m}$$

■ Objective Questions (Level 1)

1. Sound cannot travel in vacuum, as it is mechanical wave.
2. Longitudinal waves can travel through all mechanical mediums.

$$3. \sqrt{\frac{RT}{32}} \sqrt{\frac{R}{28}} \quad T \frac{32}{28} \quad 288 \text{ K} \quad \frac{8}{7} \quad 288 \text{ K} \quad 56 \text{ C}$$

4. Third overtone is 7th harmonic *ie*, there 4 nodes and 4 antinodes.



5.  $\frac{v}{l} \propto \sqrt{T}$  so with increase in temperature, frequency increases.

6. For sound water is rarer medium and air is denser medium so, it bends towards normal while going from water to air.

$$7. c \frac{v}{4l_c} \quad o \quad \frac{v}{2l_o} \quad \frac{l_c}{l_o} \quad \frac{2}{4} \quad 1:2$$

$$8. \frac{2}{1} \sqrt{\frac{F_2}{F_1}} \quad F_2 \frac{2}{1} F_1 \quad M_2 \frac{2}{1} M_1 \quad \frac{256}{320} \quad 10 \text{ kg} \quad 6.4 \text{ kg}$$

*OM*  $M_2$   $M_1$  6.4 10 3.6 kg  
*i.e.*, Mass has to be decreased by 3.6 kg

9. direct  $\frac{v}{v - v_s}$  and reflected  $\frac{v}{v + v_s}$   
 as  $D = R$  so there will be no beats *i.e.*, beat frequency will be zero.

$$10. \frac{2}{v} \frac{1}{v} \frac{v}{2} \frac{v}{1} \frac{v}{1} \frac{v}{2} \quad v \frac{1}{2}$$

$$v \frac{1.01 \frac{10}{3}}{0.01} \quad 337 \text{ m/s}$$

$$11. \frac{v}{v} \frac{1}{2} \quad n \quad n \quad \frac{1}{2} \quad 0.5$$

$$12. I_{\max} (\sqrt{I} \sqrt{I})^2 \quad 4I \quad NI \quad N \quad 4$$

$$13. \frac{v}{4(l_1 - e)} \frac{3v}{4(l_2 - e)} \quad l_2 \quad e \quad \frac{3l_1}{2} \quad 3e \quad e \quad \frac{3l_1}{2} \quad e \quad \frac{42 \cdot 3 \cdot 17}{2} \text{ cm} \quad 0.5 \text{ cm}$$

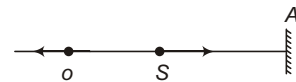
$$v \quad 4(l_1 - e) \quad 4 \quad 500(17 - 8.5) \quad 10^2 \quad 20 \quad 17.5 \quad 350 \text{ m/s}$$

14. At the moment when velocity of source is perpendicular to the line joining source and observer then there is no Doppler effect *i.e.*,  $n = n_1 = n_2 = 0$

$$15. \frac{(n - 1)v}{4l} \quad (2n - 1) \frac{340}{4} \quad 85(2n - 1)$$

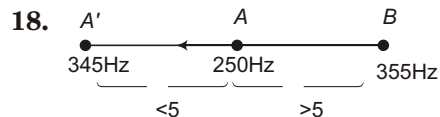
85, 255, 425, 595, 765, 935  
 6 frequencies below 1 kHz.

$$16. \frac{v - v_0}{v - v_s} \frac{v - v_0}{v - v_s} \quad 1 \quad \frac{v - v_0}{v - v_s}$$



$$\frac{v_s - v_0}{v - v_s} \frac{10}{360} \quad 180 \quad 5 \text{ Hz}$$

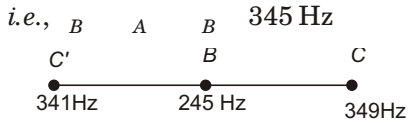
$$17. n \quad 1 \quad 2 \quad \frac{v}{1} \quad \frac{v}{2} \quad v \frac{(2 - 1)}{1} \quad v \frac{n - 1}{2}$$





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As beat frequency between A and B decreases on loading A.



After loading A,  $f_A = 345 \pm 2 = 247 \text{ Hz}$  and  $f_C = 349 \pm 6 = 341 \text{ or } 353 \text{ Hz}$ .

As possible frequency of C are 341 Hz and 249 Hz then only 341 Hz is justified.

19.  $e \frac{l_2}{v} = \frac{3l_1}{5v} = \frac{122}{4} = \frac{3}{4} \text{ cm}$

So,  $l(l_1 - e) = \frac{5l_1}{4} = \frac{4l}{e}$

20.  $\frac{1}{2} \frac{F}{F} = \frac{1}{2} \frac{F}{F} = \frac{1}{2} \cdot 200 = \frac{1}{100} = 1 \text{ Hz}$

21.  $\frac{2n-1}{4l} v = \frac{2n-1}{4} \frac{v}{l} = \frac{2n-1}{4} \frac{340}{4} = \frac{1}{4} \text{ m}, \frac{3}{4} \text{ m}, \frac{5}{4} \text{ m}$ .

As,  $l_{\max} = 120 \text{ cm}$   $l = 25 \text{ cm}$   $75 \text{ cm}$ .

Height of water column

22.  $7 \frac{1}{4} = 105 \text{ cm}$   $\frac{105}{7} = 15 \text{ cm}$



$\frac{60}{4} = 15 \text{ cm}$

So, nodes are at,  $\frac{3}{4}l, \frac{5}{4}l$  and  $7 \frac{1}{4}l$  from closed end i.e., they are at, 15 cm, 45 cm, 75 cm and 105 cm.

23.  $c \frac{v}{4l} = 512 \text{ Hz}$ ,  $o \frac{v}{2l} = 2 \frac{v}{4l}$

$2 \frac{v}{4l} = 2 \cdot 512 \text{ Hz} = 1024 \text{ Hz}$

24.  $M_{\min} = \frac{1}{1} \frac{32}{1} \frac{1}{2} = 17$

$\frac{v_{\min}}{v_2} = \sqrt{\frac{M_{H_2}}{M_{\min}}} = \sqrt{\frac{2}{17}}$

25.  $f_a$  and  $f_r$  but  $a$  constant and  $r$  constant.

So, curve in (b) represents correctly.

26.  $u = \frac{(2n-1)v}{4l} = \frac{(m-1)v}{2l}$

How,  $\frac{(2n-1)v}{4 \cdot 2l} = \frac{(m-1)v}{4 \cdot 2l}$

$\frac{4}{2} = 2 \text{ beat/s}$

27.  $a \frac{v}{v_1} = r \frac{v}{v_1}$

$\frac{2vv_1}{(v-v_1)(v+v_1)} = \frac{2 \cdot 320 \cdot 4}{(320-4)(320+4)} = \frac{2560}{316 \cdot 324} = 6 \text{ Hz}$

28.  $c \frac{(2n-1)v}{4l_c} = \frac{320}{4} (2n-1)$

$(2n-1) = 80 \text{ Hz}$   $80 \text{ Hz}, 240 \text{ Hz}, \dots$   $400 \text{ Hz}, \dots$

$o \frac{(n-1)v}{2l_0} = \frac{320}{2 \cdot 1.6} (n-1)$

$(n-1) = 100 \text{ Hz}$   $100 \text{ Hz}, 200 \text{ Hz}, \dots$   $300 \text{ Hz}, 400 \text{ Hz}, \dots$

$c \quad o \quad 400 \text{ Hz}$

29.  $I_{\max} = 4I_0$

and  $I_{\max} = \frac{4I_{\max}}{L} = \frac{16I_0}{10 \text{ dB}} = 10 \log 16$

$10 \text{ dB} = 40 \log 2 \text{ dB} = 22 \text{ dB}$

30.  $\frac{2}{k} = \frac{2}{2} = 4 \text{ m}$

$l = 5 \frac{1}{4} = 5 \frac{4}{4} \text{ m} = 5 \text{ m}$

$$31. d = (2n-1) \frac{v}{4} = \frac{(2n-1)}{4} \frac{330}{660} \text{ m}$$

$$\frac{330}{24} (2n-1) \text{ cm} = (2n-1) \cdot 13.75 \text{ cm.}$$

13.75 cm, 41.25 cm, 68.75 cm, 96.25 cm  
etc.

$$32. \frac{v_1}{332} - \frac{v_2}{10^2} = \frac{v(2-1)}{2}$$

$$\frac{0.49}{332} - \frac{0.5}{10^2} = 13.15 \text{ Hz}$$

$$33. \frac{v_B}{300} - \frac{v_A}{30} = \frac{300}{300} - \frac{300}{300}$$

33.33 Hz and  $\frac{v_B}{300} - \frac{v_A}{30}$   
So both (a) and (b) options are wrong.

$$34. f_a = \frac{v}{v_0} f \quad 1 \quad \frac{v_0}{v} f \text{ and}$$

$$f_r = 1 \quad \frac{v_0}{v} f$$

$$\frac{f_a}{f_r} = \frac{v}{v_0} \frac{v_0}{v}$$

$$(f_a - f_r) v = (f_a - f_r) v_0$$

$$\frac{f_a}{v_0} = \frac{f_r}{f_a} \frac{f_r}{f_r}$$

and

$$f_a - f_r = \frac{2v_0}{v} f - 2 \frac{f_a}{f_a} \frac{f_r}{f_r} f$$

$$f = \frac{f_a - f_r}{2}$$

## JEE Corner

### ■ Assertion and Reason

1.  $c = (2n-1) \frac{v}{4l} = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l}, \dots$

while,  $c_0 = \frac{(n-1)v}{2l} = \frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \frac{4v}{2l}, \dots$

it can be seen that  $c = c_0$  at all situation  
and  $c = \frac{1}{2} c_0$  so assertion is true but  
reason is false.

2. Apparent frequency is constant for constant relative velocity so assertion is false.

3. At a point of minimum displacement pressure amplitude is maximum i.e., pressure difference is maximum not pressure. So assertion is false.

4. The driver receiver two sounds one direct,  $c_0$  and other  $c_R = \frac{v+u}{v-u}$  such

that he detects beats. So reason is true explanation of assertion.

5. With increase in intensity sound level increases in logarithmic order so assertion is false.

6. Speed of sound  $v = \sqrt{\frac{p}{\rho}}$ , with increase in only pressure density increases such that  $\frac{p}{\rho}$  remains constant. Again  $v = \sqrt{\frac{RT}{M}}$  so both assertion and reason are true but reason is not correct explanation of assertion.

7.  $f_A - f_B = 4$  when A is loaded with little wax then  $f_A$  slightly decreases and then beat frequency decreases, but if it is heavily loaded with wax then its frequency goes much below  $f_B$  such that beat frequency increases. So, assertion and reason are both true but reason is not correct explanation of assertion.

8.  $\frac{150}{3}, \frac{450}{5}, 750$

The frequencies are odd harmonics then the pipe is closed and fundamental frequency is also 150 Hz. So assertion and reason are both true but reason is not correct explanation of assertion.

9.  $\frac{1}{l e}$  with increase in diameter end

correction,  $e$ , increases and decreases. So reason is correct explanation of assertion.

10. With increasing length of air column, number of overtone increases and not the wavelength so assertion is false.

■ Objective Questions (Level 2)

1. At the boundary between two mediums, one part of incident wave gets reflected and other part gets transmitted or refracted.

2.  $\frac{3}{2} \frac{3.9}{1.5} \frac{3.9}{1.5} \frac{2}{3.9} \frac{3}{3.9} S_0 kB$

$S_0 \frac{0}{kB} \frac{10^2}{3.9} \frac{10^5}{1.3 (200)^2} \frac{3.9}{12} \frac{10^1}{1.3} 0.025 \text{ m } 2.5 \text{ cm}$

3.  $\frac{A_t}{A_i} \frac{2v_2}{v_1 v_2} \frac{2}{200} \frac{100}{100} \frac{2}{3}$

4.  $\frac{v}{v_2} \frac{v}{v_1} \frac{v (v_1 v_2)}{(v v_2)(v v_1)} \sim \frac{(v_1 v_2)}{v}$

$v_1 v_2 \frac{v}{1700} \frac{340}{10} 2 \text{ m/s}$

5.  $v_s \frac{gt}{v v_0} \frac{10 \text{ m/s}}{v v_s} \frac{v v_0}{v v_s} \frac{300}{300} \frac{2}{10} \frac{300}{300} \frac{2}{10} 150 \text{ Hz}$   
 $\frac{302}{290} \frac{298}{310} 150 12 \text{ Hz}$

6.  $7 \frac{L}{2} \frac{2L}{7}$   $A a \cos kx a \cos \frac{2L}{7} \frac{L}{7} a \cos a$

7. For maxima,  $n = 3$   
 $\frac{3}{n}; \frac{v}{nv} = 110 n$

110, 220, 330 Hz, ..etc. maxima will be formed so maximum will not be formed at 120 Hz and 100 Hz.

8.  $\frac{20 \text{ m/s}}{v \cos 60} \frac{v}{v \cos 60} \frac{300}{10} \frac{10}{20} 500 \text{ Hz}$   
 $\frac{310}{290} 500 534 \text{ Hz}$

9.  $R = 0 \frac{v}{v} \frac{20}{10} \frac{v}{v} \frac{20}{10} 500$   
 $\frac{360}{300} \frac{360}{350} 500 \text{ Hz } 31 \text{ Hz}$

10.  $\frac{404}{2} \frac{400}{2} 202 200 2 \text{ Hz}$   
 $\frac{I_{\max}}{I_{\min}} \frac{2}{2} \frac{1}{1} 9 : 1$

$$11. \frac{3}{4} \cdot 34 \text{ cm} = \frac{4}{3} \cdot 34 \text{ cm}$$

$$\frac{v}{v_{51}} = \frac{136}{3}$$

$$\frac{v_{16}}{v_{51}} = \frac{\sqrt{273}}{\sqrt{273}} \cdot \frac{16}{51} = \frac{\sqrt{289}}{\sqrt{324}} \cdot \frac{1}{\sqrt{1.121}} = \frac{1}{1.1}$$

$$16 \cdot \frac{51}{1.1} = \frac{136}{3} \cdot 1.1 \Rightarrow 41.21 \text{ cm}$$

$$12. 176 \frac{v}{v} \cdot \frac{v}{22} = 165 \frac{v}{v}$$

$$176v(v-v) = 165(v-v)(v-22)$$

$$176 = 330(330-v) = 165(330-v)(330-22)$$

or  $1.143(330-v) = 330-v$   
 or  $0.143 \cdot 330 = 2.143v - v = 22 \text{ m/s}$

$$13. M_{\min} = \frac{2 \cdot 32 \cdot 3 \cdot 48}{2 \cdot 3} = 41.6$$

$$\frac{2}{1} \frac{v_2}{v_1} \sqrt{\frac{1}{2}} \sqrt{\frac{32}{41.6}} = \sqrt{0.77}$$

$$= 0.875 \cdot 175 \text{ Hz}$$

$$14. v_0 = gt = 30 \text{ m/s}$$

$$1100 \frac{v}{v} = \frac{30}{v} \Rightarrow 1000, 1.1v - v = 30$$

$$0.1v = 30 \Rightarrow v = 300 \text{ m/s}$$

**Passage (Q 5 to 17)**

$$v_m = v_p = 8 \text{ m/s}, 50v_m = 150v_p$$

$$v_m = 3v_p, 4v_p = 8 \text{ m/s}$$

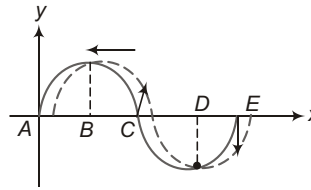
$$v_p = 2 \text{ m/s and } v_m = 6 \text{ m/s}$$

$$15. \frac{v}{v} \cdot \frac{2}{6} f_0 = \frac{332}{324} f_0 \text{ constant}$$

$$16. \frac{v}{v} \cdot \frac{2}{6} f_0 = \frac{328}{336} f_0 \text{ constant}$$

$$17. f_0 = v \text{ and graph is (a)}$$

$$18.$$



Both (a) and b are correct.

**More Than One**

$$19. \frac{(2n-1)v}{4l}$$

$$l \frac{v}{4} (2n-1) = \frac{330}{4} \frac{(2n-1)m}{264}$$

$$(2n-1) = 31.25 \text{ cm}$$

$$31.25 \text{ cm}, 93.75 \text{ cm}, 156.25 \text{ cm}$$

$$20. (a) v = p^0, (b) v = \sqrt{T} = v^2 T,$$

where  $T$  is absolute temperature.

$$(c) v = \sqrt{F} \quad (d) \frac{1}{l}$$

(c) and (d) are correct.

$$21. P_0 = BAk; B = \frac{P}{V} = \frac{P}{p}$$

$$\frac{P}{B} = BAk = \frac{P}{B} = Ak$$

Pressure and density equations are in opposite phase i.e.,  $\frac{1}{2}$  and not  $\frac{1}{4}$ .

So, (a), (b) and (c) are correct.

$$22. \frac{5v}{4l_c} = \frac{3v}{2l_o} = \frac{125}{l_c} = \frac{2}{l_o} = \frac{l_o}{l_c} = \frac{2}{1.25} = \frac{8}{5}$$

$$(a) c = \frac{v}{4l_c} = \frac{v}{4 \cdot \frac{5}{8} l_o} = \frac{2v}{5l_o}$$

$$\frac{4}{5} \frac{v}{2l_o} = \frac{4}{5} \frac{v}{l_o} = c$$

$$(b) c = \frac{3v}{4l_c} = \frac{3v}{4 \cdot \frac{5}{8} l_o} = \frac{12}{5} \frac{v}{2l_o}$$

$$\frac{6}{5} \frac{2v}{2l_o} = \frac{6}{5} \frac{v}{l_o} = c$$

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(c)  $c \frac{15v}{4l_c} \frac{15v}{4 \frac{5}{8}l_o} \frac{6v}{l_o} 12 \frac{v}{2l_o}$

$12v_o$  twelfth harmonic.

(d) Closed organ pipe cannot have tenth harmonic it only has odd harmonics.

23.  $f \frac{v}{4(l-e)} \frac{1}{4(l-e)} \sqrt{\frac{RT}{M}}$

(a) increase in  $r$  increase in  $e$   
decrease in  $f$

(b) increase in  $T$  increase in  $v$   
increase in  $f$

(c) increase in  $M$  decrease in  $v$   
decrease in  $f$

(d) increase in  $P$  increase in no  
change in  $v$  no change in  $f$

24.  $f_a \frac{v}{v-v_s} f$  and  $f_r \frac{v}{v+v_s} f$  are constants during approach and received.

■ Match the Columns

1.  $\frac{v}{2l} f$

(a)  $c \frac{v}{4 \cdot 2l} \frac{f}{4} 0.25f \quad s$

(b)  $c_2 \frac{5v}{4 \cdot 2l} \frac{5}{4} f 1.25f \quad p$

(c)  $c_1 \frac{3v}{4 \cdot 2l} \frac{3}{4} f 0.75f \quad r$

(d)  $c_1 \frac{3v}{4 \cdot 2l} 0.75f \quad r$

2.  $1 \frac{v}{v-v_s} \frac{v}{v+v_s} f \frac{2vv_f}{v^2-v_s^2} f$

$\frac{2v \frac{v}{4}}{v^2 - \frac{16}{v^2}} f \frac{16}{15} \frac{1}{2} f \frac{8}{15} f$

2  $\frac{v}{v-v_s} \frac{v}{v+v_s} 1 f \frac{2v_s}{v-v_s} f$

$\frac{2v/4}{v-v/4} f \frac{2}{3} f$

3  $\frac{v}{v-v_5} \frac{v}{v+v_5} f 0$

(a)  $1 \frac{8}{15} f \quad q$

(b)  $2 \frac{2}{3} f \quad p$

(c)  $3 0 \quad s$

(d)  $3 0 \quad s$

3.  $f \quad f_T \quad f_S$

(a) If tuning fork is loaded  $f_T$  decreases such that beat frequency may increase or decrease depending upon amount of wax  $r, s$

(b) If prongs are filed, beat frequency must increase  $p$

(c) If tension is increased beat frequency may increase or decrease depending upon the amount of change in tension.  $r, s$

(d) If tension is decreased, beat frequency must increase  $p$

4. (a) For point source,  $I \frac{1}{r}$ , and  $A \frac{1}{r} \quad r$

(b)  $q$

(c) For line source,  $I \frac{1}{r}$  and  $A \frac{1}{\sqrt{r}} \quad q$

(d)  $\frac{2}{k} \frac{2}{2} \quad 2m$

$5 - \frac{5}{4} \frac{5}{2} m \quad 2.5m$

(a)  $l \quad 2.5m \quad s$

(b)  $\frac{2}{2} m \quad r$

(c)  $\frac{2}{2}, \frac{2}{2} \quad 1m, 2m \quad p, r$

(d)  $\frac{3}{4}, 3 - \frac{3}{4} \quad 0.5m, 1.5m \quad q$

# 17. Thermometry, Thermal Expansion & Kinetic Theory of Gases

## Introductory Exercise 17.1

1. (a)  $\frac{C}{5} = \frac{F - 32}{9}$  for  $F = 0, C = \frac{5}{9} \times 32$

(b)  $\frac{K - 273.15}{5} = \frac{F - 32}{9}$  for  $K = 0,$   
 $F = \frac{9}{5} \times 273.15 + 32 = 459.67 \text{ F}$

2. (a)  $\frac{x}{5} = \frac{2x - 32}{9}$   $x = \frac{10x}{9} = 17.8$   
 $17.8 = \frac{10}{9} \times 1 \times x$

(b)  $\frac{x}{5} = \frac{x/2 - 32}{9}$   $x = \frac{5}{18} \times x = 17.8$   
 $17.8 = \frac{13}{18} \times x$   $x = 24.65 \text{ C}$

3.  $\frac{C - 5}{99} = \frac{F - 32}{52}$   $\frac{C - 5}{180} = \frac{F - 32}{94}$   
 $\frac{C - 5}{52} = \frac{F - 32}{94}$   $47 = \frac{122}{94} \text{ F}$

4.  $\frac{K - 273.15}{5} = \frac{F - 32}{9}$   
 $x = 273.15 + \frac{5}{9} \times x = 17.8$   
 $\frac{4}{9} \times x = 255.35$   $x = 574.54$

5.  $\frac{C}{5} = \frac{F - 32}{9}$   $\frac{9}{5} \times x = x - 32$   
 $\frac{4}{5} \times x = 32$   
 $x = \frac{5}{4} \times 32 = 40 \text{ C}$

6.  $t = \frac{1}{2} t$   
 $\frac{1}{2} = 1.2 \times 10^5 = 86400 = 30$   
 $1.5 = 1.2 \times 8.64 \text{ s} = 15.55 \text{ s}$  given.

7. As from  $0^\circ\text{C}$  to  $4^\circ\text{C}$ , density of water increases so the volume of wooden block above water level increases and as from  $4^\circ\text{C}$  to  $10^\circ\text{C}$  density of water decreases so the volume of block above water decreases.

8.  $V_1 = 1g$   $V_2 = 1g$  and  $V_2 = 2g$   $V_2 = 2g$

$$\frac{V_1}{V_1} = 1 = \frac{V_1}{V_1} = 1 = \frac{1}{1}$$

and  $\frac{V_2}{V_2} = 1 = \frac{2}{2}$

$$\frac{V_2}{V_2} = \frac{V_1}{V_1}$$

$$1 = \frac{2}{2} = 1 = \frac{1}{1} = \frac{1}{1} = \frac{2}{2}$$

$$\frac{1}{1} = \frac{1}{1} (1 = \frac{2}{1} T)$$

$$\frac{1}{1} = \frac{1}{1} (1 = \frac{1}{1} T)$$

$$\frac{1}{1} \frac{2}{1} \frac{1}{1} T$$

$$\frac{1}{1} \frac{1}{1} \frac{2}{1} T$$

$$\frac{1}{1} \frac{1}{1} \frac{2}{1} T$$

9. On cooling brass contracts more than iron ( $\alpha_{Br} > \alpha_{Fe}$ ) such that brass disk gets loosen from hole of iron.

10.  $V \propto T$   $V \propto kT$   $\ln V \propto \ln k \propto \ln T$

$$\frac{V}{V} \frac{T}{T} \frac{V}{V} \frac{1}{T}$$

### Introductory Exercise 17.2

1. For ideal gases,  $pV = nRT$

$$\text{Slope} = \frac{VM}{mR}$$

As slope  $\frac{1}{m} = \frac{m_2}{m_1}$

2.  $pV = nRT$   $\frac{p_2}{p_1} = \frac{T_2}{T_1}$

$$\frac{360}{300} = \frac{6}{5}$$

$$p_2 = \frac{6}{5} p_1 = \frac{6}{5} \times 10 \text{ atm} = 12 \text{ atm}$$

3.  $M_{\text{mix}} = \frac{1}{4} \times 28 + \frac{1}{4} \times 44 + \frac{7}{4} \times \frac{11}{2} = 36$

$$pV = nRT = \frac{m}{M} RT$$

$$pM = \frac{m}{V} RT = \rho RT$$

$$\frac{pM}{RT} = \rho$$

$$\frac{101 \times 10^5}{8.31 \times 290} = \rho$$

4.  $pV = nRT = \frac{N}{N_A} RT$

$$N = \frac{pVN_A}{RT}$$

$$10^6 = \frac{13.6 \times 10^3 \times 10 \times 250 \times 10^6}{8.31 \times 300}$$

$$N = \frac{13.6 \times 5 \times 6.02}{8.31 \times 6} \times 10^{15} = 8.21 \times 10^{15}$$

5.  $pV = nRT$

$$V = \frac{nR}{p} T$$

Slope  $\frac{1}{p}$

6.  $pV = nRT$   $p = (nRT) \frac{1}{V}$

$y = mx$  is a straight line passing through origin.

### Introductory Exercise 17.3

1. Average velocity depends on the direction of motion of gas molecules and as container do not move such that their net effect becomes zero, due to the reason that some molecules are moving

in one direction while other are moving in opposite direction. But in case of average speed only magnitudes are in use which do not cancel each other.

$$2. \text{ KE } \frac{3}{2} kT = \frac{3}{2} \frac{8.31}{6 \times 10^{23}} \times 300 \text{ J}$$

$$= \frac{3}{4} \times 8.31 \times 10^{21} \text{ J}$$

$$6.21 \times 10^{21} \text{ J}$$

$$3. v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{He}} = \sqrt{\frac{3 \times 8.31 \times 300}{4 \times 10^{-3}}}$$

$$= \frac{1.37 \times 10^3 \text{ m/s}}{\sqrt{\frac{3 \times 8.31 \times 300}{20.2 \times 10^{-3}}}} = 608.5 \text{ m/s}$$

$$\text{KE} = \frac{3}{2} kT = 6.21 \times 10^{21} \text{ J}$$

$$4. v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{4 \times 10^{-3} \times 10^6}{3 \times 8.31}$$

$$160.45 \text{ K}$$

$$5. \frac{n_1}{n_1} = \frac{n_2}{n_2} = \frac{(1 - n_2)}{1 - n_2} = \frac{n_2}{n_2}$$

$$\frac{1}{n_2} = \frac{n_2(1 - n_2)}{1 - n_2}$$

$$n_2 = \frac{1}{2} = \frac{1.293}{1.251 + 1.429}$$

$$= \frac{136}{178} = 0.764 = 76.4\% \text{ by mass}$$

$$6. \frac{V_2}{V_1} = \frac{p_1 T_2}{p_2 T_1} = \frac{(p_0 - h) g}{p_0} = \frac{277}{277}$$

$$= \frac{(1.01 \times 10^5 - 40 \times 10^3) \times 10}{1.01 \times 10^5 \times 277}$$

$$= \frac{5.01 \times 293}{1.01 \times 277} = 5.25$$

$$V_2 = 5.25 V_1 = 105 \text{ cm}^3$$

$$7. N = nN_A = \frac{1}{18} \times 6 \times 10^{23}$$

$$\frac{1}{3} \times 10^{23};$$

$$S = 4R^2 = 4 \times 3.14 \times (6400 \times 10^3 \times 10^2)^2$$

$$= \frac{5.14 \times 10^{18} \text{ cm}^2}{\frac{N}{S} = \frac{10^{23}}{3 \times 5.14 \times 10^{18}}}$$

$$(a) nC_V = \frac{6.5 \times 10^3 \text{ molecules/cm}^2}{\frac{3}{2} nR} = \frac{35 \text{ J/K}}{2}$$

$$n = \frac{70}{3R} = 2.8 \text{ mole}$$

$$(b) U = \frac{3}{2} nRT = \frac{35 \text{ J/K} \times 273 \text{ K}}{2} = 9555 \text{ J}$$

$$(c) C_p = C_V + R = \frac{5}{2} R = 20.8 \text{ J/K mole}$$

$$8. (a) n(C_p - C_V) = nR = 29.1 \text{ J/K}$$

$$n = \frac{29.1}{8.314} = 3.5 \text{ mole}$$

$$(b) C_V = nc_V = n \frac{3}{2} R = 3.5 \times 1.5 \times 8.314$$

$$43.65 \text{ J/K}$$

$$C_p = nc_p = n \frac{5}{2} R = C_V + nR$$

$$43.65 + 3.5 \times 8.314$$

$$72.75 \text{ J/K}$$

$$(c) C_V = nc_V = n \frac{5}{3} R = 72.75 \text{ J/K}$$

$$C_p = nc_p = n \frac{7}{2} R$$

$$72.75 + 3.5 \times 8.314$$

$$101.85 \text{ J/K}$$

$$10. v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ and } v_{\text{av}} = \sqrt{\frac{8RT}{M}}$$

$$\text{Here } 3 \frac{8}{3} v_{\text{rms}} = v_{\text{av}},$$

*i.e.*, the statement is true.



## AIEEE Corner

## ■ Subjective Questions (Level 1)

$$1. \frac{C}{5} \frac{68}{9} \frac{32}{9} \frac{36}{9} 4$$

$$C \quad 20 \text{ C}; \frac{K}{5} \frac{273}{9} \frac{68}{9} \frac{32}{9} 4$$

$$\frac{K}{C} \frac{293 \text{ K}}{5} \frac{32}{9} \frac{27}{9} 3$$

$$C \quad 15 \text{ C}; \frac{K}{5} \frac{273}{9} \frac{5}{9} \frac{32}{9} 3$$

$$\frac{K}{C} \frac{258 \text{ K}}{5} \frac{176}{9} \frac{32}{9} \frac{144}{9} 16$$

$$C \quad 80 \text{ C}; \frac{K}{5} \frac{273}{9} 16$$

$$2. \frac{30}{5} \frac{F}{9} \frac{32}{9} F \quad 54 \quad 32 \quad 86 \text{ F}$$

$$\frac{5}{5} \frac{546 \text{ R}}{9} \frac{F}{9} \frac{32}{9} F \quad 9 \quad 32 \quad 41 \text{ F} \quad 501 \text{ R}$$

$$\frac{20}{5} \frac{F}{9} \frac{32}{9} F \quad 36 \quad 32 \quad 41 \text{ F}$$

$$3. \frac{x}{5} \frac{x}{9} \frac{32}{9} 32 \quad x \quad \frac{9}{5}x \quad \frac{4}{5}x$$

$$x \quad \frac{5}{4} \quad 32 \quad 40$$

$$4. \frac{C}{5} \frac{F}{9} F \quad \frac{9}{5} C \quad \frac{9}{5} 40 \quad 72$$

$$5. \frac{F_2}{80} \frac{F_1}{20} \frac{72}{100} \frac{140.2 \text{ F}}{0}$$

$$\frac{12}{60} \frac{C}{100} C \quad \frac{12}{60} \frac{100}{60} 20 \text{ C}$$

$$6. \frac{T_2}{T_1} \frac{p_2}{p_1} \frac{160}{80} 2 \quad T_2 \quad 2T_1$$

$$T_2 \quad 2 \quad 273.15 \text{ K} \quad 546.30 \text{ K}$$

$$7. R_t \quad R_0(1 \quad )$$

$$3.50 \quad 250(1 \quad 100 \quad ) \quad 1 \quad 250 \text{ K}$$

$$\text{or} \quad \frac{10}{250} \quad 4 \quad 10^3 / ^\circ\text{C}$$

$$650 \quad 250(1 \quad 4 \quad 10^3 \quad )$$

$$4 \quad 10^2$$

$$400 \quad 2 \quad 400^\circ\text{C}$$

$$400 \quad 2 \quad 400 \text{ C}$$

*i.e.*, boiling point of sulphur is  $400^\circ\text{C}$ .

$$8. \frac{T_2}{T_1} \frac{p_2}{p_1} \frac{75}{75} \frac{45}{5} \frac{120}{80} \frac{3}{2}$$

$$T_2 \quad \frac{3}{2} T_1 \quad \frac{3}{2} \quad 300.15 \text{ K}$$

$$450 \quad 225 \text{ K} \quad 177.08 \text{ C}$$

$$9. \quad ( \text{Br} \quad \text{Fe} )$$

$$\frac{1}{\text{Br} \quad \text{Fe}}$$

$$\frac{0.01}{6} \frac{10^3}{10^2} \frac{1}{\text{Br} \quad \text{Fe}}$$

$$\frac{10^3}{6( \text{Br} \quad \text{Fe} )}$$

$$2 \quad 1 \quad \frac{10^3}{6( \text{Br} \quad \text{Fe} )}$$

$$30 \text{ C} \quad \frac{10^3}{6( \text{Br} \quad \text{Fe} )} \quad 30 \text{ C} \quad \frac{100}{6} \quad 0.63$$

$$57.78 \text{ C.}$$

$$10. (a) \quad l \quad l \quad \sim 88.42 \quad 2.4 \quad 10^5 \quad 30$$

$$0.064 \text{ cm}$$

$$(b) \quad l \quad l( \text{Al} \quad \text{St} )$$

$$88.42(2.4 \quad 1.2) \quad 10^5 \quad 30$$

$$0.032 \text{ cm}$$

$$l_S \quad l \quad l \quad 88.42 \quad 0.032 \text{ cm}$$

$$88.45 \text{ cm}$$

$$11. \frac{l}{l} \quad 100\% \quad 100\%$$

$$1.2 \times 10^5 \quad 35 \quad 100\%$$

$$0.042\%$$

$$12. F \quad YA \frac{l}{l} \quad YA$$

$$2 \times 10^{11} \quad 2 \times 10^6 \quad 1.2 \times 10^5 \quad 40$$

$$4 \quad 1.2 \quad 40 \text{ N} \quad 160 \quad 1.2 \text{ N} \quad 192 \text{ N}$$

$$13. V \quad g \quad (50 \quad 45) \quad 10^3 \text{ kg}$$

$$5 \times 10^3 \text{ kg}$$

$$V \quad g \quad (50 \quad 45.1) \quad 10^3 \text{ kg}$$

$$4.9 \times 10^3 \text{ kg}$$

$$V(1 \quad s) \frac{g}{l} \quad 4.9 \times 10^3$$

$$\frac{1 \quad s}{1 \quad l} \quad \frac{4.9}{5}$$

$$5 \quad 5 \quad \frac{4.9}{0.1 \quad 5 \quad s} \quad \frac{4.9}{1} \quad \frac{5}{5}$$

$$l \quad \frac{4.9}{4.9} \quad \frac{5}{4.9} \quad s$$

$$s \quad \frac{1}{49} \quad \frac{5}{75} \quad \frac{5}{49} \quad 12 \times 10^6$$

$$272.1 \times 10^6 \quad 12.2 \times 10^6$$

$$2.84 \times 10^4 \text{ C}$$

$$14. M \quad 14 \quad 3 \quad 17 \text{ g/mole}$$

$$17 \times 10^3 \text{ kg/mole}$$

$$M \quad \frac{17 \times 10^3}{6033 \times 10^{23}} \text{ kg/molecule}$$

$$282 \times 10^{26} \text{ kg/molecule}$$

$$15. n \quad \frac{pV}{RT} \quad \frac{1.52 \times 10^6 \times 10^2}{8.314 \times 298.15} \quad 6.13$$

$$\frac{m}{V} \quad \frac{nM}{V} \quad \frac{6.13 \times 2 \times 10^3}{10^2}$$

$$1.23 \text{ kg/m}^3$$

$$\frac{m}{V} \quad \frac{nM}{V} \quad \frac{16 nM}{V} \quad 16$$

$$19.62 \text{ kg/m}^3$$

$$16. p_2 \quad p_1 \frac{V_1}{V_2} \quad 1 \text{ atm} \quad \frac{76}{6}$$

$$12.7 \text{ atm}$$

$$17. V_2 \quad \frac{p_1 V_1}{T_1} \frac{T_2}{p_2} \quad \frac{p_1}{p_2} \frac{T_2}{T_1} \quad V_1$$

$$\frac{1}{0.5} \quad \frac{270}{300} \quad 500 \text{ m}^3 \quad 900 \text{ m}^3$$

$$18. \frac{p_1 V_1}{T_1} \quad \frac{p_2 V_2}{T_2}$$

$$\frac{\frac{mg}{A}}{p_0} \quad A \quad h_i \quad \frac{\frac{mg}{A}}{p_0} \quad A h_f$$

$$\frac{293}{293} \quad \frac{373}{293} \quad h_i \quad \frac{373}{293} \quad 4 \text{ cm} \quad 50.9 \text{ cm}$$

$$19. p_1 \quad p_2 \quad \frac{n_1}{V_1} \quad \frac{n_2}{V_2} \quad \frac{25/28}{L_1 A} \quad \frac{40/4}{L_2 A}$$

$$\frac{L_1}{L_2} \quad \frac{25}{28} \quad \frac{1}{10} \quad \frac{5}{56} \quad 0.089$$

$$\frac{n_1}{n_2} \quad \frac{25/28}{40/4} \quad \frac{25}{280} \quad \frac{5}{56} \quad 0.089$$

$$20. n \quad n_1 \quad n_2$$

$$p(V_1 \quad V_2) \quad p_1 V_1 \quad p_2 V_2$$

$$p \quad \frac{p_1 V_1}{V_1} \quad \frac{p_2 V_2}{V_2}$$

$$p \quad \frac{1.38 \quad 0.11 \quad 0.69 \quad 0.16}{0.11 \quad 0.16} \text{ MP}_a$$

$$\frac{0.1518 \quad 0.1104}{0.27} \quad \frac{0.2622}{0.27} \quad 0.97 \text{ MP}_a$$

$$21. \frac{pV_1}{T} \quad \frac{pV_2}{T} \quad \frac{p_1 V_1}{T_1} \quad \frac{p_1 V_2}{T_2}$$

$$\frac{1 \text{ atm}}{293 \text{ K}} \quad 600 \text{ cm}^3$$

$$p_1 \quad \frac{400 \text{ cm}^3}{373 \text{ K}} \quad \frac{200 \text{ cm}^3}{273 \text{ K}}$$

$$p_1 \quad \frac{600/293}{\frac{400}{373} \quad \frac{200}{273}} \text{ atm}$$

$$p_1 \quad \frac{3}{293 \quad \frac{2}{373} \quad \frac{1}{273}} \text{ atm}$$

$$\frac{3}{1.57 \quad 1.07} \quad \frac{3}{2.64} \text{ atm}$$

$$1.136 \text{ atm}$$

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22.  $V = \frac{nRT}{p} = \frac{1 \times 8.314 \times 273.15}{1.013 \times 10^5} \text{ m}^3$

$0.02242 \text{ m}^3 = 22.42 \text{ litre}$

23.  $p_2 = \frac{p_1 V_1}{T_1} \frac{T_2}{V_2} = p_1 \frac{V_1}{V_2} \frac{T_2}{T_1}$   
 $1.5 \times 10^5 \times \frac{0.75}{0.48} \times \frac{430}{300}$   
 $3.36 \times 10^5 \text{ Pa}$

24.  $\frac{p_1}{n_1 RT} = \frac{p_2}{n_2 RT} = \frac{p_1 V_1}{n_1 RT} = \frac{p_2 V_2}{n_2 RT}$   
 $\frac{0.7}{20} \times \frac{1.4}{10} = \frac{0.3}{10} \times \frac{1.4}{5} \times \frac{0.4}{10} \times \frac{RT}{1500}$   
 $\frac{0.7}{20} \times \frac{0.3}{10} = \frac{1}{10} \times \frac{8.314}{5} \times \frac{1500}{10^3}$   
 $\frac{3.3}{20} \times 8.314 \times 3 \times 10^5 \text{ Pa}$   
 $4.11 \times 10^5 \text{ Pa}$

25. RKE  $= 2 \times \frac{1}{2} kT = \frac{1}{2} I \omega^2$   
 $\sqrt{\frac{2kT}{I}} = \sqrt{\frac{2 \times 1.38 \times 10^{23} \times 300}{8.28 \times 10^{38} \times 10^{-7}}}$   
 $10^{12} \sqrt{\frac{6 \times 1.38}{8.28}} = 10^{12} \text{ rad/s}$

26.  $v = \sqrt{\frac{p}{\rho}}$   
 $\frac{v^2}{p} = \frac{1.3 \times (330)^2}{1.013 \times 10^5} = 1.398$   
 $\frac{f}{f} = \frac{2}{0.398} \sim 5$

27.  $\frac{n_1 C_{p1}}{n_1 C_{V1}} = \frac{n_2 C_{p2}}{n_2 C_{V2}} = \frac{3 \times \frac{5}{2} \times 2 \times \frac{7}{2}}{3 \times \frac{3}{2} \times 2 \times \frac{5}{2}}$   
 $\frac{15}{9} \times \frac{14}{10} = \frac{29}{19} \times 1.53$

28.  $K = \frac{3}{2} pV$   
 $\frac{K_2}{K_1} = \frac{\frac{3}{2} p_2 V_2}{\frac{3}{2} p_1 V_1} = \frac{3}{2} \times \frac{15}{5} = 4.5$

29.  $C_p = \frac{f}{2} R = \frac{K_2}{29} = \frac{4.5 \text{ K}}{29} \times \frac{58}{R} = 2 \times 5$

$pT = p \frac{pV}{nR}$

$p^2 V = \text{constant}$

$pV^{1/2} = \text{constant} \Rightarrow a = \frac{1}{2}$

$c = \frac{f}{2} R = \frac{R}{1} \times \frac{1}{2} = \frac{f}{2} \times 4 R = 29$

$f = \frac{58}{R} \times 4 \times 3$

30. TKE  $= \frac{3}{5}$  of total energy and RKE  $= \frac{2}{5}$  of total energy, so the gas is diatomic.

TKE  $= \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{23} \times 300 \text{ J}$

$6.21 \times 10^{21} \text{ J/molecule}$

$Q = n C_V T = 1 \times \frac{5}{2} \times 8.314 \times 1 \times 20.8 \text{ J}$

31.  $C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 + n_2} = \frac{2.5R + 3.5R}{1 + 1} = 3R$

$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{1.5R + 2.5R}{1 + 1} = 2R$

$\frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$

32.  $\frac{n_1 C_{p1}}{n_1 C_{V1}} = \frac{n_2 C_{p2}}{n_2 C_{V2}} = \frac{(n_1 + n_2) C_{p1}}{(n_1 + n_2) C_{V1}}$   
 $\frac{C_{p1}}{C_{V1}}$

33.  $p = aV^b \Rightarrow pV^b = \text{constant}$

$C = \frac{Q}{nT} = 0$  for adiabatic process for

which  $pV^b = \text{constant}$  comparing, we get,  $b$

$$34. p \propto \frac{1}{V} \quad pV = \text{constant}$$

$$C_V = \frac{R}{\gamma - 1} = \frac{R}{1 - 1} = \frac{R}{2}$$

$$35. v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 373.15}{2 \times 10^3}}$$

$$2.16 \times 10^3 \text{ m/s}$$

$$2.16 \text{ km/s}$$

$$36. v_{\text{rms}} = \sqrt{\frac{(500)^2 + (600)^2 + (700)^2 + (800)^2 + (900)^2}{5}}$$

$$\frac{100}{\sqrt{5}} \sqrt{25 + 36 + 49 + 64 + 81}$$

$$714 \text{ m/s}$$

$$500 \quad 600 \quad 700 \quad 800 \quad 900$$

 $v_{\text{av}}$ 

$$20(5 + 6 + 7 + 8 + 9) = 700 \text{ m/s}$$

 $v_{\text{rms}}$ 

$$37. \text{KE} = \frac{3}{2} pV$$

$$N = 6 \times 10^{26} \quad 1.5 \times 2 \times 10^5 \quad 100 \times 10^3 \quad 10^3$$

$$N = \frac{3 \times 10}{6 \times 10^{26}} = 5 \times 10^{26}$$

$$\frac{5000}{6.023} = 830.15 \text{ Na}$$

$$n = 830.15 \text{ moles}$$

$$38. \text{Frequency of collision, } \frac{v}{2\sqrt{3}l} = \frac{v}{2\sqrt{3}V}$$

$$\frac{1}{2\sqrt{3}V} \sqrt{\frac{3RT}{M}}$$

$$\sqrt{\frac{RT}{4VM}} = \sqrt{\frac{RT}{4 \frac{nRT}{p} M}} = \sqrt{\frac{p}{4nM}}$$

$$\sqrt{\frac{2 \times 10^5}{4 \times 1 \times 46 \times 10^3}}$$

$$41.04 \times 10^3 / \text{s}$$

$$39. \text{KE} = \frac{3}{2} pV = \frac{3}{2} \times 10^5 \times 2 \times 10^6 = 0.3 \text{ J}$$

$$N = \frac{m}{m_1} = \frac{50 \times 10^6}{8 \times 10^{26}} = 6.25 \times 10^{20}$$

$$K_1 = \frac{K}{N} = \frac{0.3}{6.25 \times 10^{20}} = \frac{30}{6.25} \times 10^{22} \text{ J}$$

$$4.8 \times 10^{22} \text{ J}$$

$$40. v_0 = \sqrt{\frac{3RT_0}{M_0}}$$

$$(a) \frac{v}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{573}{293}} = 1.4 \quad v = 1.4 v_0$$

(b)  $v \propto v_0$  as RMS speed changes with temperature and not with pressure.

$$(c) \frac{v}{v_0} = \sqrt{\frac{M_0}{M}} = \sqrt{\frac{M_0}{3M_0}} = \frac{1}{\sqrt{3}}$$

$$v = \frac{v_0}{\sqrt{3}} = 0.58 v_0$$

$$41. \sqrt{\frac{RT}{M_{\text{H}_2}}} = \sqrt{\frac{RT}{M_{\text{O}_2}}} \quad T = \frac{M_{\text{H}_2}}{M_{\text{O}_2}} T$$

$$\frac{2}{32} = \frac{320}{20 \text{ K}} = \frac{253 \text{ C}}{253 \text{ C}}$$

$$42. \frac{1}{2} m v_e^2 = \frac{GMm}{R_e} = g_e R_e m$$

$$v_e = \sqrt{2g_e R_e} = v_{\text{H}_2} = \sqrt{\frac{3RT}{M}}$$

$$T_e = \frac{2g_e R_e M}{3R} = \frac{2 \times 9.8 \times 6367 \times 10^6}{3 \times 8.314} = 2 \times 10^3$$

$$\text{and } T_m = \frac{10007 \text{ K}}{2} = \frac{2g_m R_m M}{3R} = \frac{2 \times 1.6 \times 1.75 \times 10^6}{3 \times 8.314} = 2 \times 10^3$$

$$449 \text{ K}$$

$$43. (a) \text{KE} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{23} \times 300 \text{ J}$$

$$6.21 \times 10^{21} \text{ J}$$

$$(b) \text{KE} = \frac{3}{2} kT N_a = 6.023 \times 10^{23}$$

$$6.21 \times 10^{21} \text{ J}$$

$$3740 \text{ J}$$

$$(c) v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32 \times 10^3}}$$

483.6 m/s

■ Objective Questions (Level 1)

1.  $v_{\text{av}} = \sqrt{\frac{8RT}{M}}$

2.  $v_{\text{rms}} = \sqrt{\frac{1^2 + 0^2 + 2^2 + 3^2}{4}}$   
 $= \sqrt{\frac{14}{4}} = \sqrt{3.5} \text{ m/s}$

3.  $\frac{l}{l} = \frac{12 \times 10^6}{50}$

$\frac{600 \times 10^6}{6 \times 10^4}$

4.  $V T = \frac{V_2}{V_1} \frac{T_2}{T_1} = 2$ ;  
 $\frac{V}{V} = \frac{V_2}{V_1} = \frac{2}{1} = 2$  100%

5.  $\text{KE} = T \frac{K_2}{K_1} = \frac{T_2}{T_1} = \frac{2E}{E} = 2$

$\frac{T_2}{T_1} = \frac{2T_1}{T_1} = 2$  283 K  
 $\frac{T_2}{T_1} = \frac{566 \text{ K}}{293 \text{ C}}$

6.  $\text{TE} = \frac{f}{2} kT = \frac{n}{2} kT$

7.  $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1200}{300}} = 2$   $v_2 = 2v_1$

8. (a)  $p_{\text{av}} = m_1 v$  is different for different  $m_1$

(b)  $(\text{KE})_{\text{molecule}} = \frac{3}{2} kT$  is same for any gas.

(c)  $(\text{KE})/V = \frac{3}{2} \frac{pV}{V} = \frac{3}{2} p$  is different as  $p$  is different for different.

(d)  $(\text{KE})_m = \frac{3}{2} \frac{pV}{m} = \frac{3}{2} \frac{p}{\rho}$  is different as  $\frac{p}{\rho}$  is different.

9.  $\frac{p_1 V_1}{RT_1} = \frac{p_2 V_2}{RT_2} = \frac{p V_1}{RT} = \frac{p V_2}{RT} = \frac{p(V_1 + V_2)}{RT}$

$T = \frac{p(V_1 + V_2)}{p_1 V_1 + p_2 V_2} = \frac{T_1 T_2 (p_1 V_1 + p_2 V_2)}{p_1 V_1 T_2 + p_2 V_2 T_1}$

10.  $\frac{l}{l} = \frac{0.08 \times 10^3}{10 \times 10^2 + 100}$   
 $= \frac{8 \times 10^6}{8 \times 10^6 + 100}$

$V = 3V = 3 \times 100 \text{ cc} = 300 \text{ cc}$   
 $10^6 \times 100$

0.24 cc

$V = 100 \text{ cc} + 0.24 \text{ cc} = 100.24 \text{ cc}$

11.  $T = T_0 \tan 45^\circ = V = T_0$   
 $pV = nRT = nR(T_0 + V) = nRT_0 + nRV$   
 or  $p = nR \frac{nRT_0}{V} + p = a + \frac{b}{V}$

ie,  $p$  versus  $V$  graph will be hyperbola.

12.  $p^2 V = \text{constant}$   
 $\frac{nRT}{V} = \text{constant}$

$T^2 = V$

$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{3V_0}{V_0}} = \sqrt{3}$

$T_2 = \sqrt{3} T_1 = \sqrt{3} T_0$

13.  $p = \frac{1}{3} \rho v_{\text{rms}}^2 = \frac{1}{3} \frac{m}{V} v_{\text{rms}}^2$

$\frac{mT}{m_1} = \frac{\text{constant}}{T_2} = \frac{310}{280} = 1.1$

14. As temperature of vessels A and B are same so is average velocity of  $\text{O}_2$ , i.e.,  $u$ .

15.  $N = nN_a = \frac{pV}{RT} N_a$   
 $\frac{10^{13}}{8.314} = \frac{10^6}{300} \times 6.023 \times 10^{23}$

$$16. \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = mgh$$

$$h \sim \frac{v^2}{2g} = \frac{3RT}{2gM}$$

$$h = \frac{3 \times 8.314 \times 273}{2 \times 10 \times 28 \times 10^3} \text{ m}$$

$$12.16 \times 10^3 \text{ m} = 12 \text{ km}$$

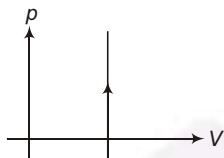
$$17. \frac{l_1}{l_2} = \frac{2}{1} \frac{19}{11} \text{ and } l_1 = l_2 = 30 \text{ cm}$$

$$l_1 = \frac{11}{19} l_2 = 30 \text{ cm}$$

$$l_1 = \frac{19}{8} \times 30 \text{ cm} = 71.25 \text{ cm}$$

$$\text{and } l_2 = \frac{11}{19} l_1 = 41.25 \text{ cm}$$

$$18. p \propto V^{-1} \quad T \propto V \quad \text{constant}$$



$$19. V = V_0 \tan \theta \quad T, pV = nRT = \frac{m}{M} RT$$

$$p(V_0 \tan \theta) = \frac{m}{M} RT$$

$$\tan \theta = \frac{1}{pT} \frac{m}{M} RT = \frac{pV_0}{pT}$$

$$\text{or } \tan \theta = \frac{mR}{pM} \frac{V_0}{T}$$

$\tan \theta$  remains same when  $m = 2m$  and  $p = 2p$

$$20. n_1 = \frac{p_1 V}{RT_1} \text{ and } n_2 = \frac{p_2 V}{RT_2}$$

$$\frac{n_1}{n_2} = \frac{p_1}{T_1} \frac{T_2}{p_2} = \frac{10}{5} \frac{300}{330} = \frac{600}{330} = \frac{20}{11}$$

$$n_2 = \frac{11}{20} n_1$$

$$m = m_1 = m_2 = m_1 \frac{11}{20} = \frac{11}{20} m_1$$

$$21. \frac{m}{V} = \frac{m}{(n_1 \frac{m}{n_2}) \frac{RT}{p}} = \frac{mp}{(n_1 \frac{m}{n_2}) RT}$$

$$\frac{12}{(2 \times 2)} \frac{1.01 \times 10^5}{8.314 \times 300} = \frac{10^3}{5} \times 0.12 \text{ kg/m}^3$$

$$22. p \propto k \frac{kM}{V}$$

$pV = \text{constant}$  is for isothermal process, i.e.,  $T = \text{constant}$

$$23. \frac{p^2}{T} = \text{constant}$$

$$p^2 V = \text{constant} \quad \frac{pT}{T} = \text{constant}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{2}{1}} \quad \sqrt{\frac{1/2}{1}} = \frac{1}{\sqrt{2}}$$

$$p_2 = \frac{p_1}{\sqrt{2}}$$

$$\frac{T_2}{T_1} = \frac{p_1}{p_2} = \sqrt{2}$$

$$T_2 = \sqrt{2} T_1 = \sqrt{2} T$$

$$\text{as } pT = \text{constant} \quad p \propto \frac{1}{T}$$

i.e.,  $p - T$  graph is hyperbola.

$$24. p^2 V = \text{constant}$$

$$PT = \text{constant and } T^2 V^{-1} = \text{constant.}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$$

$$p_2 = \frac{p_1}{2} = \frac{p}{2}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{4V}{V}} = 2$$

$$T_2 = 2T_1 = 2T$$

as  $p \propto \frac{1}{T}$   $p - T$  graph is hyperbola.

$$25. \frac{C}{100} = \frac{0}{0} \quad \frac{F}{212} = \frac{32}{32} \quad \frac{F}{BP} = \frac{MP}{MP}$$

ice point = 32 F and steam point = 212 F

26.  $p \propto V$  at 'a'  $p = p_0$  and  $V = V_0$  and at 'b',  
 $p = 2p_0$  and  $V = 2V_0$ ,

$$\frac{p_b}{p_a} = \frac{V_b}{V_a} = \frac{2V_0}{V_0} = 2$$

$$p_b = 2 \text{ Pa}$$

$$\frac{T_b}{T_a} = \frac{p_b V_b}{p_a V_a} = \frac{2p_0 \cdot 2V_0}{p_0 V_0} = 4$$

$$T_b = 4T_a$$

$$\text{as } \frac{U}{P} = \frac{T}{V} \Rightarrow U_b = 4U_a$$

Parabola passing through origin

27. (a)  $\frac{3}{2} nRT$  is independent of type of gas true.  
 (b) In one degree of freedom for one mole of gas,  $V = \frac{1}{2} RT$

(c) false

(d) false

28.  $V \propto T \Rightarrow V \tan T$

$$pV = nRT \Rightarrow \frac{m}{M} RT$$

$$p \propto T \tan \frac{mRT}{M} \Rightarrow \tan \frac{mR}{MP}$$

$$\tan \theta_1 = \tan \theta_2 \Rightarrow \frac{m_1}{p_1} = \frac{m_2}{p_2}$$

all  $a, b, c$  and  $d$  are possible.

29.  $pV = nRT$

$$p = \frac{n}{V} RT = \frac{N_a m/V}{M} RT = \frac{m}{M} RT$$

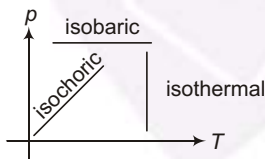
$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kN_a T}{M}} = \sqrt{\frac{3kT}{M}}$$

(a) and (d) are correct.

## JEE Corner

### ■ Assertion & Reason

1. Assertion is false.



2. Assertion and reason are both true but reason is not correct explanation of assertion. As at low temperature atoms in molecules are tightly bound such that they cannot oscillate.
3.  $pV = nRT = \frac{2}{3} \text{ KE}$

$$p = \frac{2 \text{ KE}}{3 V} \Rightarrow p = \frac{2}{3} E.$$

Assertion and reason are both true but reason cannot explain assertion.

4. Internal energy remains same in train frame of reference, so temperature do not change, but KE of gas molecules in ground frame increases.
5. According to equipartition theory, energy is equally distributed for each degree of freedom, so assertion is false.
6. At high temperature and low pressure intermolecular distance is much larger than size of the molecules and intermolecular forces can be neglected. So, assertion and reason are both true but not correct explanation.
7. At  $4^\circ\text{C}$ , volume is minimum or density is maximum *i.e.*, liquid will overflow on increasing or decreasing temperature. This reason is false.

8. Temperature remains constant as pressure is double and volume is halved, so internal energy remains constant. So reason partially explains assertion.
9. Assertion and reason are both true but not correct explanation.

10.  $V \propto \frac{nR}{Mp} T$  slope  $m$ ; reason is correct explanation.

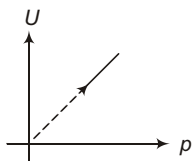
### Match The Columns

1.

- (a) TKE  $\frac{3}{2}nRT$   $\frac{3}{2} 2RT$   $3RT$   $r$
- (b) RKE  $\frac{2}{2}nRT$   $\frac{2}{2} 2RT$   $2RT$   $p$
- (c) PE  $s$
- (d) TKE  $\frac{5}{2}nRT$   $5RT$   $s$

- (d)  $T \propto \frac{1}{2} T$  increases with  $p$  increasing temperature

2.



$U \propto p \quad T \propto \frac{1}{V} \quad VT = \text{constant}$

$pT^2 = \text{constant}$  and  $pV^2 = \text{constant}$

- (a)  $U$  increases  $T$  increases  $P$  decreases  $r$
- (b)  $p$  increase  $V$  decreases  $r$
- (c)  $U$  increases  $T$  increases  $q$
- (d)  $\frac{T}{V} \propto \frac{TV}{V^2} = \frac{\text{constant}}{V^2}$  increase as  $V$  decreases  $q$

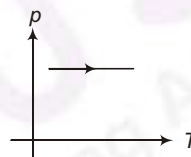
3.  $x_1 = 3, x_2 = \frac{8}{-}, x_3 = 2$  and  $x_4 = r$

- (a)  $r$ , (b)  $s$ , (c)  $q$ , (d)  $s$

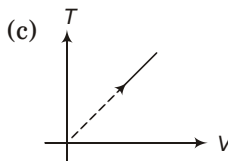
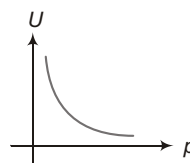
4.

- (a) density of water is maximum of  $4^\circ\text{C}$   $s$
- (b) depends of change in density of solid and liquid  $s$
- (c) depends of change in density of solid and liquid  $s$

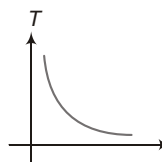
5. (a)  $p$  constant  $q$



- (b)  $V \propto \frac{1}{U} T$



- (d)  $V \propto \frac{1}{T} T$





# 18. First Law of Thermodynamics

## Introductory Exercise 18.1

1. (a)  $W = p \Delta V = 1.7 \times 10^5 (1.2 - 0.8) \text{ J}$   
 $6.8 \times 10^4 \text{ J}$   
 (b)  $V = 1.1 \times 10^5 \text{ J}$   
 $Q = U - W = 17.8 \times 10^4 \text{ J}$   
*i.e.*,  $1.78 \times 10^5 \text{ J}$  of heat has flown out of the gas.  
 (c) No, it is independent of the type of the gas.
2. (a) In  $p$ - $V$  graph of cyclic process, clockwise rotation gives positive work and anticlockwise gives negative work. And as loop 1 has greater area than loop 2, that is why total work done by the system is positive.  
 (b) As in cyclic process change in internal energy is zero, that's why for positive work done by the system, heat flows into the system.  
 (c) In loop '1' work done is positive so, heat flows into the system and in loop '2' work done is negative so heat flows out of the system.
3. As the box is insulated *i.e.*, no heat exchange takes place with surrounding and as the gas expands against vacuum *i.e.*, zero pressure that's why no work has been done and there is no change in internal energy. Thus, temperature do not change, internal energy and gas does not do any work.
4.  $U = \frac{f}{2} nRT = \frac{3}{2} nRT$   
 $n = \frac{2U}{3RT} = \frac{2 \times 100}{3 \times 8.314 \times 300}$   
 $0.0267 \text{ mole.}$
5.  $Q = ms = 1 \times 387 \times 30 \text{ J} = 11610 \text{ J}$   
 $W = p \Delta V = 1.01 \times 10^5 \times 7.06 \times 10^{-8}$   
 $7.13 \times 10^{-3} \text{ J}$   
 $U = Q - W = 11609.99 \text{ J}$

## Introductory Exercise 18.2

1. (a) At constant volume,  
 $U = 0$     $W = 0$   
 $Q = nC_V \Delta T = \frac{Q}{nC_V} = \frac{200}{1 \times \frac{3}{2} \times 8.314} = 16.04 \text{ K}$   
 $T_f = T_i + \Delta T = 300 + 16.04 = 316.04 \text{ K}$
- (b) At constant pressure,  
 $T = \frac{Q}{nC_p} = \frac{200}{1 \times \frac{5}{2} \times 8.314} = 9.62 \text{ K}$   
 $T_f = 300 + 9.62 \text{ K} = 309.62 \text{ K}$
2. For adiabatic process,  
 $pV^c = \text{constant}$     $c$  (say)  
 $\int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{c}{V^{c+1}} dV = \frac{c}{-c} \left[ V^{-c} \right]_{V_i}^{V_f} = - \frac{c}{c} \left[ \frac{V_f^{-c}}{V_i^{-c}} - 1 \right]$

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$$c \frac{V_f - V_i}{V_i} \left| \frac{V_f}{V_i} \right. \quad c \frac{V_f - V_i}{V_i} \left| \frac{V_f}{V_i} \right.$$

$$\frac{p_f V_f - p_i V_i}{1} \quad \frac{p_i V_i - p_f V_f}{1} \quad \text{(Proved)}$$

3.  $W_{AB} = 500 \text{ J}, Q_{AB} = 250 \text{ J}$   
 $U_{AB} = 250 \text{ J}$   
 $W_{AC} = 700 \text{ J}, Q_{AC} = 300 \text{ J}$   
 $U_{AC} = 400 \text{ J}$

(a) Path BC is isochoric process, i.e.,

$$W_{BC} = 0$$

$$Q_{BC} = U_{BC} - U_{AC} = U_{AB} - 150 \text{ J}$$

(b)  $W_{CDA} = W_{CD} + W_{DA}$   
 $800 \text{ J} = 0 + 800 \text{ J}$

(Work is negative as volume is decreasing)

$$U_{CDA} = U_{AC} - U_{AC} = 400 \text{ J}$$

$$Q_{CDA} = W_{CDA} - U_{CDA} = 400 \text{ J} - 400 \text{ J} = 0$$

4. (a)  $T = \frac{pV}{nR} = \frac{1 \times 10^2 \times 2 \times 10^5}{1 \times 8.314}$

(b)  $W = \frac{p}{1} \left[ \frac{V_2}{5} - \frac{V_1}{3} \right] = 10^3 \text{ J}$   
 $\frac{2}{3} \text{ J}$

5. (a)

$$K = \frac{p^2}{2m_i} - \frac{p^2}{2m_f} = \frac{p^2}{2} \left[ \frac{1}{m_i} - \frac{1}{m_f} \right]$$

$$= \frac{(10 \times 10^3 \times 200)^2}{2} \left[ \frac{1}{10 \times 10^3} - \frac{1}{2.01} \right]$$

$$= 2 \times 100 \times \frac{1}{2} \times 199 \text{ J}$$

(b)  $Q = nC_V T - T = \frac{Q}{nC_V} - \frac{Q}{\frac{m}{M} C_V}$

$$\frac{M}{m} = \frac{Q}{3R} = \frac{200}{2010} = \frac{199}{3} = 8.314$$

0.8 C

6.  $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$nRT_1 m \frac{p_1}{p_2} - p_2 (V_C - V_{BC})$$

$$nRT m \frac{p_2}{p_1} - p_1 (V_1 - V_2)$$

$$nR(T_2 - T_1) \ln \frac{p_2}{p_1} - p_1 V_2 - p_1 V_1$$

$$p_1 V_1 - p_1 V_2$$

$$(p_2 V_2 - p_1 V_1) \ln \frac{p_2}{p_1}$$

7.  $W_{ABCA} = (+ve) W_{AB} + (+ve),$

$$W_{BC} = 0, W_{CA} = (-ve)$$

For BC,  $Q = (-ve) U_{BC} = (-ve)$  and

$$W_{BC} = 0$$

For CA,  $U = (-ve) Q_{CA} = (-ve)$  as  $W_{CA} = (-ve).$

	U	W	Q
AB	+	+	+
BC		0	
CA			
Total	0	+	+

For AB, as  $U_{ABCA} = 0$  and

$$U_{BC} = (-ve),$$

$$U_{CA} = (-ve)$$

$$U_{AB} = (-ve)$$

As  $Q_{ABCA} = W_{ABCA} = (-ve)$  and

$$Q_{BC} = (-ve)$$

$$Q_{CA} = (-ve) \quad Q_{AB} = (-ve)$$

In isobaric process,  $W = p \Delta V = nR \Delta T$

$$= 0.2 \times 8.314 \times (300 - 200) = 166.3 \text{ J}$$

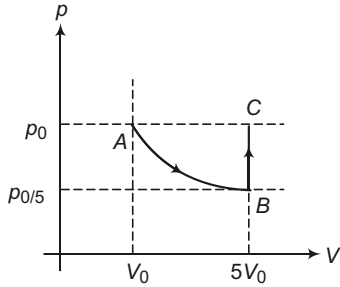
9.  $W = \int p dV = \int V^2 dV = \frac{1}{3} V^3$

$$= \frac{1}{3} (5 \times 1.01 \times 10^5)^3 - (2^3 \times 1^3)$$

$$= 1.18 \times 10^6 \text{ J}$$

### Introductory Exercise 18.3

1.



$$W_{BB} = nRT \ln \frac{V_B}{V_A} = 3R \cdot 273 \ln 5$$

$$10959 \text{ J}$$

$$W_{BC} = 0$$

$$Q = U = W$$

$$U = Q = W$$

$$80000 \quad 10959$$

$$69041$$

$$T_f = 5T_i = 5 \cdot 273 \text{ K} = 1365 \text{ K}$$

$$Q_{ABC} = Q_{AB} + Q_{BC} = W_{BC} = 0$$

$$Q_{BC} = nC_V T + C_V \frac{Q_{BC}}{n T}$$

$$\frac{69041}{3 \cdot 4 \cdot 273} = 21.07$$

$$C_p \quad C_V \quad R \quad 29.39$$

$$\frac{C_p}{C_V} = \frac{29.39}{21.07} = 1.4$$

2.  $Q = U = W; Q = nC_p T$

$$1600 = 1 \cdot C_p \cdot 72$$

$$C_p = 22.22$$

$$C_V = C_p - R = 13.9 \quad \frac{C_p}{C_V} = 1.6$$

$$W = Q = U = 1600 = nC_V T$$

$$1600 = 1 \cdot 13.9 \cdot 72$$

$$1600 = 1000.8 \text{ J}$$

$$599.2 \text{ J}$$

and  $U = nC_V T = 1 \cdot 13.9 \cdot 72$

$$1001$$

$$1 \text{ kJ}$$

3.  $W = \frac{1}{2} p V$

$$\frac{1}{2} \cdot 20 \cdot 1.01 \cdot 10^5 = 1 \cdot 10^3$$

$$10 = 101 \cdot 1010 \text{ J}$$

$$p = \frac{n W}{t} = \frac{100 \cdot 1010 \text{ J}}{60 \text{ s}}$$

$$1.68 \text{ kW}$$

### AIEEE Corner

#### ■ Subjective Questions (Level 1)

1.  $U = Q = W = 254 \text{ J} = 73 \text{ J}$

$$327 \text{ J}$$

2. (a)  $T = \frac{Q}{nC_V} = \frac{2 Q}{3nR} = \frac{2 \cdot 200}{2 \cdot 1 \cdot 8.314}$

$$16 \text{ K}$$

$$T_f = T_i = T = 316 \text{ K}$$

(b)  $T = \frac{Q}{nC_p} = \frac{2 Q}{5nR} = \frac{2 \cdot 200}{5 \cdot 1 \cdot 8.314}$

$$9.6 \text{ K}$$

$$T_f = T_i = T = 309.6 \text{ K}$$

3.  $U = nC_V T$ , in adiabatic process,

$$Q = 0 \text{ and } U = W$$

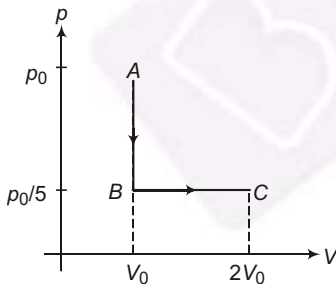
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where,  $W = \frac{nR \Delta T}{1}$   
 $U = \frac{nR \Delta T}{1}$  for all process.

4.  $V = 0$   $W = 0$   
 $Q = U = nC_V \Delta T = n \frac{5}{2} R \Delta T$   
 $\frac{5}{2} (p_f V_f - p_i V_i) = \frac{5}{2} (p_f - p_i) V$   
 $\frac{5}{2} (5 \times 10^5 - 10^5) = 10 \times 10^3$   
 $\frac{5}{2} \times 4 \times 10^5 = 10^2 \times 10^4 \text{ J}$

5.  $Q_1 = U_1 = nC_V \Delta T = n \frac{5}{2} R (3T_i - T_i)$   
 $5 nRT_i$   
 $Q_2 = nC_p \Delta T = n \frac{5}{2} R (6T_i - 3T_i)$   
 $7.5 nRT_i$   
 $c = \frac{Q}{n \Delta T} = \frac{12.5 nRT_i}{n (6T_i - 3T_i)} = \frac{12.5R}{3} = 2.5R$

6.  $W_{AB} = 0$ ,  $W_{BC} = \frac{p_0}{2} V_0 - \frac{1}{2} p_0 V_0$   
 $\frac{1}{2} nRT_0 - 300 R$



$Q = (U_2 - U_1) = (U_C - U_A) = (U_{BC} - U_{AB}) = (U_C - U_A) - (U_B - U_A) = U_C - U_B = W_{BC}$   
 $0 - 300 R$   
 (As  $T_A = T_C$ )  
 $2.49 \times 10^3 \text{ J} - 2.49 \text{ kJ}$

7.  $U = Q = W = 1200 \text{ J} - 2100 \text{ J} = 900 \text{ J}$

$T = \frac{U}{nC_V} = \frac{900}{5 \times \frac{3}{2}} = 14.43$

$T_f - T_i = T = 127 \text{ C} - 14.43 \text{ C} = 112.6 \text{ C}$

8. When gas expands it does positive work on the surrounding and for this purpose heat has to be supplied into the system.

9.  $W = -V \Delta p = -V (p_f - p_i)$   
 $m \frac{1}{\rho_f} - \frac{1}{\rho_i} = m \frac{1}{1000} - \frac{1}{999.9}$   
 $\frac{10^5}{1000} - \frac{0.1}{999.9} = 0.02 \text{ J}$

(work done is negative as volume decreases)

$Q = ms = 2 \times 4200 = 8400 \text{ J}$   
 $33600 \text{ J}$   
 $U = Q - W = 33600 - 0.02 \text{ J}$

10.  $W = p \Delta V = p(V_f - V_i) = p \frac{m}{\rho}$   
 $\frac{10^5}{0.6} (10 - 10^3) = 1666.67 \text{ J}$

$Q = ms = mL = 10^2 \times 4200 = 420000 \text{ J}$   
 $100 \times 10^2 = 10^6 \text{ J}$   
 $25 \times 10^6 = 250000 \text{ J}$   
 $29200 \text{ J}$   
 $U = Q - W = 29200 \text{ J} - 1666.67 \text{ J} = 27533.33 \text{ J}$   
 $2.75 \times 10^4 \text{ J}$

11.  $W = p \Delta V = 1.013 \times 10^5 (1670 - 10^6) = 1.013 \times 167 \text{ J} = 169.2 \text{ J}$

$Q = mL = 10^3 \times 2.256 = 2256 \text{ J}$   
 $10^6 \text{ J}$   
 $U = Q - W = (2256 - 169.2) \text{ J} = 2086.8 \text{ J}$   
 $2087 \text{ J}$

12.  $W = p \Delta V = 2.3 \times 10^5 (0.5 - 1.15) = -1.15 \times 10^5 \text{ J}$   
 $U = 1.4 \times 10^5 \text{ J}$   
 $Q = U - W = 1.4 \times 10^5 \text{ J} + 1.15 \times 10^5 \text{ J} = 2.55 \times 10^5 \text{ J}$

$$(1.4 + 1.15) \cdot 10^5 \text{ J}$$

$$2.55 \cdot 10^5 \text{ J}$$

Thus,  $2.55 \cdot 10^5 \text{ J}$  of heat flows out of the system and it is independent of the type of the gas.

13. In a cyclic process,  $U = 0$   $Q = W$

$$(a) \quad W = (Q_1 \quad Q_2 \quad Q_3 \quad Q_4)$$

$$(W_1 \quad W_2 \quad W_3)$$

$$(5960 \quad 5585 \quad 2980 \quad 3645)$$

$$(2200 \quad 825 \quad 1100)$$

$$1040 \quad 275 \quad 765 \text{ J}$$

$$(b) \quad \frac{\text{work done}}{\text{heat supplied}} = \frac{1040}{9605} = 10.83\%$$

14. (a)  $W = \frac{1}{2} AB \quad AC = \frac{1}{2} \cdot 2p_0 \cdot V_0$

$$(b) \quad T_C = \frac{p_0 V_0}{nR} \quad \text{and} \quad T_A = \frac{p_0 V_0}{nR}$$

$$Q_{CA} = nC_p T = nC_p \frac{p_0 V_0}{nR}$$

$$= \frac{5}{2} R \frac{p_0 V_0}{R} = \frac{5}{2} p_0 V_0$$

$$T_B = \frac{3p_0 V_0}{nR}, \quad Q_{AB} = nC_V T$$

$$= n \cdot \frac{3}{2} R \frac{3p_0 V_0}{nR} = \frac{9}{2} p_0 V_0$$

$$\frac{3}{2} \cdot 2 p_0 V_0 = 3 p_0 V_0$$

$$(c) \quad Q_{AB} = Q_{BC} = Q_{CA} = W$$

$$3 p_0 V_0 \quad Q_{BC} = \frac{5}{2} p_0 V_0 \quad p_0 V_0$$

$$Q_{BC} = \frac{p_0 V_0}{2}$$

(d) Temperature is maximum at a point  $D$  lying somewhere between  $B$  and  $C$  where the product  $pV$  is maximum.

$$p = \frac{2p_0}{V_0} = 5p_0$$

$$pV = \frac{2p_0}{V_0} V = 5p_0 V$$

$$\frac{2p_0}{V_0} V^2 = 5p_0 V$$

For  $pV$  maximum  $\frac{d}{dV}(pV) = 0$

$$2V \frac{2p_0}{V_0} = 5p_0 \quad 0$$

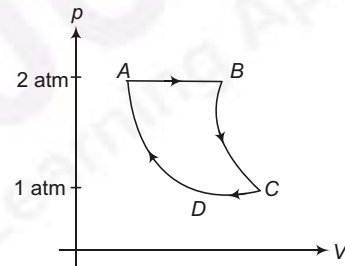
$$V = \frac{5V_0}{4}$$

$$p = \frac{2p_0}{V_0} \frac{5V_0}{4} = 5p_0 \quad \frac{5}{2} p_0$$

$$T_{\text{max}} = \frac{(pV)_{\text{max}}}{nR}$$

$$= \frac{\frac{5}{2} p_0 \cdot \frac{5}{4} V_0}{1 R} = \frac{25 p_0 V_0}{8R}$$

15.  $V_A = \frac{nRT_A}{p_A} = \frac{2R \cdot 300}{2 \cdot 10^5} = 3 \cdot 10^{-3} R$



$$V_B = \frac{2R \cdot 400}{2 \cdot 10^5} = 4 \cdot 10^{-3} R,$$

$$V_C = \frac{2R \cdot 400}{10^5} = 8 \cdot 10^{-3} R$$

$$V_0 = \frac{2R \cdot 300}{10} = 6 \cdot 10^{-3} R$$

$$W = 2 \cdot 10^5 (4 - 3) = 10^3 R$$

$$2R \cdot 400 \ln \frac{8}{4} = 1 \cdot 10^5 (6 - 8) = 10^3 R$$

$$K = 2R \cdot 300 \ln \frac{3}{6}$$

$$W = 200R - 800R \ln 2 = 200R - 600R \ln 2$$

$$= 2000R \ln 2 - 1153 J$$

As  $Q = W = 1153 \text{ J}$  and  $U = 0$  cyclic process.

$$16. \quad W = \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} (p_B - p_0) - \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} (p_0 - p_0) + \frac{1}{2} V_0 (p_B - p_0) - \frac{1}{2} V_0 (p_0 - p_D) - \frac{1}{2} V_0 (p_B - p_D)$$

where,  $p_B = p_0 \frac{p_0}{V_0} \frac{V_0}{2} = \frac{3p_0}{2}$

and  $p_D = p_0 \frac{p_0}{V_0} \frac{V_0}{2} = \frac{p_0}{2}$

$$W = \frac{1}{2} V_0 \left( \frac{3}{2} p_0 - \frac{1}{2} p_0 \right) - \frac{1}{2} p_0 V_0$$

$$W_{ABC} = p_0 \left( \frac{3V_0}{2} - \frac{V_0}{2} \right) - \frac{1}{2} \frac{3V_0}{2} \frac{V_0}{2} - \frac{V_0}{2}$$

$$(p_B - p_0) p_0 V_0 - \frac{1}{2} V_0 \left( \frac{3}{2} p_0 - p_0 \right)$$

$$= \frac{5}{4} p_0 V_0$$

$$U_{ABC} = nC_V \frac{(T_C - T_A)}{3V_0} \frac{V_0}{2}$$

$$= n \frac{3}{2} R \frac{p_0}{nR} - \frac{p_0}{nR} \frac{V_0}{2} = \frac{3}{2} p_0 V_0$$

$$Q_{\text{supplied}} = \frac{5}{4} p_0 V_0 - \frac{11}{4} p_0 V_0$$

$$\frac{\frac{1}{2} p_0 V_0}{\frac{11}{4} p_0 V_0} = \frac{2}{11} = 0.1818 = 18.18\%$$

17. (a) As the cyclic process is clockwise *i.e.*, work done is positive, so heat is absorbed by the system.

(b) In cyclic process work done is equal to the net heat absorbed (as change in internal energy is zero) so, work done in one cycle is 7200 J.

(c) In anticlockwise rotation, work done is negative and heat is liberated by the system, and its magnitude is 7200 J.

18. (a) As area under clockwise loop is more than that at anticlockwise loop, so network done is positive.

(b) In loop I work done is positive and in loop II work done is negative.

(c) As network done in one cycle is positive so heat flows into the system.

(d) In loop I heat flows into the system and in loop II heat flows out of the system.

$$19. \quad T_A = \frac{p_A V_A}{nR} = \frac{1.01 \times 10^5 \times 22.4}{10^3 \times 8.314}$$

$$= 273 \text{ K}$$

$$T_B = \frac{p_B V_A}{nR} = \frac{2p_A V_A}{nR} = 2T_A$$

$$= 546 \text{ K}$$

$$V_c = \frac{nRT_c}{p_c} = \frac{T_c}{p_A} \frac{nRT_B}{p_A}$$

$$= \frac{2nRT_A}{p_A}$$

$$= 2V_A = 44.8 \text{ m}^3$$

20. (a)  $W_{AB} = BC$

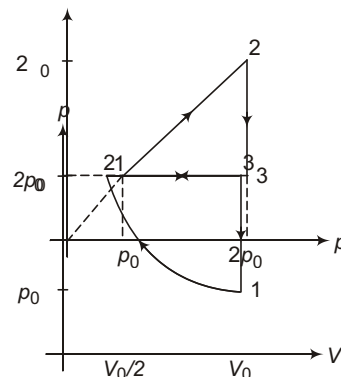
$$(4 \times 1.5) \times 10^6 - (4 \times 2) \times 10^5$$

$$= 2.5 \times 10^6 - 0.8 \times 10^6 = 1.7 \times 10^6 \text{ J}$$

(b)  $Q = W$  as  $U = 0$  in a cycle

$$Q = 0.5 \text{ J}$$

21. As  $\frac{1}{V}$



$$(a) W_{12} = nRT_0 \ln \frac{V_0/2}{V_0} = p_0 V_0 \ln 2 = p_0 \frac{M}{\rho} \ln 2$$

$$W_{23} = 2p_0 V_0 \left[ \frac{V_0}{2} - V_0 \right] = -p_0 V_0$$

$$p_0 \frac{M}{\rho}; W_{31} = 0$$

$$(b) Q_{231} = Q_{23} + Q_{31} = nC_V T_{23} + W_{23} + nC_V T_{31}$$

$$n \left[ \frac{3}{2}R \frac{2p_0 V_0}{nR} - \frac{2p_0 V_0}{nR} + \frac{2p_0 V_0}{nR} \right] = p_0 V_0$$

$$n \left[ \frac{3}{2}R \frac{p_0 V_0}{nR} - \frac{2p_0 V_0}{nR} + \frac{2p_0 V_0}{nR} \right]$$

$$p_0 V \left[ \frac{3}{2} p_0 V_0 - p_0 V_0 + \frac{5}{2} p_0 V_0 \right]$$

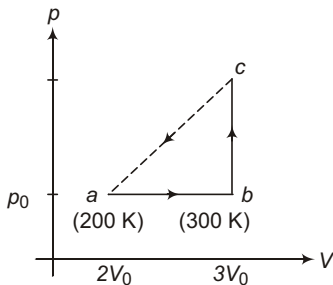
Heat rejected  $Q_{231} = W$

$$\frac{5}{2} p_0 V_0 - p_0 V_0 - p_0 V_0 \ln 2 = \frac{3}{2} p_0 V_0 - p_0 V_0 \ln 2$$

$$p_0 V_0 \left[ \frac{5}{2} \ln 2 - \frac{p_0 M}{p_0} \frac{3}{2} \ln 2 \right]$$

$$(c) \frac{\text{work done}}{\text{heat supplied}} = \frac{W}{Q_{231}} = \frac{2}{3} (1 - \ln 2)$$

22.  $W_{AB} = p_0(3V_0 - 2V_0) = p_0 V_0;$



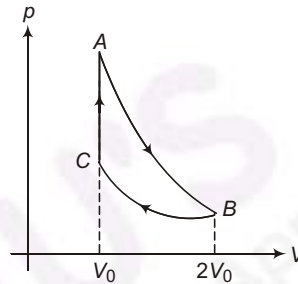
$$W_{BC} = 0, W_{CA} = ?$$

$$Q = W_{AB} + W_{BC} + W_{CA} = 800 \text{ J} + p_0 V_0 + 0 = W_{CA}$$

$$W_{CA} = 800 \text{ J} + p_0 V_0 = 800 \text{ J} + \frac{1}{2} nRT_A$$

$$W_{CA} = 800 \text{ J} + 200R = 2463 \text{ J}$$

23.  $W_{AB} = \frac{p_B V_B - p_A V_A}{1}$



$$\frac{3}{2} (p_A V_A - p_B V_B) = \frac{3}{2} nR(T_A - T_B) = \frac{3}{2} nRT_B \left( \frac{T_A}{T_B} - 1 \right); TV^{-1}$$

$$\frac{3}{2} nRT_B \left( \frac{2}{T} \right)^{\frac{5}{3}-1} = \frac{3}{2} nRT_B (2^{2/3} - 1)$$

$$W_{BC} = nRT_B \ln \frac{V_0}{2V_0} = -nRT_B \ln 2$$

and  $W_{CA} = 0$

Heat Supplied

$$Q_{CA} = U_{CA} = \frac{3}{2} nR(T_A - T_C) = \frac{3}{2} nR(T_A - T_B)$$

$$\frac{3}{2} nRT_B \left( \frac{T_A}{T_D} - 1 \right) = \frac{3}{2} nRT_B (2^{2/3} - 1)$$

$$\frac{W}{Q_{CA}} = \frac{-nRT_B \ln 2}{\frac{3}{2} nRT_B (2^{2/3} - 1)}$$

$$1 \frac{2}{3} \frac{\ln 2}{2^{2/3}} \frac{1}{1} \quad 1 \quad 0.7867 \quad 0.213$$

$$21.3\%$$

■ Objective Questions (Level 1)

1.  $U = nC_V T = \frac{3}{2} RT = \frac{3}{2} RT$

$$T = \frac{2U}{3R}$$

$$T_D = \frac{2V_0}{3R} = 300 \text{ K} \quad U_0 = 450R,$$

$$T_A = \frac{4V_0}{3R} = 600 \text{ K}$$

$$W = W_{AB} = W_{CD} = nRT_A \ln \frac{2V_0}{V_0}$$

$$nRT_D \ln \frac{V_0}{2V_0} = nR(T_A - T_D) \ln 2$$

$$1 \quad R \quad (600 - 300) \ln 2$$

$$300R \ln 2 = Q$$

2.  $W_{12} = p \Delta V = nR \Delta T$

$$= \frac{2R}{300} \cdot 600R$$

As,  $Q = W_{12} + W_{23} + W_{31}$

$$300 \text{ J} = 600R + W_{23} + 0$$

$$W_{23} = 300 \text{ J} - 600R = 5288 \text{ J}$$

3.  $nC_p T_1 = nC_V T_2$

$$\frac{7}{2} \cdot 30 = \frac{5}{2} T_2$$

$$T_2 = 42 \text{ K}$$

4.  $TV^{n-1} = \text{constant}$

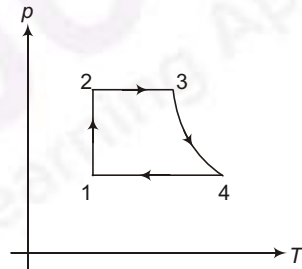
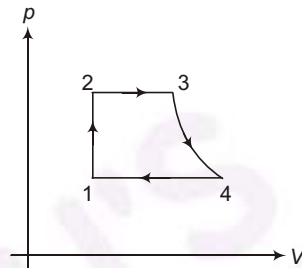
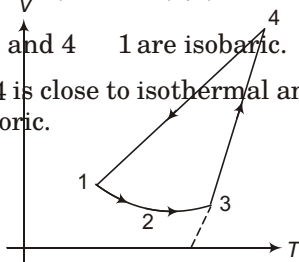
$$pV = V^n \cdot \frac{1}{V} = pV^n = \text{constant}$$

$$\ln p + n \ln V = \ln c$$

$$\frac{p}{p} + n \frac{V}{V} = \frac{p}{V/V} = np = B$$

5. 2, 3 and 4, 1 are isobaric.

3, 4 is close to isothermal and 1, 2 is isochoric.



6.  $W = \int p dV = \int kV dV = \frac{k}{2} V^2$

$$= \frac{1}{2} pV = \frac{1}{2} nR(T_2 - T_1) = \frac{R}{2}(T_2 - T_1)$$

7.  $p = V^2, W = \int p dV = \int kV^2 dV$

$$= \frac{1}{3} kV^3 = \frac{1}{3} pV$$

$$= \frac{1}{3} nR(T_f - T_i) \quad ( ) \text{ ve}$$

8.  $W = nRT \ln \frac{V_f}{V_i}$

$$= nRT \ln \frac{1}{2} = nRT \ln 2$$

9.  $U = 600 \text{ J} - 150 \text{ J} = 450 \text{ J}$

$$nC_V T = \frac{3}{2} R n T$$



$$C \frac{Q}{nT} = \frac{600 \text{ J}}{450 \text{ J}} = \frac{3}{2} R = \frac{600}{450} \frac{3}{2} R$$

$$\frac{3}{2} R = \frac{4}{3} 2R$$

10.  $W_1$  ( ) ve,  $W_2 = 0$ ,  $W_3$  ( ) ve  
and  $U_1 = U_2 = U_3$   
as  $Q = U - W$   $Q_1 = Q_2 = Q_3$

11.  $U = 2p_0 2V_0 = 2p_0 V_0 = 2p_0 V_0$   
and  $W = p_0(2V_0 - V_0) = p_0 V_0$   
 $Q = U - W = 3p_0 V_0$

12. In adiabatic compression, temperature of the gas increases and as  $pV = T$  so,  $pV$  increases.

13. As  $W_1 = W_2$  while  $U_1 = U_2$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{C_1}{C_2} = 1$$

14.  $W = nR(4T - T) = \frac{nR(5T - 4T)}{1}$   
 $nR(3T - 5T) = \frac{nR(T - 3T)}{1}$   
 $3nRT - 2nRT = \frac{nRT}{1} - \frac{2nRT}{1}$   
 $nRT = \frac{nRT}{1}$   
 $\frac{nRT}{1} (1 - 1) = -\frac{nRT}{1}$   
 $\frac{5/3}{5/3 - 1} 1RT = 2.5RT$

15.  $U_p$  constant

$$\frac{3}{2} nRT = \frac{nM}{V} = \frac{3}{2} n^2 MR \frac{T}{V}$$

$T = V$  i.e., isobaric process.

$$\frac{U}{W} = \frac{U}{Q - U} = \frac{3/2}{5/3 - 3/2} = \frac{3}{2}$$

$$\frac{C_V}{C_p} = \frac{C_V}{R}$$

16.  $W = 50(0.4 - 0.1) = \frac{1}{2} 50(0.2 - 0.1)$   
 $15 = 2.5 = 27.5 \text{ J}$   
 $U = 2.5 \text{ J}$

17.  $W_1 = \int_{V_0}^{2V_0} p dV = p(2V_0 - V_0) = pV_0$   
 $W_2 = \int_{V_0}^{2V_0} kV dV = \frac{1}{2} kV^2$   
 $= \frac{1}{2} k(4V_0^2 - V_0^2) = \frac{3}{2} kV_0^2 = \frac{3}{2} pV_0$

18.  $W = \frac{W_1}{r_1 r_2} = \frac{W_2}{ab}$   
 $\frac{r_2}{2} = \frac{r_1}{2} \left( \frac{p_2}{p_1} \right)$   
 $-(p_2 - p_1)(V_2 - V_1)$

19.  $W = \int_{x=V}^{x=b} PdV = \frac{nRT}{V} dV = nRT \frac{dx}{x}$   
 $x = V$  to  $b$   
 $dx = dV$   
 $nRT \ln x = nRT \ln(V/b) \Big|_V^{2V}$   
 $nRT [\ln(2V/b) - \ln(V/b)]$   
 $nRT \ln \frac{2V}{V} = RT \ln \frac{2V}{V}$

as  $n = 1$  mole

20.  $AB$  is isochoric process, so,  $W_{AB} = 0$

$BC$  is isothermal process, so,

$$W_{BC} = nRT_2 \ln \frac{V_2}{V_1} = RT_2 \ln \frac{V_2}{V_1}$$

$CA$  is close to isobaric process, so,

$$W_{CA} = nRT = nR(T_1 - T_2) = R(T_1 - T_2)$$

21.  $Q = U - W$   $Q = W$   
 $W = 2Q$   
 $U = nC_V T = n \frac{f}{2} R T = \frac{n}{1} R T$

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$$f = \frac{2}{1} = 2$$

$$W = \int p dV = 2 Q = \frac{2n}{1} R T$$

$$\frac{2nR}{1} \frac{T}{1} = \frac{nR}{1} \frac{T}{a} \text{ for polytropic process with } pV^a \text{ constant}$$

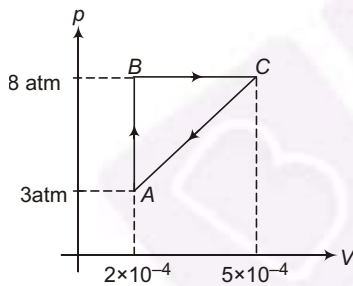
$$\frac{2}{1} = \frac{1}{1} \frac{1}{a} = 1 - a \Rightarrow \frac{1}{2} = -a$$

or  $a = -\frac{1}{2}$

$$pV^{-1/2} = \text{constant}$$

$$TV^{1/2} = \text{constant}$$

22.  $W_{AB} = 0, U_{AB} = 600 \text{ J}$



$$W_{BC} = 8 \times 10^5 (5 - 2) \times 10^{-4} = 240 \text{ J}$$

$$U_{BC} = Q_{BC} = W_{BC} = 200 + 240 = 40 \text{ J}$$

$U_{CA} = U_{AB} + U_{BC} + U_{CA} = 0$  in cyclic process.

$$U_{CA} = U_{AB} + U_{BC} = 600 \text{ J} + 40 \text{ J} = 560 \text{ J}$$

23. Starting and ending points along x-axis in graph are not clear, so nothing can be said about the magnitude of work.

It can only be said that work done in ABC is negative and that in DEF is

positive. Looking at the graph, area can be assumed to be equal so,

$$W_{DEF} = W_{ABC}$$

24.  $W_{\text{isobaric}} = p(V_f - V_i) = pV \ln 2$

$$W_{\text{isothermal}} = nRT \ln \frac{2V}{V} = pV \ln 2$$

$$W_{\text{adiabatic}} = \frac{0.693 pV}{1} = \frac{p_f V_f - p_i V_i}{1}$$

$$= \frac{p_i \frac{V}{2V} - p_i V}{1} = \frac{p_i V (2^{-1/r} - 1)}{1}$$

$$= pV \frac{1 - 2^{1/r}}{r}$$

$$= 0.55 pV$$

So, work done is minimum in adiabatic process.

25.  $Q = \Delta U = W$

$$\frac{7}{2} RT_0 = 10 \left( \frac{5}{2} R T - 10R T + 35R T \right)$$

$$T_0 = 100T = 10(T - T_0)$$

$$11T_0 = 10T$$

$$\frac{T}{pV_0} = \frac{1.1T_0}{pV}$$

$$V = \frac{11}{10} V_0 = 1.1V_0$$

26.  $W = (3p_0 - p_0)(2V_0 - V_0) = 2p_0 V_0$

$$Q_{\text{supplied}} = n \frac{3}{2} R \frac{3p_0 V_0}{nR} - \frac{p_0 V_0}{nR}$$

$$= n \frac{5}{2} R \frac{3p_0 \cdot 2V_0}{nR} - \frac{3p_0 V_0}{nR}$$

$$= \frac{3}{2} nR \frac{2p_0 V_0}{nR} - \frac{5}{2} nR \frac{3p_0 V_0}{nR}$$

$$= 3p_0 V_0 - \frac{15}{2} p_0 V_0 = \frac{21}{2} p_0 V_0$$

$$\frac{W}{r} = \frac{2p_0 V_0}{\frac{21}{2} p_0 V_0} = \frac{4}{21}$$

27.  $W_{12}$   $W_{13}$  can be seen from area under the curve, while  $V_1$   $V_2$

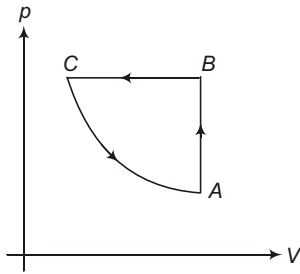
$$Q_{12} = Q_{13} = Q_2 = Q_1 \text{ or } Q_1 = Q_2$$

28.  $W_{CA} = p_0(V_0 - 2V_0) = -p_0V_0$

and  $U_{CA} = \frac{3}{2}p_0V_0$

$$Q_{CA} = \frac{5}{2}p_0V_0$$

29.  $Q_{AB} = 200 \text{ kJ} = nC_V T$ ;



$U_{BC} = 100 \text{ kJ}$  and  $W_{BC} = 50 \text{ kJ}$   
 $W_{AB} = 0$   $U_{AB} = 200 \text{ kJ}$ ,  $Q_{CA} = 0$   
 $U_{ABC} = U_{AB} + U_{BC} = U_{CA} = 0$   
 or  $200 \text{ kJ} = 100 \text{ kJ} + U_{CA} = 0$

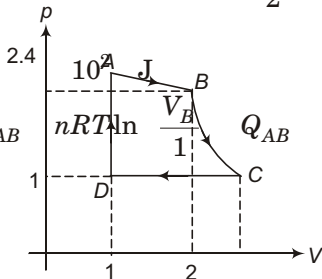
$U_{CA} = 100 \text{ kJ}$   
 $Q_{AB} = Q_{BC} = Q_{CA} = 200 \text{ kJ} = (100 \text{ kJ} + 50 \text{ kJ}) = 0$   
 $50 \text{ kJ}$

$W_{AB} = W_{BC} = W_{CA} = 0$   $200 \text{ kJ}$   $W_{CA}$

$Q_{ABC} = 50 \text{ kJ}$

$W_{CA} = 150 \text{ kJ}$

30.  $Q = W = ab = \frac{20 \cdot 10^3}{2} = \frac{20 \cdot 10^3}{2}$



31.  $W_{AB} = nRT \ln \frac{V_B}{V_A} = 9 \cdot 10^4 \text{ J}$

$$800 T \ln V_B = 9 \cdot 10^4 \text{ J}$$

$$T \ln V_D = \frac{225}{2}$$

$$W_{ABCD} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$nRT \ln \frac{V_B}{V_A} = \frac{p_C p_C}{p_B p_B} = \frac{p_C(V_D - V_C)}{nRT_B} = 0$$

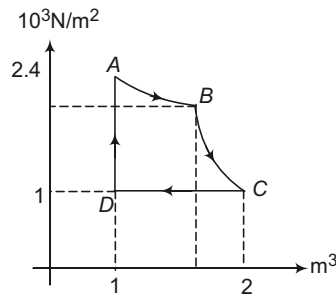
$$9 \cdot 10^4 = \frac{10^5}{1} \cdot \frac{5}{3} = 10^5 (2 - 1)$$

$$19 \cdot 10^4 = \frac{3}{2} (10^5 - 800 T_B)$$

$$4 \cdot 10^4 = 1200 T_B$$

$$4 \cdot 10^4 = 1200 \cdot \frac{2.4 \cdot 10^5}{100 \cdot 8}$$

31.  $W = W_{AB} = \frac{p_C V_C - p_B V_B}{1} = W_{CD}$



$$9 \cdot 10^4 = \frac{2 \cdot 10^5}{1} \cdot \frac{9 \cdot 10^4}{5/3} = 1 \cdot 10^5$$

$$9 \cdot 10^4 = \frac{3}{2} \cdot 11 \cdot 10^4 = 10 \cdot 10^4$$

$$\frac{33}{2} = 1 \cdot 10^4 = 15.5 \cdot 10^4$$

32.  $W = \int p dV = \int kV dV = \frac{1}{2} kV^2$

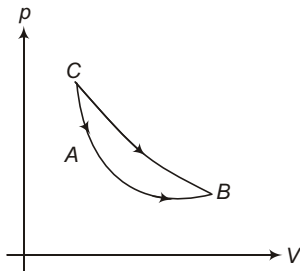
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$$\frac{1}{2} pV = \frac{1}{2} nRT_0 = \frac{1}{2} RT_0$$

$$U = nC_V T = \frac{3}{2} RT_0$$

$$Q = \frac{3}{2} \left( \frac{1}{2} RT_0 + 2RT_0 \right)$$

33.  $pT = \text{constant}$   $p \frac{pV}{nR} = \frac{p^2 V}{nR}$   
 $p^2 V = \text{constant}$



$$p_0^2 V_0 = \frac{p_0^2}{2} V = V = 4V_0$$

$$T = \frac{p_0}{2} \frac{4V_0}{nR} = 2 \frac{p_0 V_0}{nR} = 2T_0$$

$$U = nC_V T = 2 \left( \frac{3}{2} R (2T_0 - T_0) \right)$$

$$3R \frac{p_0 V_0}{2R} = \frac{3}{2} p_0 V_0$$

35.  $W_{BC} = nRT_0 \ln \frac{V_C}{V_B}$

$$nRT_0 \ln \frac{p_B}{p_C} = 2 nRT_0 \ln \frac{V_B}{V_A}$$

$$2nRT_0 \ln \frac{p_A}{p_B}$$

$$\ln \frac{p_B}{p_C} = \ln \frac{p_0}{p_0/2} = \ln 4$$

$$\frac{p_B}{p_C} = \frac{4p_C}{p_C} = \frac{p_0}{8}$$

36. As,  $W_a = W_b = W_1 = W_2$   
 while,  $U_1 = U_2 = Q_1 = Q_2$

37.  $1 - \frac{T_{\text{sink}}}{T_{\text{source}}} = 1 - \frac{300}{600}$   
 $1 - \frac{1}{2} = 0.5 = 50\%$

38. As the volume is adiabatically decreased, temperature of the gas increases and as the time elapsed, temperature normalizes i.e., decreases and so pressure also decreases.

39. As the compression is quick, the process is adiabatic while leads to heating of the gas.

40.  $pV = \text{constant}$   
 $\frac{nRT}{V} V = nRTV = \text{constant}$   
 $TV = \text{constant}$   
 $\frac{T_1}{T_2} = \frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{L_2}{L_1} = \frac{2}{3}$

41.  $pV = \text{constant}$   $p = \frac{nT}{V}$   
 $p^1 T = \text{constant}$   
 $p = T^{-1}$

As  $\frac{7/5}{1} = \frac{7}{2}$  for diatom gases.

$$p = T^{3.5} = 3.5$$

42.  $pV^x = \text{constant}$ ,  $W = \frac{nR T}{1-x}$ ,

$$U = n \frac{5}{2} R T$$

$$C = \frac{Q}{n T} = \frac{\frac{nR T}{1-x} \frac{5}{2} nR T}{n T}$$

$$\frac{5}{2} R \frac{R}{x} = \frac{R}{1-x} \frac{1}{x} = \frac{2}{5}$$

$x = \frac{7}{5}$   $x = 1.4$  but  $x = 1$  as for  $x = 1$ ,

$C$  will become positive.

43.  $C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{13}{6}R$

(a)  $\frac{2 \cdot \frac{5}{2}R + 4 \cdot \frac{5}{2}R}{2 + 4} = \frac{15}{6}R$

(b)  $\frac{2 \cdot \frac{5}{2}R + 4 \cdot \frac{3}{2}R}{2 + 4} = \frac{11}{6}R$

(c)  $\frac{2 \cdot \frac{3}{2}R + 4 \cdot \frac{5}{2}R}{2 + 4} = \frac{13}{6}R$  and

(d)  $\frac{2 \cdot \frac{6}{2}R + 4 \cdot \frac{3}{2}R}{2 + 4} = \frac{12}{6}R$

Passage 44 & 45

44.  $W_{ABCA} = \frac{1}{2} p V = \frac{pV}{2} = Q_{net}$

45.  $CA$  isobaric and  $BC$  isochoric,

$$\frac{C_p}{C_v} = \frac{5}{3}$$

46.  $pV = \text{constant} \Rightarrow p = \frac{nRT}{V}$

$p^1 T = \text{constant}$

$$T = \frac{1}{p}$$

$$T = p^{-1/3} \Rightarrow T = p^{-2/5}$$

$$\frac{T_B}{T_A} = \frac{p_B}{p_A} = \frac{2p_c}{3p_c} = 0.85$$

$T_B = 0.85T_A = 850 \text{ K}$

47.  $W_{AB} = \frac{nRT}{1} \left[ \frac{1}{\frac{25}{3}} - \frac{1}{\frac{5}{3}} \right] = 150 - 75 = 75 \text{ J}$

$1875 \text{ J}$

48.  $W_{BC} = 0, Q_{BC} = U_{BC}$

$$n \cdot \frac{3}{2}R(T_C - T_B)$$

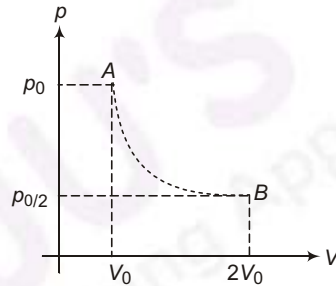
$$n \cdot \frac{3}{2}R \frac{p_C V}{nR} = \frac{p_B V}{nR}$$

$$\frac{3}{2} \cdot \frac{1}{3} p_A = \frac{2}{3} p_A \Rightarrow V$$

$$\frac{1}{2} p_A V = \frac{1}{2} \cdot \frac{3}{2} p_B V = \frac{3}{4} nRT_B$$

$$\frac{3}{4} \cdot 1 \cdot \frac{25}{3} = 850 \Rightarrow 5312.5 \text{ J}$$

49.  $W_{AB} = (\ ) \text{ ve}, T_A = T_B$



$$p = \frac{p_0}{2V_0} V = \frac{3}{2} p_0 \frac{nRT}{V}$$

$$\text{or } T = \frac{p_0}{2nRV_0} V^2 = \frac{3p_0}{2nR} V_0$$

$y = ax^2 + bx$  is parabola.

Again,  $p = \frac{p}{2V_0} = \frac{nRT}{p} = \frac{3}{2} p_0$

is also equation of parabola.

While going from A to B temperature first increases and then decreases.

50.  $pV^2 = \text{constant}$

$$W = \int p dV = \int \frac{k}{V^2} dV = k \left[ -\frac{1}{V} \right]$$

$$pV \Big|_i^f = p_i V_i = p_f V_f$$

$$nR(T_i - T_f) = nR(T_f - T_i) \quad (\ ) \text{ ve}$$

as  $T_f > T_i$

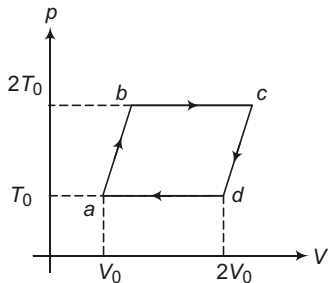
as  $T_i > T_f \Rightarrow U_i > U_f$

$U = (\ ) \text{ ve}$

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$Q = nC_V \Delta T = nR \Delta T = n(C_V + R) \Delta T$   
 ( ) ve as  $C_V > R$   
 i.e., heat is given to the system.

51. In cyclic process,  $U = 0$



$$W = 0 = nR2T_0 \ln \frac{2V_0}{V_0} + nRT_0 \ln \frac{V_0}{2V_0}$$

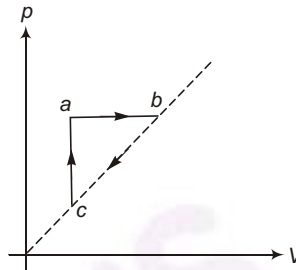
$$= 2nRT_0 \ln 2 - nRT_0 \ln 2 = nRT_0 \ln 2 \text{ ( ) ve}$$

i.e.,  $W > 0$

$$Q_{\text{supplied}} = U_{ab} + W_{bc} = nC_V(2T_0 - T_0) + nR2T_0 \ln \frac{2V_0}{V_0}$$

$$= 2 \left( \frac{3}{2} RT_0 + 4RT_0 \ln 2 \right) - 3RT_0 - 4RT_0 \ln 2$$

52. *ab* isochoric, *bc* isobaric and *ca* isothermal.



$$W_{ab} = 0, U_{ca} = 0$$

as in *ca* density is increasing, so volume is decreasing i.e.,

$$W_{ca} \text{ ( ) ve, i.e., } W_{ca} < 0$$

in isochoric process  $Q_{ab}$  is positive for increase in temperature.

53. In isochoric process  $W = 0$ .

and in adiabatic process

$$Q = 0 \quad Q_3 \text{ to be minimum} \quad Q_2 < Q_1 < Q_3$$

## JEE Corner

### ■ Assertion & Reasons

1. In adiabatic expansion,  $W < 0$  ve while  $Q = 0$  and as according to first law of thermodynamics,

$$Q = \Delta U + W \quad \Delta U = -W$$

i.e.,  $\Delta U < 0$  ve this implies decrease in temperature. So, Assertion and reason are both true but not correct explanation.

2. Assertion is false, as work done is a path function and not a state function i.e., it

depends on the path through which the gas was taken from initial to final state.

3. Assertion is false, as first law can be applied for both real and ideal gases.

4. During melting of ice its volume decreases, so work done by it is negative and that by atmosphere is positive. So, reason is true explanation of assertion.

5. As  $Q = \Delta U + W = U_2 - U_1 + Q = W$ , where  $U$  is state function while  $Q$  and  $W$  are path function as for definite

initial and final state  $U$  is constant and so is  $Q = W$ . Thus assertion and reason are both true but not correct explanation.

6. Carnot's engine is ideal heat engine with maximum efficiency but it is not also 100%. So assertion and reason are both true but not correct explanation.

7.  $pT = \text{constant}$   $p = \frac{pV}{R} = \frac{p^2V}{nR}$

$$p^2V = \text{constant}$$

$$W = \int p dV = \int \sqrt{k} \frac{dV}{\sqrt{V}} = \sqrt{k} \frac{V^{1/2}}{1/2}$$

$$= 2\sqrt{k}\sqrt{V} = 2\sqrt{kV} = 2\sqrt{p^2/V}$$

$$= \frac{2pV}{2nR(T_f - T_i)} = \frac{2nRT}{2nRT} T$$

$W$  ( ) ve for  $T$  ( ) ve  
and  $\frac{nRT}{V} = T = \text{constant}$ .

### ■ Match the Columns

1. (a)  $W = \int p dV = \int pV = nR(T_f - T_i)$

(b)  $U = nC_V T = 2 \cdot \frac{3}{2} R(2T - T)$

(c)  $W = \frac{3RT}{1 - 5/3} = \frac{3}{2} \cdot 2RT$

(d)  $U = nC_V T = 3RT = pV$

2. (a) In  $ab$  slope is more so, pressure is less as  $V = \frac{nR}{p} T$ , but is constant and in

isobaric process.  $W = p \Delta V = nR \Delta T$  and as  $T$  is same in both process so,  $W$  is same for both  $r$

(b) As  $U = nC_V T$  is same for both process  $r$

(c) As  $Q = U - W$ , it is also same for both process  $s$

$$T^2 \propto V$$

or,  $V \propto T^2$

Thus assertion is true but reason is false.

8. In adiabatic changes for free expansion,  $Q = 0, W = 0$  and  $U = 0$

as in free expansion no work is done against any force.

For ideal gases  $pV = \text{constant}$  as  $U = 0$   $T = \text{constant}$  So, assertion and reason are both true but not correct explanation.

9. Assertion and reason are both true and correct explanation.

10. Assertion and reason are both true and correct explanation.

(d) Nothing can be said about molar heat capacity  $s$

3. (a)  $W = \int p dV$

$$= \int \frac{k}{V} dV = \sqrt{k} \frac{dV}{\sqrt{V}}$$

$$= 2\sqrt{kV} = 2pV = 2nR T = pV$$

(b)  $U = nC_V T = \frac{3}{2} nR T = s$

(c)  $Q = 2nR T = \frac{3}{2} nR T$

$$= \frac{7}{2} nR T = s$$

(d)  $s$

4. (a)  $W = \int p dV = nR \Delta T$  and  $U = nC_V \Delta T$

$$W = U = q$$

(b)  $W = 0$   $Q = U$ ,  $U$  ( ) ve  $p, r$

(c)  $W$  ( ) ve,  $U$  ( ) ve,  $Q = 0$   $p$

(d)  $W$  ( ) ve,  $U = 0$ ,  $Q$  ( ) ve  $p$

5. (a)  $W_{AB} = p_0 V_0 - \frac{1}{2} p_0 V_0$

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$$\frac{3}{2} p_0 V_0 \quad s$$

$$(b) \quad U_{AB} \quad Q \quad W$$

$$6 p_0 V_0 \quad \frac{3}{2} p_0 V_0 \quad \frac{9}{2} p_0 V_0 \quad s$$

$$(c) \quad Q \quad 6 p_0 V_0$$

$$nC \quad \frac{4 p_0 V_0}{nR} \quad \frac{p_0 V_0}{nR}$$

$$\frac{3 p_0 V_0}{R} C$$

$$(d) \quad U \quad C \quad 2R \quad p$$

$$nC_V \quad \frac{4 p_0 V_0}{nR} \quad \frac{p_0 V_0}{nR}$$

$$3C_V \quad \frac{p_0 V_0}{R} \quad \frac{9}{2} p_0 V_0$$

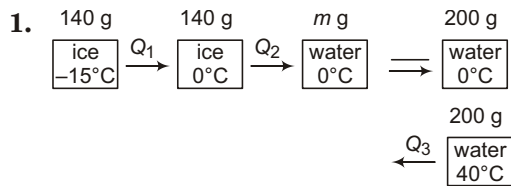
$$C_V \quad \frac{3}{2} R \quad s$$





# 19. Calorimetry and Heat Transfer

## Introductory Exercise 19.1.



As Heat gain Heat loss

$$Q_1 \quad Q_2 \quad Q_3$$

$$140 \quad 0.53 \quad 15 \quad m \quad 80$$

$$m \frac{8000}{80} = \frac{1113}{80} \quad 200 \quad 1 \quad 40$$

86 g is the mass of ice melt

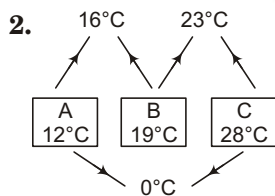
Mass of water

$$200 \text{ g} + 86 \text{ g} = 286 \text{ g}$$

and mass of ice

$$140 \text{ g} - 86 \text{ g} = 54 \text{ g}$$

while final temperature of mixture is  $0^\circ\text{C}$ .



$$ms_A (16 \quad 12) \quad ms_B (19 \quad 16)$$

$$4s_A \quad 3s_B$$

$$ms_B (23 \quad 19) \quad ms_C (28 \quad 23)$$

$$4s_B \quad 5s_C$$

$$ms_A ( \quad 12) \quad \frac{4}{5} ms (28 \quad )$$

or  $\frac{3}{4} s_B ( \quad 12) \quad \frac{4}{5} s_B (28 \quad )$

or  $15 ( \quad 12) \quad 16(28 \quad )$

or  $31 \quad 448 \quad 180$

$$20.26 \text{ C}$$

3.  $mL \quad ms$

$$80 \text{ cal} \quad 1 \text{ cal}/^\circ\text{C} ( \quad 0 \text{ C})$$

$$80 \text{ C}$$

4. As Heat gain Heat loss

$$(100 \quad m) \quad 529 \quad m \quad 80$$

$$\frac{100 \quad 529}{100 \quad 529} \quad 609 \text{ m}$$

$$m \frac{609}{609} \text{ g} \quad 86.86 \text{ g of ice will}$$

be formed.

5.  $P \frac{d}{dt} \frac{d}{dt} (ms \quad ) \quad \frac{dm}{dt} s$

$$\frac{dm}{dt} \quad \frac{P}{s}$$

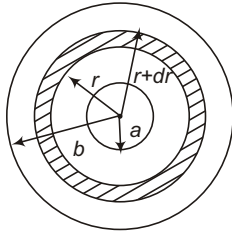
$$\frac{dm}{dt} = \frac{500 \cdot 10^6 \text{ J/s}}{4200 \text{ J/kg} \cdot \text{C} \cdot 10 \text{ C}}$$

$$\frac{5}{42} \cdot 10^4 \text{ kg/s} \quad 12 \quad 10^4 \text{ kg/s}$$

### Introductory Exercise 19.2

1. Rest of the liquid will be heated due to conduction and not convection.

$$2. \frac{dQ}{dt} = \frac{k \cdot 4 \cdot r^2 \cdot (d)}{dr}$$



$$\frac{dQ}{dt} = \frac{dr}{r^2} \cdot 4 \cdot k \cdot d$$

$$\text{or } \frac{dQ}{dt} = \frac{b \cdot dr}{a \cdot r^2} \cdot 4 \cdot k \cdot \frac{T_2}{T_1} \cdot d$$

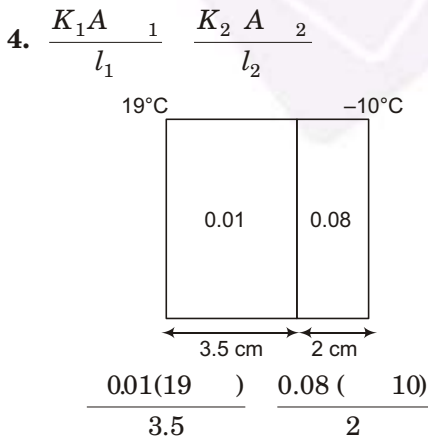
$$\text{or } \frac{dQ}{dt} = \frac{1}{a} \cdot \frac{1}{b} \cdot 4 \cdot k \cdot (T_2 - T_1)$$

$$\frac{dQ}{dt} = \frac{4 \cdot k \cdot (T_1 - T_2)}{\frac{1}{a} - \frac{1}{b}} = 4 \cdot kab \cdot \frac{T_1 - T_2}{b - a}$$

$$3. \frac{dQ}{dt} = \frac{kA}{t}$$

$$k = \frac{dQ}{dt} \cdot \frac{t}{A}$$

Unit of  $k$  = watt  $\frac{m}{m^2 \cdot K}$  = W/m-K



or  $2(19 - (-10)) = 28(19 - (-10))$   
 or  $38 = 280 - 30$

or  $\frac{242}{30} = 8.07 \text{ C}$

$$\frac{dQ}{dt} = \frac{0.01 \cdot 1 \cdot (19 - 8.1)}{3.5 \cdot 10^{-2}}$$

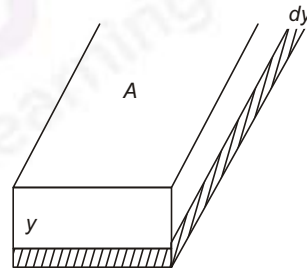
$$5. \frac{dQ}{dt} = \frac{dm}{dt} \cdot L = \frac{0.44 \text{ kg}}{300 \text{ s}} \cdot 2.256 \cdot 10^6 \text{ J/kg}$$

$$= \frac{3308.8 \text{ J/s}}{kA} = \frac{50.2 \cdot 0.15 \cdot (100)}{t \cdot 1.2 \cdot 10^{-2}}$$

$$627.5 \cdot (100) = 100 \cdot \frac{3308.8}{627.5} \cdot 5.27$$

$$105.27 \text{ C}$$

$$6. \frac{dQ}{dt} = \frac{kA [0 - ( )]}{y} = \frac{dm}{dt} \cdot L$$



$$\frac{dV}{dt} = L \cdot A \cdot \frac{dy}{dt}$$

$$\frac{kA}{y} = \frac{AL}{dt} \cdot \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{k}{L y} \quad (\text{Proved})$$

$$7. \frac{dQ}{dt} = e \cdot AT^4$$

$$4 \cdot 5.67 \cdot 10^8 = 4 \cdot (4 \cdot 10^2)^2 \cdot (3000)^4$$

$$0.4 \cdot 4 \cdot 5.67 \cdot 4^2 \cdot 3^4 \text{ J/s}$$

$$3.7 \cdot 10^4 \text{ watt}$$

$$8. \frac{dQ}{dt} = \frac{1}{R_{th}} = R_{th} \cdot \frac{K}{W} \text{ KW}^{-1}$$

## AIEEE Corner

### ■ Subjective Questions (Level-1)

1.  $\begin{matrix} Q_1 \\ 0^\circ\text{C} \end{matrix}$  Water  $\begin{matrix} Q_2 \\ 0^\circ\text{C} \end{matrix}$  Water  $\begin{matrix} Q_3 \\ 100^\circ\text{C} \end{matrix}$  steam  $\begin{matrix} \\ 100^\circ\text{C} \end{matrix}$

$Q$   $Q_1$   $Q_2$   $Q_3$   $mL_f$   $ms$   $mL_v$

10 [80 1 100 540]

10 720 cal 7200 cal

2. 10 g of water at  $40^\circ\text{C}$  do not have sufficient heat energy to melt 15 g of ice at  $0^\circ\text{C}$ , so there will be a mixture of ice-water at  $0^\circ\text{C}$ . Let the mass of ice left is  $mg$ .

$$\begin{matrix} (15 & m) & 80 & 10 & 1 & 40 \\ & 15 & m & 5 & m & 10 \text{ g} \end{matrix}$$

Mass of ice 10 g

and mass of water (10 5) g 15 g

3.  $4 s_P$  (60 55) 1  $s_R$  (55 50)

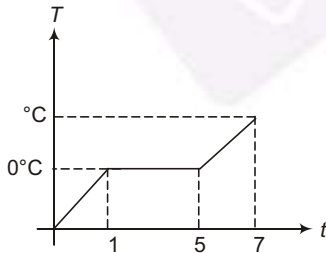
$$\begin{matrix} 4s_P & s_R \\ 1 & s_P & (60 & 55) & 1 & s_Q & (55 & 50) \end{matrix}$$

$$\begin{matrix} s_P & s_Q \\ 1 & s_Q & (60 & ) & 1 & s_R & ( & 50) \end{matrix}$$

or  $s_P$  (60 )  $4s_P$  ( 50)

$$\begin{matrix} 260 & 5 & \frac{260}{5} & 52 \text{ C} \end{matrix}$$

4.  $\frac{dQ}{dt} = \frac{m}{4} \frac{336}{60 \text{ s}} \frac{10^3 \text{ J/kg}}{60 \text{ s}}$



$$\begin{matrix} 1400 \text{ J/kg} \\ 1400 \text{ mW/kg} \\ \frac{m}{t} = \frac{m}{2} \frac{4200 ( \quad ) \text{ c}}{60 \text{ s}} \\ \frac{1400}{2} \frac{2}{60} \\ 4200 \end{matrix}$$

5.  $Q = \frac{1}{2} \frac{1}{2} mv^2 = ms \quad mL$

$$v = \sqrt{\frac{4(s \quad L)}{4(125 \quad 300 \quad 25 \quad 10^4)}}$$

$$v = \sqrt{\frac{4(3.75 \quad 2.5) \quad 10^4}{4 \quad 6.25 \quad 10^4 \quad 500 \text{ m/s}}}$$

6.  $mg = h \quad ms$

$$\frac{g \quad h}{s} = \frac{0.4 \quad 10 \quad 0.5}{800} \frac{1}{400} \text{ C}$$

7.  $\frac{K_1 A ( \quad )}{l} = \frac{K_2 A (100 \quad )}{l}$

$$\begin{matrix} (K_1 & K_2) & 100 & K_2 \\ \frac{100 & K_2}{K_1 & K_2} & \frac{100 & 46}{390 & 46} \end{matrix} \quad 1055 \text{ C}$$

8.  $i_{CD} = \frac{i_{AC} \quad i_{CB}}{KA( \quad )} = \frac{KA(100 \quad )}{l/2} = \frac{KA( \quad )}{l/2}$

or  $\frac{25 \quad 2(100 \quad ) \quad 2}{5 \quad 225 \quad 45 \text{ C}}$

or  $i_{CD} = \frac{45 \quad 25}{R_{th} \quad 5} = 4 \text{ W}$

9.  $\frac{i_A \quad i_C \quad i_D}{KA(T_1 \quad )} = \frac{KA( \quad T_3)}{3l/2} = \frac{KA( \quad T_2)}{3l/2}$

$$T_1 = \frac{2}{3} ( T_3 ) = \frac{2}{3} ( T_2 )$$

or  $T_1 = \frac{2}{3} (T_2 \quad T_3) = 1 \quad \frac{4}{3}$

$$T_1 = \frac{2}{3} (T_2 \quad T_3)$$

$$\frac{3T_1 \quad 2(T_2 \quad T_3)}{7}$$

10.  $\frac{KA(200 \quad )}{l} = \frac{2KA( \quad )}{l}$

$$\frac{3KA(200 - 100)}{l}$$

$$200 \times \frac{3}{1} \times \frac{2}{2} \times \frac{100}{2} = 3(200 - 100)$$

$$\frac{3 \times 1 \times 2 \times 200}{1 \times 3 \times 2} = \frac{200}{500}$$

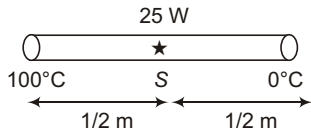
$$\frac{11 \times 2 \times 1300}{11} = 1300$$

$$\frac{2 \times 1300}{11} = 118.2 \text{ C}$$

$$\frac{1}{3} [200 \times 2 + 2 \times 2] = 145.45 \text{ C}$$

11.  $25 \times \frac{400 \times 10^4 (100)}{1/2}$

$$\frac{400 \times 10^4 (100)}{1/2}$$



or  $25 \times 8 \times 10^2 [100]$

$$312.5 \times 2 \times \frac{100}{412.5} = 206.25$$

$$\frac{1}{l} \times \frac{106.25 \text{ C}}{1/2 \text{ m}} = 212.5 \text{ C/m}$$

and  $\frac{2}{l} \times \frac{206.25 \text{ C}}{1/2 \text{ m}} = 412.5 \text{ °C/m}$

12.  $\frac{dQ}{dt} = e AT^4 = 0.6 \times 5.67 \times 10^8$

$$= 0.6 \times 5.67 \times (10.73)^4 \times 10^2 = 2 \times 902 \text{ W}$$

13.  $\frac{dQ}{dt} = e AT^4$  and  $\frac{dQ}{dt} = AT^4$

$$e \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{210}{700} = 0.3$$

14.  $\frac{(80 - 50)_c}{5} = \frac{80 - 50}{2} = 20 \text{ c}$

$$K = \frac{6}{45};$$

$$\frac{(60 - 30)}{t} = \frac{6}{45}$$

$$\frac{60 - 30}{2} = 20 \quad t = 9 \text{ min}$$

■ Objective Questions (Level-1)

1.  $\frac{3KA(35 - 0)}{10} = \frac{KA(0 - 0)}{20}$

$$\frac{6(35 - 0)}{6 \times 35} = \frac{0}{7} = 30 \text{ C}$$

2.  $\frac{T_S}{T_N} = \frac{N}{S} = \frac{350}{510} = 0.69$

According to Wien's law

3.  $\frac{dQ}{dt} \propto \frac{1}{K} \propto \frac{4}{l^2} \propto \frac{4}{K}$

4.  $\frac{dm}{dt} \propto \frac{dQ}{dt} \propto \frac{1}{K} \propto \frac{4}{l^2} \propto \frac{4}{K}$

$$\frac{dm}{dt} \propto \frac{4}{K} \propto \frac{4}{3a^2} \propto \frac{4}{3a^2}$$

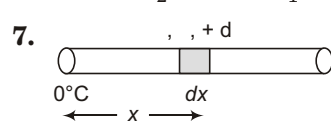
5.  $\frac{K_1 A (T_2 - T_1)}{d} = \frac{K_2 A (T_3 - T_2)}{3d}$

$$K_1 (T_2 - T_1) = \frac{1}{3} K_2 (T_3 - T_2)$$

$$K_1 = \frac{1}{3} K_2 \quad K_1 : K_2 = 1 : 3$$

6.  $\frac{dQ}{dt} \propto \frac{dQ}{dt} \propto \frac{2K}{2l} \propto \frac{KA}{l}$

$$\frac{dQ}{dt} \propto \frac{dQ}{dt} \propto \frac{2K}{2l} \propto \frac{KA}{l} = 2$$



$$P \frac{dQ}{dt} = \frac{K Ad}{dx} \frac{K_0(1 - \alpha x) A d}{dx}$$

$$l \frac{dx}{dt} = \frac{K_0 A}{P} \frac{10^2 \cdot 10^4}{1} \frac{10^3}{1} \frac{1}{1}$$

$$\frac{1}{a} \ln(1 - \alpha x) \Big|_0^l = \frac{10^2 \cdot 10^4}{1} \frac{10^3}{1} \frac{1}{1}$$

$$\ln(1 - \alpha l) = \ln 1 - 1$$

$$\ln(1 - \alpha l) = \ln 1 - 1$$

$$\ln(1 - \alpha l) = 1$$

or  $1 - \alpha l = e^{-1}$   
 or  $l = \frac{1}{\alpha} (e - 1) = e - 1 = 1.7 \text{ m}$

8.  $\frac{2}{1} \frac{T_1}{T_2} = \frac{2}{3} = 2 \frac{2}{3} \text{ m}$

9. Heat required to boil 1 g of ice is 180 cal while 1 g of steam can release 540 cal during condensation. So, temperature of the mixture will be 100°C with 2/3 g steam and 4.3 g water.

10.  $T_1, T_2, T_3$  as temperature of a body decreases in rate of cooling also decreases such that time increases for equal temperature difference.

11. Conduction is maximum for which thermal resistance is minimum, as  $R_{th} = \frac{l}{r^2}$  then for

(a) 50 (b) 25 (c) 100 (d) 33.33,  
 So option 'b' has minimum resistance.

12. Slope of temperature *versus* heat graph gives increase of specific heat or heat capacity and the portion DE is the gaseous state.

13.  $dQ = m s dt = maT^3 dT$   
 $\frac{Q}{m} = \frac{a}{4} T^4 \Big|_1^2 = \frac{a}{4} (16 - 1) = \frac{15a}{4}$

14. Resistance becomes 1/4th in parallel of that in series, so times taken will also become 1/4th i.e., 12/4 = 3 min.

15.  $ms_1 = 12, ms_2 = 8, s_1 : s_2 = 2 : 3$

16.  $\frac{KA(T - T_c)}{\sqrt{2}l} = \frac{KA(T_c - \sqrt{2}T)}{l}$   
 $\frac{T}{\sqrt{2}} = \frac{\sqrt{2}T - T_c}{1}$   
 $\frac{3}{\sqrt{2}} T = \frac{1}{\sqrt{2}} T_c$   
 $T_c = \frac{3}{1} \frac{1}{\sqrt{2}} T$

17.  $P = \frac{(1000 - 160) W}{2} = \frac{840 W}{2} = 4200 \text{ W}$

$t = \frac{42 \cdot 10^4}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$

18.  $\frac{dQ}{dt} = \frac{KA(T_2 - T)}{x} = \frac{2KA(T - T_1)}{4x}$   
 $T_2 - T = \frac{1}{2} T = \frac{1}{2} T_1$   
 $T_2 = \frac{1}{2} T_1 + \frac{3}{2} T$   
 $T = \frac{2}{3} T_2 = \frac{1}{2} T_1 + \frac{1}{3} (2T_2 - T_1)$   
 $\frac{dQ}{dt} = \frac{KA}{x} T_2 = \frac{1}{3} (2T_2 - T_1)$   
 $\frac{KA}{x} [3T_2 - 2T_2 - T_1] = \frac{1}{3}$   
 $\frac{KA}{x} (T_2 - T_1) = \frac{1}{3}$   
 $f = \frac{1}{3}$

19.  $\frac{1}{K} = \frac{A}{B} \frac{K_B}{K_A} = \frac{1}{2}$   
 $A = \frac{1}{2}, B = 18 \text{ C}$

■ More than One Correct Options

20. Amount of heat radiated or absorbed depends upon. Surface type, surface area, surface temperature and temperature of surrounding, so (a) and (b) are correct.

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21.  $\frac{KA(40)}{l} \quad \frac{KA(30)}{l} \quad \frac{KA(20)}{l}$   
 $\frac{40}{3} \quad \frac{2}{90} \quad \frac{50}{3}$

or  $30 \text{ C}$

So, (b) and (d) are correct.

22.  $m \quad s \quad (2 \quad 0) \quad m \quad 2s \quad (0 \quad )$   
 $4 \quad 3 \quad 0 \quad 0 \quad \frac{4}{3}$

$c_1 : c_2 \quad m_1 : s_2 \quad s_1 : s_2 \quad 1 : 2$

So, (b) and (c) are correct.

23. In series rate of  $R \quad R_1 \quad R_2$   
 $\frac{1}{q} \quad \frac{1}{q_1} \quad \frac{1}{q_2} \quad q \quad \frac{q_1 q_2}{q_1 \quad q_2}$

In parallel  $\frac{1}{R} \quad \frac{1}{R_1} \quad \frac{1}{R_2}$

$q \quad q_1 \quad q_2$  as  $q \quad \frac{1}{R}$

So, (b) and (c) are correct.

24. (a), (c) and (d) are correct.

## JEE Corner

### ■ Assertion and Reason

- Assertion is false.
- According to Wien's law assertion and reason are correct.
- Assertion and reason are true but not correct explanation.
- Assertion is true but reason is false as resistance becomes 1/4th.
- Assertion and reason are both false.
- Assertion is false as this statement was not given by Newton.
- Assertion and reason are both true with correct explanation.
- Both are true but not correct explanation.
- Assertion is false as temperature at different points become different.
- As mass of follow sphere is less so cooling will be faster. So, both are true with correct explanation.

### ■ Match the Columns

1.

(a)	$\frac{(dQ/dt)}{AT^4} \quad \frac{ML^2T^{-2}T^{-1}}{L^2T^{-4}}$	s
(b)	$b \quad T \quad L$	p

(c)	$\frac{E}{At} \quad \frac{[ML^2T^{-2}]}{[L^2T]} \quad [MT^{-3}]$	r
(d)	$R_{th} \quad \frac{d}{dQ/dt} \quad \frac{[ML^2T^{-2}T^{-1}]}{[M^{-1}L^2T^3]}$	s

2.

(a)	Slope of line $ab$	s
(b)	Length of line $bc$	m
(c)	Solid liquid $bc$	s
(d)	Only liquid $cd$	q

3.  $\frac{KA(100 \quad b)}{l} \quad \frac{KA( \quad b \quad d)}{l}$   
 $\frac{KA( \quad d \quad 80)}{l}$

$100 \quad b \quad b \quad d$  and

$100 \quad b \quad d \quad 80$

$d \quad 2 \quad b \quad 100 \quad 3 \quad b \quad 120$   
 $d \quad b \quad 20$

$b \quad 40 \text{ C} \quad d \quad 20 \text{ C}$

$c \quad f \quad \frac{40 \quad 20}{2} \quad 10 \text{ C}$

(a) q, (b) p, (c) p, (d) r

4. (a)  $ms( \quad 1 \quad ) \quad 2ms(2 \quad 1)$

$3 \quad 1 \quad 5 \quad 1 \quad \frac{5}{3} \quad q$

$$(b) \frac{ms}{4} \left( \frac{2}{10} \right) + \frac{3ms}{2} \left( \frac{3}{2} \right) = p$$

$$(c) \frac{2ms}{5} \left( \frac{3}{13} \right) + \frac{3ms}{3} \left( \frac{3}{2} \right) = s$$

$$(d) \frac{ms}{6} \left( \frac{4}{14} \right) + \frac{2ms}{4} \left( \frac{4}{2} \right) + \frac{3ms}{4} \left( \frac{4}{3} \right) = r$$

5.

(a)	$s$	$\frac{1}{m} \frac{dQ}{d}$	$\frac{J}{kg \text{ } ^\circ C}$	$q$
(b)	$c$	$ms$	$m \frac{dQ}{md}$	$J/^\circ C$
(c)	$i$	$\frac{dQ}{dt}$	$J/s$	$r$
(d)	$L$	$\frac{E}{m}$	$J/kg$	$s$

