Answers & Explanations

1. Option: b)

Let circumcenter be G then $GA = GB$

Hence $\sqrt{(x-4)^2+(y-6)^2} = \sqrt{(x-0)^2+(y-b)^2}$

Here values of (x, y) is value of coordinate of circumcenter $= (3,3)$.Putting these values in place

We get, $10 \ = \ 9+(3-b)^2 \rightarrow \ b = 3+1 \ = 4$

2. Option: a)

$3x \ + \ 4y \ = \ 5\sqrt{2}$ Dividing both sides of equation by $\sqrt{3^2+4^2} = 5$

$\frac{3}{5}\, x + \frac{4}{5} y = \frac{5\sqrt{2}}{5}$

$cos\ \theta = \frac{3}{5}$ and $sin\ \theta = \frac{4}{5}$

$tan\ \theta \ = \frac{sin}{cos} \ = \frac{4}{3}$

$\Rightarrow \theta = \tan^{-1} \ \frac{4}{3}$

3. Option : d)

$x^2+y^2-2x-14y = 119 \ \rightarrow \ (x-1)^2+(y-7)^2 - 50 \ = \ 119$

$(x-1)^2 + \ (y- \ 7)^2 = 169 \ \rightarrow \ (x-1)^2 + \ (y- \ 7)^2 = \ 13^2$

$So\ center\ is\ at\ (1,7)$

4. Option: c)

$(x-0)^2+(y+3)^2$

$= \left(\frac{0x+y+2}{0+\sqrt{1}}\right)^2$

$\rightarrow x^2+2y+5 = 0$

5. Option: c)

$25x^2 + 16y^2 = 400$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

$A = 4\,;\ b = 5$

$E = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{16}}$

$E = -\frac{3}{4}$

6. Option: a)

$A.B = A \times B \times cos\theta = 40$

$\rightarrow\ cos\theta = \frac{40}{(20\times4)} = \frac{1}{2}$

$\rightarrow \theta = 60$

$|a| \times |b| = A \times B \times sin\theta = 20 \times 4 \times \sqrt{\frac{3}{2}} = 40\sqrt{3}$

7. Option: b)

$a = i - j = 1i - 1j + 0k$

$b = i + j = 1i + 1j + 0k$

Projector of a on $b = \frac{1}{|b|}(a\,.\,b)$

$(a\,.\,b) = (1 \times 1) + (-1 \times 1) + (0 \times 0) = 1 + (-1) + 0 = 0$

$|b| = \sqrt{1} + 1 + 0 = \sqrt{2}$

$\frac{1}{|b|}(a\,.\,b) = \frac{1}{\sqrt{2}} \times (0) = 0$

8. Option: a)

$a_1 = 1,\ b_1 = 2,\ c_1 = -8$

$a_2 = 3,\ b_2 = 4,\ c_2 = 6$

$$cos\alpha = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{(a_1^2+b_1^2+c_1^2}\sqrt{(a_1^2+b_1^2+c_1^2)}})$$

$$cos\alpha = \frac{|3+8-48|}{\sqrt{(1^2+2^2+(-8)^2)\sqrt{3^2+4^2+6^2}}}$$

$$cos\alpha = \frac{37}{\sqrt{69\sqrt{61}}}$$

$$cos\alpha = \frac{37}{\sqrt{4209}}$$

$$\alpha = \cos^{-1}\left(\frac{37}{\sqrt{4209}}\right)$$

9. Option: d)

$$\lim_{x\to 2}\frac{2x^2+5x-8}{x^2-4}$$

$$= \frac{2(2^2)+5(2)-8}{4-4}$$

$$= \frac{10}{0} = \infty$$

10. Option: b)

$Put\ a = \ tan\theta$

$$y = \cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$y = \ \cos^{-1}(cos2\theta)$$

$$y = 2\theta\ \{\ since\ 0 < a < 1;\ 0 < \ tan\theta < 1 \to 0 < 2\theta < \frac{\pi}{2}$$

$$y = 2\tan^{-1}a \ = \frac{2}{1+a^2}$$

11. Option: b)

Given, $f(x) = sin(x)\ , x\ \in\ [0,\pi]\ Now\ f(0)\ = \ f(\pi)\ = \ 0$

$f'(c)\ = \ cos(c)\ = \ 0$

$c = \frac{\pi}{2}.$

12. Option: c)

$Let\ a = x^3$

$\frac{da}{dx} = 2x$

$dx = \left(\frac{1}{3x}\right)\ da\ substituting\ this\ value$

$\int x. cos\ a. \left(\frac{1}{3x}\right) da$

$\frac{1}{3} \int cos\ a\ da$

$\rightarrow \frac{1}{3}\ sina\ + c \rightarrow \frac{1}{3}\ sin\ x^3 + c$

13. Option: d)

Slope of tangent is derivative of equation

$y' = 2\ x\ \{now\ x = 4\}$

So slope $= 2 \times 4 = 8$

14. Option: d)

Highest order is 3 but the lower order derivative has a power of ¼ and making the equation with power in whole number

The highest order derivative takes power 4 hence $\frac{d^3g}{dx^3} + \left(\frac{dg}{dx}\right)^{\frac{1}{4}} + g = 0.$

15. Option: a)

Since, the mango is dropped, therefore u=0.

Applying, $v^2 - u^2 = 2as$

Where $a = 10m/s^2$ (taking downward direction as positive).

$s = 45\ m$ and v is the velocity with which mango hits the ground.

$v^2 - 0 = 2 \times 10 \times 45$

$v^2 = 900$

Thus, $v = \sqrt{900}$

$v = 30\ m/s.$

SECTION-B

16. Coordinates of x axis is $(x, 0)$

Distance is 6 units from $(3, -3)$

So $(3 - x)^2 + (-3 - 0)^2 = 36$

$(3 - x)^2 = 25 \rightarrow 3 - x = 5 \rightarrow x = -2$

Hence coordinates are $(-2,0)$

17. Let $R(x, y)$ be any point of the circle. Connecting the points A and B with the point R and makes an angle 90∘ between them.

Let us find the slopes of the lines RA and RB as:

Slope of the line $m1 = RA = \frac{y-y1}{x-x1}$

Slope of the line $m2 = RB = \frac{y-y2}{x-x2}$

Since ∠APB=90∘ product of their slopes is $m1 \times m2 = -1$.

$\frac{y-y1}{x-x1} \times \frac{y-y2}{x-x2} = -1$

$(y - y1)(y - y2) = -(x - x1)(x - x2)$

$(x - x1)(x - x2) + (y - y1)(y - y2) = 0$ is the equation of the circle.

equation of line : $5x + 7y = 21$

at Y axes,$x = 0$; therefore $y = 5 = (0,5)$

at X axes, $y = 0$; therefore $x = 7 = (7,0)$

Using these in above equation of circle we get,

$(x - 0)(x - 7) + (y - 5)(y - 0) = 0$

$X^2 - 7x + y^2 - 5y = 0$

18. $y^2 + 8y + 10x - 4 = 0$
$y^2 + 8y = -10x + 4$
$y^2 + 8y + 16 = -10x + 4 + 16$
$(y + 3)^2 = -10x + 20$
$(y + 3)2 = -10(x - 2)$
Comparing with equation $(y - b)^2 = 2p(x - a)$

The focus is $(a + \frac{p}{2}, b)$

$(2 - \frac{5}{2}, 3)$

$(-\frac{1}{2}, 3)$

19. The equation of the hyperbola, is $(x + 2y + 5)(2x + 5y + 3) = k$, k being a constant.

This passes through the point $(1,3)$ $\rightarrow (1 + 2(3) + 5)(2(1) + 5(3) + 3) = k \rightarrow k = 240$ $\therefore$ The equation of the hyperbola is $(x + 2y + 5)(2x + 5y + 3) = 240$

20. Unit vector in direction of $\rightarrow a$ is given by
$\rightarrow \hat{a} = \rightarrow \frac{a}{|a|}$
So for $\rightarrow a = 1i + 3j + 4k$

$\rightarrow a = \left(\frac{1}{\sqrt{1^2+3^2+4^2}}\right)(1i + 3j + 4k) = \left(\frac{1}{\sqrt{26}}\right)(1i + 3j + 4k)$

21. Area = ½ (product of diagonals)

$$\frac{1}{2}\begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$\frac{1}{2}\{(0-4)i-(6-2)j+(4-0)k\}$

$\frac{1}{2}\{-4i-4j+4k\}$

$\frac{1}{2}\{\sqrt{16+16+16}\}=\frac{1}{2}\sqrt{48}$

22. $\frac{6-x}{3}=\frac{2-y}{1}=\frac{7-Z}{4}=t$

$\frac{6-x}{3}=t\rightarrow x=6-3t$

$\frac{2-y}{1}=t\rightarrow y=2-t$

$\frac{7-Z}{4}=t\rightarrow z=7-4t$

$R=xi+yj+zk$

$R=(6i+2j+7k)+t(-3i-j-4k)$

23. Equation of sphere is, $(x-x1)(x-x2)+(y-y1)(y-y2)+(z-z1)(z-z2)$

Since $(x1,y1,z1)$ and $(x2,y2,z2)$ be $(1,1,-1)$ and $(0,0,2)$ respectively

$(x-1)(x-0)+(y-1)(y-0)+(z+1)(z-0)$ or

$x^2+y^2+z^2-x-y+z-1=0$ is the equation of sphere when its extremities is given.

24. $f(x)=(5x-3)^{\frac{1}{2}}$

L.H.L. $=\lim_{x\to a^-} f(x)$

$=\lim_{x\to\frac{3}{5}}(5x-3)^{\frac{1}{2}}$

$=\left(5\times\frac{3}{5}-3\right)^{\frac{1}{2}}=0$

R.H.L. $=\lim_{x\to a^+} f(x)$

$f(x) = \lim_{x \to \frac{3}{5}^+} (5x - 3)^{\frac{1}{2}}$

$= \left(5 \times \frac{3}{5} - 3\right)^{\frac{1}{2}} = 0$

$L.H.L = R.H.L = f(a) = 0.$

Thus the function is continuous at about the point $x = \frac{3}{5}$

25.

$f(x) = \int f'(x)$

$\int 5x^4 + \frac{2}{x^3} + 2x$

$\rightarrow 5(\frac{x^5}{5}) + \frac{2(x^{-2})}{-2} + 2\left(\frac{x^2}{2}\right) + c$

$x^5 - \frac{1}{x^2} + x^2 + c \rightarrow x^5 + \frac{1}{x^2} + c$

$f(1) = 1 + 1 + c = 0 \rightarrow c = -2$

$f(x) = x^5 + \frac{1}{x^2} - 2$

26. $I = \int \frac{sin8\theta}{cos4\theta}\, d\theta$

$\int \frac{(2sin4\theta cos4\theta)}{cos4\theta}\, d\theta$

$2\int sin4\theta\, d\theta$

$-2\frac{cos4\theta}{4} + c$

$-\frac{cos4\theta}{2} + c$

27. $\int \frac{x^8+x^4+1}{x^4-x^2+1} dx$

$\int (x^4 + 1)^2 - \frac{x^4}{x^4-x^2+1} dx$ s

$\rightarrow \int \frac{(x^4+x^2+1)(x^4-x^2+1)}{x^4-x^2+1}\, dx$

$\rightarrow \int (x^4 + x^2 + 1) \rightarrow \frac{x^5}{5} + \frac{x^3}{3} + x + c$

28. Let point of contact of tangent line parallel to given line be $Q(x1, y1)$

Equation of curve is $y = (\sqrt{6x} - 2) - 3$[on differentiating]

$\frac{dy}{dx} = \frac{6}{2(\sqrt{6x}-2)}$

$\left(\frac{dy}{dx}\right)_{x1,y1}$ = slope of line $3x - 2y + 1$

$\frac{6}{2}(\sqrt{6x_1} - 2) = -\frac{3}{-2}$

$\left(\sqrt{6x} - 1 - 2\right) = ½$

$(6x - 1 - 2) = \frac{1}{4}$

$6x_1 = \frac{9}{4}$

$x1 = \frac{3}{8}$

$y1 = y = (\sqrt{6x - 1 - 2}) - 3$

$y1 = \left(\frac{1}{2}\right) - 3 = -\frac{5}{2}$

$So\ (x1, y1) = \left(\frac{3}{8}, -\frac{5}{2}\right)$

And equation becomes $y - \left(-\frac{5}{2}\right) = 2\left(x - \frac{3}{8}\right)$

29. Initial speed of train, $u = 90\ km/hr \rightarrow\ 90 \times \frac{5}{18} = 25\ m/s\ \ (since,\ \frac{1000}{3600} m/s = 1\ km/hour)$

Final speed of train, $v = 54 km/hr \rightarrow 54 \times \frac{5}{18} = 15 m/s$

$V = u + at \rightarrow a = \frac{v-u}{t} \rightarrow \frac{15-25}{15} = \left(-\frac{2}{3}\right)\ m/s^2$and negative (-)sign shows retardation

30. The green colored tennis ball will have minimum velocity at the highest point.

So, $x - coordinate$ will be $\frac{80}{2}\ m$, i.e. $40\ m$

Again, $y = h_{\max} = v^2 \sin^2 \frac{45}{2} g$

$\rightarrow \frac{v^2}{4} g$

$\rightarrow \frac{1}{4} \times 80 = 20m$

SECTION – C

31. Let point $P(a, b)$ be any point on locus and $A(7,0)$ and $B(5,0)$

$PA + PB = 8$

$\sqrt{(a-7)^2 + (b-0)^2} + \sqrt{(a-5)^2 + (b-0)^2} = 8$

$\sqrt{(a-7)^2 + (b-0)^2} = 8 - \sqrt{(a-5)^2 + (b-0)^2}$

$= -2a^2 - 2b^2 + 24a + 40$

$a^2 + b^2 - 12a + 20 = 0$

Hence locus is

$a^2 + b^2 - 12a + 20 = 0$

Hence the locus of the given point represents a circle.

32. Shifting the origin at $(-2, 2)$ we have $X_1 = x - 2$; $Y_1 = y + 2$

Using the given expression,

The parabola becomes $Y_1^2 = 4X_1$

The coordinates of the endpoints of latus rectum are $(X_1 = 1, Y_1 = 2)$ *and* $(X_1 = 1, Y_1 = -2)$

Now using value of X_1 *and* Y_1 from first expression

The coordinates of the end point of the latus rectum are $(3,0)$ and $(3,-4)$

33. Let S and S' be the two foci and their coordinates $(2, 4)$ and $(-2,4)$

SS'= 4

Let 2a and 2b be length of the axes

$SS' = 2ae \ \rightarrow 2ae = 4 \ \rightarrow ae = 2$

$B^2 \ = \ a^2(1 - e^2) \ \rightarrow \ 3 = \ a^2 \ - 2^2 \rightarrow \ a = 3$

$SP + S'P \ = 2a$

$\sqrt{(x-2)^2 \ + \ (y-4)^2} \ + \sqrt{(x+2)^2 + (y-4)^2} \ = \ 6$

On solving we get

$8x^2 + 9y^2 + 18x - 72y + 99 = 0$

34. One asymptotes of the rectangular hyperbola is $3x + y - 7 = 0$. the other asymptote is $\perp$ to it. The equation is of the form $2x - y + l = 0$

The combined equation is $(3x + y - 7)\ (2x - y + l) \ = 0$

Thus its equation form becomes $(3x + y - 7)\ (2x - y + l) \ = k$

The rectangular hyperbola passes through $(8, 0)$ and $(-5,0)$.

Substituting

$(24 - 7)(16 + L) = k$

$272 \ + \ 17L = k$

$k - 17L = 272 \ \rightarrow 1$

$(-15 - 7)(-10 + l) = k$

$220 - 22L \ = k$

$K + 22L = 220 \ \rightarrow 2$

$L = 249\frac{1}{3} \ ; \ k = 1\frac{1}{3}$

OR

$4x^2 - 25y^2 - 8x + 50y - 121 = 0$

$4(x^2 - 2x) - 25(y^2 - 2y) \ - 121 = 0$

$4(x-1)^2 - 25(y-1)^2 = 100$

$\frac{(x-1)^2}{25} - \frac{(y-1)^2}{4} = 1$

$Let\ (x-1) = X\ and\ (y-1) = Y$

$\frac{X^2}{5^2} - \frac{Y^2}{2^2} = 1$ which is equation of hyperbola

Coordinates of center w.r.t. new axis is $(0,0)$ so coordinates w.r.t. old axes $(1,1)$

Length of latus rectum $= \frac{2b^2}{a} = \frac{8}{5}$

35. $a = i + 2j - 3k$

$|a| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

$A = \frac{a}{|a|} = \frac{i + 2j - 3k}{\sqrt{14}}$

Here direction ratios is $(1,2,-3)$

And direction cosines are $(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}})$

36. $(2i + 8j + 12k)(i + \gamma j + \beta k) = 0$

So $\begin{vmatrix} i & j & k \\ 2 & 8 & 12 \\ 1 & \gamma & \beta \end{vmatrix}$

$I(8\beta - 12\gamma) - j(2\beta - 12) + k(2\gamma - 8) = 0$

On equating coefficients

$8\beta - 12\gamma = 0$;

$(2\beta - 12) = 0$;

$(2\gamma - 8) = 0$

$B = 6, \gamma = 4$

37. We use the general formula for the equation of a sphere with center (h, k, l) and radius r,

$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

$(x - 3)^2 + (y + 3)^2 + (z - 2)^2 = 25$

$Since\ it\ is\ xz\ plane, y = 0$

$(x - 3)^2 + 9 + (z - 2)^2 = 25$

$(x - 3)^2 + (z - 2)^2 = 16$

From the above equation it is clear that, the circle on xz plane has center $(3,0,2)$ and radius is 4

38. $f(x) = \{x + 5 + a\ , for\ x \leq 1$

$bx + 4\ , for\ x > 1$

Since the polynomial function is differentiable everywhere

$f(x)$ is differentiable at $x = 1$ and continuous at $x = 1$

$lim_{x\to1+} f(x) = lim_{x\to1-} f(x) = f(1)$

$1 + 5 + a = b + 4$

$6 + a = b + 4$

$a - b + 2 = 0$

Again $f(x)$ is differentiable at $x = 1$

$lim_{x\to1-} \frac{f(x)-f(1)}{x-1} = lim_{x\to1+} \frac{f(x)-f(1)}{x-1}$

$\lim_{x\to1} \frac{x+5+a-(6+a)}{x} - 1 = \lim_{(x\to1)} \frac{(bx+4-(6+a))}{x} - 1$

$\lim_{x\to1} \frac{x+5-6}{x-1} = \lim_{x\to1} \frac{bx+4-6-a}{x-1}$

$= \lim_{x\to1} \frac{bx-2-a}{x-1}$

$b = 1;\ a = -1$

OR

$y = a^{(cos^{-1} x)^2}$

$\frac{dy}{dx} = a^{(cos^{-1} x)^2} \; log\, a \,.\frac{d}{dx}(\cos^{-1} x)^2$

$\frac{dy}{dx} = a^{(cos^{-1} x)^2} \; log\, a. 2\cos^{-1} x\,. - \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{-2loga.cos-1x.a^{(cos^{-1} x)^2}}{(\sqrt{1-x^2})}$

39. Equation of curve = $x^2 - 7x + 8$

Differentiating we get

$\frac{dy}{dx} = 2x - 7$

Now, $m1\ (slope\ of\ tangent\ at\ (4,0)) = 2 \times 4 - 7 = 1$

$m1\ (slope\ of\ tangent\ at\ (3,0)) = 2 \times 3 - 7 = -1$

For tangents to be at right angles $m1 \times m2$ should be -1.

$m1 \times m2 = 1 \times -1 = -1$

40. $\frac{sin2x}{sin(2x-r)}\ dx$

$let\ 2x = \theta \rightarrow x = \frac{\theta}{2} \rightarrow dx = \frac{1}{2}\ d\theta$

$\int \frac{1}{2} \left(\frac{sin\theta}{sin(\theta - r)}\right) d\theta$

$\frac{1}{2} \int \left(\frac{\sin(\theta-r)}{sin(\theta-r)} + r\right) d\theta$

$\frac{1}{2} \int \frac{sin(\theta-r) \cos\ r + cos(\theta-r) \sin\ r}{sin(\theta - r)} d\theta$

$\frac{1}{2} \int (\cos\ r + cot\ (\theta - r) sin\, r)\ \ d\theta$

$\frac{1}{2} \left(\theta \cos\ r + sin\, r. log\left|sin(\theta - r)\right)\right)$

$\frac{1}{2}\ (2xcos\, r + sin\, r. log|sin\ (2x - r)$

OR

$\int \sqrt{\frac{1-\sqrt{t}}{1+\sqrt{t}}}\, dt$

$Let\ t = x^2 \ \rightarrow\ dt\ =\ 2xdx$

$\rightarrow \int 2x\, \frac{(1-x)}{(\sqrt{(1-x^2)})}\, dx$

$\rightarrow \int\ \frac{2x}{\sqrt{(1-x^2\,)}}\, dx\ +\ 2\int\ \frac{1-x^2}{\sqrt{1-x^2}} dx$

$\rightarrow -\int\ \frac{2x}{\sqrt{(1-x^2\,)}}\ \ dx\ +\ 2\int \sqrt{1-x^2} dx\ -2\int\ \frac{1}{\sqrt{(1-x^2)}} dx$

$\rightarrow -2\sqrt{1-x^2} + x\sqrt{1-x^2} - sin^{-1}x\ + c$

$(1-t)(\sqrt{t}\ -2\,) - sin^{-1}\sqrt{t}\ + c$

41. Since $x\ and\ y$ lie between 0 and $\frac{\pi}{4}$

$-\frac{\pi}{4} < x - y < \frac{\pi}{4}\ and\ 0 < x + y < \frac{\pi}{2}$

$So\ cos(x-y)\ and\ sin\ (x+y)\ are\ positive$

$Sin\ x + y\ = \sqrt{1 - cos^2 x + y}\ \rightarrow sin\ (x+y) = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$Cos(x-y)\ = \sqrt{1 - sin^2(x-y)} \rightarrow cos(x-y)\ = \sqrt{1 - \frac{144}{225}} = \frac{9}{15}$

$tan(x+y) = \frac{sin(x+y)}{cos(x+y)}$

$\rightarrow \frac{\frac{4}{5}}{\frac{3}{5}}\ = \frac{4}{3}$

$tan(x-y) = \frac{sin(x-y)}{cos(x-y\,)}$

$\rightarrow \frac{\frac{12}{15}}{\frac{9}{15}} = \frac{4}{3}$

$tan2x\ = \ tan(x+y+x-y) = \frac{tan(x+y)+tan(x-y)}{1-\ tan(x+y)tan(x-y)}$

$\rightarrow \left(\frac{4}{3} + \frac{\frac{4}{3}}{1-\frac{16}{9}}\right)$

$\rightarrow \frac{\frac{8}{3}}{-\frac{7}{9}}$

$\rightarrow -\frac{24}{7}$

42. $D(2,3,4);\ E(5,6,9);\ F(3,4,\frac{17}{3})$

Let ratio be $m:n$

$\frac{n\times2+m\times5}{m+n} = 3 \rightarrow 2n+5m = 3m+3n \rightarrow n = 2m \rightarrow \frac{m}{n} = \frac{1}{2}$

Now putting this value in other equations to verify

$\frac{2\times3+1\times6}{3} = 4$

$\frac{2\times4+1\times9}{3} = \frac{17}{3}$

Hence true

OR

points L and M are $L(2,3,1)$; $M(1,3,7)$

The coordinates are $\left(\frac{4\times2+3\times1}{4+3}\right)X, \left(\frac{4\times3+3\times3}{4+3}\right)Y, \left(\frac{4\times1+3\times7}{4+3}\right)Z$

$\left(\frac{11}{7}, 3, \frac{25}{7}\right)$

43. $P = 5i+2j+3k$ and $Q = 3i-2j+5k$

$$\begin{vmatrix} i & j & k \\ 5 & 2 & 3 \\ 3 & -2 & 5 \end{vmatrix}$$

$P\times Q = (10+6)I + (25-9)j + (-10-6)k$

$\rightarrow 16i + 16j - 16k$

$\rightarrow \sqrt{16^2+16^2+16^2} = 16\sqrt{3}$

OR

$W = F.S = (4i + 2j + 3k).(8i + j + 12k)$

$\{S = r2 - r1\}$

$= (4 \times 8 + 1 \times 2 + 3 \times 12) = 70J.$

44. $\theta = \frac{1}{2} sin^{-1}\left(\frac{9}{4}\right) \rightarrow Sin2\theta = \frac{9}{4}$

$\frac{x-1}{cos\theta} = \frac{y+1}{sin\theta}$

$Since\ distance\ is\ 2\ units$

$\frac{x-1}{cos\theta} = \frac{y+1}{sin\theta} = 2$

$X = 2cos\theta + 1\ ;\ y = 2sin\theta - 1$

$Point\ lies\ on\ line\ x + y = a$

$2cos\theta + 1 + 2sin\theta - 1 = a$

$cos\theta + sin\theta = \frac{a}{2}$

$(cos\theta + sin\theta)^2 = \frac{a^2}{4}$

$Sin2\theta = \frac{a^2}{4} \rightarrow \frac{3}{2} = \frac{a}{2}$

$\rightarrow a = 3$

45. Coordinates of any point on line $x - y = 1$ can be taken as $(t, t - 1)$

So equation can be written as

$(x - t)^2 + \left(y - (t - 1)\right)^2 = 4^2$

It passes through (3,4)

$(3 - t)^2 + \left(4 - (t - 1)\right)^2 = 16$

$(3 - t)^2 + (5 - t)^2 = 16$

$9 + t^2 - 6t + 25 + t^2 - 10t - 16 = 0$

$2t^2 - 16t + 18 \ = 0$

$t^2 - 8t + 9 = 0$

$t = -9, +1$

Substituting

$(x-9)^2 + (y+2)^2 = \ 4^2$

46. Rationalizing since it takes form of $0/0$

$\frac{((\sqrt{4+x})-2)\left((\sqrt{4+x})+2\right)}{x(\sqrt{4+x}+2)}$

$\frac{4+x-4}{x}\left(\sqrt{4}+x\right)+2)$

$\frac{x}{x}\left(\sqrt{4}+x\right)+2)$

Putting the limit

$= \frac{1}{4}$

47. $V = \frac{4\pi}{3}\left\{\frac{9}{4}(2x+5)\right\}^3 = \frac{243\pi}{16}(2x+5)^3$

$\frac{dV}{dx} = \frac{243\pi}{16}.3(2x+5)^2.2$

$\frac{dV}{dx} = \frac{729\pi}{8}(2x+5)^2$

OR

$Y^2 = x^2 + (110)^2$

$Y = \sqrt{(60)^2 + (110)^2} = 125.29 \approx 125$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\frac{dx}{dt} = -5.8\ km/hr$

$\frac{dy}{dt} = \frac{-5.8x}{y}$

$\frac{dy}{dt} = \frac{-5.8 \times 60}{125} = 2.78$

48. $\int \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^4 \, dx$

$\int \; x^{\frac{4}{2}} \; + \; x^{-\frac{4}{2}} \, dx$

$\frac{x^3}{3} + \frac{x^{-1}}{-1} + c$

$\frac{1}{3}x^3 \; - \frac{1}{x} \; + c$

49. $\int \sqrt[3]{\frac{si\,n^{(-4)} \;\; x}{co\,s^{(8)}\,x}}$

$\int \sin^{-\frac{8}{3}} x \cos^{-\frac{4}{3}} \; x \, dx$

Sum of exponents is -4 , dividing both numerator and denominator by $cos^4 x$

$\int \frac{1+\tan^2(x)}{\tan^{\frac{8}{3}} x} \sec^2 \;\; xdx$

$Let \; tan \, x \; = \; u$ and $sec^2 xdx = \; du$

$\int \frac{1+u^2}{u^{\frac{8}{3}}} \, du$

$- \; \frac{3}{5} \; u^{-\frac{5}{3}} \; + \; 3 \, u^{\frac{1}{3}} \; + \; c$

$- \; \frac{3}{5} \; tan^{-\frac{5}{3}} x \; + \; 3 \, tan^{\frac{1}{3}} \; x \; + \; c$

50.

$ax^2 + bx + c = 0$

$a = 2 \, , b = 4i \, , c = 3$

$x_1 \;\; = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \; ; \; x_2 \;\; = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$x_1 \;\; = \frac{-4i + \sqrt{(-4i)^2 - 4.2.3}}{2.2} \;\rightarrow\; (-4i \;\; + \frac{\sqrt{(16-24\,)}}{4} = \frac{-2i + \sqrt{2i}}{2}$

$x_2 \;\; = \frac{-4i - \sqrt{(-4i)^2 - 4.2.3}}{2.2} \;\rightarrow\; (-4i \;\; + \frac{\sqrt{(16-24\,)}}{4} = \frac{-2i - \sqrt{2i}}{2}$

SECTION - E

51. Area of triangle with vertices are $A(2,3)$; $B(2,7)$; $(-3,5)$

$= \frac{1}{2}\begin{vmatrix} 2+3 & 2+3 \\ 3-5 & 7-5 \end{vmatrix}$

$= \frac{1}{2}\,(5 \times 2\ - 5 \times -2)$

$= 10$

52.

$f'(x)\ =\ lim_{h\to 0}\, f(x+h) - \frac{f(x)}{h}$

$f'(x)\ =\ lim_{h\to 0}\left\{[(x+h)^2 - 6(x+h) + 8] - \frac{[x^2-6x+8]}{h}\right\}$

$f'(x)\ = lim_{h\to 0}\left\{\frac{2hx-6h+h^2}{h}\right\}$

$f'(x)\ =\ lim_{h\to 0}\,\{2x - 6 + h)$

$f'(x) =\ 2x - 6$

$f'(5) - 3f'(2)\ =\ (2\times 5 - 6) -\ 3(2\times 2 - 6)\ =\ 4 + 6\ = 10$

$Now\ for\ f'(8)\ =\ 2\times 8 - 6 = 10$

$Hence\ f'(5)\ - 3f'(2)\ =\ f'(8)$

53. $\frac{d}{dx}(e^{3x})$

$f(x) = e^{3x} then\ f(x+h)\ =\ e^{3(x+h)}$

$lim\ h_{(\to 0)} f(x+h) - \frac{f(x)}{h}$

$lim\ h_{\to 0} e^{3(x+h)} - \frac{e^{3x}}{h}$

$lim\ h_{\to 0} e^{3x}.e^{3h} - \frac{e^{3x}}{h}$

$3\ e^{3x}\ lim\ h_{\to 0}\ e^{2h} - \frac{1}{3h}$

$Let\ 3h = y$

$3\ e^{3x}\ lim\ y_{\to 0}\ e^{y} - \frac{1}{y} = 3e^{3x}.1 = 3e^{3x}$

OR

$let\ x = tan\ c$

$y = \cos^{-1}\left(\frac{2tanc}{1+tan2c}\right)$

$y = \cos^{-1}(\ sin\ 2c)$

$y = \cos^{-1}\left(cos\left(\frac{\pi}{2} - 2c\right)\right)$

$y = \left(\frac{\pi}{2} - 2c\right)$

$y = \left(\frac{\pi}{2} - 2tan - 1x\right)$

$\frac{dy}{dx} = -\frac{2}{1+x^2}$

54.

$\int \frac{1}{t\sqrt{(ut-t^2)}}\ dt$

Let $\frac{1}{t} = x \to dt = -\frac{1}{x^2}\ dx$

$\to \int \frac{1}{\frac{1}{x}\sqrt{\frac{ux-1}{x^2}}}\ x\left(-\frac{1}{x^2}\right)\ dx$

$\to -\int \frac{1}{\sqrt{(ux-1)}} dx$

$\to -\frac{(ux-1)^{\frac{1}{2}}}{\frac{1}{2}u} + c$

$\to -\frac{2}{a}\sqrt{\frac{u-t}{t}} + c$

OR

$$\int \frac{x+1}{4+x^2}\ dx$$

$$= \int \frac{x}{(4+x^2)}\ dx + \int \frac{1}{4+x^2}\ dx$$

$Let\ u = 4 + x^2$

$du = 2xdx$

$$\int \frac{x}{4+x^2}\ dx = \int \frac{1}{2u}\ du$$

$Ln\ |2u| + c = ln|8 + 2x^2| + c$

$Let\ x = 2tan\ c\ so\ 2\ sec^2cdc = dx$

$$So \int \left\{\frac{1}{4+4tan^2c}\right\}\ 2\ sec^2cdc$$

$$\int \frac{1}{2}\ dc = \frac{c}{2} + constant$$

$$\frac{1}{2}\frac{tan^{-1}x}{2}$$

$So\ answer\ is\ ln|8 + 2x^2| + ½\frac{tan^{-1}x}{2} + c$