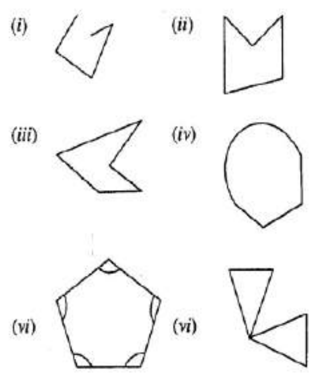
CHAPTER 16 - UNDERSTANDING SHAPES

Question 1.

State which of the following are polygons:



If the given figure is a polygon, name it as convex or concave.

Solution:

In given Fig. (ii), (iii) and (v) are polygons.

Fig. (ii) and (iii) are concave polygons while

Fig. (v) is convex.

Question 2.

Calculate the sum of angles of a polygon with:

(i) 10 sides

Solution:

No. of sides n=10

Sum of angles of polygon $=(n-2)\times 180°$

$=(10-2)\times 180° = 1440°$

(ii) 12 sides

Solution:

No. of sides n=12

Sum of angles $= (n-2) \times 180°$

$= (12-2) \times 180° = 10 \times 180° = 1800°$

(iii) 20 sides

Solution:

n =20

Sum of angles of Polygon = $(n-2) \times 180°$

$= (20-2) \times 180° = 3240°$

(iv)25 sides

Solution:

(iv) n = 25

Sum of angles of polygon $= (n-2) \times 180°$

$= (25-2) \times 180° = 4140°$

Question 3.

Find the number of sides in a polygon if the sum of its interior angles is:

(i) 900°

Solution:

Let no. of sides = n

Sum of angles of polygon = $900°$

$(n-2) \times 180° = 900$

$n-2 = \frac{900}{180}$

$n-2 = 5$

$n = 5+2$

$n = 7$

(ii) 1620°

Solution:

Let no. of sides = n

Sum of angles of polygon = 1620°

$(n-2) \times 180° = 1620°$

$n-2 = \frac{1620}{180}$

$n-2 = 9$

$n = 9+2$

$n = 11$

(iii) 16 right-angles

Solution:

Let no. of sides =n

Sum of angles of polygon $= 16 \text{ right angles } = 16 \times 90 = 1440°$

$(\mathrm{n}-2) \times 180° = 1440°$

$\mathrm{n}-2 = \frac{1440}{180}$

$\mathrm{n}-2 = 8$

$\mathrm{n} = 8+2$

$\mathrm{n} = 10$

(iv) 32 right-angles.

Solution:

Let no. of sides =n

Sum of angles of polygon =32 right angles $= 32 \times 90 = 2880°$

$(\mathrm{n}-2) \times 180° = 2880$

$\mathrm{n}-2 = \frac{2880}{180}$

$\mathrm{n}-2 = 16$

$\mathrm{n} = 16+2$

$n = 18$

Question 4.

Is it possible to have a polygon; whose sum of interior angles is?

(i) 870°

Solution:

(i) Let no. of sides = n

Sum of angles $= 870°$

$(n-2) \times 180° = 870°$

$n - 2 = \frac{870}{180}$

$n - 2 = \frac{20}{6}$

$n = \frac{29}{6} + 2$

$n = \frac{41}{6}$

which is not a whole number.

Hence it is not possible to have a polygon, the sum of whose interior angles is 870°

(ii) 2340°

Solution:

Let no. of sides =n

Sum of angles $= 2340°$

$(n-2) \times 180° = 2340°$

$n - 2 = \frac{2340}{180}$

$n - 2 = 13$

$n = 13 + 2 = 15$

which is a whole number.

Hence it is possible to have a polygon, the sum of whose interior angles is 2340°.

(iii) 7 right-angles

Solution:

Let no. of sides =n

Sum of angles =7 right angles $= 7 \times 90 = 630°$

$(n-2) \times 180° = 630°$

$n - 2 = \frac{630}{180}$

$n - 2 = \frac{7}{2}$

$n = \frac{7}{2} + 2$

$n = \frac{11}{2}$

which is not a whole number. Hence it is not possible to have a polygon, the sum of whose interior angles is 7 right-angles.

(iv) 4500°

Solution:

Let no. of sides = n

$(n-2) \times 180° = 4500°$

$n - 2 = \frac{4500}{180}$

$n - 2 = 25$

$n = 25 + 2$

$n = 27$

which is a whole number.

Hence it is possible to have a polygon, the sum of whose interior angles is 4500°.

Question 5.

(i) If all the angles of a hexagon are equal; find the measure of each angle.

Solution:

No. of sides of hexagon, $n = 6$

Let each angle be $= x°$

Sum of angles $= 6x°$

$(n-2) \times 180° =$ Sum of angles

$(6-2) \times 180° = 6x°$

$4 \times 180 = 6x$

$x = \frac{4 \times 180}{6}$

$x = 120°$

$\therefore$ Each angle of hexagon $= 120°$

(ii) If all the angles of a 14 -sided figure are equal; find the measure of each angle.

Solution:

No. of sides of polygon, $n = 14$

Let each angle $= x°$

Sum of angles $= 14x°$

$\therefore (n-2) \times 180° =$ Sum of angles of polygon

$\therefore \quad (14-2) \times 180° = 14x$

$12 \times 180° = 14x$

$x = \frac{12 \times 180}{14}$

$x = \frac{1080}{7}$

Ans: $x = \left(154\frac{2}{7}\right)^{\circ}$

Question 6.

Find the sum of exterior angles obtained on producing, in order, the sides of a polygon with:

(i) 7 sides

(ii) 10 sides

(iii) 250 sides.

(i) Solution:

No. of sides n=7

Sum of interior exterior angles at one vertex $=180°$

Sum of all interior exterior angles $=7 \times 180°$

$=1260°$

Sum of interior angles $= (n-2) \times 180°$

$= (7-2) \times 180°$

$=900°$

$\therefore$Sum of exterior angles $=1260°-900°$

$=360°$

(ii)Solution

No. of sides n=10

Sum of interior and exterior angles $=10° \times 180°$

$=1800°$

But sum of interior angles $= (n-2) \times 180°$

$= (10-2) \times 180°$

$=1440°$

$\therefore$Sum of exterior angles $= 1800 - 1440$

Sum of exterior angles $= 360°$

(iii) Solution:

No. of side n=250

Sum of all interior and exterior angles

$=250 \times 180°$

=45000°

But sum of interior angles = (n-2) × 180°

= (250-2) ×180°

=248 × 180°

=44640°

∴ Sum of exterior angles =45000-44640

=360°

Question 7 .

The sides of a hexagon are produced in order. If the measures of exterior angles so obtained are

(6x-1)° , (10 x+2)°,(8 x+2)°(9 x-3)°,(5 x+4)° and (12 x+6)°;Find each exterior angle.

Solution:

Sum of exterior angles of hexagon formed by producing sides of order =360°

$\therefore (6x-1)° + (10x+2)° + (8x+2)° + (9x-3)° + (5x+4)° + (12x+6)° = 360°$

$50x + 10° = 360°$

$50x = 360° - 10°$

$50x = 350°$

$x = \frac{350}{50}$

$x = 7$

∴ Angles are (6x-1)°:(10x+2)°:(8x+2)°;(9x-3)° (5x+4)° and (12x+6)°

i.e., (6×7-1)° ; (10×7+2)° ; (8×7+2)° ; (9×7-3)° ; (5×7+4)°; (12×7+6)°)

41° ; 72° ; 58° ; 60° ; 39° and 90°

Question 8.

The interior angles of a pentagon are in the ratio 4:5:6:7:5. Find each angle of the pentagon.

Solution:

Let the interior angles of the pentagon be 4 x, 5 x, 6 x, 7 x, 5 x

Their sum = $4\,x + 5\,x + 6\,x + 7\,x + 5\,x = 27\,x$

Sum of interior angles of polygon = $(n-2) \times 180° = (5-2) \times 180° = 540°$

$27x = 540\,x = \frac{540}{27} = 20°$

∴Angles are $4 \times 20° = 80°$

$5 \times 20° = 100°$

$6 \times 20° = 120°$

$7 \times 20° = 140°$

$5 \times 20° = 100°$

Question 9

Two angles of a hexagon are 120°and 160°. If the remaining four angles are equal, find each equal angle.

Solution:

Two angles of a hexagon are 120°, 160°

Let remaining four angles be x, x, x and x.

Their sum = $4\,x + 280°$

But sum of all the interior angles of a hexagon = $(6-2) \times 180° = 4 \times 180° = 720°$

∴ $4x + 280° = 720°$

⇒ $4x = 720° - 280° = 440°$ ⇒ $x = 110°$

∴Equal angles are 110° (each)

Question 10

The figure, given below, shows a pentagon ABCDE with sides AB and ED parallel to each other, and

<B : <C : <D:5:6:7.

(i) Using formula, find the sum of interior angles of the pentagon.

(ii) Write the value of ∠A+∠E

(iii) Find angles B, C and D .

Solution:

(i) Sum of interior angles of the pentagon $= (5-2) \times 180°$

$=3 \times 180°=540°$ (∵sum for a polygon of x sides$=(x-2)\times180°$

(ii) Since AB || ED

$\therefore \angle A + \angle E = 180°$

(iii) Let ∠B=5 x ∠C=6 x ∠D=7x

$\therefore 5x + 6x + 7x + 180° = 540°$

$\angle A + \ \angle E = 180°\ Proved\ in\ (ii)$

$18x = 540° - 180°$

$\Rightarrow \ \ 18x = 360° \ \ \Rightarrow \ \ x = 20°$

$\therefore \angle B = 5 \times 20° = 100°, \angle C = 6 \times 20 = 120°$

$\angle D = 7 \times 20 = 140°$

Question 11.

Two angles of a polygon are right angles and the remaining are 120° each. Find the number of sides in it.

Solution:

Let number of sides = n

Sum of interior angles$= (n-2) \times 180°$

$= 180n - 360°$

Sum of 2 right angles $= 2 \times 90°$

$= 180°$

$\therefore$ Sum of other angles $= 180n - 360° - 180°$

$= 180\ n - 540$

No. of vertices at which these angles are formed

$= n - 2$

$\therefore$ Each interior angle $=\frac{180n-540}{n-2}$

$\therefore$ $\frac{180n-540}{n-2}=120°$

$180\,n-540=120\,n-240$

$180\,n-120\,n=-240+540$

$60\,n=300$

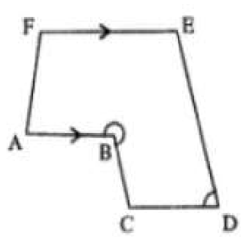
$n=\frac{300}{60}$

$n=5$

Question 12.

In a hexagon ABCDEF, side AB is parallel to side FE and $\angle B:\angle C:\angle D:\angle E=6:4:2:3.\,find\ \angle B\ and\ \angle D.$

Solution:



Given: Hexagon ABCDEF in which AB II EF

and ∠B:∠C:∠D:∠E=6:4:2:3

To find: $\angle B$ and $\angle D$

Proof: No of sides $n=6$

$\therefore$ Sum of interior angles $=(n-2)\times 180°$

$=(6-2)\times 180°=720°$

$\because$ AB||EF(Given)

$\therefore \mathrm{A}+\angle \mathrm{F}=180°$

But $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F=720°$

(proved)

$\angle B + \angle C + \angle D + \angle E + \angle 180° = 720°$

$\therefore\ \angle B + \angle C + \angle D + \angle E = 720° - 180° = 540°$

Ratio $= 6:4:2:3$

Sum of parts $= 6 + 4 + 2 + 3 = 15$

$\therefore\ \angle B = \frac{6}{15} \times 540 = 216°$

$\angle \mathrm{D} = \frac{2}{15} \times 540° = 72°$

Hence $\angle B = 216° : \angle D = 72°$

Question 13.

the angles of a hexagon are x+10°,2x+20°,2x-20°,3x-50°,x+40° and x+20°. Find x.

Solution: Angles of a hexagon are $x + 10°, 2x + 20°,$

$2x - 20°, 3x - 50°, x + 40°$ and $x + 20°$

$\therefore$ But sum of angles of a hexagon $= (x - 2) \times 180°$

$= (6 - 2) \times 180° = 4 \times 180° = 720°$

But sum $= x + 10 + 2x + 20° + 2x - 20° + 3x - 50° + x + 40 + x + 20$

$= 10x + 90 - 70 = 10x + 20$

$\therefore 10x + 20 = 720° \Rightarrow 10x = 720 - 20 = 700$

$\Rightarrow x = \frac{700°}{10} = 70°$

$\therefore x = 70°$

Question 14.

In a pentagon, two angles are 40° and 60° and the rest are in the ratio 1:3:7. Find the biggest angle of the pentagon.

Solution:

In a pentagon, two angles are 40° and 60° Sum of remaining 3 angles $= 3 \times 180°$

$= 540° - 40° - 60° = 540° - 100° = 440°$

Ratio in these 3 angles $= 1:3:7$

Sum of ratios $= 1 + 3 + 7 = 11$

Biggest angle $= \frac{440 \times 7}{11} = 280°$

Question 15

Fill in the blanks:

In case of regular polygon, with:

No. of sides	Each exterior angle	Each interior angle
(i).....8.....	....	....
(ii)....12....	....	....
(iii)..........	......72°.....	....
(iv)......	......45°....	....
(v).......	....	......150°.....
(vi)..........	....	......140°.....

Solution:

No. Of sides	Each exterior angle	Each interior angle
(i)8	45°	135°
(ii)12	30°	150°
(iii)5	72°	108°
(iv)8	45°	135°
(v)12	30°	150°
(vi)9	40°	140°

Explanation:

(i) Each exterior angle= $\frac{360°}{8} = 45°$

Each interior angle= $180° - 45° - 135°$

(ii) Each exterior angle= $\frac{360°}{12} = 30°$

Each interior angle $= 180° - 30° = 150°$

(iii) Since each exterior $= 72°$

$\therefore$ Number of sides $= \frac{360°}{72°} = 5$

Also interior angle $= 180° - 72° = 108°$

(iv) Since each exterior angle $= 45°$

$\therefore$ Number of sides $= \frac{360°}{45°} = 8$

Interior angle $= 180° - 45° = 135°$

(v) Since interior angle $= 150°$

$\therefore$ Exterior angle $= 180° - 150° = 30°$

$\therefore$ Number of sides $= \frac{360°}{30°} = 12$

(vi) Since interior angle $= 140°$

$\therefore$ Exterior angle $= 180° - 140° = 40°$

$\therefore$ Number of sides $= \frac{360°}{40°} = 9$