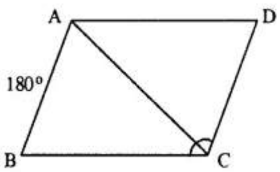
CHAPTER 17 – SPECIAL TYPES OF QUADRILATERALS

Question 1.

In parallelogram ABCD, ∠A = 3 times ∠B. Find all the angles of the parallelogram. In the same parallelogram if AB = 5x - 7 and CD = 3x + 1; find the length of CD.

Solution:



Let $\angle B = x$

$\angle A = 3\angle B = 3x$

$AD||BC$

$\angle A + \angle B = 180°$

$3x + x = 180°$

$\Rightarrow 4x = 180°$

$\Rightarrow x = 45°$

$\angle B = 45°$

$\angle A = 3x = 3 \times 45 = 135°$

and $\angle B = \angle D = 45°$

Opposite angles of parallelogram are equal.

$\angle A = \angle C = 135°$

Opposite sides of parallelogram are equal.

$AB = CD$

$5x - 7 = 3x + 1$

$\Rightarrow 5x - 3x = 1 + 7$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

$CD = 3 \times 4 + 1 = 13$

Hence $135°, 45°, 135°$ and $45°: 13$

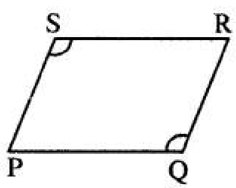
Question 2.

In parallelogram PQRS, ∠Q = (4x - 5)° and ∠S = (3x + 10)° . Calculate: ∠Q and ∠R.

Solution:

In parallelogram PQRS,

∠Q = (4x - 5)° and ∠S = (3x + 10)°



Opposite ∠s of parallelogram are equal

$\angle Q = \angle S$

$4x - 5 = 3x + 10$

$4x - 3x = 10 + 5$

$x = 15$

$\angle Q = 4x - 5 = 4 \times 15 - 5 = 55°$

Also $\angle Q + \angle R = 180°$

$55° + \angle R = 180°$

$\angle R = 180° - 55° = 125°$

$\angle Q = 55°: \angle R = 125°$

Question 3.

In rhombus ABCD:

(i) if ∠A=74° ; find ∠B and ∠C .

(ii) if AD=7.5cm ; find BC and CD .

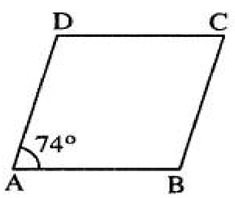
Solution:

$AD || BC$

$\angle A + \angle B = 180°$

$74° + \angle B = 180°$

$\angle B = 180° - 74° = 106°$



Opposite angles of Rhombus are equal.

$\angle A = \angle C = 74°$

Sides of Rhombus are equal.

$BC = CD = AD = 7.5\text{cm}$

(i) $\angle B = 106°; \angle C = 74°$

(ii)Ans: $BC = 7.5\text{cm}$ and $CD = 7.5\text{cm}$

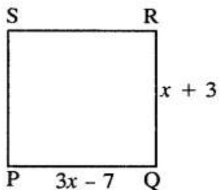
Question 4

In square PORS:

(i) if PQ = 3x - 7 and QR = x + 3; find PS

Solution:

(i) Sides of square are equal.



PQ=QR

$3x-7=x+3$

$3x-x=3+7$

$2x=10$

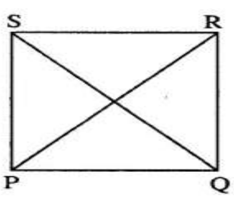
$x=5$

$PS=PQ=3x-7=3\times 5-7=8$

(ii) If PR = 5x and QR = 9x - 8. Find OS

Solution:

(ii) PR = 5x and QR = 9x - 8.



As diagonals of square are equal.

$\mathrm{PR}=\mathrm{QS}$

$5\mathrm{x}=9\mathrm{x}-8$

$\Rightarrow 5x-9x=-8$

$\Rightarrow -4x=-8$

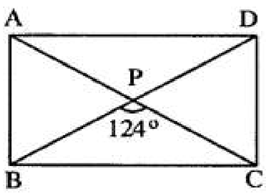
$\Rightarrow \quad \Rightarrow x=2$

$QS=9x-8=9\times 2-8=10$

Question 5.

ABCD is a rectangle, if ∠BPC = 124°

Calculate: (1)∠BAP(ii) ∠ADP



Solution:

Diagonals of rectangle are equal and bisect each other.

$\angle PBC = \angle PCB = x$(say)

But $\angle BPC + \angle PBC + \angle PCB = 180°$

$124° + x + x = 180°$

$2x = 180° - 124°$

$2x = 56°$

$\Rightarrow x = 28°$

$\angle PBC = 28°$

But $\angle PBC = \angle ADP$[Alternate ∠S]

$\angle ADP = 28°$

Again $\angle APB = 180° - 124° = 56°$

$\angle BAP = \frac{1}{2}(180° - \angle APB)$

$\angle BAP = \frac{1}{2}(180° - \angle APB)$

$= \frac{1}{2} \times (180° - 56°) = \frac{1}{2} \times 124° = 62°$

Hence (i) $\angle BAP = 62°\big($ ii) $\angle ADP = 28°$

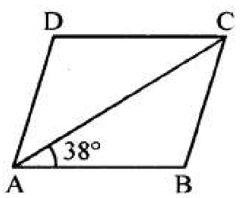
Question 6.

ABCD is a rhombus. If ∠BAC = 38°, find:

(i) ∠ACB

(ii) ∠DAC

(iii) ∠ADC



Solution:

ABCD is Rhombus (Given)

AB = BC ∠BAC = ∠ACB (∠S opp. to equal sides)

But ∠BAC = 38° (Given)

∠ACB = 38° In ΔABC, ∠ABC + ∠BAC + ∠ACB = 180°

∠ABC + 38°+ 38° = 180°

∠ABC = 180°- 76° = 104°

∠ADC = ∠ADC (Opp.∠s of rhombus)

∠ADC = 104°∠DAC = ∠DCA (AD = CD)

∠DAC= ½ [180° - 104°]

∠DAC = ½ × 76° = 38°

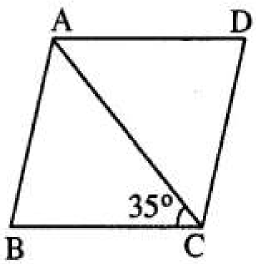
Hence $(i) \angle ACB = 38^\circ$(ii) $\angle DAC = 38^\circ$ (iii) $\angle ADC = 104^\circ$ Ans.

Question 7.

ABCD is a rhombus. If ∠BCA = 35°. Find ∠ADC.

Solution:

Given: Rhombus ABCD in which ∠BCA = 35°



To find: ∠ADC

Proof: AD II||BC

∠DAC = ∠BCA (Alternate ∠S)

But ∠BCA = 35° (Given)

∠DAC = 35°

But ∠DAC = ∠ACD (AD = CD) & ∠DAC + ∠ACD + ∠ADC = 180°

∠B° + 35° + ∠ACD = 180°

∠ADC = 180° - 70° = 110°

Hence ∠ADC = 110°

Question 8.

PQRS is a parallelogram whose diagonals intersect at M.

$\angle PMS = 54°, \angle OSR = 25°$ and $\angle SOR = 30°$;

(i) $\angle RPS$

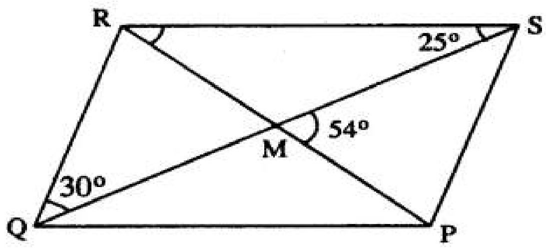
(ii) $\angle PRS$

(iii) $\angle PSR$

Solution:

Given: Parallelogram PQRS in which diagonals PR 8 OS intersect at M.

$\angle PMS = 54°; \angle OSR = 25°$ and $\angle SQR = 30°$



To find: :(1)$\angle RPS$(ii) $\angle PRS$(iii) $\angle PSR$

Proof: $QR\ IIPS$

⇒> $PSQ = \angle SQR$(Alternate ∠S)

But $\angle SOR = 30°$

$\angle PSQ = 30°$

In ΔSMP,

$\angle PMS + \angle PSM + \angle MPS = 180°$ or $54° + 30° + \angle RPS = 180°$

$\angle RPS = 180° - 84° = 96°$

Now, $\angle PRS + \angle RSQ = \angle PMS$

$\angle PRS + 25° = 54°$

$\angle PRS = 54° - 25° = 29°$

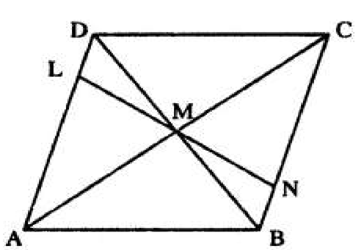
$\angle PSR = \angle PSQ + \angle RSQ = 30° + 25° = 55°$

Hence, (i)$\angle RPS = 96°$ (ii) $\angle PRS = 29°$ (iii) $\angle PSR = 55°$

Question 9.

Given: Parallelogram ABCD in which diagonals AC and BD intersect at M. Prove: M is mid-point of LN.

Solution:



Proof: Diagonals of parallelogram bisect each other

MD= MB

Also $\angle ADB = \angle DBN$(Alternate $\angle S$)

&$\angle DML = \angle BMN$(ert. opp. $\angle S$)

ΔDML = ΔBMN

LM=MN

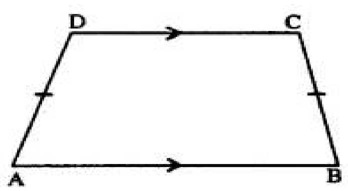
M is mid-point of LN.

Hence proved.

Question 10

In an isosceles-trapezium, show that the opposite angles are supplementary.

Solution:



Given: ABCD is isosceles trapezium in which AD = BC

To Prove: (i) ∠A + ∠C = 180

(ii) ∠ B + ∠D = 180°

Proof: AB //||CD

⇒∠A + ∠D = 180

But ∠A = ∠B [Trapezium is isosceles)]

∠B + ∠D = 180°

Similarly ∠A + ∠C = 180°

Hence the result.

Question 11.

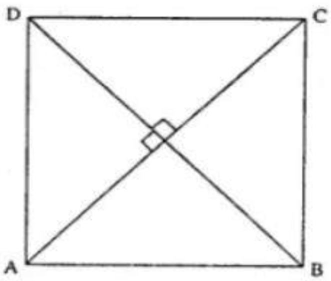
ABCD is a parallelogram. What kind of quadrilateral is it if:

(i) AC=BD and AC is perpendicular to BD?

(ii) AC is perpendicular to BD but is not equal to it?

(iii) AC=BD but AC is not perpendicular to BD?

Solution:



$\mathrm{AC} = \mathrm{BD}$(Given)

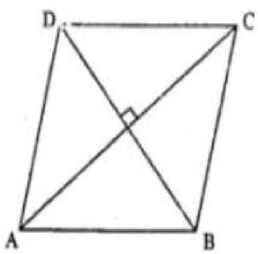
&$\mathrm{AC} \perp \mathrm{BD}$(Given)

i.e. Diagonals of quadrilateral are equal and they

are perpendicular to each other.

∴ABCD is square

(ii)

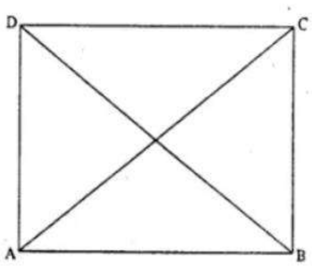


$\mathrm{AC} \perp \mathrm{BD}$ (Given)

But AC&BD are not equal

∴ABCD is a Rhombus.

(iii)



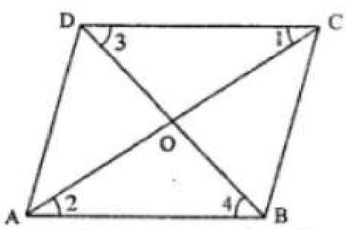
AC=BD but AC&BD are not ⊥r to each other.

∴ABCD is a Rectangle.

Question 12.

Prove that the diagonals of a parallelogram bisect each other.

Solution:



Given: Parallelogram ABCD in which diagonals AC and BD bisect each other.

To Prove: $\mathrm{OA}=\mathrm{OC}$ and $\mathrm{OB}=\mathrm{OD}$

Proof : $\mathrm{AB} \| \mathrm{CD}$(Given)

$\angle 1=\angle 2$(Alternate ∠S)

$\angle 3=\angle 4=$ (Alternate $\angle S$

and $\mathrm{AB}=\mathrm{CD}$(opposite sides of parallelogram)

$\Delta \mathrm{COD}=\Delta \mathrm{AOB}$(A.S. rule)

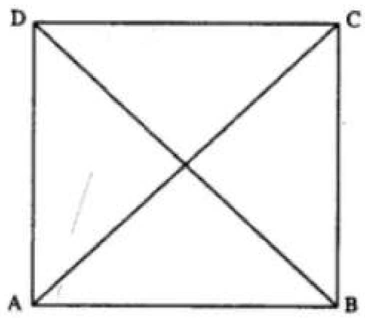
$\mathrm{OA}=\mathrm{OC}$ and $\mathrm{OB}=\mathrm{OD}$

Hence the result.

Question 13.

If the diagonals of a parallelogram are of equal lengths, the parallelogram is a rectangle. Prove it.

Solution:



Given: parallelogram ABCD in which AC=BD

To Prove: ABCD is rectangle

Proof : In ΔABC and ΔABD

$AB = AB$(Common)

$AC = BD$(Given)

$BC = AD$ (Opposite sides of parallelogram)

$\Delta ABC = \Delta ABD$(S.S.S. Rule)

$\angle A = \angle B$

But $\mathrm{AD} // BC$ (opp. sides of llgm are

$\angle A + \angle B = 180°$

$\angle A = \angle B = 90°$

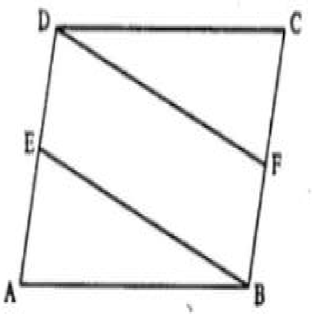
Similarly $\angle D = \angle C = 90°$

Hence ABCD is a rectangle.

Question 14.

In parallelogram ABCD, E is the mid-point of AD and F is the mid-point of BC. Prove that BFDE is a parallelogram.

Solution:



Given: Parallelogram ABCD in which E and F are mid−points of AD and BC

To Prove: BFDE is a parallelogram.

Proof: E is mid-point of AD. (Given)

$\mathrm{DE} = \frac{1}{2}\mathrm{AD}$

Also F is midpoint of BC (Given)

$\mathrm{BF} = \frac{1}{2}\mathrm{BC}$

But AD=BC (opp. sides of ∥gm)

$\mathrm{BF} = \mathrm{DE}$

Again $\mathrm{AD} \| \mathrm{BC}$

$\mathrm{DE} \| \mathrm{BF}$

Now $\mathrm{DE} \| \mathrm{BF}$ and $\mathrm{DE} = \mathrm{BF}$

Hence BFDE is a parallelogram.

Question 15.

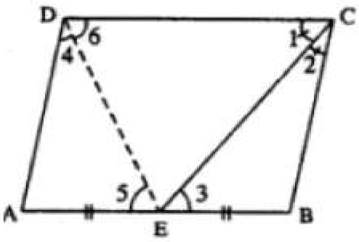
In parallelogram ABCD, E is the mid-point of side AB and CE bisects angle BCD. Prove that:

(i) AE=AD.

(ii) DE bisects and ∠ADC and

(iii) Angle DEC is a right angle.

Solution:



Given: ****gm ABCD in which E is mid-point of AB and CE bisects ZBCD.

To Prove: (i) AE=AD

(ii) DE bisects ∠ADC

(iii) ∠DEC=90°

Const. Join DE

Proof: (i) AB||CD (Given)

And CE bisects it.

$\angle 1 = \angle 3$ (Alternate $\angle S$) … … (i)

But $\angle 1 = \angle 2$ (Given) ……… (ii)

From (i) & (ii)

∠2=∠3

$\mathrm{BC} = \mathrm{BE}$(Sides opp. to equal angles)

But $\mathrm{BC} = \mathrm{AD}$(opp. sides of ||gm)

and $\mathrm{BE} = \mathrm{AE}$(Given)

AD=AE

$\angle 4 = \angle 5$(∠S opp. to equal sides)

But $\angle 5 = \angle 6$ (alternate ∠S

$\Rightarrow \angle 4 = \angle 6$

DE bisects $\angle \mathbf{ADC}$.

NOW $\mathrm{AD} // \mathrm{BC}$

$\Rightarrow \angle D + \angle C = 180°$

$2\angle 6 + 2\angle 1 = 180°$

DE and CE are bisectors.

$\angle 6 + \angle 1 = \frac{180°}{2}$

$\angle 6 + \angle 1 = 90°$

But $\angle DEC + \angle 6 + \angle 1 = 180°$

$\angle DEC + 90° = 180°$

$\angle DEC = 180° - 90°$

$\boldsymbol{\angle DEC = 90°}$

Hence the result.