BYJU'S The Learning App

CHAPTER 6- SETS

Exercise – 6(A)

Question 1.

Write the following sets in roster (Tabular) form:

(i) $\boldsymbol{A_1 = \{x{:}2x + 3 = 11\}}$

Solution:

$\therefore \; 2x + 3 = 11$

Shifting the terms $2x = 11 - 3$ (Subtracting)

$2x = 8$

$x = \frac{8}{2} \Rightarrow x = 4$

∴ Given set in roster (Tabular) Form is $A_1 = \{4\}$

(ii) $\boldsymbol{A_2 = \{x{:}x^2 - 4x - 5 = 0\}}$

Solution:

Given $x^2 - 4x - 5 = 0$

$\Rightarrow \; x^2 - 5x + x - 5 = 0$

$\Rightarrow \; x(x-5) + 1(x-5) = 0$

$\therefore$ Either $x - 5 = 0$ or $x + 1 = 0$

$\Rightarrow \; x = 5 \;\; \Rightarrow \; x = -1$

∴ Given set in roster (Tabular) Form is $A_2 = \{5, -1\}$

(iii) $\boldsymbol{A_3 = \{x{:}x \in Z, -3 \le x < 4\}}$

Solution:

Given $-3 \le x < 4$

$\therefore \quad x = -3, -2, -1, 0, 1, 2, 3$

∴ Given set in roster (Tabular) form is

$A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$

(iv) $A_4 = \{x : x$ is a two digit number and sum of digits of x is $7\}$

Solution:

∵ x is a two digit number and sum of digits of x is 7

$\therefore \quad x = 16, 25, 34, 43, 52, 61, 70$

∴ Given set in roster (Tabular) form is $A_4 = \{16, 25, 34, 43, 52, 61, 70\}$

(v) $A_5 = \{x : x = 4n, n \in W \text{ and } n < 4\}$

Solution:

Given $x = 4n$

When $n = 0,\ x = 4 \times 0$

$\Rightarrow \quad x = 0$

$n = 1,\ x = 4 \times 1 = 4$

$\Rightarrow \quad x = 4$

$n = 2,\ x = 4 \times 2 = 8$

$\Rightarrow \quad x = 8$

$n = 3,\ x = 4 \times 3$

$\Rightarrow \quad x = 12$

∴ Given set in roster (Tabular) form is $A_5 = \{0, 4, 8, 12\}$

(vi) $A_6 = \left\{x : x = \frac{n}{n+2}; n \in N \,\&\, n > 5\right\}$

Solution:

Given $x = \frac{n}{n+2}$

When $n = 6,\ x = \frac{6}{6+2}$ [∵ n > 5]

$\Rightarrow\ x = \frac{6}{8} \Rightarrow x = \frac{3}{4}$

When, $n = 7,\ x = \frac{7}{7+2} \Rightarrow x = \frac{7}{9}$

When, $n = 8,\ x = \frac{8}{8+2} \Rightarrow x = \frac{8}{10}$

$\Rightarrow\ x = \frac{4}{5}$

When, $n = 9,\ x = \frac{9}{9+2} \Rightarrow x = \frac{9}{11}$

∴ Given set in roster (Tabular) form is

$A_6 = \left\{\frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}, \dots\right\}$

Question 2.

Write the following sets in set-builder (Rule Method) form:

(i) $\boldsymbol{B_1 = \{6, 9, 12, 15, \dots\}}$

Solution:

$\{x : x = 3n + 3; n \in \mathrm{N}\}$

(ii) $\boldsymbol{B_2 = \{11, 13, 17, 19\}}$

Solution:

{x: x is a prime number between 10 and 20}

(iii) $\boldsymbol{B_3 = \left\{\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \dots\right\}}$

Solution:

$\{x : x = \frac{n}{n+2}, where\ n\ is\ an\ odd\ natural\ number\}$

(iv) $\boldsymbol{B_4 = \{8, 27, 64, 125, 216\}}$

Solution:

$= \{x : x = n^3; n \in N$ and $2 \le n \le 6\}$

(v) $\boldsymbol{B_5 = \{-5, -4, -3, -2, -1\}}$

Solution:

$= \{x : x \in Z, -5 \le x \le -1\}$

(vi) $B_6 = \{\dots, -6, -3, 0, 3, 6, \dots\}$

Solution:

$= \{x : x = 3n, n \in \mathbf{Z}\}$

Question 3.

(i) Is {1, 2, 4, 16, 64} = {x : x is a factor of 32} ? Give reason.

Solution:

No, $\{1, 2, 4, 16, 64\} \ne \{x :\ x\ is\ a\ factor\ of\ 32\}$. Because 64 is not a factor of 32.

(ii) Is $\boldsymbol{\{x :\ x\ is\ a\ factor\ of\ 27\} \ne \{3, 9, 27, 54\}}$**? Give reason.**

Solution:

Yes, $\{x : x\ is\ a\ factor\ of\ 27\} \ne \{3, 9, 27, 54\}$ Because 54 is not a factor of 27

(iii) Write the set of even factors of 124.

Solution:

$1 \times 124 = 124$

$2 \times 62 = 124$

$4 \times 31 = 124$

Factors of $124 = 1, 2, 4, 31, 62, 124$

Set of even factors of $124 = \{2, 4, 62, 124\}$

(iv) Write the set of odd factors of 72.

Solution:

$1 \times 72 = 72$

$2 \times 36 = 72$

$3 \times 24 = 72$

$4 \times 18 = 72$

$6 \times 12 = 72$

$8 \times 9 = 72$

Factors of $72 = 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72$

Set of odd factors of $72 = \{1, 3, 9\}$

(v) Write the set of prime factors of 3234.

Solution:

2	3234
3	1617
7	539

7	77
	11

$3234 = 2 \times 3 \times 7 \times 7 \times 11$

$\therefore$ Set of prime factors of $3234 = \{2, 3, 7, 11\}$

(vi) Is $\{x : x^2 - 7x + 12 = 0\} = \{3, 4\}$?

Solution:

$x^2 - 7x + 12 = 0$

$\Rightarrow \; x^2 - 4x - 3x + 12 = 0$

$\Rightarrow \; x(x-4) - 3(x-4) = 0$

$\Rightarrow \; (x-4)(x-3) = 0$

$\therefore$ Either $x - 4 = 0$ or $x - 3 = 0$

$x - 4 = 0 \; \Rightarrow \; x = 4$

$x - 3 = 0 \Rightarrow \; x = 3$

$\therefore \; \{x : x^2 - 7x + 12 = 0\} = \{3, 4\}$ is true.

(vii) Is $\{x : x^2 - 5x - 6 = 0\} = \{2, 3\}$?

Solution:

$x^2 - 5x - 6 = 0$

$\Rightarrow \; x^2 - 6x + x - 6 = 0$

$\Rightarrow \; x(x-6) + 1(x-6) = 0$

$\Rightarrow \; (x-6)(x+1) = 0$

$\therefore$ Either $x-6=0$ Or $x+1=0$

$x-6=0 \Rightarrow x=6$

$x+1=0 \Rightarrow x=-1$

$\therefore \{x: x^2-5x-6=0\} \neq \{2,3\}$

i.e. $\{x: x^2-5x-6=0\} = \{2,3\}$ is not true.

Question 4.

Write the following sets in Roster form:

(i) The set of letters in the word 'MEERUT'

(ii) The set of letters in the word 'UNIVERSAL'

(iii) $A = \{x: x = y+3, y \in N \text{ and } y > 3\}$

(iv) $B = \{p: p \in W \text{ and } p^2 < 20\}$

(v) $C = \{x: x$ is composite number and $5 < x < 21\}$

Solution:

(i) Roster form of the set of letters in the word "MEERUT"= {m, e, r, u, t}

(ii) Roster form of the set of letters in the word "UNIVERSAL"= {u, n, i, v, e, r, s, a, l}

(iii) $A = \{x: x = y+3, y \in N \text{ and } y > 3\}$

$x = y+3$

$y = 4, 5, 6, 7, 8, 9, \ldots$ $[\because y > 3]$

When $y=4,\ x=4+3=7$

When $y=5,\ x=5+3=8$

When $y=6,\ x=6+3=9$

when $y=7,\ x=7+3=10$

when $y=8,\ x=8+3=11$

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∴ Roster form of the given set $A = \{7, 8, 9, 10, 11, \dots \dots \dots\}$

(iv) $B = \{P : P \in W \text{ and } P^2 < 20\}$

$P^2 = 0, 1, 4, 9, 16$ $\quad [\because P^2 < 20]$

When $P^2 = 0 \Rightarrow P = \sqrt{0} = 0$

When $P^2 = 1 \Rightarrow P = \sqrt{1} = 1$

When $P^2 = 4 \Rightarrow P = \sqrt{4} = 2$

When $P^2 = 9 \Rightarrow P = \sqrt{9} = 3$

When $P^2 = 16 \Rightarrow P = \sqrt{16} = 14$

∴ Roster form of the given set B = {0, 1, 2, 3, 4}

(v) $C = \{x : x \text{ is composite number and } 5 \le x \le 21)$

$5 \le x \le 21$ means $x = 5, 6, 7, 8, 9, 10, \dots \dots, 21$

But we are given that x is a composite number, so we need to ignore prime numbers in between 5 and 21.

$\therefore x = 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21$

∴ Roster form of the given set C = {6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21}

Question 5.

List the elements of the following sets:

(i) $\{x : x^2 - 2x - 3 = 0\}$

Solution:

Given $x^2 - 2x - 3 = 0$

$\Rightarrow\ x^2 - 3x + x - 3 = 0$

$\Rightarrow\ x(x-3) + 1(x-3) = 0$

$\Rightarrow\ (x+1)(x-3) = 0$

$\therefore$ Either $x - 3 = 0$ or $x + 1 = 0$

$x - 3 = 0 \Rightarrow x = 3$

$x + 1 = 0 \Rightarrow x = -1$

$\therefore$ Elements of the set $\{x: x^2 - 2x - 3 = 0\}$ are 3 and -1

(ii) $\{x: x = 2y + 5; y \in N \text{ and } 2 \le y < 6\}$

Solution:

$\{x: x = 2y + 5; y \in \text{N and } 2 \le y < 6\}$

$x = 2y + 5$

$y = 2, 3, 4, 5$ $\quad [\because 2 \le y < 6]$

When $y = 2,\ x = 2 \times 2 + 5 = 4 + 5 = 9$

When $y = 3,\ x = 2 \times 3 + 5 = 6 + 5 = 11$

When $y = 4\ x = 2 \times 4 + 5 = 8 + 5 = 13$

When $y = 5,\ x = 2 \times 5 + 5 = 10 + 5 = 15$

$\therefore$ Elements of the given set $\{x: x = 2y + 5y \in N \text{ and } 2 \le y < 6\}$ are 9, 11, 13, 15.

(iii) $\{x: x \text{ is a factor of } 24\}$

Solution:

Given $\{x: x \text{ is a factor of } 24\}$

$24 = 1 \times 24$

$24 = 2 \times 12$

$24 = 3 \times 8$

$24 = 4 \times 6$

$\therefore$ Elements of the given set $\{x : x\ is\ a\ factor\ of\ 24\}$ are 1, 2, 3, 4, 6, 8, 12, 24.

(iv) $\{\boldsymbol{x : x \in Z \text{ and } x^2 \leq 4}\}$

Solution:

Given $\{x : x \in Z \text{ and } x^2 \leq 4\}$

$x = 4, 1, 0$ $\quad [\because x^2 \leq 4]$

When $x^2 = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2$

When $x^2 = 1 \Rightarrow x = \pm\sqrt{1} = \pm 1$

When $x^2 = 0 \Rightarrow x = \sqrt{0} = 0$

$\therefore$ Elements of the given set $\{x : x \in \mathrm{Z} \text{ and } x^2 \leq 4\}$ are $-2, -1, 0, 1, 2$

(v) $\{\boldsymbol{x : 3x - 2 \leq 10, x \in N}\}$

Solution:

Given $3x - 2 \leq 10$

$\Rightarrow\ 3x \leq 10 + 2$

$\Rightarrow\ 3x \leq 12$

$\Rightarrow\ x \leq \frac{12}{3}$

$\Rightarrow\ x \leq 4$

$\therefore x = 1, 2, 3, 4$

∴ Elements of the given set $\{x: 3x - 2 \leq 10, x \in \mathrm{N}\}$ are $1, 2, 3$ and 4.

(vi) $\{x: 4 - 2x > -6, x \in Z\}$

Solution:

Given $4 - 2x > -6$

Subtracting 4 from both sides, we get

$-4 + 4 - 2x > -6 - 4$

$-2x > -10$

Adding $2x + 10$ to both sides, we get

$-2x + 2x + 10 > -10 + 2x + 10$

$+10 > 2x$

$\frac{10}{2} > x$

$5 > x$

∴ Elements of the given set $\{x: 4 - 2x > -6, x \in \mathrm{Z}\}$ are $4, 3, 2, 1, 0, -1, -2, \ldots \ldots$

<u>Exercise – 6 (B)</u>

Question 1.

Find the cardinal number of the following sets:

(i) $A_1 = \{-2, -1, 1, 3, 5\}$

Solution:

$A_1 = \{-2, -1, 1, 3, 5\}$

Cardinal number of a set is the number of elements in a set.

The number of elements in the set A_1 is 5.

∴ Cardinal Number of a set $A_1 = 5$

(ii) $A_2 = \{x : x \in N \text{ and } 3 \le x < 7\}$

Solution:

$A_2 = \{x : x \in N \text{ and } 3 \le x < 7\}$

$= \{3, 4, 5, 6\}$

The number of elements in the set A_2 is 4.

$\therefore$ Cardinal number of set $A_2 = 4$

(iii) $A_3 = \{P : P \in \mathbb{W} \text{ and } 2P - 3 < 8\}$

Solution:

Given $2P - 3 < 8$

Adding 3 to both sides, we get

$\Rightarrow\ 2P - 3 + 3 < 8 + 3$

$\Rightarrow\ 2\text{P} < 11$

Dividing both sides by 2, we get

$P < \frac{11}{2}$

$\Rightarrow\ P < 5.5$

$\therefore A_3 = \{0, 1, 2, 3, 4, 5\}$

The number of elements in the set A_3 is 6.

$\therefore$ Cardinal number of set $A_3 = 6$

(iv) $A_4 = \{b : b \in Z \text{ and } -7 < 3b - 1 \le 2\}$

Solution:

Given $-7 < 3b - 1 \le 2$

Let us take $-7 < 3b - 1$ first,

Adding 1 to both sides, we get

$\Rightarrow \quad -7 + 1 < 3b - 1 + 1$

$\Rightarrow \quad -6 < 3b$

Dividing both sides by 3, we get

$\Rightarrow \quad -\frac{6}{3} < b$

$\Rightarrow \quad -2 < b$

Now we take, $3b - 1 \leq 2$

Adding 1 to both sides, we get

$\Rightarrow \quad 3b - 1 + 1 \leq 2 + 1$

$\Rightarrow \quad 3b \leq 3$

Dividing both sides by 3, we get

$\Rightarrow \quad b \leq \frac{3}{3}$

$\Rightarrow \quad b \leq 1$

Hence, we get $-2 < b \leq 1$

$\therefore$ Given set $A_4 = \{-1, 0, 1\}$

The number of elements in the set A_4 is 3.

$\therefore$ Cardinal Number of a set $A_4 = 3$

Question 2:

If P={P:P is a letter in the word “PERMANENT”}. Find n(P).

Solution:

Given P=(P:P is a letter in the word “PERMANENT”}

I.e. P = {p, e, r, m, a, n, t)

The number of elements in the set P is 7

Therefore, n(p) =7

Question 3.

State, which of the following sets are finite and which are infinite:

(i) $A = \{x : x \in Z \text{ and } x < 10\}$

Solution:

$A = \{x : x \in Z \text{ and } x < 10\}$

$= \{\ldots\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$= \{9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, \ldots\ldots\}$

We know that, a set which is not finite is called an infinite set.

$\therefore\ A = \{x : x \in Z \text{ and } x < 10\}$ is an infinite set.

(ii) $B = \{x : x \in W \text{ and } 5x - 3 \leq 20\}$

Solution:

Given $5x - 3 \leq 20$

Adding 3 to both sides, we get

$\Rightarrow\ 5x - 3 + 3 \leq 20 + 3$

$\Rightarrow\ 5x \leq 23$

Dividing both sides by 5, we get

$\Rightarrow\ x \leq \frac{23}{5}$

$\Rightarrow\ x \leq 4.6$

$\therefore\ B = \{0, 1, 2, 3, 4\}$

We know that a set with finite number of elements is called a finite set.

$\therefore\ B = \{x : x \in W \text{ and } 5x - 3 \le 20\}$ is a finite set.

(iii) $\mathrm{P} = \{y : y = 3x - 2, x \in \mathrm{N} \& x > 5\}$

Solution:

Given $y = 3x - 2$

$x = 6, 7, 8, 9, \dots\dots$ $[\because x > 5]$

When $x = 6,\ y = 3 \times 6 - 2 = 18 - 2 = 16$

When $x = 7,\ y = 3 \times 7 - 2 = 21 - 2 = 19$

When $x = 8,\ y = 3 \times 8 - 2 = 24 - 2 = 22$

When $x = 9,\ y = 3 \times 9 - 2 = 27 - 2 = 25$

$\therefore\ \mathrm{P} = \{16, 19, 22, 25, \dots\dots\}$

We know that, a set which is not finite is called an infinite set.

$\therefore \mathrm{P} = \{y : y = 3x - 2, x \in \mathrm{N}\ \&\ x > 5\}$ is an infinite set.

(iv) $\mathrm{M} = \left\{r : r = \frac{3}{n}; n \in \mathrm{W} \text{ and } 6 < n \le 15\right\}$

Solution:

Given $r = \frac{3}{n}$

$n = 7, 8, 9, 10, 11, 12, 13, 14, 15$ $[\because 6 < n \le 15]$

When $n = 7,\ r = \frac{3}{7}$

When $n = 8,\ r = \frac{3}{8}$

When $n = 9,\ r = \frac{3}{9}$

When $n = 10,\ r = \frac{3}{10}$

When $n = 11,\ r = \frac{3}{11}$

When $n = 12,\ r = \frac{3}{12}$

When $n = 13,\ r = \frac{3}{13}$

When $n = 14,\ r = \frac{3}{14}$

When $n = 15,\ r = \frac{3}{15}$

$\therefore\ \mathrm{M} = \left\{\frac{3}{7}, \frac{3}{8}, \frac{3}{9}, \frac{3}{10}, \frac{3}{11}, \frac{3}{12}, \frac{3}{13}, \frac{3}{14}, \frac{3}{15}\right\}$

We know that a set with finite number of elements is called a finite set.

$\therefore \mathrm{M} = \left\{r : r = \frac{3}{n}; n \in \mathrm{W} \text{ and } 6 < n \leq 15\right\}$ is a finite set.

Question 4.

Find, which of the following sets singleton sets are:

(i) The set of points of intersection of two non-parallel straight lines in the same plane

A set, which has only one element in it, is called a SINGLETON or unit set.

Solution:

The set of points of intersection of two non-parallel straight lines in the same plane.

Therefore, the set has only one element in it.

Then, the given set is singleton set.

(ii) $\boldsymbol{A = \{x : 7x - 3 = 11\}}$

Solution:

Given $7x - 3 = 11$

$\Rightarrow\ 7x = 11 + 3$

$\Rightarrow\ 7x = 14$

$\Rightarrow \ x = \frac{14}{7} = 2$

$\therefore \ A = \{2\}$

The set A has only one element in it.

Hence the given set A is a singleton set.

(iii) $B = \{y: 2y + 1 < 3 \text{ and } y \in W\}$

Solution:

Given $2y + 1 < 3$

Subtracting 1 from both sides, we get

$\Rightarrow \ 2y + 1 - 1 < 3 - 1$

$\Rightarrow \ 2y < 2$

(Dividing both sides by 2)

$\Rightarrow \ y < \frac{2}{2}$

$\Rightarrow \ y < 1$

$\therefore \ B = \{0\}$

The set B has only one element in it.

Hence the given set B is a singleton set.

Question 5.

Find, which of the following sets are empty:

(i) The set of points of intersection of two parallel lines.

Solution:

“The set of points of intersection of two parallel lines" is an empty set because two parallel lines do not intersect anywhere.

(ii) $A = \{x : x \in N \text{ and } 5 < x < 6\}$

Solution:

$A = \{x : x \in \mathrm{N} \text{ and } 5 < x \leq 6\}$

As, $5 < x \leq 6$

$\therefore \; x = 6$

$\therefore \; A = \{6\}$

The set A has one element in it.

Hence the given set A is not an empty set.

(iii) $B = \{x : x^2 + 4 = 0, x \in N\}$

Solution:

Given $x^2 + 4 = 0$

$\Rightarrow \; x^2 = -4$

$\Rightarrow \; x = \sqrt{-4}$ which is not a natural number.

But $x \in \mathbf{N}$

$\therefore \; \mathbf{B} = \{\}$

$\therefore$ Given set B is an empty set.

(iv) $C = \{$Even numbers between 6 & $10\}$

Solution:

Given C = {Even numbers between 6 and 10}

∴ C = {8}

The set C has one element in it.

Hence, the given set C is not an empty set.

(v) $\mathbf{D = \{ prime\ numbers\ between\ 7\ \&\ 11\}}$

Solution:

$D = \{$Prime numbers between 7 and $11\}$

There is no prime number between 7 and 11.

$\therefore\ D = \{\}$

The set D has no element in it.

Hence, the given set D is an empty set or null set.

Question 6.

(i) Are the sets A = {4, 5, 6} and B=$\{x: x^2 - 5x - 6 = 0\}$ disjoint?

(ii) Are the sets A= $\{b, c, d, e\}$ and B= $\{x$:x is a letter in the word 'MASTER'} joint?

Note:

(i) Two sets are said to be joint sets, if they have atleast one element in common.

(ii) Two sets are said to be disjoint, if they have no element in common.

Solution:

(i) $A = \{4,5,6\}$

$B = \{x: x^2 - 5x - 6 = 0\}$

$x^2 - 5x - 6 = 0$

$\Rightarrow\ x^2 - 6x + x - 6 = 0$

$\Rightarrow\ x(x-6) + 1(x-6) = 0$

$\Rightarrow\ (x-6)(x+1) = 0$

$\therefore$ Either $x - 6 = 0$ Or $x + 1 = 0$

$x-6=0 \Rightarrow \ x=6$

$x+1=0 \Rightarrow \ x=-1$

$\therefore \ B=\{6,-1\}$

Set A and B have the element 6 in common. So these sets are joint sets.

Hence set A and set B are not disjoint.

(ii) $\mathrm{A}=\{b,c,d,e\}$

$\mathrm{B}=\{x{:}\,x$ is a letter in the word "MASTER"}

$\therefore \ \mathrm{B}=\{m,a,s,t,e,r\}$

The element 'e' is common in both the sets A and B.

Hence set A and set B are joint sets.

Question 7.

State, whether the following pairs of sets are equivalent or not:

(i) $A=\{x{:}\,x\in N \ and \ 11\geq 2x-1\}$ and $B=\{y{:}\ y\in W \ and \ 3\leq y\leq 9\}$

(ii) Set of integers and set of natural numbers.

(iii) Set of whole numbers and set of multiples of 3.

(iv) P={5,6,7,8} and $M=\{x{:}\,x\in W \ and \ x<4\}$

Note: Two sets are said to be equivalent, if they contain the same number of elements.

Solution:

(i) $\mathrm{A}=\{x{:}\,x\in\mathrm{N}$ and $11\geq 2x-1\}$

$11\geq 2x-1$

Adding 1 on both sides, we get

$\Rightarrow \ 11+1\geq 2x-1+1$

$\Rightarrow\ 12 \geq 2x$

Dividing both sides by 2, we get

$\Rightarrow\ \frac{12}{2} \geq x$

$\Rightarrow\ 6 \geq x$

$\therefore\ A = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ Cardinal number of set A, $n(A) = 6$

$B = \{y: y \in W$ and $3 \leq y \leq 9\}$

$\because\ 3 \leq y \leq 9$

B = {3, 4, 5, 6, 7, 8, 9}

Cardinal number of set B, n(B) = 7

Set A and set B are not equivalent.

(ii) We know that, Set of integers has infinite number of elements.

Set of natural numbers has infinite number of elements.

Set of integers and set of natural numbers are equivalent because both sets have infinite number of elements.

(iii) Set of whole numbers has infinite number of elements.

Set of multiples of 3, has infinite number of elements.

Set of whole numbers and set of multiples of 3 are equivalent because both sets have infinite number of elements.

(iv) P = {5, 6, 7, 8}

Cardinal number of set P, n(P) = 4

$M = \{x: x \in W\ and\ x \leq 4\}$

M = {0, 1, 2, 3, 4}

Cardinal number of set M, n(M) = 5

These sets are not equivalent, because both sets do not have same number of elements.

Question 8.

State, whether the following pairs of sets are equal or not:

(i) $A = \{2,4,6,8\}$ and

$B = \{2n: n \in N$ and $n < 5\}$

(ii) $M = \{x : x \in W$ and $x + 3 < 8\}$ and

$N = \{y: y = 2n - 1, n \in N$ and $n < 5\}$

(iii) $\mathrm{E} = \{x: x^2 + 8x - 9 = 0\}$

$F = \{1, -9\}$

(iv) $A = \{x: x \in N, x < 3\}$

$B = \{y: y^2 - 3y + 2 = 0\}$

Note: Two sets are equal, if both the sets have same (identical) elements.

Solution:

(i) $A = \{2,4,6,8\}$

$B = \{2n: n \in N$ and $n < 5\}$

$n = 1, 2, 3, 4$ $[\because n < 5]$

When $n = 1, 2n = 2 \times 1 = 2$

When $n = 2, 2n = 2 \times 2 = 4$

When $n = 3, 2n = 2 \times 3 = 6$

When $n = 4, 2n = 2 \times 4 = 8$

$\therefore\ B = \{2,4,6,8\}$

Now we see that elements of sets A and B are same (identical)

∴ Sets A and B are equal.

(ii) $M = \{x: x \in W$ and $x + 3 < 8\}$

$x + 3 < 8$

Subtracting 3 from both sides, we get

$\Rightarrow x < 8 - 3$

$\Rightarrow\ x < 5$

$\therefore\ M = \{0, 1, 2, 3, 4\}$

$N = \{y: y = 2n - 1, n \in N$ and $n < 5\}$

$y = 2n - 1$

$n = 1, 2, 3, 4 \qquad [\because n < 5]$

When $n = 1,\ y = 2 \times 1 - 1$

$\Rightarrow\ y = 2 - 1 = 1$

When $n = 2,\ y = 2 \times 2 - 1$

$\Rightarrow\ y = 4 - 1 = 3$

When $n = 3,\ y = 2 \times 3 - 1$

$\Rightarrow\ y = 6 - 1 = 5$

When $n = 4,\ y = 2 \times 4 - 1$

$\Rightarrow\ y = 8 - 1 = 7$

$\therefore\ N = \{1, 3, 5, 7\}$

The elements of sets M and N are not same (identical).

∴ Sets M and N are not equal.

(iii) $E = \{x: x^2 + 8x - 9 = 0\}$

$x^2 + 8x - 9 = 0$

$\Rightarrow \; x^2 + 9x - x - 9 = 0$

$\Rightarrow \; x(x+9) - 1(x+9) = 0$

$\Rightarrow \; (x-1)(x+9) = 0$

$\therefore$ Either $x + 9 = 0$ or $x - 1 = 0$

$x + 9 = 0 \Rightarrow \; x = -9$

$x - 1 = 0 \Rightarrow \; x = 1$

$\therefore E = \{-9,1\}$

$F = \{1,-9\}$

The elements of sets E and F are same (identical)

$\therefore$ Sets E and F are equal.

(iv) $A = \{x: x \in N, x < 3\}$

$A = \{1,2\}$

$B = \{y: y^2 - 3y + 2 = 0\}$

$y^2 - 3y + 2 = 0$

$\Rightarrow \; y^2 - 2y - y + 2 = 0$

$\Rightarrow \; y(y-2) - 1(y-2) = 0$

$\Rightarrow \; (y-2)(y-1) = 0$

$\therefore$ Either $y - 2 = 0$ or $y - 1 = 0$

$y - 2 = 0 \Rightarrow \; y = 2$

$y - 1 = 0 \Rightarrow y = 1$

$\therefore\ B = \{1,2\}$

The elements of sets A and B are same (identical).

∴ Sets A and B are equal.

Question 9.

State whether each of the following sets is a finite set or an infinite set:

(i) The set of multiples of 8.

(ii) The set of integers less than 10.

(iii) The set of whole numbers less than 12.

(iv) $\{x: x = 3n - 2, n \in W, n \leq 8\}$

(v) $\{x: x = 3n - 2, n \in Z, n \leq 8\}$

(vi) $\left\{x: x = \frac{n-2}{n+1}, n \in w\right)$

Solution:

(i) The set of multiples of 8

= {8, 16, 24, 32, …}

It is an infinite set.

(ii) The set of integers less than 10

= {9, 8, 7, 6, 5, 4, 3, 2, 1, -1, -2, …}

It is an infinite set.

(iii) The set of whole numbers less than 12

= {11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0}

It is a finite set.

(iv) $\{x: x = 3n - 2, n \in \mathbb{W}, n \leq 8\}$

Substituting the value of n = (0, 1, 2, 3, 4, 5, 6, 7, 8) we get

= {-2, 1, 4, 7, 10, 13, 16, 19, 22}

It is a finite set.

(v) $\{x: x = 3n - 2, n \in Z, n \leq 8\}$

Substituting the value of n = (0, 1, 2, 3, 4, 5, 6, 7, 8) we get

x = {22, 19, 16, 13, 10, 7, 4, 1, -2, -5, ...}

It is an infinite set.

(vi)$\left\{x: x = \frac{n-2}{n+1}, n \in \mathbb{W}\right\}$

$\left\{-2, -\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5} \ldots \ldots\right\}$

It is an infinite set.

Question 10.

Answer, whether the following statements are true or false. Give reasons.

(i) The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are equivalent sets.

(ii) If E = {factors of 16} and F = {factors of 20}, then E=F .

(iii) The set A = {integers less than 20} is a finite set.

(iv) If A={x:x is an even prime number}, then set A is empty.

(v) The set of odd prime numbers is the empty set.

(vi) The set of squares of integers and the set of whole numbers are equal sets.

(vii) In n(P)=n(M), then P→M

(viii) If set P=set M, then n(P)=n(M)

(ix) n(A)=n(B)=>A=B)

Solution:

(i) Set of even natural number less than 21 = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

∴ Cardinal Number of this set = 10

Set of odd natural numbers less than 21 = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

∴ Cardinal number of this set =10

The cardinal numbers of both these sets = 10

∴ The given statement is True.

(ii) E= $\{ Factors\ of\ 16\}$

$1 \times 16 = 16$

$2 \times 8 = 16$

$4 \times 4 = 16$

$E = \{1, 2, 4, 8, 16\}$

$F = \{ Factors\ of\ 20\}$

$1 \times 20 = 20$

$2 \times 10 = 20$

$4 \times 5 = 20$

$F = \{1, 2, 4, 5, 10, 20\}$

The elements of set E and set F are not same (identical)

Therefore, the given statement is false.

(iii) A = {Integers less than 20}

A = {19, 18, 17, 16,........0,-1,-2,-3,...}

The set A has infinite elements, so it is not a finite set.

Therefore, the given statement is false.

(iv) $A = \{x: x \text{ is an even prime number}\} = \{2\}$

The given set A has only one element in it, so it is a singleton set.

Therefore, it is not an empty set.

Hence, the given statement is false.

(v) Set of odd prime numbers = {3, 5, 7, 11, 13, 17, 19, 23, …}

The given set has infinite number of element, so it is an infinite set.

Therefore it is not an empty set.

Hence, the given statement is false.

(vi)

Integer	Square of Integer	Whole No.
0	$(0)^2 = 0$	0
±1	$(\pm1)^2 = 1$	1
±2	$(\pm2)^2 = 4$	2
±3	$(\pm3)^2 = 9$	3
±4	$(\pm4)^2 = 16$	4
±5	$(\pm5)^2 = 25$	5
. . . .	. . . .	. . . .

Set of squares of integers = {0, 1, 4, 9, 16, 25, …}

Set of whole number = {0, 1, 2, 3, 4, 5, 6, 7, …}

Therefore, these two sets are not equal sets.

Hence, the given statement is false.

(vii) $n(\mathrm{P}) = n(\mathrm{M})$ means the number of elements of set P = Number of elements of set M.

∴ Sets P and M are equivalent.

Hence, the given statement is true.

(viii) Set P = Set M

It means sets P and M are equal. Equal sets are equivalent also.

∴ Number of elements of set P = Number of elements of set M

Hence, the given statement is true.

(xi) $n(\mathrm{A}) = n(\mathrm{B})$

i.e. Number of elements of set A = Number of elements of set B

∴ Given sets are equivalent but not equal.

Hence, the given statement is false.