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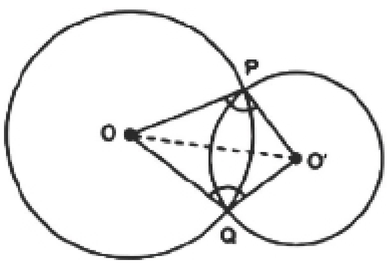
## Exercise: 10.6

**1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.**

**Solution:**

Consider the following diagram




In ΔPOO' and ΔQOO'

OP = OQ (Radius of circle 1)

O'P = O'Q (Radius of circle 2)

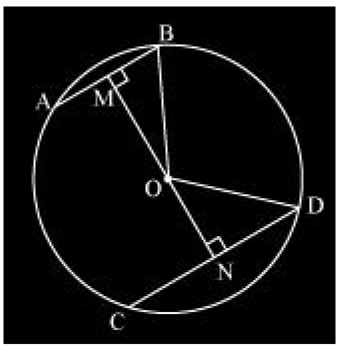
OO' = OO' (Common arm)

So, by SSS congruency, ΔPOO' ≅ ΔQOO'

Thus, ∠OPO' = ∠OQO' (proved).

**2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 , find the radius of the circle.**

**Solution:**




Here, OM ⊥ AB and ON ⊥ CD. is drawn and OB and OD are joined.
We know that AB bisects BM as the perpendicular from the centre bisects chord.
Since AB = 5 so,
BM = AB/2
Similarly, ND = CD/2 = 11/2

Now, let ON be x.
So, OM = 6− x.

Consider ΔMOB,
$OB^2 = OM^2 + MB^2$
Or,

$$OB^2 = 36 + x^2 - 12x + \frac{25}{4} \quad \ldots (1)$$

Consider ΔNOD,
$OD^2 = ON^2 + ND^2$
Or,

$$OD^2 = x^2 + \frac{121}{4} \quad \ldots (2)$$

We know, OB = OD (radii)
From equation 1 and equation 2 we get

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4}$$

$$12x = \frac{48}{4} = 12$$

$$x = 1$$

Now, from equation (2) we have,

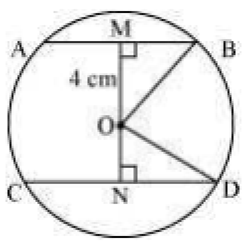
$OD^2 = 1^1 + (121/4)$

Or OD = (5/2) × √5

**3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre?**

**Solution:**

Consider the following diagram




Here AB and CD are 2 parallel chords. Now, join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

So, OM = 4 cm

MB = AB/2 = 3 cm

Consider ΔOMB

$OB^2 = OM^2 + MB^2$

Or, OB = 5cm

Now, consider ΔOND,

OB = OD = 5 (since they are the radii)

=> ND = CD/2 = 4 cm

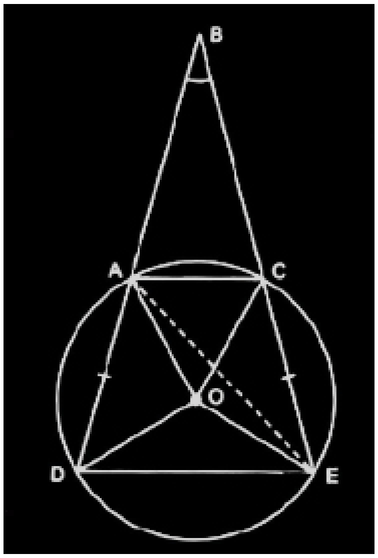
Now, $OD^2 = ON^2 + ND^2$

Or, ON = 3.

**4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.**

**Solution:**

Consider the diagram




Here AD = CE

We know, any exterior angle of a triangle is equal to the sum of interior opposite angles.

So,

$\angle DAE = \angle ABC + \angle AEC$ (in $\Delta BAE$) --------------------(i)

DE subtends $\angle DOE$ at the centre and $\angle DAE$ in the remaining part of the circle.

So,

$\angle DAE = (½)\angle DOE$ --------------------(ii)

Similarly, $\angle AEC = (½)\ \angle AOC$ --------------------(ii)

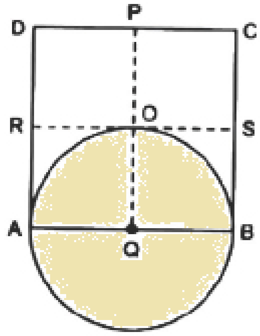
Now, from equation (i), (ii), and (iii) we get,

$\angle DOE = \angle ABC + (½)\ \angle AOC$

Or, $\angle ABC = (½)\ [\angle DOE - \angle AOC]$ (hence proved).

**5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.**

**Solution:**




**To prove:** A circle drawn with Q as centre, will pass through A, B and O (i.e. QA = QB = QO)

Since all sides of a rhombus are equal,

AB = DC

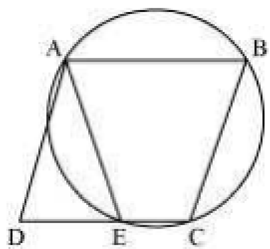
Now, multiply (½) on both sides

(½)AB = (½)DC
So, AQ = DP
=> BQ = DP
Since Q is the midpoint of AB,
AQ= BQ
Similarly,
RA = SB
Again, as PQ is drawn parallel to AD,
RA = QO

Now, as AQ = BQ and RA = QO we get,
QA = QB = QO (hence proved).

**6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE, = AD.**

**Solution:**



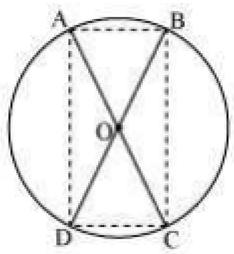

Here, ABCE is a cyclic quadrilateral. In a cyclic quadrilateral, the sum of the opposite angles is 180°.
So, ∠AEC + ∠CBA = 180°
As ∠AEC and ∠AED are linear pair,
∠AEC + ∠AED = 180°
Or, ∠AED = ∠CBA ... (1)

We know in a parallelogram, opposite angles are equal.
So, ∠ADE = ∠CBA ... (2)

Now, from equations (1) and (2) we get,
∠AED = ∠ADE
Now, AD and AE are angles opposite to equal sides of a triangle,
∴ AD = AE (proved).

**7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.**

**Solution:**




Here chords AB and CD intersect each other at O.

Consider ΔAOB and ΔCOD,
∠AOB = ∠COD (They are vertically opposite angles)
OB = OD (Given in the question)
OA = OC (Given in the question)
So, by SAS congruency, ΔAOB ≅ ΔCOD

Also, AB = CD (By CPCT)
Similarly, ΔAOD ≅ ΔCOB
Or, AD = CB (By CPCT)

In quadrilateral ACBD, opposite sides are equal.
So, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.
So, ∠A = ∠C

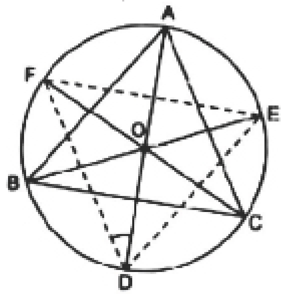
Also, as ABCD is a cyclic quadrilateral,
∠A + ∠C = 180°
⇒ ∠A + ∠A = 180°
Or, ∠A = 90°

As ACBD is a parallelogram and one of its interior angles is 90°, so, it is a rectangle.

∠A is the angle subtended by chord BD. And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

**8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are 90° – (½)A, 90° – (½)B and 90° – (½)C.**

**Solution:**
Consider the following diagram

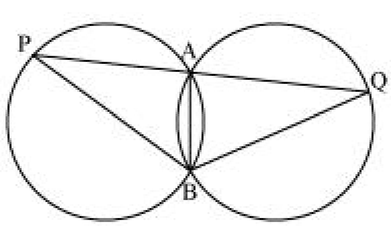



Here, ABC is inscribed in a circle with center O and the bisectors of ∠A, ∠B and ∠C intersect the circumcircle at D, E and F respectively.
Now, join DE, EF and FD
As angles in the same segment are equal, so,
∠FDA = ∠FCA -------------(i)
∠FDA = ∠EBA -------------(i)
By adding equations (i) and (ii) we get,
∠FDA + ∠EDA = ∠FCA + ∠EBA
Or, ∠FDE = ∠FCA + ∠EBA = (½)∠C + (½)∠B

We know, ∠A + ∠B + ∠C = 180°

So, ∠FDE = (½)[∠C + ∠B] = (½)[180° - ∠A]
=> ∠FDE = [90 - (∠A/2)]
In a similar way,
∠FED = [90 - (∠B/2)]
And,
∠EFD = [90 - (∠C/2)]

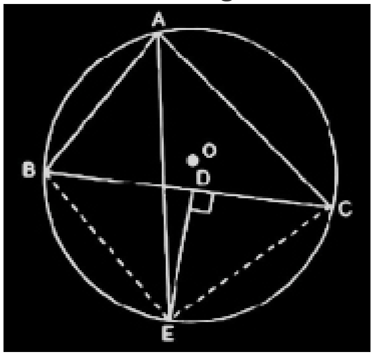
**9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.**
**Solution:**
The diagram will be




Here, ∠APB = ∠AQB (as AB is the common chord in both the congruent circles.)
Now, consider ΔBPQ,
∠APB = ∠AQB
So, the angles opposite to equal sides of a triangle.
∴ BQ = BP

**10. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.**

**Solution:**
Consider this diagram




Here, join BE and CE.
Now, since AE is the bisector of ∠BAC,
∠BAE = ∠CAE
Also,
∴ arc BE = arc EC
This implies, chord BE = chord EC

Now, consider triangles ΔBDE and ΔCDE,
DE = DE (It is the common side)
BD = CD (It is given in the question)
BE = CE (Already proved)

So, by SSS congruency, ΔBDE ≌ ΔCDE.
Thus, ∴∠BDE = ∠CDE
We know, ∠BDE = ∠CDE = 180°
Or, ∠BDE = ∠CDE = 90°
∴ DE ⊥BC (hence proved).