Exercise 12.1 Page No: 197

1. Evaluate:

(i) $\mathbf{3^{-2}}$ **(ii)** $(-4)^{-2}$ **(iii)** $\left(\frac{1}{2}\right)^{-5}$

Solution:

(i) $3^{-2} = \left(\frac{1}{3}\right)^2$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

= 1/9

(ii) $(-4)^{-2} = \left(\frac{1}{-4}\right)^2$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

= 1/16

(iii) $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5$

$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

= 2^5

= 32

2. Simplify and express the result in power notation with positive exponent:

(i) $(-4)^5 \div (-4)^8$

(ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $-(3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Solution:

(i) $(-4)^5 \div (-4)^8$

= $(-4)^5 / (-4)^8$
$\left[\because \; a^m \div a^n = a^{m-n}\right]$

= $(-4)^{5-8}$

=$1/(-4)^3$

(ii) $\left(\frac{1}{2^3}\right)^2$

= 1^2 / (2^3)^2

$\left[\because \; \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$

= 1/ 2 ^(3x2) = $1/2^6$

$\left[\because \; (a^m)^n = a^{m\times n}\right]$

(iii)

$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4} \quad \left[\because \; \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$$

= (-1)^4 x 3^4 x (5^4 / 3^4)

$\left[\because \ (ab)^m = a^m b^m\right]$

= 3^(4-4) x 5^4

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

= 3^0 x 5^4   = 5^4

$\left[\because \ a^0 = 1\right]$

iv)

$\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

= $(3^{-7}/ 3^{-10}) \times 3^{-5}$

= $3^{-7-(-10)} \times 3^{-5}$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

= 3^(-7 + 10) x 3 ^(-5)

= $3^3 \times 3^{-5}$

= 3^(3 + (-5))

$\left[\because \ a^m \times a^n = a^{m+n}\right]$

= 3^{-2}

= $1/3^2$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

v) $2^{-3} \times (-7)^{-3}$

$= (2\times -7)^{-3}$

(Because a^m x b^m = (ab)^m)

$= 1 / (2 \times -7)^3$

$$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$$

$= 1/ (-14)^3$

3. Find the value of :

(i) $(3^0 + 4^{-1}) \times 2^2$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{\left(-\frac{2}{3}\right)^{-2}\right\}^2$

Solution:

(i)$(3^0 + 4^{-1}) \times 2^2$ = (1+1/4) × 2^2

$$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$$

= ((4+1)/4) x 2^2

= (5/4) x 2^2

= (5/2^2) x 2^2

= 5 x 2^(2 - 2)

$\left[\because \quad a^m \div a^n = a^{m-n}\right]$

= 5 x 2^0

= 5 x 1 = 5

$\left[\because \quad a^{-m} = \frac{1}{a^m}\right]$

(ii)$(2^{-1} \times 4^{-1}) \div 2^{-2}$

= [(1/2) × (1/4)] ÷ (1/4)

$\left[\because \quad a^{-m} = \frac{1}{a^m}\right]$

= (1/2 x 1/ 2^2) ÷ 1/4

= 1/2 ^3 ÷ 1/4

= (1/8) × (4)
= 1/2

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

= $(2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$

$\left[\because \quad a^{-m} = \frac{1}{a^m}\right]$

= 2^ (-1 x -2) + 3^ (-1 x -2) + 4^ (-1 x -2)

$\left[\because \quad \left(a^m\right)^n = a^{m \times n}\right]$

= $2^2 + 3^2 + 4^2$

= 4 + 9 + 16

= 29

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$
= 1

$\left[\because \; a^0 = 1\right]$

(v) $\left\{\left(-\frac{2}{3}\right)^{-2}\right\}^2 = \left(-\frac{2}{3}\right)^{-2\times 2}$

$\left[\because \; \left(a^m\right)^n = a^{m\times n}\right]$

$= \left(-\frac{2}{3}\right)^{-4}$

$= \left(-\frac{3}{2}\right)^{4}$

$\left[\because \; a^{-m} = \frac{1}{a^m}\right]$

= 81/16

4. Evaluate

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Solution:

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}$

$\frac{8^{-1}\times 5^3}{2^{-4}} = \frac{\left(2^3\right)^{-1}\times 5^3}{2^{-4}} = \frac{2^{-3}\times 5^3}{2^{-4}} \quad \left[\because \; \left(a^m\right)^n = a^{m\times n}\right]$

$= 2^{-3-(-4)} \times 5^3 = 2^{-3+4} \times 5^3 \quad \left[\because \; a^m \div a^n = a^{m-n}\right]$

= 2 x 125 = 250

(ii) $(\mathbf{5^{-1}} \times \mathbf{2^{-1}}) \times \mathbf{6^{-1}}$

$\left(5^{-1} \times 2^{-1}\right) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6} \quad \left[\because \; a^{-m} = \frac{1}{a^m}\right]$

= (1/10) x 1/6

= 1/60

5. Find the value of *m* for which $5^m \div 5^{-3} = 5^5$

Solution:

$5^m \div 5^{-3} = 5^5$

$5^{(m-(-3))} = 5^5$

$\left[\because \; a^m \div a^n = a^{m-n}\right]$

$5^{m+3} = 5^5$

Comparing exponents both sides, we get

m + 3 = 5

m = 5 - 3

m = 2

6. Evaluate

(i)

$\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(ii)

$$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$$

Solution:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\} = \left\{\left(\frac{3}{1}\right)^{1} - \left(\frac{4}{1}\right)^{1}\right\} \quad \left[\because \; a^{-m} = \frac{1}{a^{m}}\right]$$

= 3 - 4

= -1

(ii)

$$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\because \; \left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}\right]$$

$$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because \; a^{m} \div a^{n} = a^{m-n}\right]$$

$$= 5^{-7+4} \times 8^{-4+7}$$

$$= 5^{-3} \times 8^{3} = \frac{8^{3}}{5^{3}} \quad \left[\because \; a^{-m} = \frac{1}{a^{m}}\right]$$

= 512/125

7. Simplify.

(i)

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

Solution:

(i)

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$$

$$= \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$$

$$\left[\because \ a^m \div a^n = a^{m-n}\right]$$

$$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^{3}}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^{3}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$\left[\because \ (ab)^{m} = a^{m}b^{m}\right]$

$\Rightarrow$

$$\frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$\left[\because \ a^{m} \times a^{n} = a^{m+n}\right]$

$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)}$ $\left[\because \ a^{m} \div a^{n} = a^{m-n}\right]$

$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^{0} \times 2^{0} \times 5^{5}$

$= 1 \times 1 \times 3125$ $\left[\because \ a^{0} = 1\right]$

= 3125