

Exercise 14.2

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1. Factorise the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

Solution:

(i) $a^2 + 8a + 16$

$$= a^2 + 2 \times 4 \times a + 4^2$$

$$= (a + 4)^2$$

$$\text{Using identity: } (x + y)^2 = x^2 + 2xy + y^2$$

(ii) $p^2 - 10p + 25$

$$= p^2 - 2 \times 5 \times p + 5^2$$

$$= (p - 5)^2$$

$$\text{Using identity: } (x - y)^2 = x^2 - 2xy + y^2$$

(iii) $25m^2 + 30m + 9$

$$= (5m)^2 + 2 \times 5m \times 3 + 3^2$$

$$= (5m + 3)^2$$

$$\text{Using identity: } (x + y)^2 = x^2 + 2xy + y^2$$

(iv) $49y^2 + 84yz + 36z^2$

$$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$$

$$= (7y + 6z)^2$$

$$\text{Using identity: } (x + y)^2 = x^2 + 2xy + y^2$$

(v) $4x^2 - 8x + 4$

$$= (2x)^2 - 2 \times 2x \times 2 + 2^2$$

$$= (2x - 2)^2$$

$$\text{Using identity: } (x - y)^2 = x^2 - 2xy + y^2$$

$$\begin{aligned} \text{(vi)} \quad & 121b^2 - 88bc + 16c^2 \\ &= (11b)^2 - 2 \times 11b \times 4c + (4c)^2 \\ &= (11b - 4c)^2 \\ &\text{Using identity: } (x - y)^2 = x^2 - 2xy + y^2 \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad & (l + m)^2 - 4lm \text{ (Hint: Expand } (l + m)^2 \text{ first)} \\ &\text{Expand } (l + m)^2 \text{ using identity: } (x + y)^2 = x^2 + 2xy + y^2 \\ &(l + m)^2 - 4lm = l^2 + m^2 + 2ml - 4lm \\ &= l^2 + m^2 - 2ml \\ &= (l - m)^2 \\ &\text{Using identity: } (x - y)^2 = x^2 - 2xy + y^2 \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad & a^4 + 2a^2b^2 + b^4 \\ &= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2 \\ &= (a^2 + b^2)^2 \\ &\text{Using identity: } (x + y)^2 = x^2 + 2xy + y^2 \end{aligned}$$

2. Factorise.

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$ differ

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution:

$$\text{(i)} \quad 4p^2 - 9q^2$$

$$= (2p)^2 - (3q)^2$$

$$= (2p - 3q)(2p + 3q)$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$(ii) \quad 63a^2 - 112b^2$$

$$\begin{aligned} &= 7(9a^2 - 16b^2) \\ &= 7((3a)^2 - (4b)^2) \\ &= 7(3a + 4b)(3a - 4b) \end{aligned}$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$(iii) \quad 49x^2 - 36$$

$$\begin{aligned} &= (7a)^2 - 6^2 \\ &= (7a + 6)(7a - 6) \end{aligned}$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$\begin{aligned} (iv) \quad &16x^5 - 144x^3 \\ &= 16x^3(x^2 - 9) \\ &= 16x^3(x^2 - 9) \\ &= 16x^3(x - 3)(x + 3) \end{aligned}$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$\begin{aligned} (v) \quad &(1 + m)^2 - (1 - m)^2 \\ &= \{(1 + m) - (1 - m)\} \{(1 + m) + (1 - m)\} \end{aligned}$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$\begin{aligned} &= (1 + m - 1 + m)(1 + m + 1 - m) \\ &= (2m)(2l) \\ &= 4ml \end{aligned}$$

$$(vi) \quad 9x^2y^2 - 16$$

$$\begin{aligned} &= (3xy)^2 - 4^2 \\ &= (3xy - 4)(3xy + 4) \end{aligned}$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$(vii) \quad (x^2 - 2xy + y^2) - z^2$$

$$= (x - y)^2 - z^2$$

Using Identity: $(x - y)^2 = x^2 - 2xy + y^2$

$$= \{(x - y) - z\}\{(x - y) + z\}$$

$$= (x - y - z)(x - y + z)$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$$(viii) \quad 25a^2 - 4b^2 + 28bc - 49c^2$$

$$= 25a^2 - (4b^2 - 28bc + 49c^2)$$

$$= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$$

$$= (5a)^2 - (2b - 7c)^2$$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$, we have

$$= (5a + 2b - 7c)(5a - 2b - 7c)$$

3. Factorise the expressions.

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Solution:

(i) $ax^2 + bx = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = m^2(a + b) + n^2(a + b) = (a + b)(m^2 + n^2)$

$$(v) \quad (lm + 1) + m + 1 = lm + m + 1 + 1 = m(1 + 1) + (1 + 1) = (m + 1)(1 + 1)$$

$$(vi) \quad y(y + z) + 9(y + z) = (y + 9)(y + z)$$

$$(vii) \quad 5y^2 - 20y - 8z + 2yz = 5y(y - 4) + 2z(y - 4) = (y - 4)(5y + 2z)$$

$$(viii) \quad 10ab + 4a + 5b + 2 = 5b(2a + 1) + 2(2a + 1) = (2a + 1)(5b + 2)$$

$$(ix) \quad 6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6 = 3x(2y - 3) - 2(2y - 3) = (2y - 3)(3x - 2)$$

4. Factorise.

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Solution:

(i) $a^4 - b^4$

$$\begin{aligned} &= (a^2)^2 - (b^2)^2 \\ &= (a^2 - b^2)(a^2 + b^2) \\ &= (a - b)(a + b)(a^2 + b^2) \end{aligned}$$

(ii) $p^4 - 81$

$$\begin{aligned} &= (p^2)^2 - (9)^2 \\ &= (p^2 - 9)(p^2 + 9) \\ &= (p^2 - 3^2)(p^2 + 9) \\ &= (p - 3)(p + 3)(p^2 + 9) \end{aligned}$$

(iii) $x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$

$$\begin{aligned} &= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\} \\ &= \{(x - (y + z))(x + (y + z))\}\{x^2 + (y + z)^2\} \\ &= (x - y - z)(x + y + z)\{x^2 + (y + z)^2\} \end{aligned}$$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - \{(x - z)^2\}^2$

$$= \{x^2 - (x - z)^2\}\{x^2 + (x - z)^2\}$$

$$= \{x - (x - z)\}\{x + (x - z)\}\{x^2 + (x - z)^2\}$$

$$= z(2x - z)(x^2 + x^2 - 2xz + z^2)$$

$$= z(2x - z)(2x^2 - 2xz + z^2)$$

$$(v) \quad a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$$

$$= (a^2 - b^2)^2$$

$$= ((a - b)(a + b))^2$$

5. Factorise the following expressions.

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Solution:

(i) $p^2 + 6p + 8$

We observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$p^2 + 6p + 8$ can be written as $p^2 + 2p + 4p + 8$

Taking Common terms, we get

$$p^2 + 6p + 8 = p^2 + 2p + 4p + 8 = p(p + 2) + 4(p + 2)$$

Again $p + 2$ is common in both the terms.

$$= (p + 2)(p + 4)$$

This implies: $p^2 + 6p + 8 = (p + 2)(p + 4)$

(ii) $q^2 - 10q + 21$

Observed that, $21 = -7 \times -3$ and $-7 + (-3) = -10$

$$q^2 - 10q + 21 = q^2 - 3q - 7q + 21$$

$$= q(q - 3) - 7(q - 3)$$

$$= (q - 7)(q - 3)$$

This implies $q^2 - 10q + 21 = (q - 7)(q - 3)$

(iii) $p^2 + 6p - 16$

We observed that, $16 = -2 \times 8$ and $8 + (-2) = 6$

$$p^2 + 6p - 16 = p^2 - 2p + 8p - 16$$

$$= p(p - 2) + 8(p - 2)$$

$$= (p + 8)(p - 2)$$

So, $p^2 + 6p - 16 = (p + 8)(p - 2)$

