

Exercise 14.3

1. Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

(i)

$$28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$$

$$56x = 2 \times 2 \times 2 \times 7 \times x$$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii)

$$-36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

(iii)

$$66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

(iv)

$$34x^3y^3z^3 \div 51xy^2z^3 = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2}{3}x^2y$$

(v)

$$12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2a^2b^4$$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3 q^6 - p^6 q^3) \div p^3 q^3$

Solution:

(i) $5x^2 - 6x = x(5x - 6)$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) = 8x^2 y^2 z^2(x + y + z)$

$$8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2 = \frac{8x^2 y^2 z^2(x + y + z)}{4x^2 y^2 z^2} = 2(x + y + z)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^2 + 2x + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

(v) $p^3 q^6 - p^6 q^3 = p^3 q^3(q^3 - p^3)$

$$(p^3 q^6 - p^6 q^3) \div p^3 q^3 = \frac{p^3 q^3(q^3 - p^3)}{p^3 q^3} = q^3 - p^3$$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

$$(iv) 9x^2y^2(3z - 24) \div 27xy(z - 8)$$

$$(v) 96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$$

Solution:

$$(i) (10x - 25) \div 5 = \frac{5(2x-5)}{5} = 2x - 5$$

$$(ii) (10x - 25) \div (2x - 5) = \frac{5(2x-5)}{2x-5} = 5$$

$$(iii) 10y(6y + 21) \div 5(2y + 7) = \frac{10y \times 3(2y+7)}{5(2y+7)} = 6y$$

$$(iv) 9x^2y^2(3z - 24) \div 27xy(z - 8) = \frac{9x^2y^2 \times 3(z-8)}{27xy(z-8)} = xy$$

$$(v) 96 abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6) = \frac{96 abc \times 3(a - 4) \times 5(b - 6)}{144(a - 4)(b - 6)} = 10abc$$

4. Divide as directed.

$$(i) 5(2x + 1)(3x + 5) \div (2x + 1)$$

$$(ii) 26xy(x + 5)(y - 4) \div 13x(y - 4)$$

$$(iii) 52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$$

$$(iv) 20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$$

$$(v) x(x + 1)(x + 2)(x + 3) \div x(x + 1)$$

Solution:

$$(i) \ 5(2x + 1)(3x + 5) \div (2x + 1) = \frac{5(2x + 1)(3x + 1)}{(2x + 1)}$$

$$= 5(3x + 1)$$

$$(ii) \ 26xy(x + 5)(y - 4) \div 13x(y - 4) = \frac{2 \times 13 \times xy(x + 5)(y - 4)}{13x(y - 4)}$$

$$= 2y(x + 5)$$

$$(iii) \ 52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$$

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p + q) \times (q + r) \times (r + p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q + r) \times (r + p)}$$

$$= \frac{1}{2}r(p + q)$$

$$(iv) \ 20(y + 4)(y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)(y^2 + 5y + 3)$$

$$20(y + 4)(y^2 + 5y + 3) \div 5(y + 4) = \frac{2 \times 2 \times 5 \times (y + 4) \times (y^2 + 5y + 3)}{5 \times (y + 4)}$$

$$= 4(y^2 + 5y + 3)$$

$$(v) \ x(x + 1)(x + 2)(x + 3) \div x(x + 1) = \frac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)}$$

$$= (x + 2)(x + 3)$$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Solution:

(i) $(y^2 + 7y + 10) \div (y + 5)$

First solve for equation, $(y^2 + 7y + 10)$

$$(y^2 + 7y + 10) = y^2 + 2y + 5y + 10 = y(y + 2) + 5(y + 2) = (y + 2)(y + 5)$$

$$\text{Now, } (y^2 + 7y + 10) \div (y + 5) = (y + 2)(y + 5) / (y + 5) = y + 2$$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

Solve for $m^2 - 14m - 32$, we have

$$m^2 - 14m - 32 = m^2 + 2m - 16m - 32 = m(m + 2) - 16(m + 2) = (m - 16)(m + 2)$$

$$\text{Now, } (m^2 - 14m - 32) \div (m + 2) = (m - 16)(m + 2) / (m + 2) = m - 16$$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

Step 1: Take 5 common from the equation, $5p^2 - 25p + 20$, we get

$$5p^2 - 25p + 20 = 5(p^2 - 5p + 4)$$

Step 2: Factorize $p^2 - 5p + 4$

$$p^2 - 5p + 4 = p^2 - p - 4p + 4 = (p - 1)(p - 4)$$

Step 3: Solve original equation

$$(5p^2 - 25p + 20) \div (p - 1) = 5(p - 1)(p - 4) / (p - 1) = 5(p - 4)$$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

Factorize $z^2 + 6z - 16$,

$$z^2 + 6z - 16 = z^2 - 2z + 8z - 16 = (z - 2)(z + 8)$$

$$\text{Now, } 4yz(z^2 + 6z - 16) \div 2y(z + 8) = 4yz(z - 2)(z + 8) / 2y(z + 8) = 2z(z - 2)$$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

$p^2 - q^2$ can be written as $(p - q)(p + q)$ using identity.

$$5pq(p^2 - q^2) \div 2p(p + q) = 5pq(p - q)(p + q) / 2p(p + q) = 5/2 q (p - q)$$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

Factorize $9x^2 - 16y^2$, we have

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x + 4y)(3x - 4y) \text{ using identity: } p^2 - q^2 = (p - q)(p + q)$$

$$\text{Now, } 12xy(9x^2 - 16y^2) \div 4xy(3x + 4y) = 12xy(3x + 4y)(3x - 4y) / 4xy(3x + 4y) = 3(3x - 4y)$$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

First solve for $50y^2 - 98$, we have

$$50y^2 - 98 = 2(25y^2 - 49) = 2((5y)^2 - 7^2) = 2(5y - 7)(5y + 7)$$

$$\text{Now, } 39y^3(50y^2 - 98) \div 26y^2(5y + 7) = \frac{3 \times 13 \times y^3 \times 2(5y - 7)(5y + 7)}{2 \times 13 \times y^2(5y + 7)} = 3y(5y - 7)$$