Exercise 14.1 Page No: 208

1. Find the common factors of the given terms.
(i) 12x, 36
(ii) 2y, 22xy
(iii) 14 pq, 28 p^2 q^2
(iv) 2x, $3x^2$, 4
(v) 6 abc, $24ab^2$, 12 a^2 b
(vi) 16 x^3, - $4x^2$, 32 x
(vii) 10 pq, 20qr, 30 rp
(viii) $3x^2$ y^3 , $10x^3$ y^2 , 6 x^2 y^2 z

Solution:
(i) Factors of 12x and 36
12x = 2 × 2 × 3 × x
36 = 2 × 2 × 3 × 3
Common factors of 12x and 36 are 2, 2, 3
and , 2 × 2 × 3 = 12

(ii) Factors of 2y and 22xy
2y = 2 x y
22xy = 2 × 11 × x × y
Common factors of 2y and 22xy are 2, y
and , 2 x y = 2y

(iii) Factors of 14 pq and 28 p^2 q
14 pq = 2 x 7 x p x q
28 p^2 q = 2 x 2 x 7 x p x p x q

Common factors of 14 pq and 28 p^2 q are 2, 7 , p , q
and, 2 x 7 x p x q = 14pq

(iv) Factors of 2x, $3x^2$ and 4
2x = 2 × x
$3x^2$ = 3 × x × x
4 = 2 x 2
Common factors of 2x, $3x^2$ and 4 is 1.

(v) Factors of 6 abc, $24ab^2$ and 12 a^2 b

6 abc = 2 × 3 × a × b × c
$24ab^2$ = 2 × 2 × 2 × 3 × a × b × b
12 a^2 b = 2 × 2 × 3 × a × a × b

Common factors of 6 abc, $24ab^2$ and 12 a^2b are 2, 3, a, b
and, 2 × 3 × a × b = 6ab

(vi) Factors of 16 x^3 , - $4x^2$ and 32 x

16 x^3 = 2 × 2 × 2 × 2 × x × x × x
- $4x^2$ = -1 × 2 × 2 × x × x
32 x = 2 × 2 × 2 × 2 × 2 × x
Common factors of 16 x^3 , - $4x^2$ and 32 x are 2,2, x
and, 2 × 2 × x = 4x

(vii) Factors of 10 pq, 20qr and 30 rp

10 pq = 2 × 5 × p × q
20qr = 2 × 2 × 5 × q × r
30 rp = 2 × 3 × 5 × r × p

Common factors of 10 pq, 20qr and 30 rp are 2, 5
and, 2 × 5 = 10

(viii) Factors of $3x^2 y^3$, $10x^3 y^2$ and 6 $x^2 y^2$ z

$3x^2 y^3$ = 3 × x × x × y × y × y
$10x^3 y^2$ = 2 × 5 × x × x × x × y × y
6 $x^2 y^2$ z = 3 × 2 × x × x × y × y × z

Common factors of $3x^2 y^3$, $10x^3 y^2$ and 6 $x^2 y^2$ z are x^2 , y^2
and, $x^2 \times y^2 = x^2 y^2$

2. Factorise the following expressions

(i) 7x – 42
(ii) 6p – 12q
(iii) $7a^2 + 14a$
(iv) $- 16 z + 20 z^3$
(v) $20 l^2 m + 30 a l m$
(vi) $5 x^2 y - 15 xy^2$
(vii) $10 a^2 - 15 b^2 + 20 c^2$
(viii) $- 4 a^2 + 4 ab – 4 ca$
(ix) $x^2 y z + x y^2 z + x y z^2$
(x) $a x^2 y + b x y^2 + c x y z$

Solution:

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7

$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3

$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$

$= 2 \times 3\ [p - (2 \times q)]$

$= 6(p - 2q)$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a

$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$

$= 7 \times a\ [a + 2] = 7a\ (a + 2)$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$

$= (2 \times 2 \times z)\ [-(2 \times 2) + (5 \times z \times z)]$

$= 4z(-4 + 5z^2)$

(v) $20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30alm = 2 \times 3 \times 5 \times a \times l \times m$

The common factors are 2, 5, l and m

$\therefore\ 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$

$= (2 \times 5 \times l \times m)[(2 \times l) + (3 \times a)]$

$= 10lm(2l + 3a)$

(vi) $5x^2y = 5 \times x \times x \times y$

$15xy^2 = 3 \times 5 \times x \times y \times y$

The common factors are 5, x, and y

$\therefore\ 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$

$= 5 \times x \times y[x - (3 \times y)]$

$= 5xy(x - 3y)$

(vii) $10a^2 - 15b^2 + 20c^2$

$10a^2 = 2 \times 5 \times a \times a$

$-15b^2 = -1 \times 3 \times 5 \times b \times b$

$20c^2 = 2 \times 2 \times 5 \times c \times c$

Common factor of $10a^2$, $15b^2$ and $20c^2$ is 5

$10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$

(viii) $-4a^2 + 4ab - 4ca$

$-4a^2 = -1 \times 2 \times 2 \times a \times a$

$4ab = 2 \times 2 \times a \times b$

$-4ca = -1 \times 2 \times 2 \times c \times a$

Common factor of $-4a^2$, $4ab$, $-4ca$ are 2, 2, a i.e. 4a

So,

$-4a^2 + 4ab - 4ca = 4a(-a + b - c)$

(ix) $x^2yz + xy^2z + xyz^2$

$x^2yz = x \times x \times y \times z$

$xy^2z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

Common factor of x^2yz, xy^2z and xyz^2 are x, y, z i.e. xyz

Now, $x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

(x) $ax^2y + bxy^2 + cxyz$

$ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

Common factors of ax^2y, bxy^2 and $cxyz$ are xy

Now, $ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$

3. Factorise.
(i) $x^2 + xy + 8x + 8y$
(ii) $15xy - 6x + 5y - 2$
(iii) $ax + bx - ay - by$
(iv) $15pq + 15 + 9q + 25p$
(v) $z - 7 + 7xy - xyz$

Solution:

(i) $x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$

$= x(x + y) + 8(x + y)$

$= (x + y)(x + 8)$

(ii) $15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5xy - 2$

$= 3x(5y - 2) + 1(5y - 2)$

$= (5y - 2)(3x + 1)$

(iii) $ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$

$= x(a + b) - y(a + b)$

$= (a + b)(x - y)$

(iv) $15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$

$= 3q(5p + 3) + 5(5p + 3)$

$= (5p + 3)(3q + 5)$

(v) $z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$

$= z(1 - xy) - 7(1 - xy)$

$= (1 - xy)(z - 7)$

Exercise 14.2

1. Factorise the following expressions.
(i) $a^2 + 8a + 16$
(ii) $p^2 – 10 p + 25$
(iii) $25m^2 + 30m + 9$
(iv) $49y^2 + 84yz + 36z^2$
(v) $4x^2 – 8x + 4$
(vi) $121b^2 – 88bc + 16c^2$
(vii) $(l + m)^2 – 4lm$ (Hint: Expand $(l + m)^2$ first)
(viii) $a^4 + 2a^2b^2 + b^4$

Solution:

(i) $a^2 + 8a + 16$
$= a^2 + 2 \times 4 \times a + 4^2$
$= (a + 4)^2$
$Using\ identity: (x + y)^2 = x^2 + 2xy + y^2$

(ii) $p^2 – 10 p + 25$
$= p^2 - 2 \times 5 \times p + 5^2$
$= (p - 5)^2$
$Using\ identity: (x - y)^2 = x^2 - 2xy + y^2$

(iii) $25m^2 + 30m + 9$
$= (5m)^2 - 2 \times 5m \times 3 + 3^2$
$= (5m + 3)^2$
$Using\ identity: (x + y)^2 = x^2 + 2xy + y^2$

(iv) $49y^2 + 84yz + 36z^2$
$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$
$= (7y + 6z)^2$
$Using\ identity: (x + y)^2 = x^2 + 2xy + y^2$

(v) $4x^2 – 8x + 4$
$= (2x)^2 - 2 \times 4x + 2^2$
$= (2x - 2)^2$
$Using\ identity: (x - y)^2 = x^2 - 2xy + y^2$

(vi) $121b^2 - 88bc + 16c^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$= (11b - 4c)^2$

$Using\ identity: (x-y)^2 = x^2 - 2xy + y^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

Expand $(l + m)^2$ $using\ identity: (x+y)^2 = x^2 + 2xy + y^2$

$(l + m)^2 - 4lm = l^2 + m^2 + 2ml - 4ml$

$= l^2 + m^2 - 2ml$

$= (l - m)^2$

$Using\ identity: (x-y)^2 = x^2 - 2xy + y^2$

(viii) $a^4 + 2a^2b^2 + b^4$

$= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$

$= (a^2 + b^2)^2$

$Using\ identity: (x+y)^2 = x^2 + 2xy + y^2$

2. Factorise.

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$ differ

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2 y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution:

(i) $4p^2 - 9q^2$

$= (2p)^2 - (3q)^2$

$= (2p - 3q)(2p + 3q)$

Using Identity: $x^2 - y^2 = (x+y)(x-y)$

(ii) $63a^2 - 112b^2$

$= 7(9a^2 - 16b^2)$
$= 7((3a)^2 - (4b)^2)$
$= 7(3a + 4b)(3a - 4b)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(iii) $49x^2 - 36$

$= (7a)^2 - 6^2)$

$= (7a + 6)(7a - 6)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(iv) $16x^5 - 144x^3$
$= 16x^3(x^2 - 9)$

$= 16x^3(x^2 - 9)$

$= 16x^3(x - 3)(x + 3)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(v) $(l + m)^2 - (l - m)^2$

$= \{(l + m) - (l - m)\}\{(l + m) + (l - m)\}$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

$= (l + m - l + m)(l + m + l - m)$

$= (2m)(2l)$

$= 4ml$

(vi) $9x^2y^2 - 16$

$= (3xy)^2 - 4^2$

$= (3xy - 4)(3xy + 4)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(vii) $(x^2 - 2xy + y^2) - z^2$

$= (x - y)^2 - z^2$

Using Identity: $(x - y)^2 = x^2 - 2xy + y^2$

$= \{(x - y) - z\}\{(x - y) + z\}$

$= (x - y - z)(x - y + z)$

Using Identity: $x^2 - y^2 = (x + y)(x - y)$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$

$= (5a)^2 - (2b - 7c)^2$

Using Identity: $x^2 - y^2 = (x + y)(x - y), we\ have$

$= (5a + 2b - 7c)(5a - 2b - 7c)$

3. Factorise the expressions.
(i) $ax^2 + bx$
(ii) $7p^2 + 21q^2$
(iii) $2x^3 + 2xy^2 + 2xz^2$
(iv) $am^2 + bm^2 + bn^2 + an^2$
(v) $(lm + l) + m + 1$
(vi) $y(y + z) + 9(y + z)$
(vii) $5y^2 - 20y - 8z + 2yz$
(viii) $10ab + 4a + 5b + 2$
(ix) $6xy - 4y + 6 - 9x$

Solution:

(i) $ax^2 + bx = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = m^2(a + b) + n^2(a + b) = (a + b)(m^2 + n^2)$

(v) (lm + l) + m + 1 = lm + m + l + 1 = m(l + 1) + (l + 1) = (m + 1) (l + 1)

(vi) y (y + z) + 9 (y + z) = (y + 9) (y + z)

(vii) $5y^2 – 20y – 8z + 2yz = 5y (y – 4) + 2z (y – 4) = (y – 4)(5y + 2z)$

(viii) 10ab + 4a + 5b + 2 = 5b (2a +1) + 2 (2a + 1) = (2a + 1) (5b + 2)

(ix) 6xy – 4y + 6 – 9x = 6xy – 9x – 4y + 6 = 3x (2y – 3) – 2(2y – 3) = (2y – 3) (3x – 2)

4. Factorise.
(i) $a^4 – b^4$
(ii) $p^4 – 81$
(iii) $x^4 – (y + z)^4$
(iv) $x^4 – (x – z)^4$
(v) $a^4 – 2a^2b^2 + b^4$

Solution:

(i) $a^4 – b^4$

$= (a^2)^2 - (b^2)^2$
$= (a^2 - b^2)(a^2 + b^2)$
$= (a – b)(a + b)\ (a^2 + b^2)$

(ii) $p^4 – 81$

$= (p^2)^2 - (9)^2$
$= (p^2 - 9)(p^2 + 9)$
$= (p^2 - 3^2)(p^2 + 9)$
$= (p - 3)(p + 3)\ (p^2 + 9)$

(iii) $x^4 – (y + z)^4 = (x^2)^2 - [(y + z)^2]$^2

$= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\}$

$= \{(x – (y + z)\ (x + (y + z)\}\ \{x^2 + (y + z)^2\}$

$= (x – y – z)(x + y + z)\ \{x^2 + (y + z)^2\}$

(iv) $x^4 – (x – z)^4 = (x^2)^2 - \{(x - 2)^2\}^2$

$= \{x^2 - (x - z)^2\}\{x^2 + (x - z)^2\}$

$= \{x-(x-z)\}\{x+(x-z)\}\{x^2+(x-z)^2\}$

$= z(2x-z)(x^2+x^2-2xz+z^2)$

$= z(2x-z)(2x^2-2xz+z^2)$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$

$= (a^2-b^2)^2$

$= ((a - b)(a+b))^2$

5. Factorise the following expressions.

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Solution:

(i) $p^2 + 6p + 8$

We observed that, $8 = 4 \times 2$ and $4 + 2 = 6$

$p^2 + 6p + 8$ can be written as $p^2 + 2p + 4p + 8$
Taking Common terms, we get

$p^2 + 6p + 8 = p^2 + 2p + 4p + 8 = p(p + 2) + 4(p + 2)$

Again p + 2 is common in both the terms.

$= (p + 2)(p + 4)$

This implies: $p^2 + 6p + 8 = (p + 2)(p + 4)$

(ii) $q^2 - 10q + 21$
Observed that, $21 = -7 \times -3$ and $-7 + (-3) = -10$

$q^2 - 10q + 21 = q^2 - 3q - 7q + 21$

$= q(q - 3) - 7(q - 3)$

$= (q – 7) (q – 3)$

This implies $q^2 – 10q + 21 = (q – 7) (q – 3)$

(iii) $p^2 + 6p – 16$

We observed that, $16 = -2 \times 8$ and $8 + (-2) = 6$

$p^2 + 6p – 16 = p^2 – 2 p + 8 p – 16$

$= p (p – 2) + 8(p – 2)$

$= (p + 8) (p – 2)$

So, $p^2 + 6p – 16 = (p + 8) (p – 2)$

Exercise 14.3 Page No: 227

1. Carry out the following divisions.

(i) $28x^4 \div 56x$
(ii) $-36y^3 \div 9y^2$
(iii) $66pq^2r^3 \div 11qr^2$
(iv) $34x^3y^3z^3 \div 51xy^2z^3$
(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

(i)
$28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$
$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

(ii) $$-36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

(iii) $$66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

(iv) $$34x^3y^3z^3 \div 51xy^2z^3 = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = 2/3\ x^2\ y$$

(v)
$$12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2\ a^2\ b^4$$

2. Divide the given polynomial by the given monomial.

(i)$(5x^2 - 6x) \div 3x$

(ii)$(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2$

(iv)$(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 – p^6q^3) \div p^3 q^3$

Solution:

$(i)\ 5x^2 - 6x = x(5x - 6)$

$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$

$(ii)\ 3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$

$(iii)\ 8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$

$(iv)\ x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^3 + 2x^2 + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$

$(v)\ p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$

3. Work out the following divisions.

(i) (10x – 25) ÷ 5

(ii) (10x – 25) ÷ (2x – 5)

(iii) 10y(6y + 21) ÷ 5(2y + 7)

(iv) $9x^2y^2(3z – 24) \div 27xy(z – 8)$

(v) 96abc(3a – 12) (5b – 30) ÷ 144(a – 4) (b – 6)

Solution:

(i) (10x – 25) ÷ 5 = $\frac{5(2x-5)}{5} = 2x - 5$

(ii) (10x – 25) ÷ (2x – 5) = $\frac{5(2x-5)}{2x-5}$ = 5

(iii) 10y(6y + 21) ÷ 5(2y + 7) = $\frac{10y\times3(2y+7)}{5(2y+7)}$ = 6y

(iv) $9x^2y^2$(3z – 24) ÷ 27xy(z – 8) = $\frac{9x^2y^2 \times3(z-8)}{27xy(z-8)}$ = xy

(v) 96 abc(3a – 12) (5b – 30) ÷ 144(a – 4) (b – 6) = $\frac{96\ abc \times3(a-4)\times5(b-6)}{144(a-4)\ (b-6)}$ = 10abc

4. Divide as directed.

(i) 5(2x + 1) (3x + 5) ÷ (2x + 1)

(ii) 26xy(x + 5)(y – 4) ÷ 13x(y – 4)

(iii) 52pqr (p + q) (q + r) (r + p) ÷ 104pq(q + r) (r + p)

(iv) 20(y + 4) (y2 + 5y + 3) ÷ 5(y + 4)

(v) x(x + 1) (x + 2) (x + 3) ÷ x(x + 1)

Solution:

(i) $5(2x+1)(3x+5) \div (2x+1) = \dfrac{5(2x+1)(3x+1)}{(2x+1)}$

$= 5(3x+1)$

(ii) $26xy\ (x+5)(y-4) \div 13x\ (y-4) = \dfrac{2 \times 13 \times xy(x+5)(y-4}{13x(y-4)}$

$= 2y(x+5)$

(iii) $52\ pqr\ (p+q)(q+r)(r+p) \div 104\ pq\ (q+r)(r+p)$

$= \dfrac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$

$= \dfrac{1}{2} r(p+q)$

(iv) $20\ (y+4)(y^2+5y+3) = 2 \times 2 \times 5 \times (y+4)(y^2+5y+3)$

$20\ (y+4)(y^2+5y+3) \div 5\ (y+4) = \dfrac{2 \times 2 \times 5 \times (y+4) \times (y^2+5y+3)}{5 \times (y+4)}$

$= 4(y^2+5y+3)$

(v) $x\ (x+1)(x+2)(x+3) \div x\ (x+1) = \dfrac{x(x+1)(x+2)(x+3)}{x(x+1)}$

$= (x+2)(x+3)$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 – 14m – 32) \div (m + 2)$

(iii) $(5p^2 – 25p + 20) \div (p – 1)$

(iv) $4yz(z^2 + 6z – 16) \div 2y(z + 8)$

(v) $5pq(p^2 – q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 – 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 – 98) \div 26y^2(5y + 7)$

Solution:

(i) **$(y^2 + 7y + 10) \div (y + 5)$**

First solve for equation, $(y^2 + 7y + 10)$
$(y^2 + 7y + 10) = y^2 + 2y + 5y + 10 = y (y + 2) + 5(y + 2) = (y + 2)(y + 5)$

Now, $(y^2 + 7y + 10) \div (y + 5) = (y + 2)(y + 5) / (y + 5) = y + 2$

(ii) **$(m^2 – 14m – 32) \div (m + 2)$**

Solve for $m^2 – 14m – 32$, we have

$m^2 – 14m – 32 = m^2 + 2m - 16m – 32 = m(m + 2) – 16(m + 2) = (m – 16)(m + 2)$

Now, $(m^2 – 14m – 32) \div (m + 2) = (m – 16)(m + 2) /)(m + 2) = m - 16$

(iii) **$(5p^2 – 25p + 20) \div (p – 1)$**

Step 1: Take 5 common from the equation, $5p^2 – 25p + 20$, we get

$5p^2 – 25p + 20 = 5(p^2 – 5p + 4)$

Step 2: Factorize $p^2 – 5p + 4$

$p^2 – 5p + 4 = p^2 – p - 4p + 4 = (p – 1) (p – 4)$

Step 3: Solve original equation

$(5p^2 – 25p + 20) \div (p – 1) = 5(p – 1) (p – 4) / (p-1) = 5(p – 4)$

(iv) **$4yz(z^2 + 6z – 16) \div 2y(z + 8)$**

Factorize $z^2 + 6z – 16$,

$z^2 + 6z – 16 = z^2 - 2z + 8z – 16 = (z – 2)(z + 8)$

Now, $4yz(z^2 + 6z – 16) \div 2y(z + 8) = 4yz(z – 2)(z + 8) / 2y(z + 8 = 2z(z - 2)$

(v) $5pq(p^2 – q^2) ÷ 2p(p + q)$

$p^2 – q^2$ can be written as $(p – q)(p + q)$ using identity.

$5pq(p^2 – q^2) ÷ 2p(p + q) = 5pq(p – q)(p + q) / 2p(p + q) = 5/2\ q\ (p – q)$

(vi) $12xy(9x^2 – 16y^2) ÷ 4xy(3x + 4y)$

Factorize $9x^2 – 16y^2$, we have

$9x^2 – 16y^2 = (3x)^2 – (4y)^2 = (3x + 4y)(3x - 4y)$ using identity: $p^2 – q^2 = (p – q)(p + q)$

Now, $12xy(9x^2 – 16y^2) ÷ 4xy(3x + 4y) = 12xy(3x + 4y)(3x - 4y) / 4xy(3x + 4y) = 3(3x - 4y)$

(vii) $39y^3(50y^2 – 98) ÷ 26y^2(5y + 7)$

First solve for $50y^2 – 98$, we have

$50y^2 – 98 = 2(25y^2 – 49) = 2((5y)^2 – 7^2) = 2(5y – 7)(5y + 7)$

Now, $39y^3(50y^2 – 98) ÷ 26y^2(5y + 7) = \frac{3 \times 13 \times y^3 \times 2(5y - 7)(5y + 7)}{2 \times 13 \times y^2 (5y+7)} = 3y(5y – 7)$

Exercise 14.4 Page No: 228

Find and correct the errors in the following mathematical statements.

1. **$4(x - 5) = 4x - 5$**

Solution:
$4(x - 5) = 4x - 20 \neq 4x - 5$ = RHS

The correct statement is $4(x - 5) = 4x - 20$

2. **$x(3x + 2) = 3x^2 + 2$**

Solution:

LHS = $x(3x + 2) = 3x^2 + 2x \neq 3x^2 + 2$ = RHS

The correct solution is $x(3x + 2) = 3x^2 + 2x$

3. **$2x + 3y = 5xy$**

Solution:

LHS = $2x + 3y \neq$ R. H. S

The correct statement is $2x + 3y = 2x + 3y$

4. **$x + 2x + 3x = 5x$**

Solution:

LHS = $x + 2x + 3x = 6x \neq$ RHS

The correct statement is $x + 2x + 3x = 6x$

5. **$5y + 2y + y - 7y = 0$**

Solution:

LHS = $5y + 2y + y - 7y = y \neq$ RHS

The correct statement is $5y + 2y + y - 7y = y$

6. $3x + 2x = 5x^2$

Solution:
LHS = $3x + 2x = 5x \neq$ RHS

The correct statement is $3x + 2x = 5x$

7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$
Solution:
LHS = $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7 \neq$ RHS

The correct statement is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

8. $(2x)^2 + 5x = 4x + 5x = 9x$
Solution:
LHS = $(2x)^2 + 5x = 4x^2 + 5x \neq 9x$ = RHS

The correct statement is $(2x)^2 + 5x = 4x^2 + 5x$

9. $(3x + 2)^2 = 3x^2 + 6x + 4$
Solution:
LHS = $(3x + 2)^2 = (3x)^2 + 2^2 + 2 \times 2 \times 3x = 9x^2 + 4 + 12x \neq$ RHS

The correct statement is $(3x + 2)^2 = 9x^2 + 4 + 12x$

10. Substituting $x = -3$ in
(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$
(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$
(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

Solution:
(a) Substituting $x = -3$ in $x^2 + 5x + 4$, we have

$x^2 + 5x + 4 = (-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$. This is the correct answer.

(b) Substituting $x = -3$ in $x^2 - 5x + 4$

$x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$. This is the correct answer

(c) Substituting $x = -3$ in $x^2 + 5x$

$x^2 + 5x = (-3)^2 + 5(-3) = 9 - 15 = -6$. This is the correct answer

11. $(y - 3)^2 = y^2 - 9$

Solution:

LHS = $(y - 3)^2$, which is similar to $(a - b)^2$ identity, where $(a - b)^2 = a^2 + b^2 - 2ab$.

$(y - 3)^2 = y^2 + (3)^2 - 2y \times 3 = y^2 + 9 - 6y \neq y^2 - 9$ = RHS

The correct statement is $(y - 3)^2 = y^2 + 9 - 6y$

12. $(z + 5)^2 = z^2 + 25$

Solution:

LHS = $(z + 5)^2$, which is similar to $(a - b)^2$ identity, where $(a + b)^2 = a^2 + b^2 + 2ab$.

$(z + 5)^2 = z^2 + 5^2 + 2\times5 \times z = z^2 + 25 + 10z \neq z^2 + 25$ = RHS

The correct statement is $(z + 5)^2 = z^2 + 25 + 10z$

13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$

Solution:
LHS = $(2a + 3b)(a - b) = 2a(a - b) + 3b(a - b)$
$= 2a^2 - 2ab + 3ab - 3b^2$
$= 2a^2 + ab - 3b^2$
$\neq 2a^2 - 3b^2$ = RHS

The correct statement is $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$

14. $(a + 4)(a + 2) = a^2 + 8$

Solution:

LHS = $(a + 4)(a + 2) = a(a + 2) + 4(a + 2)$
$= a^2 + 2a + 4a + 8$
$= a^2 + 6a + 8$
$\neq a^2 + 8$ = RHS

The correct statement is $(a + 4)(a + 2) = a^2 + 6a + 8$

15. (a – 4) (a – 2) = a^2 – 8

Solution:

LHS = (a – 4) (a – 2) = a (a – 2) – 4 (a – 2)
= a^2 – 2a – 4a + 8
= a^2 – 6a + 8
≠ a^2 - 8 = RHS

The correct statement is (a – 4) (a – 2) = a^2 – 6a + 8

16. $\frac{3x^2}{3x^2}$ = 0

Solution:

LHS = $\frac{3x^2}{3x^2} = 1$ ≠ 0 = RHS

The correct statement is $\frac{3x^2}{3x^2}$ = 1

17. $\frac{3x^2+1}{3x^2}$ = 1 + 1 = 2

Solution:

LHS = $\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2}$ ≠ 2 = RHS

The correct statement is $\frac{3x^2+1}{3x^2}$ = 1 + $\frac{1}{3x^2}$

18. $\frac{3x}{3x+2} = \frac{1}{2}$

Solution:

LHS = $\frac{3x}{3x+2}$ ≠ 1/2 = RHS

The correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

19. $\frac{3}{4x+3} = \frac{1}{4x}$

Solution:

LHS = $\frac{3}{4x+3} \neq \frac{1}{4x}$

The correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$

20. $\frac{4x+5}{4x}$ **= 5**

Solution:

LHS = $\frac{4x+5}{4x}$ = 4x/4x + 5/4x = 1 + $\frac{5}{4x} \neq 5$ = RHS

The correct statement is $\frac{4x+5}{4x}$ = 1 + $\frac{5}{4x}$

21. $\frac{7x+5}{5}$ **= 7x**

Solution:

LHS = $\frac{7x+5}{5}$ = 7x/5 + 5/5 = $\frac{7x}{5} + 1 \neq$ 7x = RHS

The correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$