Exercise 1.2

1. Express each number as a product of its prime factors:

(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429

Solutions:

(i) 140

By Taking the LCM of 140, we will get the product of its prime factor.
Therefore, 140 = $2 \times 2 \times 5 \times 7 \times 1 = 2^2 \times 5 \times 7$

(ii) 156
By Taking the LCM of 156, we will get the product of its prime factor.
Hence, 156 = $2 \times 2 \times 13 \times 3 \times 1 = 2^2 \times 13 \times 3$

(iii) 3825
By Taking the LCM of 3825, we will get the product of its prime factor.
Hence, $3825 = 3 \times 3 \times 5 \times 5 \times 17 \times 1 = 3^2 \times 5^2 \times 17$

(iv) 5005
By Taking the LCM of 5005, we will get the product of its prime factor.
Hence, $5005 = 5 \times 7 \times 11 \times 13 \times 1 = 5 \times 7 \times 11 \times 13$

(v) 7429
By Taking the LCM of 7429, we will get the product of its prime factor.
Hence, $7429 = 17 \times 19 \times 23 \times 1 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
(i) 26 and 91
(ii) 510 and 92
(iii) 336 and 54

Solutions:

(i) 26 and 91

Expressing 26 and 91 as product of its prime factors, we get,
26 = 2 × 13 × 1

91 = 7 × 13 × 1

Therefore, LCM (26, 91) = 2 × 7 × 13 × 1 = 182

And HCF (26, 91) = 13
Verification
Now, product of 26 and 91 = 26 × 91 = 2366
And Product of LCM and HCF = 182 × 13 = 2366
Hence, LCM × HCF = product of the 26 and 91.

(ii) 510 and 92

Expressing 510 and 92 as product of its prime factors, we get,
510 = 2 × 3 × 17 × 5 × 1
92 = 2 × 2 × 23 × 1
Therefore, LCM(510, 92) = 2 × 2 × 3 × 5 × 17 × 23 = 23460
And HCF (510, 92) = 2
Verification
Now, product of 510 and 92 = 510 × 92 = 46920
And Product of LCM and HCF = 23460 × 2 = 46920
Hence, LCM × HCF = product of the 510 and 92.

(iii) 336 and 54

Expressing 336 and 54 as product of its prime factors, we get,
336 = 2 × 2 × 2 × 2 × 7 × 3 × 1
54 = 2 × 3 × 3 × 3 × 1
Therefore, LCM(336, 54) = $2^4 \times 3^3 \times 7$ = 3024
And HCF(336, 54) = 2×3 = 6
Verification
Now, product of 336 and 54 = 336 × 54 = 18,144
And Product of LCM and HCF = 3024 × 6 = 18,144
Hence, LCM × HCF = product of the 336 and 54.

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21
(ii) 17, 23 and 29
(iii) 8, 9 and 25

Solutions:
(i) 12, 15 and 21

Writing the product of prime factors for all the three numbers, we get,
12=2×2×3
15=5×3
21=7×3

Therefore,

HCF(12,15,21) = 3

LCM(12,15,21) = $2 \times 2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

Writing the product of prime factors for all the three numbers, we get,

$17=17\times1$

$23=23\times1$

$29=29\times1$

Therefore,

HCF(17,23,29) = 1

LCM(17,23,29) = $17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25

Writing the product of prime factors for all the three numbers, we get,

$8=2\times2\times2\times1$

$9=3\times3\times1$

$25=5\times5\times1$

Therefore,

HCF(8,9,25)=1

LCM(8,9,25) = $2\times2\times2\times3\times3\times5\times5 = 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solutions: As we know that,

HCF×LCM=Product of the two given numbers

Therefore,

9 × LCM = 306 × 657

LCM = $\frac{306\times657}{9}$ = 22338

Hence, LCM(306,657) = 22338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solutions: If the number 6^n ends with the digit zero (0), then it should be divisible by 5, as we know any number with unit place as 0 or 5 is divisible by 5.

Prime factorization of $6^n = (2 \times 3)^n$

Therefore, the prime factorization of 6^n doesn't contain prime number 5.

Hence, it is clear that for any natural number n, 6^n is not divisible by 5 and thus it proves that 6^n cannot end with the digit 0 for any natural number n.

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Solutions: By the definition of composite number, we know, if a number is composite, then it means it has factors other than 1 and itself. Therefore, for the given expression;
7 × 11 × 13 + 13

Taking 13 as common factor, we get,
=13(7×11×1+1) = 13(77+1) = 13×78 = 13×3×2×13

Hence, 7 × 11 × 13 + 13 is a composite number.

Now let's take the other number,
7 × 6 × 5 × 4 × 3 × 2 × 1 + 5

Taking 5 as a common factor, we get,
=5(7×6×4×3×2×1+1) = 5(1008+1) = 5×1009

Hence, 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solutions: Since, Both Sonia and Ravi move in the same direction and at the same time, the method to find the time when they will be meeting again at the starting point is LCM of 18 and 12.

Therefore, LCM(18,12) = $2\times3\times3\times2\times1=36$

Hence, Sonia and Ravi will meet again at the starting point after 36 minutes.

Exercise 1.3

1. Prove that $\sqrt{5}$ is irrational.

Solutions: Let us assume, that $\sqrt{5}$ is rational number.
i.e. $\sqrt{5} = \frac{x}{y}$ (where, x and y are co-primes)
$y\sqrt{5} = x$

Squaring both the sides, we get,
$(y\sqrt{5})^2 = x^2$
$\Rightarrow 5y^2 = x^2$……………………………….. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,
$5y^2 = (5k)^2$
$\Rightarrow y^2 = 5k^2$

$\boldsymbol{y^2}$ is divisible by 5 it means y is divisible by 5.

Therefore, x and y are co-primes. Since, our assumption about $\sqrt{5}$ is rational is incorrect.

Hence, $\sqrt{5}$ is irrational number.

2. Prove that 3 + 2$\sqrt{5}$ + is irrational.

Solutions: Let us assume 3 + 2$\sqrt{5}$ is rational.
Then we can find co-prime x and y (y ≠ 0) such that 3 + 2$\sqrt{5} = \frac{x}{y}$.
Rearranging, we get,

$2\sqrt{5} = \frac{x}{y} - 3$
$\sqrt{5} = \frac{1}{2}(\frac{x}{y} - 3)$
Since, x and y are integers, thus, $\frac{1}{2}(\frac{x}{y} - 3)$ is a rational number.
Therefore, $\sqrt{5}$ is also a rational number. But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude that 3 + 2$\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:
(i) 1/$\sqrt{2}$
(ii) 7$\sqrt{5}$
(iii) 6 + $\sqrt{2}$

Solutions: (i) $1/\sqrt{2}$

Let us assume $1/\sqrt{2}$ is rational.
Then we can find co-prime x and y (y ≠ 0) such that $1/\sqrt{2} = \frac{x}{y}$.
Rearranging, we get,
$\sqrt{2} = \frac{y}{x}$

Since, x and y are integers, thus, $\sqrt{2}$ is a rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

Hence, we can conclude that $1/\sqrt{2}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is a rational number.
Then we can find co-prime a and b (b ≠ 0) such that $7\sqrt{5}= \frac{x}{y}$.
Rearranging, we get,

$\sqrt{5}= \frac{x}{7y}$

Since, x and y are integers, thus, $\sqrt{5}$ is a rational number, which contradicts the fact that $\sqrt{5}$ is irrational.

Hence, we can conclude that $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let us assume $6 + \sqrt{2}$ is a rational number.
Then we can find co-primes x and y (y ≠ 0) such that $6 + \sqrt{2}= \frac{x}{y}$.
Rearranging, we get,

$\sqrt{2} = \frac{x}{y} - 6$

Since, x and y are integers, thus, $\frac{x}{y} - 6$ is a rational number and therefore, $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is a irrational number.

Hence, we can conclude that $6 + \sqrt{2}$ is irrational.

Exercise 1.4 Page: 17

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:
(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Solutions:

Note: If the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has terminating decimal expansion.

If the denominator has factors other than 2 and 5 then it has a non-terminating decimal expansion.

$(i)\ \frac{13}{3125}$

Factorizing the denominator, we get,

3125 = 5 × 5 × 5 = 5^5

Since, the denominator has only 5 as its factor, $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8}$

Factorizing the denominator, we get,

$\mathbf{8 = 2 \times 2 \times 2} = 2^3$

Since, the denominator has only 2 as its factor, $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455}$

Factorizing the denominator, we get,

455 = 5 × 7 × 13
Since, the denominator is not in the form of $2^m \times 5^n$, thus $\frac{64}{455}$ has a non-terminating decimal expansion.

(iv) $\frac{15}{1600}$

Factorizing the denominator, we get,

1600 = $2^6 5^2$
Since, the denominator is in the form of $2^m \times 5^n$, thus $\frac{15}{1600}$ has a terminating decimal expansion.

(v) $\frac{29}{343}$

Factorizing the denominator, we get,
343 = $\mathbf{7 \times 7 \times 7} = 7^3$
Since, the denominator is not in the form of $2^m \times 5^n$, thus $\frac{29}{343}$ has a non-terminating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$

Clearly, the denominator is in the form of $2^m \times 5^n$.
Hence, $\frac{23}{2^3 5^2}$ has a terminating decimal expansion.

(vii) $\frac{129}{2^2 5^7 7^5}$

As you can see, the denominator is not in the form of $2^m \times 5^n$.
Hence, $\frac{129}{2^2 5^7 7^5}$ has a non-terminating decimal expansion.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2}{5}$
Since, the denominator has only 5 as its factor, thus, $\frac{6}{15}$ has a terminating decimal expansion.

(ix) $\frac{35}{50}$
$\frac{35}{50} = \frac{7}{10}$

Factorising the denominator, we get,
10 = 2 × 5
Since, the denominator is in the form of $2^m \times 5^n$, thus, $\frac{35}{50}$ has a terminating decimal expansion.

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{7 \times 11}{7 \times 30} = \frac{11}{30}$

Factorising the denominator, we get,
30 = 2 × 3 × 5

As you can see, the denominator is not in the form of $2^m \times 5^n$.
Hence, $\frac{77}{210}$ has a non-terminating decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solutions:

$(i)\ \frac{13}{3125}$

```
3125)13.00000(0.00416
          0
        ---------------
         130
           0
        ---------------
         13000
       - 12500
        ---------------
           5000
          -3125
       ---------------
           18750
           18750
        --------------
            00000
        --------------
```

$\frac{13}{3125}$ = 0.00416

(ii) $\frac{17}{8}$

```
8) 17 (2.125
  -16
   --------
    10
     -8
   --------
    20
   -16
   --------
     40
    -40
   --------
     00
   --------
```

$\frac{17}{8}$ = 2.125

(iii) $\frac{64}{455}$ has a Non terminating decimal expansion

(iv) $\frac{15}{1600}$

```
1600) 15.000000 (0.009375
        0
      ------------------
      150
        0
     -------------------
      1500
         0
     -------------------
      15000
     -14400
     --------------------
       6000
      -4800
    ---------------------
      12000
     -11200
     --------------------
        8000
       -8000
    --------------------
       0000
   --------------------
```

$\frac{15}{1600}$ = 0.009375

(v) $\frac{29}{343}$ has a Non terminating decimal expansion

(vi) $\frac{23}{2^3 5^2} = \frac{23}{8\times25} = \frac{23}{200}$

200) 23.000(0.115

0

23
-0

230
-200

300
-200

1000
-1000

0000

$\frac{23}{2^3 5^2} = 0.115$

(vii) $\frac{129}{2^2 5^7 7^5}$ has a Non terminating decimal expansion

(viii) $\frac{6}{15} = \frac{2}{5}$

5) 2.0 (0.4
0

20
-20

00

(ix) $\frac{35}{50} = \frac{7}{10}$

10) 7 (0.7
0

70
-70

00

$$\frac{35}{50} = 0.7$$

(x) $\frac{77}{210}$ has a non-terminating decimal expansion.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p q what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000. . .

(iii) $43.\overline{123456789}$

Solutions:

(i) 43.123456789
Since it has a terminating decimal expansion, it is a rational number in the form of p/q and q has factors of 2 and 5 only.

(ii) 0.120120012000120000. . .
Since, it has non-terminating and non- repeating decimal expansion, it is an irrational number.

(iii) $43.\overline{123456789}$
Since it has non-terminating but repeating decimal expansion, it is a rational number in the form of p/q and q has factors other than 2 and 5.