Exercise 1.3

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1. Prove that $\sqrt{5}$ is irrational.

Solutions: Let us assume, that $\sqrt{5}$ is rational number.
i.e. $\sqrt{5} = \frac{x}{y}$ (where, x and y are co-primes)
$y\sqrt{5} = x$

Squaring both the sides, we get,
$(y\sqrt{5})^2 = x^2$
$\Rightarrow 5y^2 = x^2$………………………………….. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,
$5y^2 = (5k)^2$
$\Rightarrow y^2 = 5k^2$

$\boldsymbol{y^2}$ is divisible by 5 it means y is divisible by 5.

Therefore, x and y are co-primes. Since, our assumption about $\sqrt{5}$ is rational is incorrect.

Hence, $\sqrt{5}$ is irrational number.

2. Prove that 3 + 2$\sqrt{5}$ + is irrational.

Solutions: Let us assume 3 + 2$\sqrt{5}$ is rational.
Then we can find co-prime x and y (y ≠ 0) such that 3 + 2$\sqrt{5}$ = $\frac{x}{y}$.
Rearranging, we get,

$2\sqrt{5} = \frac{x}{y} - 3$
$\sqrt{5} = \frac{1}{2}(\frac{x}{y} - 3)$
Since, x and y are integers, thus, $\frac{1}{2}(\frac{x}{y} - 3)$ is a rational number.
Therefore, $\sqrt{5}$ is also a rational number. But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude that 3 + 2$\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:
(i) 1/$\sqrt{2}$
(ii) 7$\sqrt{5}$
(iii) 6 + $\sqrt{2}$

Solutions: (i) $1/\sqrt{2}$

Let us assume $1/\sqrt{2}$ is rational.
Then we can find co-prime x and y (y ≠ 0) such that $1/\sqrt{2} = \frac{x}{y}$.
Rearranging, we get,
$\sqrt{2} = \frac{y}{x}$

Since, x and y are integers, thus, $\sqrt{2}$ is a rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

Hence, we can conclude that $1/\sqrt{2}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is a rational number.
Then we can find co-prime a and b (b ≠ 0) such that $7\sqrt{5} = \frac{x}{y}$.
Rearranging, we get,

$\sqrt{5} = \frac{x}{7y}$

Since, x and y are integers, thus, $\sqrt{5}$ is a rational number, which contradicts the fact that $\sqrt{5}$ is irrational.

Hence, we can conclude that $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let us assume $6 + \sqrt{2}$ is a rational number.
Then we can find co-primes x and y (y ≠ 0) such that $6 + \sqrt{2} = \frac{x}{y}$.
Rearranging, we get,

$\sqrt{2} = \frac{x}{y} - 6$

Since, x and y are integers, thus, $\frac{x}{y} - 6$ is a rational number and therefore, $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is a irrational number.

Hence, we can conclude that $6 + \sqrt{2}$ is irrational.